



# The relevant bosons at the liquid-solid transition

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## ABSTRACT

Using the cubic alkali halogenides as model materials, it is shown that the cohesion of the solids up to the rather high melting temperatures,  $T_m$ , is not by the inter-atomic interactions but by a boson field. A reasonable measure of the absolute interatomic interaction strength is given by the Debye-temperature,  $\Theta_D$ , which is much lower than  $T_m$ . It is explained that in the wide temperature range  $\Theta_D < T < T_m$ , the dynamics is the dynamics of a boson field. This is evidenced by the observed universality in the temperature dependence of heat capacity and relative thermal length changes,  $\Delta L/L_0$  below  $T_m$ . The boson field orders at  $T_m$  and defines the perfect long-range atomic order of the crystalline state. Upon ordering all bosons condense in the lowest quantum state (Bose-Einstein condensation). This is the highest possible thermodynamic order, and provides a plausible entropy argument for the exclusion of the interatomic interactions at order-disorder phase transitions. Additionally, ordered boson fields contract themselves to a finite volume such as a domain. The mosaic blocks, occurring in, practically, all crystalline solids, have to be viewed as the domains of the bosons that order at  $T_m$ . Within each mosaic block, the bosons are in a stationary mode. The constricting force of the ordered boson field that compresses each mosaic block increasingly with decreasing temperature, guarantees the cohesion of the whole solid up to  $T_m$ . Plausible arguments are given that the bosons that order at  $T_m$  are elastic quadrupole radiation.

## 1. Introduction

Among the many experimental observations that cannot be explained by the atomistic theories of solid-state physics are: (1) a sharp maximum in the thermal conductivity, occurring in all non-magnetic insulators in the temperature range 10 ... 30 K [1], (2) magnetic ordering temperatures that are generally lower than conforms to the exchange interactions between neighbouring spins [2], and, (3) dispersion relations of the acoustic phonons (magnons) in three-dimensional solids as for the linear atomic (spin) chain [3]. Here we treat on two other phenomena that are beyond the atomistic concepts of solid-state physics: heat capacity values of larger than the Dulong-Petit (D-P) limit [4], and large thermal lattice contractions below the melting temperature  $T_m$  [5]. As we will show, all these observations are signatures of the hitherto not considered bosonic energy degrees of freedom. For the explanation of these phenomena, in particular for a quantitative understanding of the observed critical exponents, field theories are needed [2]. These theories are, however, not yet developed. Condition for the formulation of advanced field theories is that the dispersion relations and the densities of states of the bosons are known. This is the case only for the sound waves (elastic dipole radiation), called here Debye bosons, for which it is intuitively clear that

the dispersion is a linear function of wave vector and the density of states a quadratic function of energy [6]. A considerable simplification is that the Debye bosons do not order at a finite temperature. As we will explain below, the critical temperature of the Debye bosons is  $T = 0$ . As a consequence, in the finite critical range at  $T = 0$  the heat capacity of the Debye boson field is a function of absolute temperature, given by the famous  $T^3$  function [6]. Typical of boson dynamics is that the exponent of three is universal, i.e., it occurs in all crystalline non-magnetic insulators. For all other boson types that order at a finite temperature, the thermodynamic observables follow power functions of the distance from the critical temperature. In these cases, the dispersion relation and the density of states must be rather complicated. The two quantities can nearly not be evaluated experimentally. In order to support the development of future field theories, this communication is aimed to collect some more experimental facts that seem to be useful for the formulation of future field theories of the dynamics of solids.

For the critical spin dynamics in the vicinity of the magnetic ordering temperature, Renormalization-Group (RG) methods have been tried [7]. The main problem of the RG concept is that the bosons were not recognized as independent energy degrees of freedom and were not considered explicitly. As a consequence, the crossover from a boson-defined spin dynamics within the finite critical range to an

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exchange-defined spin dynamics outside the critical range, cannot be explained by the RG theory [2,3]. In other words, there is a clear distinction between the large length scale of the bosons and the short length scale of the interatomic interactions. This change in length scale is not a continuous process but means a qualitative difference. Since on the two length scales different interaction mechanisms are effective, different theoretical concepts are needed. In particular, on the large length scale next to the ordering transition the, up to now not considered bosons, are the relevant excitations [2]. Below, we will discuss the experimental indications that the bosons control not only the critical dynamics, but cause virtually all of the well-known order-disorder phase transitions of solid-state physics as well.

The bosons that drive the magnetic ordering transition could be identified as magnetic dipole radiation, generated by the precessing spins [2]. We have called these bosons Goldstone bosons [8]. In the conventional theories of magnetism, magnetic dipole radiation was considered as too weak to affect the spin dynamics significantly. As we have explained elsewhere, this is not the case, mainly because in the critical range, the dominant generation process of the bosons is stimulated emission [2]. Stimulated emission leads to an amplification of selected bosons with an identical wave function. This has to be considered as a beginning thermodynamic order of the boson field. Due to a finite interaction with the spins, the Goldstone bosons control not only the critical dynamics of the spins, they order via Bose-Einstein condensation in the background and drive the visible ordering transition of the spins, instead of the exchange interactions. As a consequence, it appears as if the spin system would order itself. This is a confusing situation, since the exchange interactions between the spins are, in fact, able to cause a magnetic ordering transition [9]. We will shortly discuss the main experimental indications that not only the magnetic ordering transition but most of the other ordering transitions are boson-executed [10].

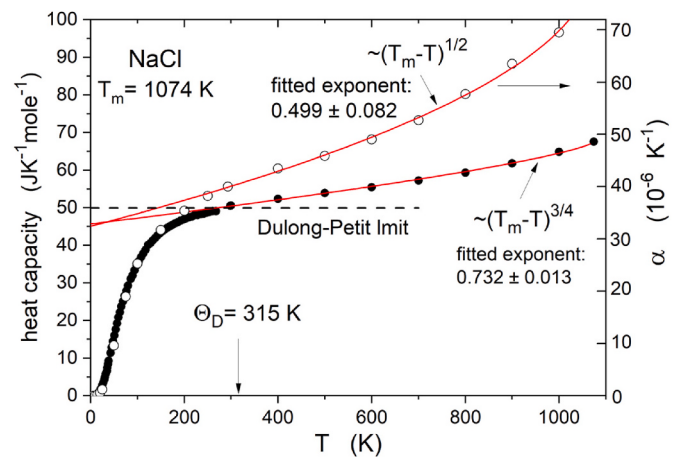
It has to be assumed that the boson-controlled spin dynamics sets in when the order of the spin system, forced by the bosons, has become higher than the order, realized by the inter-atomic interactions. Note that the perfect collinear spin order within each domain cannot be explained by the locally anisotropic exchange interactions. The crossover from the exchange-controlled to the boson-controlled spin dynamics happens at the change from the Curie-Weiss regime of the paramagnetic susceptibility to the critical paramagnetic susceptibility above the magnetic ordering transition [3]. This crossover manifests as an analytical change in the temperature dependence of all thermodynamic observables (susceptibility, heat capacity ...) [11]. Such a crossover cannot be simulated by one function of temperature [9] but indicates that two mechanisms define the spin dynamics alternately as a function of temperature [2,3]. As a consequence, different theoretical concepts are needed for the critical range and above the critical range.

The typical indication of a boson-defined spin dynamics in the critical range near the magnetic ordering transition is universality. Universality means material independent critical exponents. This is a consequence of the lattice structure independent, i.e., ballistic propagation mode of the mass-less Goldstone bosons (magnetic dipole radiation). Outside the finite critical range, the spin dynamics is defined by the exchange interactions and is material-specific [2,3]. It has to be assumed that, at the crossover into the critical paramagnetic range, thermal energy changes from the excitation system of the exchange-coupled spins to the excitation system of the bosons. The two excitation systems exist in parallel but define the dynamics of the spins alternately as a function of temperature [2,3]. In other words, the respective two dispersion relations cannot be populated thermally at the same time. The reason for the mutual exclusion of the two excitations from the thermodynamics of the spins are their different translation symmetries that is continuous (lattice-structure independent) for the bosons but discrete-periodic for the atomistic excitations. Note that for the ballistically propagating bosons there exists no atoms. The two (translational) symmetries exclude each other. The different

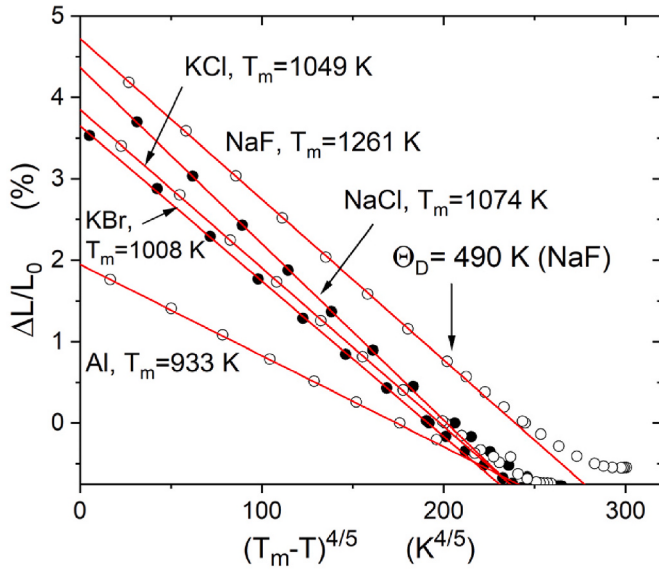
translational symmetries can be considered as the generators of the two (quasi)particles (magnons, bosons) including their different excitation spectra. Exclusion of the material-specific inter-atomic interactions in the critical range near ordering transitions is the origin of the observed universal critical behaviour. Since all of the here considered bosons have neither a mass nor a magnetic moment, they cannot be observed using conventional neutron scattering. As a consequence, using neutron scattering, the atomistic excitations (magnons, phonons) are sampled selectively. This appears to be the main reason that the bosons have not been recognized up to now. Since a direct observation of the bosons seems not possible, the bosons have to be identified by their actions. This means, one has to focus on those experimental observations that are in disagreement with the predictions of the atomistic models.

Boson dynamics is not particular to the magnetic critical range. Boson dynamics holds in the critical range of virtually all of the well-known order-disorder phase transitions of solid-state physics [10]. In particular, at the visible ordering transitions of the atomistic systems, the bosons order in the background and define the perfect order of spins, atoms, electric dipoles ... As a consequence, the atomistic concepts, that consider massive particles (atoms, electrons, spins, electric dipoles ...) and the interactions between them, exclusively, cannot give a correct description of the ordering phenomena.

The aim of this communication is to discuss the experimental indications that the liquid-solid transition of all solids is the ordering transition of a boson field as well. For this task, the alkali halide compounds were chosen as suitable model materials. This is because the inter-atomic interactions of these materials are of electrostatic nature and can be calculated theoretically using the point charge model [31]. Comparison with experiment reveals that the calculated room-temperature electrostatic binding forces are much larger than conforms to the melting temperatures. The room temperature lattice parameters, therefore, are surprisingly short. This is a consequence of a further unexplained particularity of all ordered boson fields. The ordered boson field compress the atomistic system within the generated ordered unit increasingly with decreasing temperature whereby the lattice parameter decreases strongly with decreasing temperature (Fig. 2). Further examples of boson-driven order-disorder phase transitions are the superconducting transition of the conventional superconductors [12] and the vapor-liquid transition [13].



**Fig. 1.** Temperature dependence of heat capacity (filled circles, left-hand scale) and of the coefficient of the linear thermal expansivity,  $\alpha$  (open points, right-hand scale) of NaCl [4,5,29]. Up to the Dulong-Petit limit, the two quantities are proportional to each other (Grüneisen relation). For higher temperatures, the two quantities are boson-defined and follow the typical power functions of the distance from the critical temperature,  $T_m$ .



**Fig. 2.** Relative thermal length changes,  $\Delta L/L_0$ , of NaF, NaCl, KCl, KBr and of fcc Al as a function of  $(T_m - T)^{4/5}$ , i.e., as a function of a decreasing temperature [5]. The reference value,  $L_0$ , is at  $T = 300$  K. The large critical range at  $T_m$  extends from the melting temperature  $T_m$  down to the crossover to phonon dynamics, at the Debye-temperature,  $\Theta_D$ . The crossover at  $\sim \Theta_D$ , manifests as an analytical change to a weaker temperature dependence of  $\Delta L/L_0$  for lower temperatures. Note that  $T = 0$  is at  $(T_m - T)^{4/5} = 303$  K $^{4/5}$  for NaF, at  $266$  K $^{4/5}$  for NaCl, at  $261$  K $^{4/5}$  for KCl, at  $253$  K $^{4/5}$  for KBr and at  $237$  K $^{4/5}$  for Al.

## 2. The different types of bosons in solids

For all energy degrees of freedom (elastic, electric, magnetic ...) one has to distinguish between the excitations of the continuous solid (bosons) and the excitations of the atomistic solid (phonons, electron-band states, magnons ...). The mass-less bosons are either dipole- or quadrupole radiation [10]. As a consequence, one has to distinguish between six different boson types: electric dipole radiation and electric quadrupole radiation, magnetic dipole radiation and magnetic quadrupole radiation and elastic dipole radiation and elastic quadrupole radiation. Since all types of bosons can order, i.e., can condense in the lowest quantum state (Bose-Einstein condensation), there are six different boson-executed order-disorder phase transitions possible in solids. We will shortly outline the experimental characteristics of the various order-disorder phase transitions due to the six different boson types.

The bosons generated by dipoles have considerably different properties compared to the bosons generated by quadrupoles. As is well-known, the radiation fields emitted by dipoles are linearly propagating plane-waves. The radiation fields emitted by quadrupoles are theoretically unclear but there are strong experimental indications that the quadrupole radiation fields are three-dimensional, rather spherical waves, that do virtually not propagate [10,12]. In the ordered state, the wave functions of these bosons close and form a three-dimensional case. The ordered boson field, within this case defines the order of the included atomistic system and compresses it increasingly with decreasing temperature.

Condensation of all bosons in one quantum state is the highest possible thermodynamic order and can be taken as a plausible entropy argument that the bosons prevent the locally anisotropic interatomic interactions from performing the ordering transitions. Note, that the anisotropic near-neighbour interactions would never result in the observed perfect long-range and coherent order of atoms, spins or electric dipoles. It seems to be a general principle of nature, to realize the highest possible order at order-disorder phase transitions. This is possible by Bose-Einstein condensation of bosons only. Due to a strong coupling between the ordered boson field and the corresponding

atomistic degrees of freedom (spins, atoms, electric dipoles ...) the passive atomistic systems order at the same temperature as the boson field, and receive their perfect order from the boson field. The observed order-disorder phase transitions of the spins, atoms, electric dipoles, therefore, are a direct consequence of a not directly visible order of a boson field in the background. Only thermodynamically non-relevant local details, such as a ferromagnetic or an antiferromagnetic orientation of neighbouring spins, or the coordination of the neighbouring atoms in the crystals, are determined by the atomistic near-neighbour interactions. These static details are specific to the short atomistic length scale and not relevant for the dynamics on the large length-scale of the bosons. This length scale is given by the (macroscopic) dimensions of the domains. The long-range order of the atomistic system within each domain is caused by the bosons.

A very important, additional condition of a stable, boson-induced long-range atomic or spin order is that, upon ordering, boson fields constrict themselves in space to the finite volume of a stationary unit. This process is theoretically unexplained as well but experimentally obvious. In the case of the ordered magnets, these units are the well-known domains [14,15]. The domains are the true signature of the long-range ordered magnetic state. Since the bosons generated by magnetic or electric dipoles are linearly propagating waves, the ordered units of these bosons are one-dimensional. We have to assume that upon ordering, the propagating bosons emitted by dipoles become stopped. The standing wave function of the condensed bosons within each domain seems to be phase-coherent with the lattice periodicity. Since a large number of bosons has condensed in this wave function, large local field amplitudes result that, evidently, dominate over the exchange fields and control the spin dynamics. Note that, by symmetry reasons, the spin dynamics is defined either by the bosons (in the critical range) or by the exchange interactions (outside the critical range). According to the one-dimensional boson field within each magnetic domain, the spins are perfectly collinearly aligned, irrespective of the dimensionality of the local exchange interactions that are not relevant in the magnetically ordered state. As a consequence, the dimensionality of the ordered spin system cannot be classified by the dimensionality of the (local) exchange interactions [16]. As the perfect collinear spin structures show, these interactions are no longer relevant in the magnetically ordered state. Instead, the dimensionality of the ordered magnets is given by the number inequivalently oriented domains, i.e., by the number of different spin axes [2]. The domain structure depends essentially on the crystal symmetry [2]. Since within each magnetic domain, Goldstone-bosons and magnons are in standing, one-dimensional modes, the observed magnon dispersions, commonly, are as for the linear spin chain [17]. No magnons are observed along those crystal directions along which there are no domains [18,19].

The same collinear order as for the spins in each magnetic domain, applies to the electric dipole moments in the one-dimensional ferroelectric domains [14,20]. The relevant bosons of the ferroelectric solids are, evidently, electric dipole radiation [20]. We can expect that within each ferroelectric domain there are local excitations that correspond to the magnons in the magnetic domains. One could call these excitations voltons. Since the neutron has no electric charge, these excitations cannot be observed using inelastic neutron scattering, and have, in fact, not been verified as yet.

The here postulated bosonic force that compresses the ordered boson field to the smallest possible volume is characteristic of all boson fields but is theoretically unexplained. For the ordered magnets, this phenomenon is well-known as magnetostriction [14,15]. The one-dimensional magnetostriction is particular to the individual domain. Note, that magnetostriction sets in just with the formation of domains at the magnetic ordering temperature. In fact, strong thermal lattice contractions, as a function of a decreasing temperature, are a domain effect and, therefore, specific to all kinds of ordered boson fields [5]. Here we discuss this type of lattice contraction below the liquid-solid transition (Fig. 2).

Since there are six different boson types, six different types of order-disorder phase transitions can be explained. However, the bosons generated by the elastic dipoles, the sound waves, do not order at a finite temperature. We have called these bosons Debye bosons [6]. Writing the  $T^3$  temperature dependence of the heat capacity of the Debye boson field as a critical power function according to  $\sim(T-T_c)^3$ , it becomes evident that the critical temperature of the Debye-boson field is  $T_c = 0$ . In other words,  $T = 0$  is a critical point, completely equivalent to a finite ordering temperature. As a consequence, Bose-Einstein condensation occurs for all bosons, either at a finite ordering temperature or at  $T = 0$  [22]. In the finite critical range at  $T = 0$ , the observed heat capacity is the heat capacity of the Debye boson field and not the heat capacity of the acoustic phonons. This critical heat capacity follows Debye's famous  $T^3$  function, and is not the first term of a power series expansion but holds, for all non-magnetic and insulating solids, up to a crossover that is at a temperature of  $10 \dots 30$  K [3]. At this crossover, thermal energy changes from the boson system into the phonon system and the temperature dependence of the heat capacity becomes material-specific. In fact, the  $T^3$  function of the heat capacity of the Debye boson field, observed in all non-magnetic insulators, is a typical, boson-defined critical power function, completely equivalent to the well-known critical power functions at finite ordering temperatures. In other words, the universal exponent of 3 has to be viewed as a critical exponent. By the way, the exponent of 3 is the only theoretically explained critical exponent of solid-state physics up to now [6].

Universality of the boson-defined critical exponents means that they get not changed by the interaction of the bosons with the corresponding atomistic system. Note that this interaction is a condition that the bosons can define the order of the atomistic system. The interaction of the bosons with the atomistic system affects the material-specific pre-factor of the critical power function, the critical amplitude, only. From the fact that Debye's universal  $T^3$  function is observed in all insulating solids, it follows that the dispersion of the Debye bosons remains a linear function of wave vector, irrespective of the interaction strength of the Debye bosons with the acoustic phonons. Due to this interaction, the dispersion relations of the Debye bosons and of the (thermally not activated) acoustic phonons attract each other [1]. Commonly, the interaction between Debye bosons and acoustic phonons is strong, and the two associated dispersion relations have completely attracted, and start with the same linear wave-vector dependence. Remarkably, this attraction is always such that the Debye bosons modify the wave-vector dependence of the dispersion of the acoustic phonons and not vice versa. For materials with a strong interaction between Debye bosons and acoustic phonons, it is observed that the dispersion of the acoustic phonons assumes, over a finite  $q$ -range, the linear dispersion of the Debye bosons [3]. This illustrates impressively the dominance of the bosons over the inter-atomic interactions. Nevertheless, Debye bosons and acoustic phonons remain independent energy degrees of freedom, by symmetry reasons. For a complete attraction, the stiffness constant of the dispersion of the Debye bosons agrees with the stiffness constant of the acoustic phonons, and the critical amplitude of Debye's  $T^3$  function, given by the material specific Debye temperature  $\Theta_D$ , becomes a reasonable measure of the absolute phonon energy [1,3]. The harder is the material is, the higher is  $\Theta_D$  and the higher is the velocity of the Debye bosons.

In contrast to the one-dimensionally ordered units, generated upon ordering of dipole-radiation, the ordered units generated by quadrupole radiation are three-dimensional. By novel analyses of the now available precise experimental data of the conventional superconductors, it could reasonably be evidenced that the relevant bosons are electric quadrupole radiation [12]. Note that in the metals there are no electric dipoles possible. The sources of the electric quadrupole radiation are the anisotropic charge distributions in the soft zones between the metal atoms. This view is supported by the observation that the transition temperatures of many superconductors can considerably be increased by applying an external pressure [21]. Pressure deforms the electric

quadrupoles in the soft zones between the hard metal atoms. If this deformation is to increase the electric quadrupole moments, the production rate of the bosons will increase and the superconducting transition temperature will increase correspondingly. Note that the ordering transition of boson fields occurs when the density of the coherent bosons has reached the critical value for the spontaneous onset of stimulate emission. This is a threshold-induced event that appears as a first-order transition in the heat capacity at the superconducting transition and at the liquid-solid transition as well [4,12].

We have every reason to assume that, upon ordering, many bosons generated by the electric quadrupoles condense in a spherical wave function and form an electromagnetic shell that encapsulates the two Cooper pair electrons. Moreover, the electromagnetic shell around the two Cooper pair electrons, seems to shield their charges perfectly against all other charges in the metal. In this way, the Cooper-pairs do not interact with each other and can move freely across the metal [12]. Since the Cooper-pairs do not interact with each other, there is no correlation between them and, as a consequence, no long-range order in the superconducting phase. Domains, as they are characteristic of the long-range ordered magnetic state, are absent in the superconducting state. Nevertheless, the Cooper pairs correspond, so to say, to the magnetic domains, but in contrast to the three-dimensional Cooper pairs, the one-dimensional magnetic domains are fixed weakly to specific crystallographic symmetry directions, i.e., they have a stable orientation [2, 14,15]. Note that the Cooper pairs are spin-zero bosons, having a mass. Experimental indications were obtained that the Cooper-pairs can undergo a Bose-Einstein condensation [22,23].

Under special experimental conditions, a similar condensation of magnetic quadrupole radiation appears possible. It is suggestive to consider the magnetic bubbles, observed on top of magnetic garnet films, as the ordered units generated upon condensation of magnetic quadrupole radiation [24]. The perfect spherical bubbles all have the same diameter and are surprisingly stable units. They are independent of each other and have a high mobility. In other words, the spherical shell, formed by the condensed magnetic quadrupole radiation, shields the magnetic stray fields of the encapsulated magnetic moments perfectly. It is tempting to consider the well visible magnetic bubbles as a good visualization of the not directly observable electric analogue, the Cooper-pairs. The magnetic bubbles include a huge, but finite number of antiferromagnetically ordered spins. This is in contrast to the Cooper-pairs, that include two electrons with antiparallel spins only [25]. This appears to be necessity since in the metals there are just two electrons with virtually no kinetic energy but opposite spin orientations, according to the Pauli principle. As a consequence, we have to assume that the two Cooper pair electrons can be from the bottom of the Fermi-sea only [25].

Here we will discuss the experimental indications that the liquid-solid transition is also the ordering transition of a boson field. One argument supporting this is that the melting temperatures of the solids are all much higher than conforms to the inter-atomic interactions, a reasonable measure of which is given by the Debye temperature  $\Theta_D$  (see Fig. 1). Note that this is opposite for the boson-executed magnetic ordering transitions that are all at a lower temperature than conforms to the exchange interaction strength between the spins [2]. This illustrates that the bosons determine the order-disorder phase transitions generally, quite independent of the strength of the interatomic interactions. We will call the bosons that order at the melting point of the solids MP-bosons. Since the perfect long-range atomic order of the crystalline state, induced by the MP-bosons, is three-dimensional, the MP-bosons cannot be dipole radiation but must be quadrupole radiation. It is reasonable to identify the MP-bosons as elastic quadrupole radiation [10]. The conclusion that the MP-bosons are of elastic nature, is supported by the observation that, for metals and for insulators, the heat capacity exhibits a qualitatively similar behaviour at  $T_m$  [4]. Moreover, thermal lattice contractions below  $T_m$  are also very similar for metals and for insulators (see Fig. 2) [5]. This observation is in line with the

view that the MP-bosons define not only the melting temperature,  $T_m$ , but the thermal lattice contractions below  $T_m$  as well.

Definite evidence that the MP-bosons are of elastic nature is obtained by the finding that the MP-bosons determine the elastic compliances,  $s_{ij}$  [27]. As a consequence, there are two types of elastic bosons in solids: the Debye-bosons (sound waves) that are one-dimensional elastic dipole radiation and the MP-bosons that are three-dimensional elastic quadrupole radiation. As is well-known, the elastic stiffness constants,  $c_{ij}$ , are defined by the velocities of the Debye-bosons [10,27]. The elastic compliances,  $s_{ij}$ , characterize the compressibility of the solid, i.e., a static property, and are defined by the MP-bosons [26]. The two elastic constants have different dimensions and different temperature dependencies (see Figs. 6 and 7) [26,27]. While the elastic stiffness constants,  $c_{ij}$ , have the dimension of a pressure, the elastic compliances,  $s_{ij}$ , have the dimension of a reciprocal pressure.

Commonly, the boson-defined thermodynamic observables reach a maximum, or even diverge, at the ordering temperature of the respective boson field. A diverging behaviour is known for the heat capacity and for the susceptibility at the second order magnetic ordering transition [11]. The constants  $c_{ij}$  have a finite maximum at  $T = 0$  [26,27]. This confirms, that the critical temperature of the Debye-bosons is  $T = 0$ , i.e., the Debye bosons do not order at a finite temperature. As a consequence, Debye-bosons do not create domains, and retain a finite velocity at all temperatures. Note that, if the Debye-bosons would order, there would be a conflict between the one-dimensional atomic order required by the Debye-bosons (elastic dipole radiation) and the already existing, three-dimensional atomic order of the mosaic blocks, realized by the MP-bosons (elastic quadrupole radiation). The constants  $s_{ij}$  do not diverge at  $T_m$  either but reach a finite maximum only because all liquid-solid transitions are threshold-induced first-order transitions [26, 27]. This means, the compressibility remains finite up to  $T = T_m$ . As a consequence, the crystalline lattice remains stable up to  $T_m$ , where the solids are mechanically very soft.

Like for the Debye-bosons,  $T = 0$  is the critical temperate of the bosons of the metallic degree of freedom (electric quadrupole radiation). In other words, the two bosons undergo no transition into a long-range ordered state at a finite temperature. Since the critical temperatures of the two boson types are  $T = 0$ , their heat capacities are functions of absolute temperature. Nevertheless, the bosons generated by the electric quadrupoles order at the superconducting transition but into a short-range ordered state only [12]. We have to assume that the short-range ordered objects, generated upon condensation of electric quadrupole radiation, are the Cooper-pairs. Short-range order above and below the superconducting transition is evidenced by the fact that in both regions, the heat capacity follows power functions of absolute temperature [12, 28]. This confirms that  $T = 0$  is the only critical temperature. Power functions of the argument ( $T_{SC}-T$ ), with  $T_{SC}$  as the superconducting transition temperature, are absent.

The exponent of unity is, as the exponent of 3 of the heat capacity of the Debye boson field, a critical exponent. In contrast to Debye's critical exponent of 3, the critical exponent of unity, is theoretically unexplained. This is because, as for all other bosons, the dispersion relation and the density of states of the metallic bosons are unknown. For the Debye bosons (elastic dipole radiation) it is intuitively clear that the dispersion relation is a linear function of wave vector and the density of states a quadratic function of energy. As we have mentioned, the universal exponent of 3 in the heat capacity of the Debye boson field, evaluated by P. Debye as early as in 1912, is the only theoretically explained critical exponent of solid-state physics up to now [6].

The linear-in- $T$  heat capacity of the metallic bosons should not be confused with the heat capacity of the conduction band electrons that holds beyond the critical range at  $T = 0$  and exhibits, by chance, also a linear temperature dependence, but with a much smaller pre-factor [1]. The larger bosonic heat capacity in the critical range at  $T = 0$  is a condition that the bosons become the relevant excitations. For the noble metals that get not superconducting, the  $T^3$  heat capacity of the

Debye-bosons (sound waves) and the linear-in- $T$  heat capacity of the bosons generated by the electric quadrupoles superimpose [28]. This shows that, in the noble metals, the two boson types do not interact. This favourable circumstance allows for an identification of both boson types. On the other hand, interaction between the two bosons seems to be a condition for superconductivity. Beyond the critical range at  $T = 0$ , i.e., above 10 ... 30 K, the observed heat capacity is atomistic, i.e., it is due to the phonons and the electron-band states. Now the observed heat capacity increases weaker with temperature compared to the bosonic heat capacity in the critical range at  $T = 0$  [1].

Since superconductivity results from condensation of electric quadrupole radiation, the transition temperatures of the conventional superconductors are restricted to the critical range at  $T = 0$  where these bosons are the relevant excitations, i.e., they get spontaneously generated, in thermal equilibrium with the temperature of the sample and are permanently present.

The constricting force, exerted by each ordered boson field on the created ordered unit (domains, Cooper pairs, mosaic blocks etc.) increases with decreasing temperature and lets the dimensions of the ordered unit decrease. For the magnetic solids this phenomenon is well-known as magnetostriction [14,15]. Magnetostriction is a one-dimensional effect, acting along the domain axis. For the three-dimensional domains, generated upon ordering of quadrupole radiation, the constricting force is concentric. Due to this concentric force, the diameter of the spherical Cooperpairs decreases strongly as a function of a decreasing temperature [3,12]. A measure of the thereby increasing binding energy between the two Cooperpair electrons, is the energy gap. In other words, the energy gap is also a boson-defined quantity [3,12]. In analogy to the term magnetostriction one could call the decreasing diameter of the Cooperpairs with decreasing temperature, electrostriction.

Here we will discuss the isotropic thermal lattice contractions below  $T_m$  of the cubic alkali halide crystals with NaCl structure (see Fig. 2) [3, 5,27]. One could call this type of lattice contraction, caused by the MP-bosons (elastic quadrupole radiation), elastostriktion. The concentric lattice contractions of all solids below  $T_m$  set-in precisely with the formation of mosaic blocks at the ordering temperature of the boson field at  $T_m$  (see Fig. 2) [3,5,27]. No domains or mosaic blocks, means no significant lattice contractions. Since the critical temperature of the Debye-boson field is  $T = 0$ , no domains get formed upon approaching  $T \rightarrow 0$ , and no significant lattice contractions occur [5]. The lattice parameter approaches  $T \rightarrow 0$  by a temperature independent behaviour [5].

### 3. Dispersion relations and propagation modes of the bosons

The dispersion relations of the mass-less bosons cannot be evaluated using inelastic neutron scattering. As we have mentioned, for a theoretical explanation of the critical exponents, the dispersion relation and the density of states of the bosons have to be known. These quantities are intuitively known for the Debye bosons only, however, for low dispersion energies only. For all temperatures above the critical range at  $T = 0$ , i.e., for  $T > 10 \dots 30$  K, the dispersion energy of the Debye bosons continues slightly non-linearly with wave-vector, but is no longer thermally populated. Nevertheless, Debye bosons can be excited as decaying modes, out of thermal equilibrium up to  $T_m$ . This shows that the dispersion continuous up to an energy of  $k_B T_m$ .

The dispersion of the Debye bosons can be constructed from the temperature dependence of the elastic stiffness constants,  $c_{ij}$ , that are known, for the most common solids, up to  $T_m$  [1,26]. Note, that the constants  $c_{ij}$  are defined by the velocity of the Debye bosons. Knowing the sound velocities for all temperatures up to  $T_m$ , the dispersion relation of the Debye bosons can be calculated up to an energy of  $k_B T_m$  [1], assuming that the velocity of the Debye-bosons, measured at a temperature  $T$ , gives the slope of the dispersion relation of the Debye bosons at an energy  $k_B T$ . Since for  $T > \Theta_D$  the constants  $c_{ij}$  decrease slightly with

increasing temperature [26], the as calculated dispersion of the Debye bosons is a somewhat weaker than linear function of wave vector [1,3]. In other words, the velocity of the Debye bosons has a maximum at the critical temperature of the Debye bosons at  $T = 0$ .

In contrast to the dispersion of the Debye-bosons, that extends up to an energy of  $k_B T_m$ , the upper dispersion energy of the phonons is limited by the finite inter-atomic interaction strength. Note that the term phonons is misleading since the phonons are vibrations of the massive atomic lattice but the sound waves are mass-less bosons. A reasonable quantitative measure of the highest phonon energy is given by the Debye-temperature,  $\Theta_D$ . As is well known,  $T_m$  can be much higher than  $\Theta_D$ . Because thermal energy is generally in the system with the lower dispersion energy, it is always in the phonon system, except for the narrow critical range at  $T = 0$ , where the Debye-bosons are the relevant excitations. The critical temperature range extends up to a crossover temperature of 10 ... 30 K only. We have to assume that at this crossover, thermal energy changes to the phonon system [1]. This crossover gives rise to a sharp maximum in the temperature dependence of the thermal conductivity [1]. Note that a crossover is a relatively sharp event but not as sharp as an ordering transition. The sharp thermal conductivity maximum results from the fact that in the narrow critical range at  $T = 0$ , where the Debye-bosons are the relevant excitations, the heat transport is by the ballistically propagating Debye-bosons and increases steeply with increasing temperature. Since, in harmonic approximation, the phonons are standing waves, confined to the volume of the individual mosaic block, they do nearly not contribute to the thermal conductivity of the solids [1]. As a consequence, above the maximum of the thermal conductivity, where the phonons are the relevant excitations, thermal conductivity of the insulators decreases steeply towards zero. For the metals, thermal conductivity saturates at a low but finite value at high temperatures [1]. This value is given by the thermal conductivity of the conduction band electrons.

As we have mentioned, the bosons generated by quadrupoles seem to be three-dimensional local modes that do nearly not propagate. It is questionable, whether they have a dispersion relation at all.

#### 4. The heat capacity of boson fields

The bosons determine the temperature dependence of the thermodynamic observables in the finite critical range, either at  $T = 0$  or at a finite ordering temperature, exclusively. In this range, the inter-atomic interactions are not relevant, i.e., they set the absolute energy scale only. Boson dynamics is recognized by the observed dynamic universality. Universality means that the temperature dependence of all thermodynamic observables is given by a power function of the distance from the critical temperature with an exponent that is identical for all materials of the same symmetry class (see Figs. 1 and 2) [11]. If the critical temperature is  $T = 0$ , the critical power functions are functions of absolute temperature. This holds for the Debye bosons and for the bosons of the continuous metallic solid (electric quadrupole radiation). However, the pre-factors of the critical power functions, the critical amplitudes, are material specific, i.e., they are defined by the interaction of the bosons with the corresponding atomistic background [11]. As we have explained, the pre-factor of Debye's universal  $T^3$  function is given by the material specific Debye temperature  $\Theta_D$  that is a measure of the hardness of the atomic lattice. On the one hand, it is very comfortable, that the critical power functions allow for an analytically simple, description of the dynamics near ordering transitions [3,27], but, on the other hand, a theoretical understanding of the fitted critical exponents and of the fitted critical amplitudes is a big, and, presently not solvable, problem.

Here we treat on the heat capacity values of larger than the atomistic Dulong-Petit (D-P) limit, that are beyond the atomistic models of the solid (Fig. 1). The D-P-limit is given by the total number of vibrational modes of the atomic lattice of  $3N$ , with  $N$  as the absolute number of atoms in the sample. Heat capacity values of larger than the D-P-limit

are indicative of the existence of energy degrees of freedom that exist in addition to the atomistic energy degrees of freedom, and have to be ascribed to the MP-bosons. Boson dynamics follows consistently from the fact that heat capacity and thermal length change data are given over the large temperature range  $\Theta_D < T < T_m$  by power functions of the argument  $(T_m - T)$  (see Figs. 1 and 2). In other words, the critical range below  $T_m$  extends over the large temperature interval of  $\Theta_D < T < T_m$ .

For  $T < \Theta_D$ , the observed heat capacity is the heat capacity of the lattice. According to the atomistic point of view, the heat capacity should saturate at the D-P-limit, approximately at the Debye-temperature  $\Theta_D$ , where all phonons are excited (see Fig. 1). There is no criterion for the much higher temperature of the melting process and for the further increasing heat capacity (see Fig. 1) [10,29]. The change to larger heat capacities than the D-P-limit happens as a crossover event, approximately at  $T \sim \Theta_D$ , at which the energy states of the MP-boson field get suddenly thermally activated. Note that a crossover implies an analytical change in the temperature dependence of the thermodynamic observables that cannot be simulated by a single function of temperature [9]. The different functions of temperature above and below the crossover at  $\sim \Theta_D$ , require different theoretical concepts. At the crossover in question, the dynamic symmetry changes from the discrete-periodic translational symmetry of the phonons for  $T < \Theta_D$  to the continuous translational symmetry of the MP-bosons for  $T > \Theta_D$ . Additionally, in the temperature range  $\Theta_D < T < T_m$  large thermal lattice contractions occur as a function of a decreasing temperature which also follow the typical critical power functions of the distance from  $T_m$  (see Fig. 2) [5]. As a consequence, these lattice contractions must be caused, as the heat capacity, by the ordered MP-boson field as well.

Unimportance of the inter-atomic interactions in the temperature range  $T > \Theta_D$  implies in particular that in this temperature range the cohesion of the solids cannot be by the inter-atomic interactions that are too weak [27]. In order to explain the cohesion of the solids, up to  $T_m$ , a new type of theoretically unexplained bosonic force has to be postulated. This force is specific to all ordered boson fields. Due to this force, ordered boson fields contract themselves to the finite volume a domain or mosaic block and intend to compress these ordered units to the smallest possible volume. We can assume that the compressive pressure, exerted by the ordered boson field on each ordered unit increases with decreasing temperature, whereby the lattice parameter of the solid decreases (Fig. 2) [5]. On the other hand, we can assume that the cohesive pressure has fallen to the ambient value at the melting point. As a consequence, the ordered MP-boson field is responsible not only for the perfect long-range atomic order below  $T_m$ , but for the cohesion of the solids up to  $T_m$  as well.

#### 5. Analysis of experimental data

The crossover from phonon dynamics for  $T < \Theta_D$  to the dynamics of the MP-bosons for  $T > \Theta_D$  manifests as an analytical change (crossover) in the temperature dependence of the heat capacity and of the coefficient of the linear thermal expansivity,  $\alpha$  (Fig. 1) [4,5,29]. The two quantities are rather precisely proportional to each other, however, only below a temperature of  $T \approx \Theta_D$ . This proportionality is known as Grüneisen relation [5]. Since the Grüneisen relation refers to the atomistic temperature range with a phonon-defined heat capacity, it can be understood using conventional atomistic concepts. The further increasing values of  $\alpha$  and of the heat capacity for  $T > \Theta_D$ , are beyond the atomistic concepts and have to be attributed to the MP-bosons, which are the only remaining and, therefore, relevant excitations for  $T > \Theta_D$ .

As can be seen in Fig. 1, for  $T > \Theta_D$ ,  $\alpha$  and heat capacity are no longer proportional to each other. Now, heat capacity and  $\alpha$  follow analytically different temperature dependencies with a stronger thermal increase of  $\alpha$  (right-hand scale) compared to the heat capacity (left-hand scale). This functional change indicates that something fundamental has changed. In the present case, the symmetry changes from the discrete-periodic translational symmetry of the phonons for  $T < \Theta_D$  to the continuous, i.

e., lattice structure independent, symmetry of the MP-bosons for  $T > \Theta_D$ . The stronger increase of  $\alpha$  for  $T > \Theta_D$  compared to the heat capacity shows that the MP-bosons have a strong impact on the temperature dependence of the lattice parameter (see Fig. 2).

Typical of the boson-defined critical dynamics in the range  $\Theta_D < T < T_m$  is, that heat capacity and coefficient of the linear thermal expansivity,  $\alpha$ , are given over this temperature range by power functions of the distance from the critical point  $T = T_m$  (red solid curves in Fig. 1) [5,29]. The relative thermal length change,  $\Delta L/L_0$  (see Fig. 2), follows also a critical power function of the type  $\sim (T_m - T)^\epsilon$  [5].

One should anticipate that the critical exponent  $\epsilon$ , observed for  $\Delta L/L_0$ , should have the same rational value as for the heat capacity, because both quantities should be defined by the heat capacity of the MP-boson field. This is, however, not confirmed. Note, that the heat capacity is the only thermodynamic observable of a boson field. The exponent  $\epsilon$ , observed in the heat capacity, seems to depend on the actual mosaic structure of the individual crystal, that results in a complicated way from the temperature gradient and from the cooling rate during solidification of the melt [29]. In this way, it can be explained, that the absolute values of the critical heat capacity as well as the exponent  $\epsilon$  are badly reproducible [4,29]. Nevertheless,  $\epsilon$  turns out to be always a rational number. A frequently observed exponent of the critical heat capacity at  $T_m$  is  $\epsilon = 1$  [4]. On the other hand, the  $\Delta L/L_0(T)$  data are specific to the individual mosaic block and seem to be independent of the mosaic structure (see Fig. 2).

It can be seen in Fig. 1 that the width of the critical range below  $T_m$  can be as large as  $\sim 800$  K. For lower temperatures than  $\Theta_D$ , the phonons are the relevant excitations, instead of the MP-bosons, and  $\Delta L/L_0$  decreases much weaker with decreasing temperature and assumes a temperature independent behaviour on approaching  $T \rightarrow 0$  (Fig. 2). Note that  $T \rightarrow 0$  is on the right-hand side of Fig. 2.

Fig. 2 shows  $\Delta L/L_0$  values for NaF, NaCl, KCl, KBr and for fcc aluminium as a function of  $(T_m - T)^{4/5}$  [5]. The fact that the universal exponent of  $4/5$  occurs for insulators and for metals supports the conclusion that thermal lattice contractions below  $T_m$  originate in the elastic degrees of freedom and must be due to the MP-bosons (elastic quadrupole radiation). The stability of the exponent of  $\epsilon = 4/5$  confirms that thermal lattice contraction is an effect of the individual mosaic block and, therefore, is independent of the mosaic structure.

It can be seen in Fig. 2 that the higher the melting temperature  $T_m$  is, the larger is the absolute variation of  $\Delta L/L_0$ . This supports the conclusion, that the two quantities are defined by the MP-bosons.

Since in the temperature range  $\Theta_D < T < T_m$ , the MP-bosons are the relevant excitations, instead of the phonons, it follows that in this temperature range the cohesion of the solids must be by the postulated compression, exerted by the ordered MP-boson field on each mosaic block. The cohesive energy, evidently, increases with decreasing temperature and lets the lattice parameter decrease strongly. As a consequence, the MP-boson field guarantees not only the cohesion of the solid up to  $T_m$ , it prevents its evaporation before melting. Note that the whole crystal can be one large mosaic block [30]. This is as for the single domain state of saturated ferromagnets. Evidently, domains and mosaic blocks are weakly coupled to the lattice and among each other.

In order to illustrate that the classical inter-atomic interactions are too weak to guarantee the cohesion of the solids up to  $T_m$ , we restrict the discussion to the alkali halide crystals with NaCl structure. These materials have the great advantage that ideal ionic bond can be assumed [31,32]. The atomistic part of the cohesive energy due to the electrostatic interaction between the ions, therefore, can be calculated and compared with experiment (see Fig. 5). Apparently, for the fcc lattice structure, the attractive interactions between the positive and the negative ions slightly dominate over the repulsive interactions between the equally charged ions, only [33]. The total atomistic binding energy, therefore is rather low.

According to the atomistic view, the inter-atomic binding energies are proportional to the Debye-temperature,  $\Theta_D$ . However, the Debye-

temperatures are much lower than the melting temperatures,  $T_m$ . In other words, the high melting temperatures are beyond the atomistic concepts. As a conclusion, there must be binding forces in addition to the classical inter-atomic forces.

According to the point-charge model, the cohesive energy depends on the lattice constant,  $a_0$ , alone [33]. As Fig. 3 shows, the electrostatic cohesive energies of the alkali halide crystals,  $E_{coh}$ , calculated for the room-temperature lattice parameters, are, in fact, a unique function of the lattice parameter [31,32].  $E_{coh}$  is a slightly weaker than linear function of  $1/a_0$ . The lower the lattice parameter is, the higher is the cohesive energy  $E_{coh}$ . However, the ionic bond model cannot explain by which mechanisms the room-temperature lattice parameters are defined. The temperature equivalent of the calculated room-temperature cohesive energies,  $E_{coh}$ , are of the order of  $10^5$  K [31, 32]. This is by a factor of  $\sim 100$  larger than the melting temperatures that are of the order of  $10^3$  K. As a consequence, the room-temperature lattice parameters are surprisingly short. Evidently, at room temperature, the crystals are strongly compressed due to the postulated compression of each mosaic block by the ordered MP-boson field. As a consequence, at ambient conditions, the crystals have a high hardness.

Consistent with the assumption that the inter-atomic interactions are of electrostatic nature, the experimental Debye-temperatures,  $\Theta_D$ , are, as the cohesive energies (Fig. 3), a unique function of the lattice parameter. As Fig. 4 shows, the shorter the lattice parameter  $a_0$  is, the stronger are the inter-atomic interactions and the larger is  $\Theta_D$ . Very interesting in Fig. 4 is, that for  $\Theta_D \rightarrow 0$ , i.e., for vanishing inter-atomic interactions, a finite lattice parameter of  $a_0 = 10.2$  Å is extrapolated. This value of  $a_0$  characterizes the finite range of the inter-atomic interactions. However, a finite interaction range cannot be understood within the point charge model because the electrostatic forces are of long range. Evidently, at this point, the assumptions of the point charge model, fail. In other words, the absolute values of  $a_0$  are beyond the point charge model. The limiting value of  $a_0 = 10.2$  Å is defined essentially by the size of the ions that results from the internal interactions within the ions.

The fact that the atomistic portion of the cohesive energies, can be calculated according to the point-charge model, allows for a separation between the atomistic, i.e., the electrostatic binding forces and the bosonic binding forces due to the ordered MP-boson field [3]. As can be

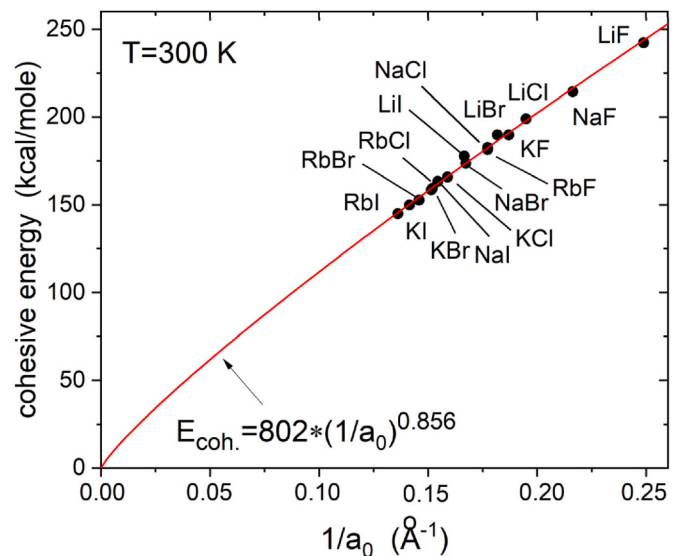
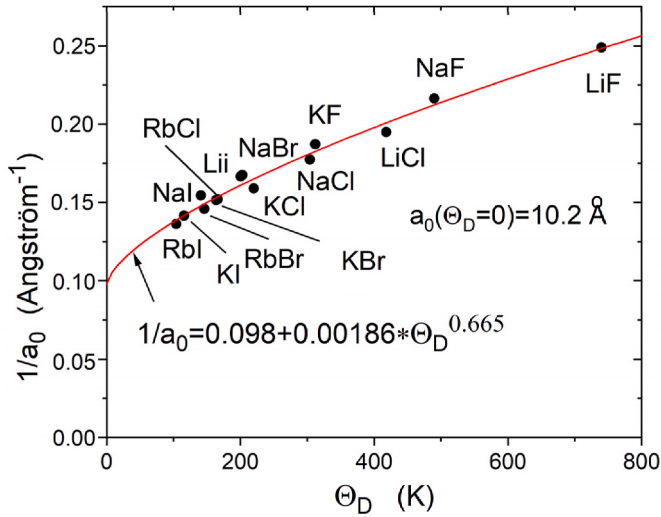
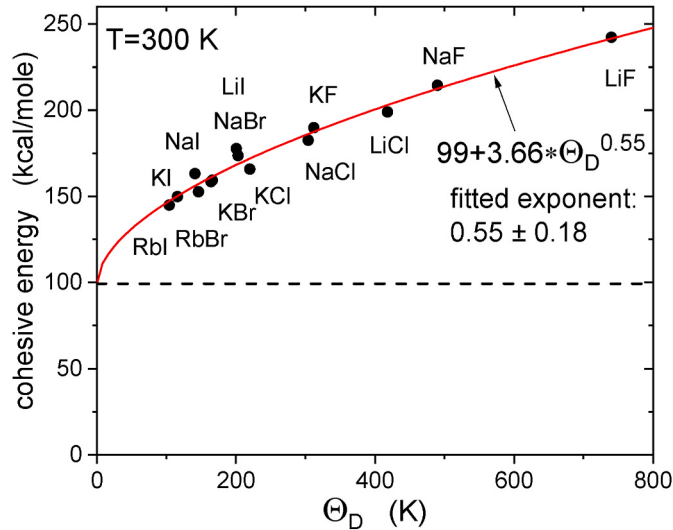


Fig. 3. Room-temperature cohesive energies of the cubic alkali halide compounds, calculated assuming ideal ionic bond [31,32], as a function of the reciprocal room-temperature lattice parameter  $a_0$ . The fact that the calculated cohesive energies are a unique function of the lattice parameter confirms the ionic bond model.  $E_{coh}$  turns out to be a slightly weaker than linear function of  $1/a_0$ .



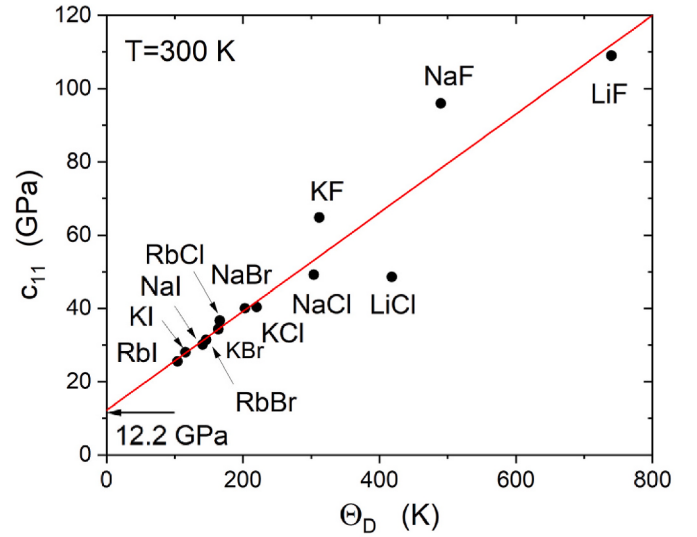
**Fig. 4.** Reciprocal lattice parameters of the alkali halide crystals as a function of the Debye-temperature  $\Theta_D$ . For  $\Theta_D \rightarrow 0$ , there are no longer inter-atomic interactions. The lattice parameter, extrapolated for  $\Theta_D \rightarrow 0$ , of  $a_0 = 10.2 \text{ Å}$  is a measure of the limited range of the inter-atomic interactions (see text).



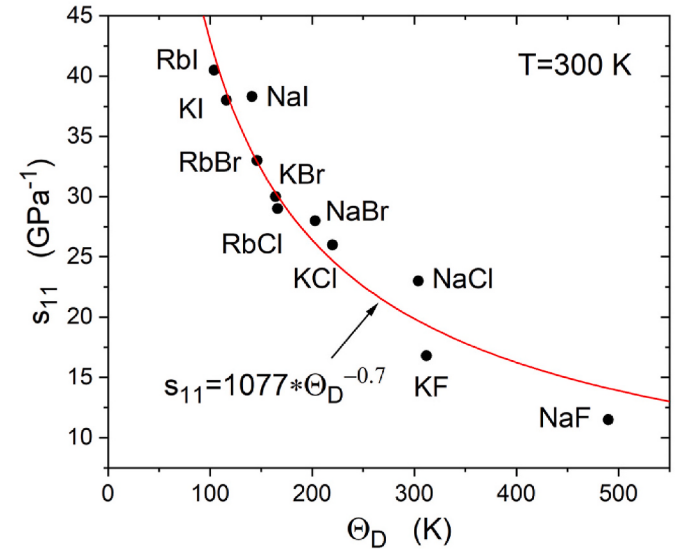
**Fig. 5.** Cohesive energies of the cubic alkali halide crystals, calculated for the room-temperature lattice parameters, using the electrostatic point charge model [31,32] as a function of the Debye-temperature,  $\Theta_D$ . The  $\Theta_D$  data visualize the contribution of the inter-atomic interactions to the cohesive energy. The finite value of  $E_{coh}$  for  $\Theta_D \rightarrow 0$  is interpreted as the contribution of the MP-bosons to the room temperature cohesive energy.

seen in Fig. 5, the electrostatic interaction energy between the atoms, given by the Debye-temperature,  $\Theta_D$ , scales reasonably with the calculated room temperature cohesive energies [31,32]. In other words, the electrostatic inter-atomic interactions contribute, in fact, to the cohesion of the alkali halide crystals. However, for  $\Theta_D \rightarrow 0$  a finite cohesive energy is extrapolated. Considering that there are no longer inter-atomic interactions for  $\Theta_D \rightarrow 0$ , the finite cohesive energy, extrapolated to  $\Theta_D \rightarrow 0$  is beyond the atomistic models and has to be interpreted as the room temperature value of the cohesive energy due to the MP-bosons.

As we have mentioned, the Debye bosons define the elastic stiffness constants,  $c_{ij}$ , but the MP-bosons define the elastic compliances,  $s_{ij}$  [10, 27]. This distinction is sharp by symmetry reasons but due to finite (and symmetry breaking) interactions of both boson fields with the atomistic background, the absolute values of the two elastic constants depend



**Fig. 6.** Room-temperature values of the elastic stiffness constants  $c_{11}$  [26] of the alkali halide crystals with cubic NaCl structure as a function of the Debye-temperature  $\Theta_D$  [29].  $\Theta_D$  and  $c_{11}$  are the larger, the harder the material is. The extrapolated finite value,  $\Theta_D \rightarrow 0$ , is considered as the small contribution of the MP-bosons to the elastic constant  $c_{11}$  at room-temperature.



**Fig. 7.** Room temperature elastic compliances,  $s_{11}$ , of the cubic alkali halide crystals as a function of the Debye temperature,  $\Theta_D$  [26]. In contrast to the elastic stiffness constants, the elastic compliances are defined by the MP-bosons, and reach a maximum for  $\Theta_D \rightarrow 0$  (compare with Fig. 6). The red solid line is a guide to the eye only.

somewhat on these interactions. As a consequence, the absolute values of the elastic stiffness constants  $c_{ij}$  are mainly due to the Debye-bosons but depend weakly on the interactions with the phonons as well.

As we have already mentioned, the Debye bosons define the elastic stiffness constants,  $c_{ij}$ , but the MP-bosons define the elastic compliances,  $s_{ij}$  [27]. As Fig. 6 shows, the higher the Debye-temperature  $\Theta_D$  is, i.e., the larger the inter-atomic interactions are, the larger are the elastic stiffness constants. However, for  $\Theta_D \rightarrow 0$  a small but finite stiffness constant is extrapolated. This is indicative of a finite interaction between Debye bosons and MP-bosons.

The dependence of the elastic compliances  $s_{11}$  on the Debye-temperature reflects the competition, i.e., the mutual exclusion of Debye-bosons and MP-bosons (Fig. 7). For the hypothetical case  $\Theta_D \rightarrow \infty$

the inter-atomic interactions dominate, and the MP-bosons are insignificant, i.e., the elastic compliances  $s_{11}$  tend to zero. On the other hand, for  $\Theta_D \rightarrow 0$  there are no inter-atomic interactions and the elastic compliances, defined by the MP-bosons predominate (Fig. 7). The qualitatively different dependencies of the two elastic constants on the Debye-temperature in Figs. 6 and 7 proves consistently, that the two elastic constants are defined by different bosons. The elastic constants reach a maximum at the critical temperature of the respective boson field. This is at  $T = 0$  for the constants  $c_{ij}$  but at  $T = T_m$  for the constant  $s_{ij}$  [10]. It appears that macroscopic quantities, such as the constants  $c_{ij}$  and  $s_{ij}$ , are defined generally by bosons.

The idea that the MP-bosons are elastic quadrupole radiation can be tested by showing that the melting temperatures of the alkali halide crystals scale reasonably with the atomic masses of anions and cations [10]. Note that the ordering transition of the MP-boson field occurs, when the density of the bosons (elastic quadrupole radiation) has reached the critical value for the spontaneous onset of stimulated emission. This is a threshold induced event, similar to the ignition of a LASER. The density of the bosons depends essentially on the production rate of the bosons, which can be assumed to be proportional to the size of the elastic quadrupole moments. Assuming that for all of the chemically very similar alkali halide crystals, the not specified elastic coupling between anions and cations are very nearly the same, the magnitude of the elastic quadrupoles should be given essentially by the atomic masses of anions and cations. As a consequence, the melting temperatures should depend somehow on the mass of the anions and of the mass of the cations. In fact, as Fig. 8 shows, the melting temperatures of the alkali halide crystals scale reasonably with the ratio between the mass of the anion and the mass of the cation. From the fact that the melting temperature has a maximum for nearly equal masses of anions and cations it can be concluded that the quantitative value of the elastic quadrupole moment has a maximum if anions and cations have similar masses.

## 6. Conclusions

The experimental indications have been discussed that bosonic energy degrees of freedom are responsible for the critical thermodynamics below the liquid-solid transitions. It is concluded that at this transition, the bosons (elastic quadrupole radiation) order in the background and

cause the visible long-range and coherent order of the crystalline lattice, instead of the atomistic near-neighbour interactions. Upon ordering, all bosons condense in the lowest energy state (Bose-Einstein condensation). This is the highest possible thermodynamic order and provides an understandable entropy argument that the bosons exclude the inter-atomic interactions from performing the ordering transition. The observed, boson-induced crystalline order within each mosaic block is more perfect than it can be expected if the locally anisotropic atomistic interactions would be relevant and would drive the ordering transition. Apparently, nature intends to realize the highest possible order at order-disorder phase transitions.

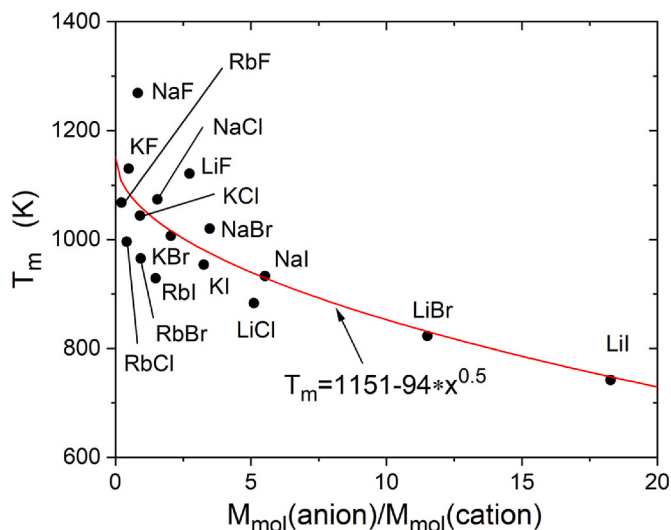
The bosons that order at the melting point of the solids (elastic quadrupole radiation) were called here MP-bosons [27]. Since the atomic order below  $T_m$  is three-dimensional, the MP-bosons cannot be dipole radiation but must be three-dimensional quadrupole radiation. It is proposed that the sources of the MP-bosons are elastic quadrupoles.

For the Rare Gas solids, it has already been shown that at the liquid-solid transition the MP-bosons order [10]. This is confirmed here using the cubic alkali halide crystals as another group of model materials. Both classes of solids have the great advantage that the interatomic interactions can be calculated theoretically and can be compared with the observed melting temperatures. To give an example, using the point charge model, a cohesive energy of  $E_{coh.} = 242.3$  kcal/mol has been calculated for LiF for the room-temperature lattice constant of  $a_0 = 4.017$  Å, [31,32]. This energy corresponds to a temperature of 122000 K which is larger by a factor of 109 compared to the melting temperature of LiF of  $T_m = 1121$  K [4,5]. The calculated high cohesive energy shows that the room temperature lattice parameter is surprisingly short, and that the solids are strongly compressed. A direct measure of the pressure  $p$  acting on the LiF crystal at room-temperature can be obtained by setting  $E_{coh.} = pV_{mol}$  with  $V_{mol} = 9.844$  cm<sup>3</sup> as the molar volume of LiF at room-temperature. It turns out that  $p = 103$  GPa or  $p \sim 10^6$  bar. For temperatures increasing above room-temperature, the compressive pressure, exerted by the ordered MP-boson field on each mosaic block, can be assumed to decrease and the lattice expands. The compressive pressure can be assumed to have fallen to the ambient value at  $T_m$ .

A reasonable measure of the atomic near-neighbour interaction energy is given by the Debye temperature  $\Theta_D$  which, commonly, is much lower than the melting temperature,  $T_m$ . As a consequence, in the wide temperature range  $\Theta_D < T < T_m$ , heat capacity and thermal lattice parameter changes have to be attributed to the MP-bosons that are the only remaining energy degrees of freedom. This idea is supported by the observed universal temperature dependence of both quantities in the temperature range  $\Theta_D < T < T_m$ .

It appears that all macroscopic properties of the solids, such as the spontaneous magnetization, are due to bosons [27]. The elastic constants of the solids are, evidently, macroscopic parameters. The bosons defining the elastic stiffness constants  $c_{ij}$  are the sound waves, called here Debye bosons (elastic dipole radiation) but the bosons defining the elastic compliances  $s_{ij}$  are the MP-bosons (elastic quadrupole radiation) [10]. In other words, there are two types of bosons of the elastic degree of freedom. While the critical temperature for the ordering transition of the Debye bosons is  $T = 0$ , the MP-bosons order at the melting temperature  $T_m$  [27]. Consistent with this view is that the constants  $c_{ij}$  reach their maximum at  $T = 0$  but the constants  $s_{ij}$  reach their maximum at  $T = T_m$  [26].

Ordering of boson fields is essentially a consequence of the fact that, in the critical range above the ordering transition, stimulated emission becomes the dominant generation process of the bosons. Due to stimulated emission, a strong amplification of particular bosons, with a distinguished wave-vector and phase, occurs. This has to be considered as a partial thermodynamic order, and is in preparation of the perfect order, reached at the ordering transition of the bosons where all bosons have condensed in this energy state. A very important additional condition of a stable and stationary ordered boson field is that, upon ordering, the bosons confine themselves in space to the finite volume of



**Fig. 8.** Melting temperatures of the alkali halide crystals,  $T_m$ , as a function of the ratio between the mass of the anion and the mass of the cation. The systematic of these data supports the conclusion that the atomic masses are decisive for the temperature of the liquid-solid transition (see text) [10]. This is consistent with the view that the bosons that order at  $T_m$ , the MP-bosons are of elastic origin (elastic quadrupole radiation).

a stationary and long-range ordered unit. In the ordered magnets these units are the magnetic domains [14], and in the crystalline solids these units are the mosaic blocks. The ordered units are the true signature of the ordered boson field. The process by which boson fields generate domains or mosaic blocks upon ordering, belongs to the many theoretically unsolved problems of solid-state physics. We must assume that upon condensation of the bosons generated by dipoles the propagation of the bosons gets stopped and the bosons become fixed to the crystalline lattice. The standing boson field resembles an interference pattern. Since a large number of coherent bosons has condensed in the same stationary wave function, the amplitude of this wave function becomes very large. In this way, high local fields result which, evidently, dominate over all interatomic interaction fields and define the critical thermodynamics of the atomistic system. Moreover, upon ordering, the bosons do not only assume the lowest possible energy, they intend to assume the smallest possible volume as well, and compress the ordered unit increasingly with decreasing temperature. This, here postulated, bosonic force is theoretically unexplained, as well. Domains and mosaic block seem to be resonators, self-organized by the ordered boson fields.

With the formation of domains, or mosaic blocks, at the ordering temperature of the respective boson field, thermal lattice contractions set in as a function of a decreasing temperature. For the ordered magnets, this phenomenon is well-known as magnetostriction that sets in sharply with the formation of domains at the magnetic ordering temperature [3,14,15]. Magnetostriction is an axial effect, along the domain axis. In fact, strong thermal lattice contractions are a domain effect and, therefore, are typical of ordered state of the boson fields. Thermal lattice contractions below  $T_m$  have to be interpreted in a similar way by an increasing compression of each mosaic block by the ordered MP-boson field. The self-constriction of the ordered MP-bosons (elastic quadrupole radiation) is concentric and compresses each mosaic block congruently [5]. This bosonic force is believed to be responsible for the cohesion of the solids, up to  $T_m$ . In other words, bosonic forces make the solids solid up to  $T_m$ .

For a theoretical explanation of the here, and previously [2,3,10], evaluated experimental critical exponents, field theories are needed. Condition for solving these field-theoretical problems is that the dispersion relations and the densities of states of the bosons are known. As we have mentioned, the two quantities can nearly not be evaluated experimentally. It is evident that for the bosons that order at a finite temperature the dispersion relation and the density of states must be rather complicated and can be assumed to be different above and below the ordering transition of the boson field. Only for the Debye bosons the situation is relatively simple since the Debye bosons do not order at a finite temperature and are, so to say, always in the paramagnetic state. The critical temperature of the Debye bosons is  $T = 0$ . As a consequence, the heat capacity in the critical range at  $T = 0$  is a function of absolute temperature. Since the Debye bosons do not order at a finite temperature, the process by which domains get formed and get increasingly compressed with decreasing temperatures is insignificant. Typical of boson dynamics is that the  $T^3$  function of the critical heat capacity at  $T = 0$  holds over a finite temperature range and that the critical exponent of three is universal, i.e., it is observed for all non-magnetic insulators. For the Debye bosons it is self-evident that the dispersion is a linear function of wave vector and the density of states a quadratic function of energy. As a consequence, the  $T^3$  function could be evaluated as early as in 1912 by P. Debye [6]. Note, that Debye's critical exponent of three is the only theoretically explained critical exponent of solid-state physics up to now. A similarly simple situation holds for the bosons of the continuous metallic solid (electric quadrupole radiation) for which the critical temperature is, as for the Debye bosons, at  $T = 0$ , and the critical heat capacity at  $T = 0$  is a linear function of absolute temperature. As a consequence, for the metals that get not superconducting (!) these bosons also do not generate domains. However, the dispersion relation and the density of states of the bosons generated by electric quadrupoles are unclear. Since the critical temperature is  $T = 0$ , it can be assumed that

the dispersion relation is a simple power function of wave vector and the density of states a simple power function of energy. As a consequence, the exponent of unity of the critical heat capacity of these bosons is, as the many other meanwhile observed universal critical exponents, theoretically unexplained.

Concluding it can be summarized that the cohesion of the solids up to  $T_m$  is by a theoretically not understood new type of bosonic force. This here postulated bosonic force is omnipresent since it guaranties the cohesion of the solids up to the occasionally very high melting temperatures and prevents their evaporation before melting.

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Submitted for publication in the journal Physics Open is my own work. I am the only responsible author. The new findings reported in this paper are my personal achievements. I got no scientific, financial or practical support by any other person or institution. I am not aware of any interference or competing interest of any colleague or institution. All analysed experimental data are from the cited references.

## Data availability

Data will be made available on request.

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