Vacuum energy in effective field theory of general relativity

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It is shown to all orders of perturbation theory that in the effective field theory of general relativity the condition of vanishing of the vacuum energy leads to the same value of the cosmological constant viewed as a parameter of the effective Lagrangian, as obtained by demanding the consistency of the effective field theory in the Minkowski background. The resulting effective action is characterized by the cosmological constant term that vanishes exactly.

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It is difficult to disentangle the philosophical and physical aspects of the cosmological constant problem. Eventually, a lot depends on the attitude one takes toward the relationship between the science and the material world around us. It seems natural to think that the vacuum state of our Universe is what remains if the whole material is removed; that is, vacuum means just "nothing." It seems even more natural that an adequate theory should assign zero energy to nothing [1]. Does this "naive" intuitive picture contradict our well-established and very successful quantum theory? Note that arguments in favor of the nonvanishing vacuum energy based on the Casimir effect should, at least, be considered as questionable; see, e.g., Refs. [2,3]. The experimental observation of the accelerating expansion of the Universe (see, e.g., Ref. [4] and references therein) is often being interpreted as evidence of the vacuum energy to have a nonzero value (for a review, see, e.g., Ref. [5]). This leads to the well-known cosmological constant problem caused by a huge mismatch between the theoretical estimations of the cosmological constant and its value suggested by the experimental data [6]. This problem is also relevant for modern cosmology, which relies on Einstein's theory of general relativity.

In the modern view, Einstein's theory of general relativity is regarded as a leading-order approximation to an effective field theory (EFT). It is widely accepted that at low energies, all fundamental interactions including gravity can be described by an EFT [7]. Not every background of general relativity represents a valid vacuum in quantum theory. Various considerations suggest that among the three maximally symmetric backgrounds, Minkowski is the only consistent vacuum with the possibility of nontrivial cosmological history [8–12]. In the EFT framework, the cosmological constant is just one of the parameters of the effective Lagrangian of interacting gravitational and matter fields [13–15]. We seem to be rather far from constructing/discovering the fundamental theory underlying this effective theory (if it exists at all); however, we can impose conditions on parameters of the low-energy EFT such that the vacuum energy is exactly zero. Even if such an underlying fundamental theory does not exist, the property of vanishing vacuum energy can equally well be realized order by order in perturbation theory within the low-energy EFT. In the realm of perturbation theory, this uniquely fixes the cosmological constant as a function of other parameters of the effective Lagrangian. On the other hand, the cosmological constant gets fixed from the consistency condition for the considered EFT in the Minkowski background. In particular, the consistency of an EFT in the Minkoswski background, enforced by demanding the absence of the massive ghost graviton degrees of freedom, was shown to uniquely determine the cosmological constant to all orders in the loop expansion for the case of an Abelian gauge theory with spontaneous symmetry breaking coupled to the metric field [16]. At the two-loop level, the conditions of vanishing vacuum energy and the absence of massive ghost modes were found to yield the same expressions for the cosmological constant, thanks to non-trivial cancellations between different contributions [17–19]. Notice that while the authors of Ref. [16] consider a particular Abelian model for demonstration, their derivation of the Ward identity and the graviton low-energy theorem used to obtain the condition of self-consistency applies to any local quantum field theory which is invariant under transformations of diffeomorphism, including the Standard Model (SM) coupled to general relativity. Below we give a general argument imposing that the two-loop-order results of Refs. [17–19] hold to all orders of perturbation theory in the EFT of general relativity. That is, the consistency of the EFT in the Minkowski background and the requirement of the vanishing vacuum energy lead to the same value of the cosmological constant as a parameter of the effective Lagrangian.

We consider the action specified by the most general effective Lagrangian of gravitational and matter fields of the SM, which is invariant under general coordinate transformations and other underlying symmetries

$$S = \int d^4x \sqrt{-g}\left\{\frac{2}{\kappa^2}(R - 2\Lambda) + \mathcal{L}_{\rm gr,ho} + \mathcal{L}_{\rm m}\right\}, \quad (1)$$

where $\kappa^2 = 32\pi G$, G is Newton's gravitational constant, g denotes the determinant of the metric field $g^{\mu\nu}$, Λ is the cosmological constant, and R refers to the scalar curvature. Further, $\mathcal{L}_{\rm gr,ho}$ represents self-interaction terms of the gravitational field with higher numbers of derivatives while $\mathcal{L}_{\rm matter}$ is the effective Lagrangian of the matter fields interacting with gravity. The success of the theory of gravitation based on the Einstein-Hilbert action suggests that the contributions of $\mathcal{L}_{\rm gr,ho}$ are heavily suppressed by some large scale(s).

The energy-momentum tensor $T_{\rm m}^{\mu\nu}$ of the matter fields coupled to the gravitational field and the pseudotensor $T_{\rm gr}^{\mu\nu}$ of the gravitational field are given by

$$T_{\rm m}^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}}{\delta g_{\mu\nu}}, \qquad T_{\rm gr}^{\mu\nu} = \frac{4}{\kappa^2}\Lambda g^{\mu\nu} + T_{\rm LL}^{\mu\nu} + \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm gr,ho}}{\delta g_{\mu\nu}}, \quad (2)$$

where $T_{\rm LL}^{\mu\nu}$ is defined via [20]

$$\begin{aligned}(-g)T^{\mu\nu}_{LL} = \frac{2}{\kappa^2}\bigg(&\frac{1}{8}g^{\lambda\sigma}g^{\mu\nu}g_{\alpha\gamma}g_{\beta\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda} - \frac{1}{4}g^{\mu\lambda}g^{\nu\sigma}g_{\alpha\gamma}g_{\beta\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda} - \frac{1}{4}g^{\lambda\sigma}g^{\mu\nu}g_{\beta\alpha}g_{\gamma\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda}\\ &+\frac{1}{2}g^{\mu\lambda}g^{\nu\sigma}g_{\beta\alpha}g_{\gamma\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda} + g^{\beta\alpha}g_{\lambda\sigma}\mathfrak{g}^{\nu\sigma}{}_{,\alpha}\,\mathfrak{g}^{\mu\lambda}{}_{,\beta} + \frac{1}{2}g^{\mu\nu}g_{\lambda\sigma}\mathfrak{g}^{\lambda\beta}{}_{,\alpha}\,\mathfrak{g}^{\alpha\sigma}{}_{,\beta}\\ &- g^{\mu\lambda}g_{\sigma\beta}\mathfrak{g}^{\nu\beta}{}_{,\alpha}\,\mathfrak{g}^{\sigma\alpha}{}_{,\lambda} - g^{\nu\lambda}g_{\sigma\beta}\mathfrak{g}^{\mu\beta}{}_{,\alpha}\,\mathfrak{g}^{\sigma\alpha}{}_{,\lambda} + \mathfrak{g}^{\lambda\sigma}{}_{,\sigma}\,\mathfrak{g}^{\mu\nu}{}_{,\lambda} - \mathfrak{g}^{\mu\lambda}{}_{,\lambda}\,\mathfrak{g}^{\nu\sigma}{}_{,\sigma}\bigg), \end{aligned} \quad (3)$$

with $\mathfrak{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$ and $\mathfrak{g}^{\mu\nu}{}_{,\lambda} = \partial\mathfrak{g}^{\mu\nu}/\partial x^\lambda$.

The full energy-momentum tensor $T^{\mu\nu} = T_{\rm m}^{\mu\nu} + T_{\rm gr}^{\mu\nu}$ gives rise to the conserved four-momentum of the matter and gravitational fields given by [20]

$$P^\mu = \int (-g)T^{\mu\nu}dS_\nu, \quad (4)$$

where the integration is carried out over any hypersurface containing the whole three-dimensional space.

The cosmological constant Λ can be uniquely fixed by imposing the condition that the energy of the vacuum is zero. This is achieved by demanding that the vacuum expectation value of the integrand in Eq. (4) vanishes order by order in perturbation theory. To show that this condition is equivalent to the self-consistency condition of Ref. [16], consider the vacuum-to-vacuum transition amplitude

$$\mathcal{I} = \int \mathcal{D}h\mathcal{D}\psi\mathcal{D}\bar{c}\mathcal{D}c\, e^{\frac{i}{\hbar}S_E(g,\psi,\bar{c},c)}, \quad (5)$$

where ψ and c, $\bar{c}$ represent the matter and ghost fields, respectively, and the metric field is decomposed as $g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}$, with $\eta^{\mu\nu}$ denoting the Minkowski background. Here, the action S_E contains the gauge fixing and ghost terms in addition to the action S specified in Eq. (1). Note that the vielbein tetrad formalism has to be used for fermionic degrees of freedom, but we suppress these details here. This integral remains unaffected by the change of the integration variables

$$h^{\mu\nu}(x) \to h^{\mu\nu}(x) + \epsilon^{\mu\nu}(x)f(g) \quad (6)$$

where $\epsilon^{\mu\nu}(x)$ are the infinitesimal local transformation parameters and $f(g)$ is an arbitrary nonsingular function of g.

Taking into account the Jacobian of the change of variables,

$$\mathcal{J} = \mathrm{Tr}\{[1 - \epsilon^{\alpha\beta}(x)g_{\alpha\beta}(x)g(x)f'(g(x))]\delta^4(x-y)\} + \mathcal{O}(\epsilon^2), \quad (7)$$

we obtain

$$0=\delta\mathcal{I}=-\int d^4y\epsilon^{\alpha\beta}(y)\int\mathcal{D}h\mathcal{D}\psi\mathcal{D}\bar{c}\mathcal{D}c\left(gf'(g)g_{\alpha\beta}(y)\delta^4(0)+\frac{i}{\hbar}f(g)\frac{\delta S_E}{\delta g^{\alpha\beta}(y)}\right)e^{\frac{i}{\hbar}S_E(g,\psi,\bar{c},c)}+\mathcal{O}(\epsilon^2). \tag{8}$$

Here, the variation of the action is given by

$$\frac{\delta S_E}{\delta g^{\alpha\beta}(y)}=\frac{\sqrt{-g}}{2}\left[T^{\mu\nu}_{\rm m}-\frac{4}{\kappa^2}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R+\Lambda g^{\mu\nu}\right)+\mathcal{T}^{\mu\nu}\right], \tag{9}$$

where $\mathcal{T}^{\mu\nu}$ denotes the contributions of the gauge fixing and ghost terms, as well as of the higher-order corrections.

Next, using $T^{\mu\nu}=T^{\mu\nu}_{\rm LL}+T^{\mu\nu}_{\rm m}-4\Lambda g^{\mu\nu}/(\kappa^2)+\mathcal{T}^{\mu\nu}$ and the identity [20]

$$(-g)\left\{\frac{4}{\kappa^2}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)+T^{\mu\nu}_{\rm LL}\right\}=\frac{\partial h^{\mu\nu\lambda}}{\partial x^\lambda}, \tag{10}$$

where

$$h^{\mu\nu\lambda}=\frac{2}{\kappa^2}\frac{\partial}{\partial x^\sigma}\{(-g)(g^{\mu\nu}g^{\lambda\sigma}-g^{\mu\lambda}g^{\nu\sigma})\}, \tag{11}$$

we obtain

$$(-g)\left\{T^{\mu\nu}_{\rm m}-\frac{4}{\kappa^2}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R+\Lambda g^{\mu\nu}\right)+\mathcal{T}^{\mu\nu}\right\}=(-g)T^{\mu\nu}-\frac{2}{\kappa^2}\frac{\partial^2}{\partial x^\sigma\partial x^\lambda}\{(-g)(g^{\mu\nu}g^{\lambda\sigma}-g^{\mu\lambda}g^{\nu\sigma})\}. \tag{12}$$

Choosing $f(g)=2\sqrt{-g}$ from Eqs. (8), (9), and (12) we obtain

$$\int\mathcal{D}h\mathcal{D}\psi\mathcal{D}\bar{c}\mathcal{D}c\left(\sqrt{-g}g^{\mu\nu}(x)\delta^{(4)}(0)-\frac{i}{\hbar}\frac{2}{\kappa^2}\frac{\partial^2}{\partial x^\sigma\partial x^\lambda}\{(-g)(g^{\mu\nu}g^{\lambda\sigma}-g^{\mu\lambda}g^{\nu\sigma})\}+\frac{i}{\hbar}(-g)T^{\mu\nu}(x)\right)e^{\frac{i}{\hbar}S(g,\psi,\bar{c},c)}=0. \tag{13}$$

Using the expansion around the Minkowski background and employing dimensional regularization, the first term in Eq. (13) involving $\delta(0)$ vanishes. In the second term the derivatives can be taken out of the path integral. Calculating the integral by perturbative expansion around the Minkowski background one obtains a result which does not depend on the spacetime coordinates due to translational invariance of the Minkowski background. By acting with derivatives on this constant expression one gets a vanishing result. The remaining third term corresponds to the vacuum energy, which hence turns out to vanish in all orders of perturbation theory.

To summarize, the cosmological constant term viewed as a parameter of the effective Lagrangian is known to be uniquely fixed from the consistency condition of the theory in the Minkowski background [16]. On the other hand, the condition of the vanishing of the vacuum energy also fixes the cosmological constant parameter uniquely. In this paper, we have shown that the resulting two values of the cosmological constant coincide to all orders in perturbation theory. Thus, one can conclude that by demanding the vanishing of the vacuum energy within perturbation theory one is uniquely led to a consistent EFT in the Minkowski background. The resulting effective action expressed in terms of local operators is free of the effective cosmological constant term.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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