



Fine-tunings in nucleosynthesis and the emergence of life: status and perspectives

Ulf-G. Meißner^{1,2,3,4,5,a} , Bernard Ch. Metsch^{1,4,b}, Helen Meyer^{1,2,c}

¹ Helmholtz-Institut für Strahlen- und Kernphysik, Rheinische Friedrich-Wilhelms Universität Bonn, 53115 Bonn, Germany

² Bethe Center for Theoretical Physics, Rheinische Friedrich-Wilhelms Universität Bonn, 53115 Bonn, Germany

³ Center for Science and Thought, Rheinische Friedrich-Wilhelms Universität Bonn, 53115 Bonn, Germany

⁴ Institute for Advanced Simulation (IAS-4), Forschungszentrum Jülich, 52425 Jülich, Germany

⁵ Peng Huanwu Collaborative Center for Research and Education, International Institute for Interdisciplinary and Frontiers, Beihang University, Beijing 100191, China

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Abstract We discuss the fine-tunings of nuclear reactions in the Big Bang and in stars and draw some conclusions on the emergence of the light elements and the life-relevant elements carbon and oxygen. We also stress how to improve these calculations in the future. This requires a concerted effort of different communities, especially in nuclear reaction theory, lattice QCD for few-nucleon systems, stellar evolution calculations, particle physics and philosophy.

1 Prologue

This viewpoint grew out of discussions with Christian Caron concerning the Advanced Grant from the European Research Council on “Emergent complexity from strong interactions”, in particular the third work package on “How fine-tuned is nucleosynthesis?”. This is a topic that can only be addressed in theory, as Nature gives us specific values for the fundamental constants that govern the emergence of nucleons and nuclei, and thus the emergence of life as we know it. The aim of this viewpoint is to get more people interested in such type of research, which also has some overlap with philosophy – for recent discussions see [1, 2].

2 Introduction

The matter we are made of consists almost completely of atomic nuclei. These are generated shortly after the Big

Bang and in stars – that is why one often says that we are made of stardust. Of particular relevance are the ^{12}C and ^{16}O nuclei, which form the basis of the life on Earth as we know it. These elements are generated in hot, old stars through the triple-alpha process and the radiative capture process $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, respectively. Both ^{12}C and ^{16}O are alpha-type nuclei, that is to a good approximation they can be described as bound states of three, respectively four, ^4He particles. ^4He is already generated abundantly shortly after the Big Bang, and it is well-known that the triple-alpha reaction features the Hoyle state [3], a resonance close to the $^4\text{He} + ^8\text{Be}$ threshold, that is required to form a sufficient amount of carbon and oxygen in stars to enable life on Earth. Similarly, in Big Bang nucleosynthesis (BBN), the deuterium bottleneck plays a crucial role, as the Universe has to cool down sufficiently so that the neutron-proton fusion to deuterium is not undone by the abundant energetic photons (with $E_\gamma > 2.2$ MeV) that disintegrate the deuteron.¹ Therefore, to understand these fine-tunings and draw conclusions on possible variations of the fundamental constants is not only interesting by itself, it also is a necessary requirement for the study of possible Beyond the Standard Model (BSM) effects in these reactions. The fundamental parameters under consideration are related to the various interactions pertinent to nuclear physics, namely the light quark masses m_u , m_d , m_s related to the strong interactions, the electromagnetic fine-structure constant α_{EM} , the coupling constant of QED, and, to a lesser extent, the Fermi coupling constant G_F that gives the strength of the weak

^a e-mail: meissner@hiskp.uni-bonn.de (corresponding author)

^b e-mail: metsch@hiskp.uni-bonn.de

^c e-mail: hmeyer@hiskp.uni-bonn.de

¹ This, by the way, is very different from high-energy heavy ion collisions on Earth, where photons play essentially no role. The photons in the Big Bang are due to particle-antiparticle annihilations happening before element synthesis.

interactions at low energies. For a study of a Universe without weak interactions, see Ref. [4]. It should be stressed that the formation of the life-enabling elements is a necessary but not sufficient condition for the emergence of life as we know it – it requires many other sciences to come to a complete picture, see e.g. [5]. Having said that, we here concentrate on the nuclear physics aspects of this topic. This paper is not a review, but we rather intend to stress some recent developments and loopholes in such type of calculations as well as discussing required improvements in this intricate interplay of effective field theories (EFTs), lattice QCD (LQCD) calculations and nuclear reaction modeling.

The paper is organized as follows: In Sect. 3 we discuss the constraints on the electromagnetic fine-structure constant α_{EM} and the light quark masses in the Big Bang. Section 4 considers similar variations for various nuclear reactions pertinent to the generation of carbon and oxygen. A discussion and outlook is given in Sect. 5.

3 Fine-tunings in the Big Bang

The light elements up to $A = 7$ are generated in BBN through an intricate interplay of nuclear reactions, the so-called reaction network, that is given in terms of coupled differential equations (the rate equations) for the abundances $Y_i = n_i/n_B$, with n_i the density of nucleus i and n_B the total baryon density.

The first point we want to stress is that one should use the different publicly available codes for BBN to get an estimate of the systematical errors in the network calculation. These codes are: NUC123 [6], AlterBBN [7,8], PArthENoPE [9, 11], PRIMAT [12] and PRyMordial [13,14]. These differ mainly in the number of reactions considered, in the parametrization of the nuclear reactions and the numerical treatment of the rate equations.

Let us first consider the variation of α_{EM} . There are essentially four different ways a dependence on α_{EM} is generated: (i) in the nuclear reactions rates, we encounter the Coulomb barrier, which leads to an energy-dependent penetration factor in the cross section [15], (ii) radiative capture reactions, (iii) in the $n \leftrightarrow p$ conversion and in β -decay rates, one has to deal with final and/or initial state interactions between charged particles, and (iv) there are various indirect effects generated by the Coulomb contribution to the nuclear binding energies and the QED contribution to the neutron-proton mass difference, $\Delta Q_n = Q_n^{\text{QED}} \cdot \Delta \alpha_{\text{EM}}/\alpha_{\text{EM}} = -0.58(16) \text{ MeV} \cdot \Delta$ and divide by $\Delta \alpha_{\text{EM}}/\alpha_{\text{EM}}$ [16]. An up-to-date calculation of the Coulomb contribution to the nuclear binding energies based on Nuclear Lattice Effective Field Theory (NLEFT) [17] compared to a parameterization $B_E^{\text{Coulomb}} = 0.6 Z(Z-1) A^{-\frac{1}{3}} [\text{MeV}]$ as in the time-

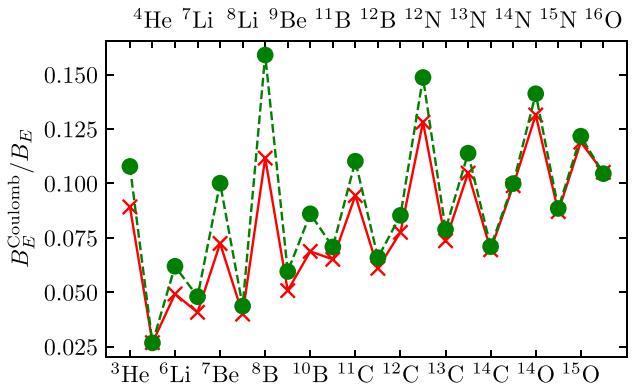


Fig. 1 Coulomb contribution to the nuclear binding energies based on NLEFT (red crosses) compared to the Bethe–Weizsäcker formula (green circles)

honored Bethe–Weizsäcker formula [18,19] is displayed in Fig. 1, showing that more and more of the quantities under consideration can be calculated *ab initio*.

The biggest source of uncertainty are indeed the reaction rates and cross sections, with the exception of the $n + p \rightarrow d + \gamma$ reaction, that can be precisely calculated in pionless EFT [20]. We remark in passing that at temperatures (or energies) relevant in primordial nucleosynthesis the relevant reaction rates can be measured. In Ref. [21], we made use of available experimental data to find novel parametrizations of the leading 17 reaction rates. Together with the pionless EFT rate for $n + p \rightarrow d + \gamma$, we implemented these new rates into the reaction network of the 5 codes mentioned above to draw conclusions on the allowed variation of α_{EM} from the reliable measurements of the d and ^4He abundances (the ^7Li abundance features the so far unsolved Lithium puzzle [22]). First, we observed that the different codes gave rather similar results. Second, we found $|\delta \alpha_{\text{EM}}| < 1.8\%$ from ^4He and $|\delta \alpha_{\text{EM}}| < 0.8\%$ from d , which are tighter constraints than found previously.

To partly overcome the modeling of the nuclear reactions, in Ref. [23], we used Halo-EFT (for a review, see [24]) to describe the reactions $n + ^7\text{Li} \rightarrow ^8\text{Li} + \gamma$ [25,26], $p + ^7\text{Be} \rightarrow ^8\text{B} + \gamma$ [27,28], $^3\text{H} + ^4\text{He} \rightarrow ^7\text{Li} + \gamma$, $^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$ [29–31]. Using these rates, one finds substantial deviations from the α_{EM} -dependence of the parameterized rates obtained for these reactions previously, however, the impact on the resulting abundances and on their α_{EM} -dependence of the light elements ^2H , ^3H + ^3He , ^4He , ^6Li is very minor only. In contrast, for the $^7\text{Li} + ^7\text{Be}$ -abundance we do find that the α_{EM} -dependence differs appreciably from that of the previous parameterized results, this α_{EM} -dependence being much more pronounced and clearly non-linear with the Halo-EFT rates. Also the nominal abundance (i.e. calculated with the current value of the fine-structure constant) of $^7\text{Li} + ^7\text{Be}$ is larger by almost 10%, whereas the other abundances remain practically unchanged. For reac-

tions involving charged particles, the Halo-EFT calculation accounts for the charged particle repulsion by inclusion of the full Coulomb propagator in all reaction steps. As shown in [23] these Coulomb effects cannot always be approximated by a universal penetration factor. It was also found that in some cases the study of the α_{EM} dependence of cross sections and the corresponding rates within the framework of Halo-EFT is limited by singularities appearing in the normalization, that enters as a factor in the resulting cross sections. This was found to be relevant for the ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$ reaction, limiting the study to relative variations of α_{EM} to less than 6%. We will come back to the microscopic description of the nuclear reactions in the following sections.

Next we consider the variations of the light quark masses m_u, m_d , which have been considered in many works using various levels of modeling. The first work that used pionless EFT to address the issue was Ref. [32], followed by the use of chiral nuclear EFT with some resonance saturation modeling of the four-nucleon contact terms in Ref. [33]. Using the Gell-Mann–Oakes–Renner relation, the quark mass dependence can be mapped onto the pion mass dependence and the leading nuclear interactions depend explicitly and implicitly on the pion mass M_π as shown in Fig. 2. These works came to the conclusion that variations of the light quark mass of about 1% are consistent with the observed abundances, where the ${}^4\text{He}$ abundance sets a tighter constraint than the d abundance (and similarly for variations of the Higgs VEV v). Remember that for constant Yukawa couplings, variations in the quark mass and v are the same).

Figure 2 already demonstrates all the ingredients and complications that such type of calculation exhibits. On the one hand, LQCD easily provides data for the nucleon mass and the pion-nucleon coupling constant g (using the Goldberger–Treiman relation) at varying quark (pion) masses, whereas the four-nucleon couplings $C_{S,T}$ can not so easily be obtained from LQCD. This is where the low-energy theorems (LETs) of Refs. [34,35] enter, as they allow to connect LQCD data for the two-nucleon system at relatively large pion masses with the small changes of the pion mass around its physical

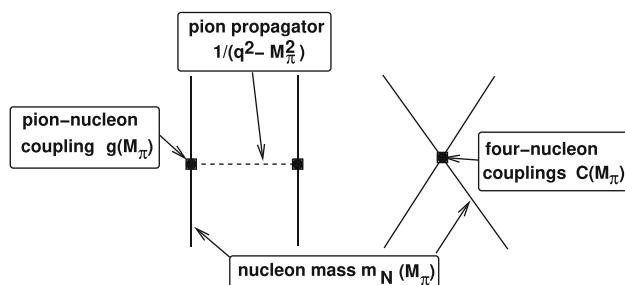


Fig. 2 Pion mass dependence of the leading order nucleon-nucleon interaction. Left: one-pion exchange. Right: Leading four-nucleon contact interactions. Solid (dashed) lines denote nucleons (pions)

value as considered e.g. in BBN.² Variations of the Higgs VEV v came back into focus recently in Ref. [38] due to the newly published EMPRESS data [39] for the ${}^4\text{He}$ abundance, using one-boson-exchange modeling. Here, the ${}^4\text{He}$ abundance led to the strongest constraints, somewhat stronger than found earlier. The variation of α_{EM} in view of the EMPRESS data was considered in Ref. [40]. The bounds on $\Delta v/v$ were reconsidered in Ref. [41], where LQCD data for $m_N(M_\pi)$ and $g_A(M_\pi)$ were used together with LQCD data on the deuteron binding energy and the S-wave scattering lengths a_s, a_t for pion masses from 300 to 800 MeV combined with the above-mentioned LETs. The most important finding in that paper was that the backwards reaction $d + \gamma \rightarrow n + p$ is influenced by a change in the deuteron binding energy. Because of the deuterium bottleneck, the rate of this reaction defines the beginning of BBN and has therefore a sizeable impact especially on the ${}^4\text{He}$ abundance. The reaction rate for $d + \gamma \rightarrow n + p$ is derived from the rate for $n + p \rightarrow d + \gamma$ through a detailed balance relation

$$\langle \sigma(d + \gamma \rightarrow n + p)v \rangle = \aleph T_9^{3/2} \exp(\kappa/T_9) \langle \sigma(n + p \rightarrow d + \gamma)v \rangle, \quad (1)$$

where T_9 is the photon temperature in units of 10^9 K . The parameter \aleph depends on the deuteron mass (and hence on the deuteron binding energy) and the neutron and proton mass through $\aleph \propto (m_n m_p/m_d)^{3/2}$, while the parameter κ corresponds to the reaction Q -value and is, for this reaction, directly proportional to the deuteron binding energy $\kappa \propto B_d$. When making changes to the deuteron binding energy (and hence its mass) and the nucleon mass, this needs to be taken into account and it has a significant effect on the d and ${}^4\text{He}$ abundances. Even more, the deuterium abundance now sets the strongest constraints, comparing with the PDG numbers leads to the 2σ bound:

$$-0.07\% \leq \Delta v/v \leq -0.02\%, \quad (2)$$

which is a much stronger fine-tuning than observed in all other earlier works. The corresponding ${}^4\text{He}$ bound is $-0.7\% \leq \Delta v/v \leq +0.4\%$.

Recently, the dependence on the strange quark mass m_s was also studied. This is more challenging, as strange quark effects are mostly indirect and chiral EFTs including baryons and kaons encounter convergence problems for a number of observables. In Ref. [42] it was argued that the dominant strangeness effect in BBN is the strange quark contribution to the nucleon mass shift, parameterized in terms of the strangeness σ -term, $\sigma_s = \langle N | m_s \bar{s}s | N \rangle$. Varying the nucleon mass in the leading eight reactions that involve neutrons,

² We are well aware that there is a discussion on the validity of some lattice data [36,37], but this does not invalidate the trend given by the LETs. Clearly, this needs to be resolved to make further progress.

protons and the four lightest nuclei, using the various BBN network codes, leads to strict limits on the allowed nucleon mass variations that translate into an upper bound on possible variations of the strange quark mass, $|\Delta m_s/m_s| \leq 5.1\%$ (assuming the strange quark condensate not to vary). It is amusing to note that a mere 2% reduction of the nucleon mass would solve the Lithium problem. For a general discussion of the primordial nuclear abundances on fundamental nuclear observables such as binding energies, scattering lengths, neutron lifetime, etc., see [43].

4 Fine-tunings in stellar nucleosynthesis

Now we consider the generation of carbon and oxygen in hot, old stars. Here, the triple- α process exhibits two fine-tunings, namely the closeness of the unstable but long-lived ^8Be nucleus to the 2α threshold and of the Hoyle state to the 3α threshold. It was speculated already by Weinberg (and others) that these two fine-tunings are correlated [44] (also, such correlations are implicit in the ground-breaking work of Ikeda and collaborators on alpha-clustering [45]). That this is indeed the case could only be shown using NLEFT in Refs. [46,47] a decade later. For a general review on nucleosynthesis, see Ref. [48].

Let us concentrate on the closeness of the Hoyle state to the 3α threshold. It is well-known that the rate of the triple-alpha process depends exponentially on the energy difference $E_R = E_{12}^* - 3E_4$, where E_{12}^* is the excitation energy of the Hoyle state and E_4 the mass of the alpha-particle. First numerical experiments by varying this energy difference in stellar simulations were performed in Refs. [49,50]. This was improved in [51], where a larger range of star masses, $M_\star = (15 - 40)M_\odot$ and also both solar and low metallicity were considered. Furthermore, these authors also investigated the generation of carbon, oxygen and heavier elements. They find values for ΔE_R that depend on the metallicity. For low metallicity, for negative values of ΔE_R , carbon production limits this to $\Delta E_R \geq -300$ keV and for positive values, oxygen production leads to $\Delta E_R \leq 300$ keV. For solar metallicity, these bounds are found to be narrower by a factor of 2, but these are still larger than found in the earlier studies [49,50]. The updated NLEFT analysis [52], which for the first time used the LETs of Refs. [34,35] in the pion mass dependence of the four-nucleon operators, gave a range of possible quark mass variations related to the treatment of the short-distance two-nucleon physics, leading to bounds between 0.4% to 5% for $\Delta m_q/m_q$ (or $\Delta v/v$). The corresponding bounds on the variation of α_{EM} , $\Delta\alpha_{\text{EM}}/\alpha_{\text{EM}} < 7.5\%$, are also less strict than found in BBN. Interestingly, the nuclear reaction rates in the Big Bang and in stars limit the mysterious QCD θ -parameter to $\theta \lesssim 0.1$ [53], which is not partic-

ular fine-tuned and far away from the experimental limit of $\theta \simeq 10^{-11}$.

These type of calculations could be improved in two aspects. First, one would like to not only vary E_R but all the masses, reactions rates etc. in the stellar burning processes, similar to what is done in BBN. That, however, is out of reach of present computational capabilities. Second, more LQCD data for the two-nucleon system at lower pion masses could certainly help to reduce the uncertainty of the allowed variations in the quark masses (the Higgs VEV) derived from the variations found for E_R in the stellar modeling.

Another venue to address the issue of parameter variations in the triple-alpha process has recently become available in the framework of NLEFT. After the pioneering work on α - α scattering at N2LO published in 2015 [54], it was possible to study the dependence on the fundamental constants of the SM of this first reaction in the triple-alpha process [55]. It was found that positive shifts in the pion mass have a small effect on the S-wave phase shift, whereas lowering the pion mass adds some repulsion in the two-alpha system. The effect on the D-wave phase shift turns out to be more pronounced as signaled by the D-wave resonance parameters. Variations of α_{EM} have almost no effect on the low-energy α - α phase shifts. This calculation can clearly be improved by going to N3LO using the high-fidelity chiral forces from [17]. In a next step, one would extend such type of study to the second reaction in the triple-alpha process, namely the fusion of ^4He with ^8Be to generate ^{12}C via the Hoyle state. This would be followed by a study of the holy grail of nuclear astrophysics, $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, using the same methods. First unpublished results on elastic α - ^{12}C scattering in S- and P-wave look indeed promising, paving the way for the calculation of the radiative capture reaction. It would also be interesting to use the pionless (cluster) EFT of Refs. [56–58] to study the dependence of α - ^{12}C on α_{EM} and the light quark masses.

Other fine-tunings observed in element generation or energy production in stars are the proton capture in ^{56}Ni to produce heavier elements in X-ray bursts [59], the $^{205}\text{Pb} - ^{205}\text{Tl}$ conversion, as ^{205}Pb plays a major role in revealing the formation history of the sun [60] and electron capture in ^{20}Ne , that has a decisive impact on the evolution of the core for stars with 7–11 solar masses [61]. To our knowledge, these have not been scrutinized along the lines discussed before.

5 Discussion and outlook

Now we are at the point to draw some conclusions on the fine-tuning (for a general discussion relating it to the anthropic principle, see [62], or the multiverse, see [63]). As can be seen, the bounds on possible variations of the Higgs VEV are much stronger from BBN, and also the theoretical uncer-

tainty on the variations deduced from carbon and oxygen production are larger. In particular, the sub-percent variations for v , cf. Eq. (2), sheds new light on the fine-tunings in BBN, and it also serves as a benchmark for BSM models applied to primordial nucleosynthesis.

There are a number of issues that need to be tackled to make these calculations more conclusive and/or can lead to novel insights:

- It would be very valuable to use more reaction theory calculations in BBN to study their parameter dependence. In case of the largely analytical Halo-EFT, this can be done by other researchers than the authors of the various papers, as discussed before. This is very different from the no-core-shell model coupled to the continuum, that offers a different *ab initio* approach the nuclear reactions in the BBN network to what has been discussed before, see e.g. Refs. [64–67]. The proponents of this approach should consider performing the pertinent calculations. Of course, NLEFT will also contribute to these developments.
- Cluster models based on EFT approaches can also be used to get a handle on possible parameter variations. In particular, the recent work of Refs. [56–58] on the holy grail of nuclear astrophysics should be mentioned, where the α_{EM} dependence is explicit and the one on the quark masses implicit, in terms of the pertinent scattering parameters and nuclear masses.
- As already stated, it would be very valuable to have LQCD simulations for the two-nucleon system at lower quark masses, which would help to reduce the uncertainty generated from the quark mass expansion of the four-nucleon contact terms, that so far is least constrained.
- Finally, we also mention that so far most calculations keep the Yukawa couplings constant. It would be very interesting to perform more work along the lines of Ref. [68], where the interrelations between the fundamental parameters arising in unified theories were considered.

We hope that with the discussion presented here, more researchers from the different fields mentioned will be contributing to this highly interesting topic. Finally, it would also be important to bring the somewhat disjoint communities of philosophers and physicists closer together to deepen our understanding of this topic.

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