# PROBING THE PHYSICS OF D-WAVE ANNEALERS: INSIGHTS FROM STANDARD, REVERSE, AND FAST ANNEALING

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# CONTENTS

# I. OVERVIEW

# 1.1 STANDARD ANNEALING

- To use **quantum annealing** for solving a problem → Map it to a Hamiltonian

$$\boldsymbol{H_P} = -\sum_{i=1}^{N} h_i^z \sigma_i^z - \sum_{\{i,j\}} J_{i,j}^z \sigma_i^z \sigma_j^z$$

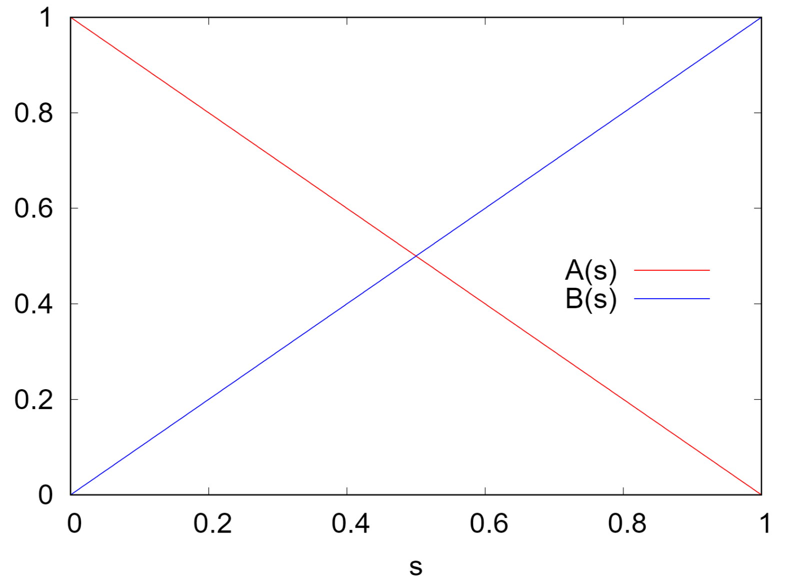
- **Annealing Hamiltonian** $H = A(s)H_I + B(s)H_P$

where,

$$s = t/T_A$$

$T_A$ = Total annealing time

$$H_I = -\sum_{i=1}^{N} \sigma_i^x$$




$$\boxed{\begin{array}{c} A(0) \gg B(0) \\ A(1) \ll B(1) \end{array}}$$



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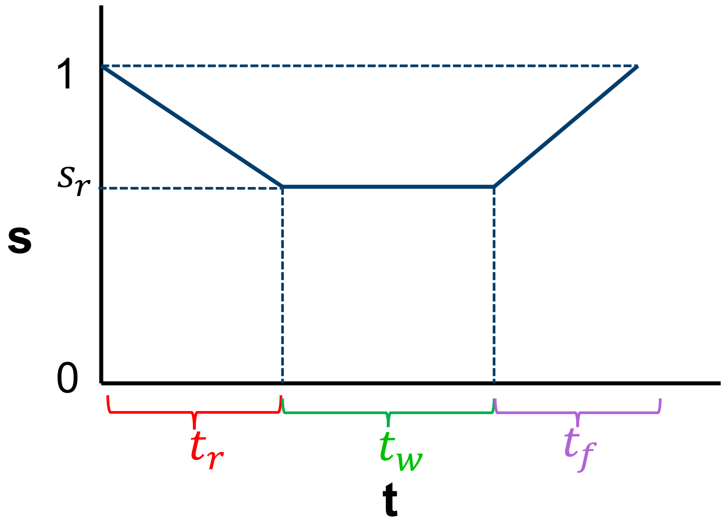
# 1.2 FAST ANNEALING

- **Standard annealing:** Shortest annealing time possible = 500 ns

- **Fast annealing:**
  - Can explore shorter annealing times (up to 5 ns)
  - However, it is not possible to have non-zero fields ($h_i$) in $H_P$

- **Way around for having effective $\boldsymbol{h_i}$ fields using flux bias offset:**
  - Select an unused spin k on D-Wave
  - Bias the spin by a large flux offset (h) ➔ Fixes the extra spin to ↑ or ↓ depending on sign of h
  - Set the field of spin i to 0 and $J_{i,k} = h_i$

# 1.3 REVERSE ANNEALING




- **Description:**
  - Start in a (low-energy) classical state
  - Increase the strength of quantum fluctuations (increase A(s) and decrease B(s)) till certain reversal distance $s_r$
  - Optional wait at $s_r$
  - Decrease the strength of quantum fluctuations (decrease A(s) and increase B(s))
- **Schemes:**
  - Waiting time scan (**WTS**): Do different runs with fixed $t_r = t_f = 1\mu s$, and varying $t_w$, collect the final p($t_{end}$),
  - Annealing times scan (**ATS**): Do different runs with $t_w = 0$ and varying $t_r = t_f$, collect the final p($t_{end}$),

where $t_{end} = t_r + t_w + t_f$

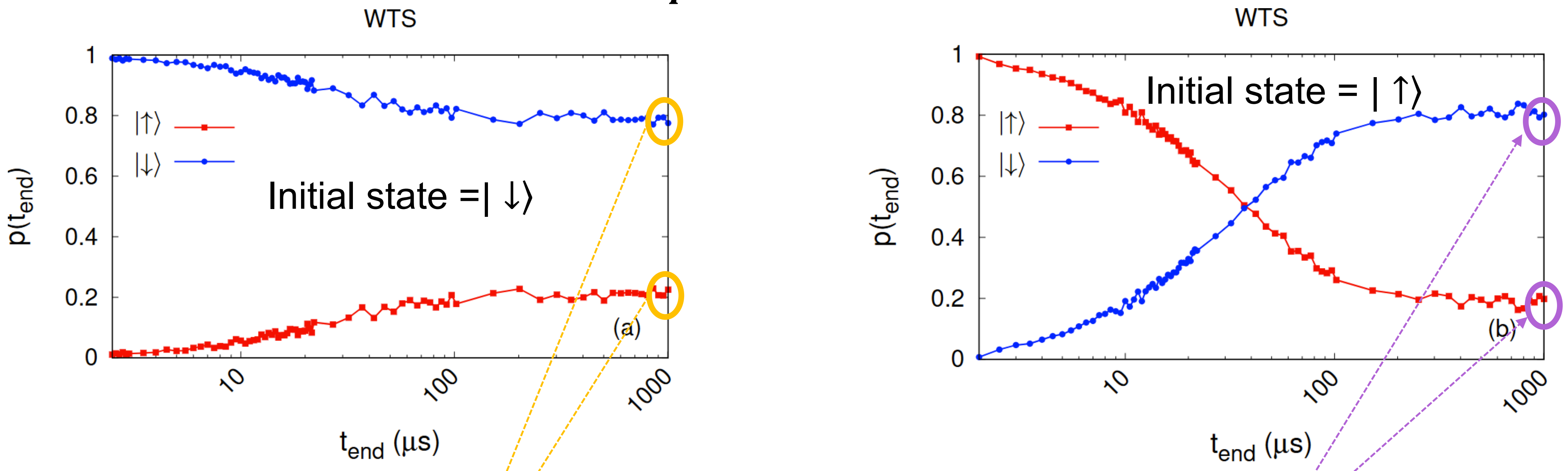
# II. D-WAVE RESULTS

# 2.1 REVERSE ANNEALING

$s_r = 0.7$

## 2.1.1 1-spin

$$H_P = 0.1\,\sigma^z$$




Fit to Boltzmann distribution: $p_i = \frac{g_i\, e^{-\beta E_i}}{\sum_j g_j\, e^{-\beta E_j}}$

$\beta = 6.93\ (T = 29.7\ mK)$

$p_\uparrow = 0.2$
$p_\downarrow = 0.8$



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[1] V. Mehta, H. De Raedt, K. Michielsen, F. Jin, arXiv:2502.08575v1

# 2.1 REVERSE ANNEALING

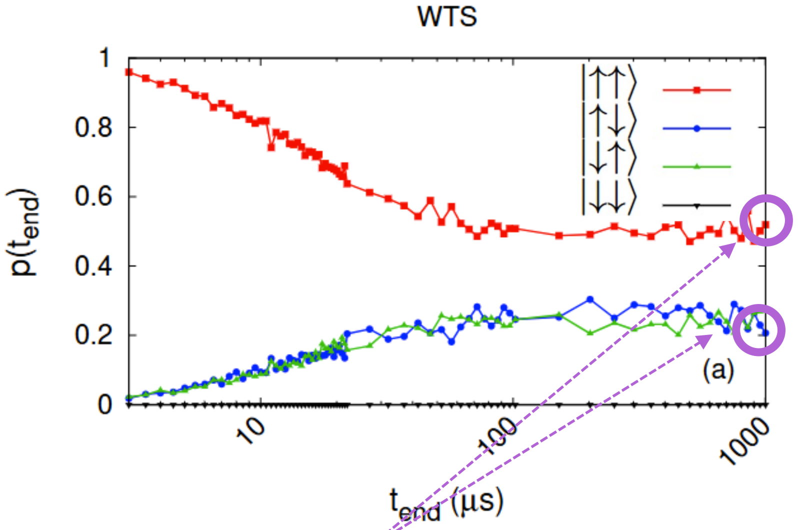
## 2.1.2 2-spin

$$s_r = 0.7$$

$$\beta = 6.93\ (T = 29.7\ mK)$$

**2S1**

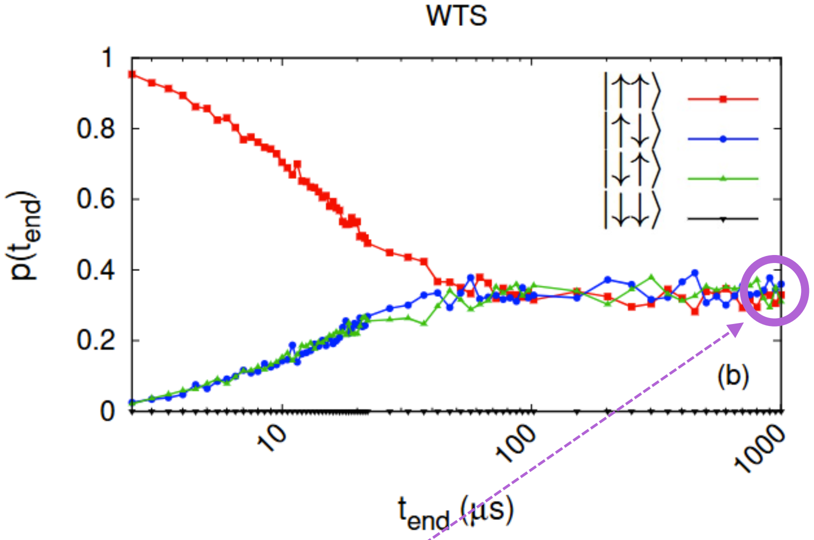
$$H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$$




$$p_{\uparrow\uparrow} = 0.5$$
$$p_{\uparrow\downarrow} = 0.25$$
$$p_{\downarrow\uparrow} = 0.25$$
$$p_{\downarrow\downarrow} = 0$$

**2S2**

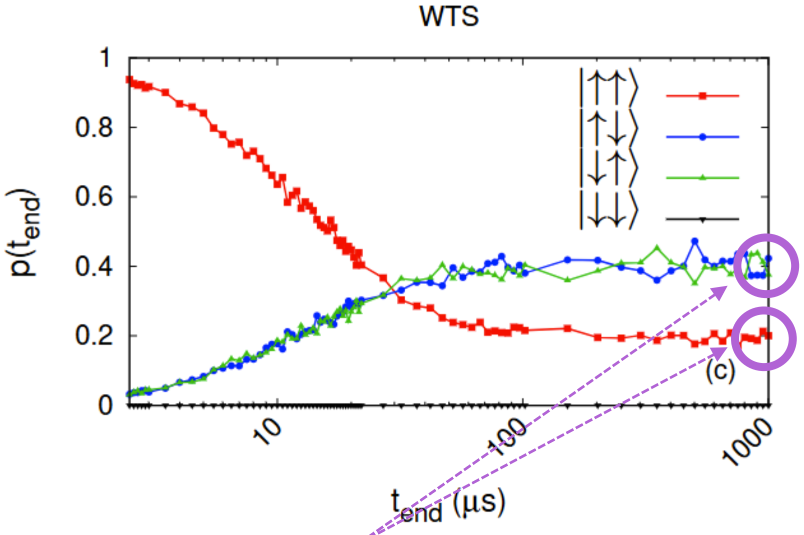
$$H_P = -\sigma_1^z - \sigma_2^z + \sigma_1^z\sigma_2^z$$




$$p_{\uparrow\uparrow} = 0.33$$
$$p_{\uparrow\downarrow} = 0.33$$
$$p_{\downarrow\uparrow} = 0.33$$
$$p_{\downarrow\downarrow} = 0$$

**2S3**

$$H_P = -0.95\sigma_1^z - 0.95\sigma_2^z + \sigma_1^z\sigma_2^z$$




$$p_{\uparrow\uparrow} = 0.2$$
$$p_{\uparrow\downarrow} = 0.4$$
$$p_{\downarrow\uparrow} = 0.4$$
$$p_{\downarrow\downarrow} = 0$$



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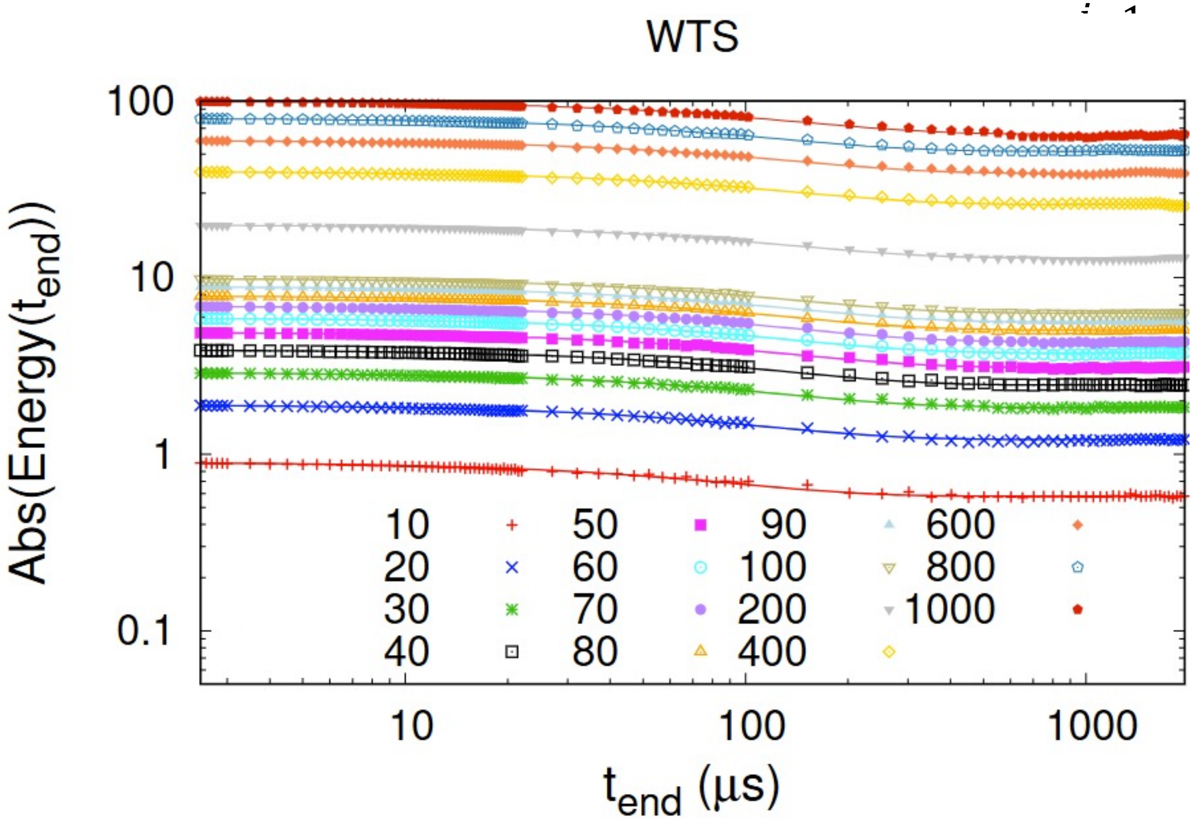
[1] V. Mehta, H. De Raedt, K. Michielsen, F. Jin, arXiv:2502.08575v1

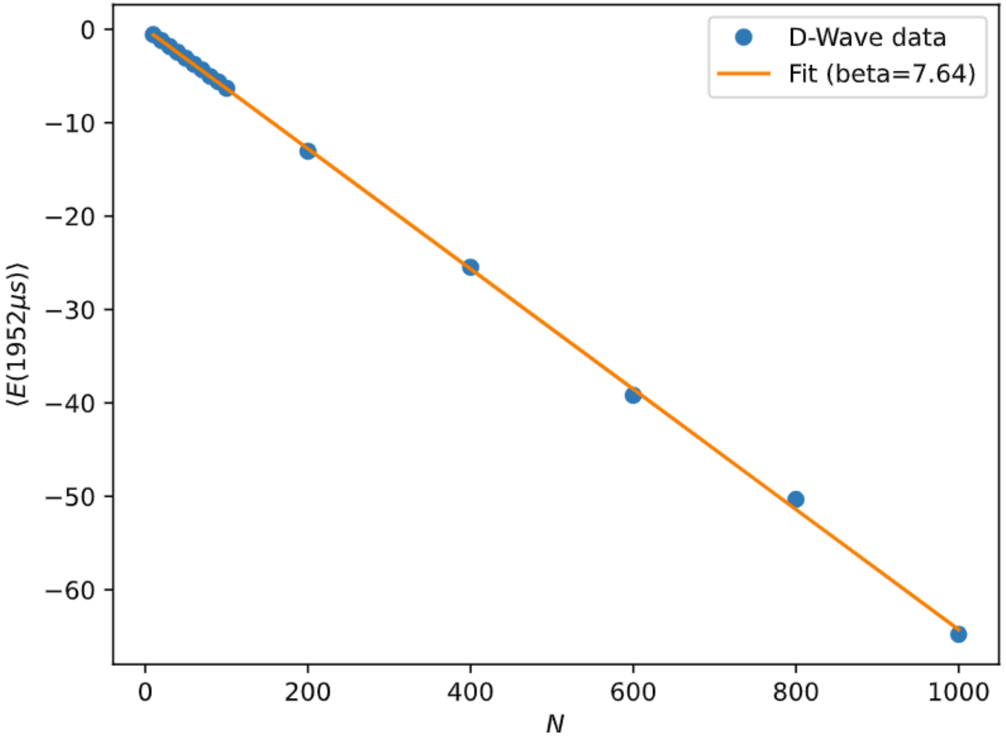
# 2.1 REVERSE ANNEALING

## 2.1.3 Ferromagnetic spin chain

$$H_P = -0.1 \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z$$




$s_r = 0.7$




$$< E > = \frac{\sum_i g_i E_i e^{-\beta E_i}}{\sum_i g_i e^{-\beta E_i}} = -J(N-1) \tanh \beta J$$

$$\boxed{\beta = 7.64 \; (T = 27 \; mK)}$$




[1] V. Mehta, H. De Raedt, K. Michielsen, F. Jin, arXiv:2502.08575v1

# 2.2 STANDARD ANNEALING

$s_r = 0.7$

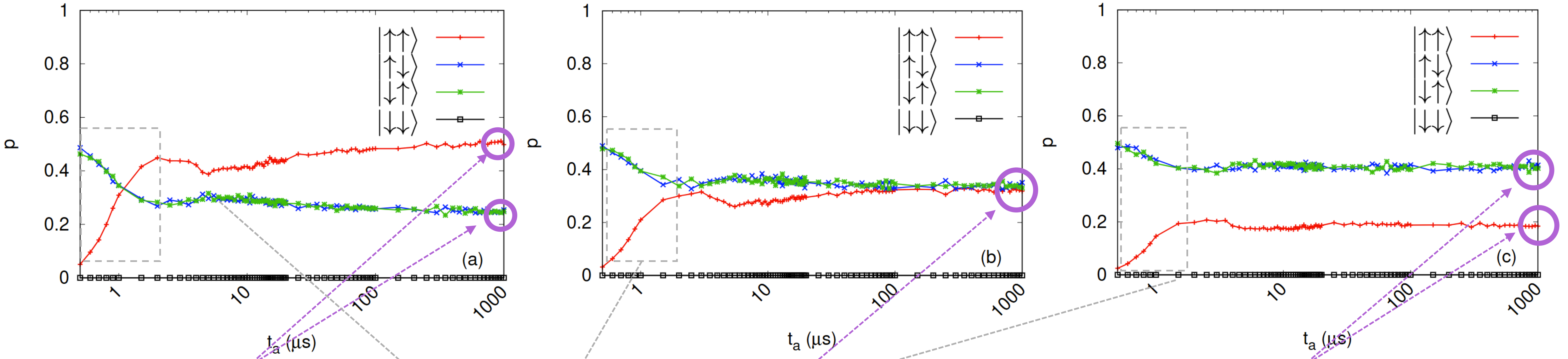
$\beta = 6.93\ (T = 29.7\ mK)$

## 2-spin

**2S1** $H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$

**2S2** $H_P = -\sigma_1^z - \sigma_2^z + \sigma_1^z\sigma_2^z$

**2S3** $H_P = -0.95\sigma_1^z - 0.95\sigma_2^z + \sigma_1^z\sigma_2^z$




$p_{\uparrow\uparrow} = 0.5$
$p_{\uparrow\downarrow} = 0.25$
$p_{\downarrow\uparrow} = 0.25$
$p_{\downarrow\downarrow} = 0$

Dips & Bumps

$p_{\uparrow\uparrow} = 0.33$
$p_{\uparrow\downarrow} = 0.33$
$p_{\downarrow\uparrow} = 0.33$
$p_{\downarrow\downarrow} = 0$

$p_{\uparrow\uparrow} = 0.20$
$p_{\uparrow\downarrow} = 0.40$
$p_{\downarrow\uparrow} = 0.40$
$p_{\downarrow\downarrow} = 0$



[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# 2.3 FAST ANNEALING

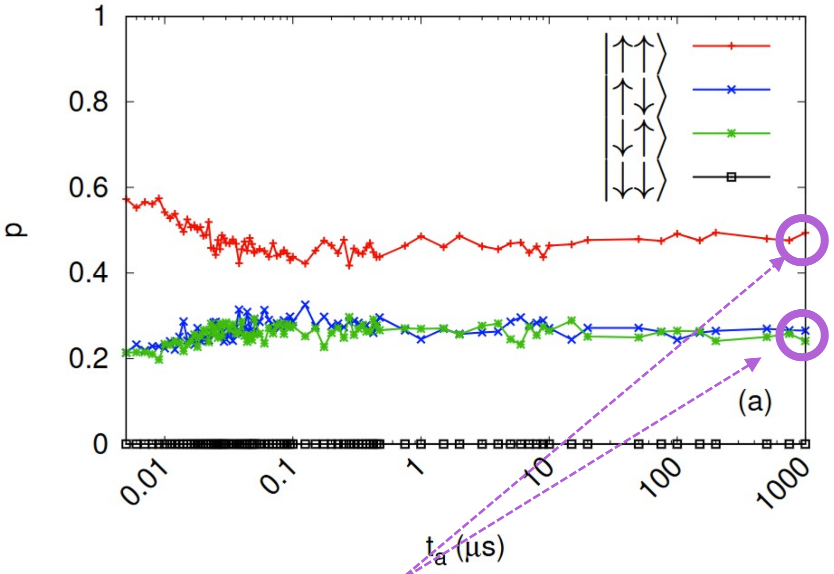
$s_r = 0.7$

$\beta = 6.93\ (T = 29.7\ mK)$

## 2-spin

**2S1**

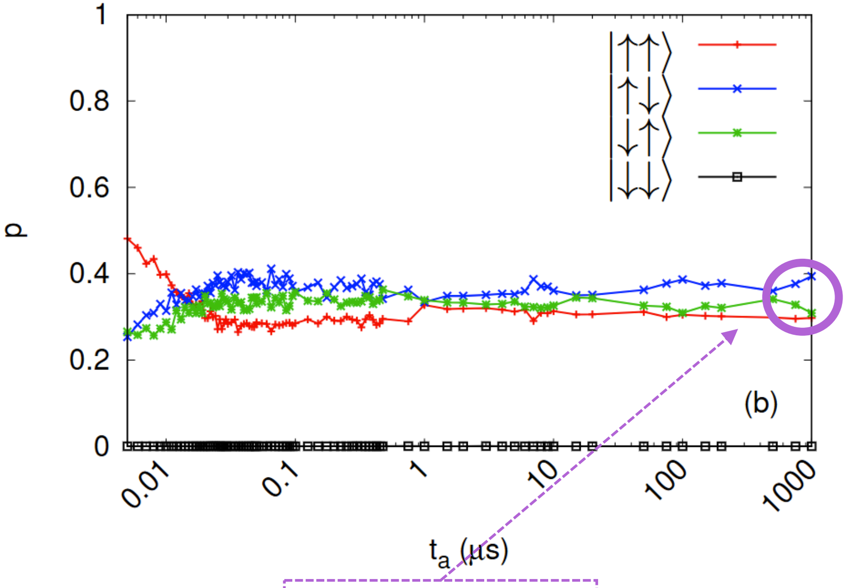
$H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$




$p_{\uparrow\uparrow} = 0.5$
$p_{\uparrow\downarrow} = 0.25$
$p_{\downarrow\uparrow} = 0.25$
$p_{\downarrow\downarrow} = 0$

**2S2**

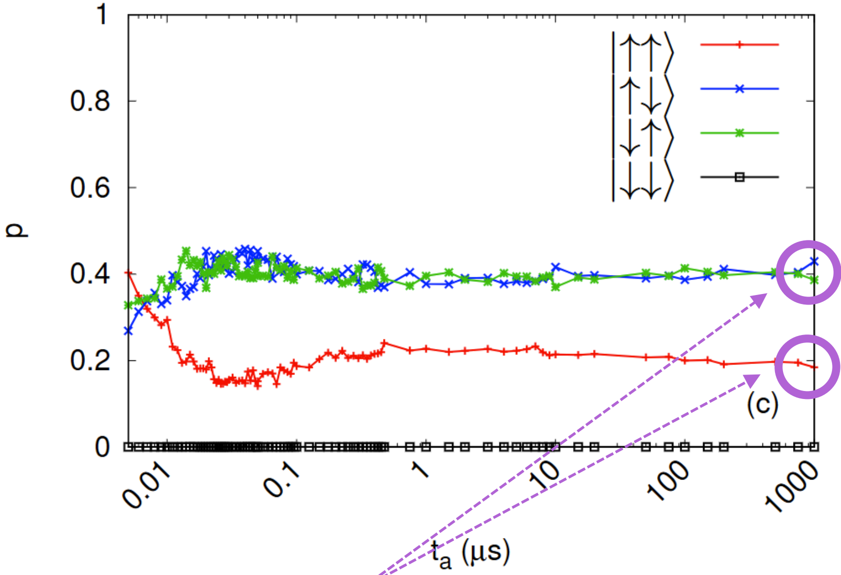
$H_P = -\sigma_1^z - \sigma_2^z + \sigma_1^z\sigma_2^z$




$p_{\uparrow\uparrow} = 0.33$
$p_{\uparrow\downarrow} = 0.33$
$p_{\downarrow\uparrow} = 0.33$
$p_{\downarrow\downarrow} = 0$

**2S3**

$H_P = -0.95\sigma_1^z - 0.95\sigma_2^z + \sigma_1^z\sigma_2^z$




$p_{\uparrow\uparrow} = 0.20$
$p_{\uparrow\downarrow} = 0.40$
$p_{\downarrow\uparrow} = 0.40$
$p_{\downarrow\downarrow} = 0$

No Dips & Bumps




[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# III. SIMULATIONS

# 3.1 TIME-DEPENDENT SCHRÖDINGER EQUATION

- The time evolution of an ideal closed quantum system is governed by the time-dependent Schrödinger equation (TDSE)

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t)|\psi(t)\rangle$$

$$|\psi(t)\rangle = \sum_{i=0}^{D} c_i |i\rangle$$

  - $H(t)$: time-dependent Hamiltonian modeling the quantum system
  - $|\psi(t)\rangle$ describes the state of the whole quantum system at time t
  - $|i\rangle$: basis states
  - $c_i$: complex valued coefficients
  - D: Dimension of the Hilbert space

# 3.2 LINDBLAD MASTER EQUATION

- Master equation that approximates the time evolution of a density matrix $\rho(t)$ of a system interacting with an environment

$$\frac{d\rho(t)}{dt} = \underbrace{-i[H,\rho(t)]}_{\text{Coherent}} + \frac{1}{2}\sum_j \underbrace{\gamma_j\left(2L_j\rho(t)L_j^\dagger - L_j^\dagger L_j\rho(t) - \rho(t)L_j^\dagger L_j\right)}_{\text{Dissipative}} \coloneqq \mathcal{L}\rho(t)$$

  - $\rho$: density matrix
  - $L_j$: dissipation operators
  - $\gamma_j$: dissipation rates

# 3.3 BLOCH EQUATIONS

- For a general 1-spin system, $H = -\frac{1}{2}\boldsymbol{B}\cdot\boldsymbol{\sigma}$
- Density matrix can be written as $\rho(t) = \frac{1}{2}\left(I + \sum_{i=1}^{3} S_i(t)\sigma_i\right)$
- Choosing $L_1 = \sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, L_2 = \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, L_3 = \sigma^z$
- Lindblad equation reduces to **Bloch equations**

$$\frac{d}{dt}\begin{pmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \end{pmatrix} = \left[\begin{pmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{pmatrix} - \begin{pmatrix} 1/T_2 & 0 & 0 \\ 0 & 1/T_2 & 0 \\ 0 & 0 & 1/T_1 \end{pmatrix}\right]\begin{pmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ M_0 \end{pmatrix}$$

where $S_k(t) = \mathbf{Tr}\rho(t)\sigma_k,\ T_1 = \frac{1}{\gamma_1+\gamma_2}, T_2 = \frac{2}{\gamma_1+\gamma_2+4\gamma_3}, M_0 = \frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}$

# 3.4 MARKOVIAN MASTER EQUATION

- Lindblad master equation

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \frac{1}{2}\sum_j \gamma_j\left(2L_j\rho(t)L_j^\dagger - L_j^\dagger L_j\rho(t) - \rho(t)L_j^\dagger L_j\right) \coloneqq \mathcal{L}\rho(t)$$

$$H(t) = \cancel{A(t)H_I} + B(t)H_P$$

and $\rho_{i,j} \to 0\ (i \neq j)$

$\Rightarrow [H,\rho(t)] = 0$

- Markovian master equation: $\frac{dP(t)}{dt} = WP(t)$
  - 1-spin: $\frac{d}{dt}\begin{pmatrix} p_\uparrow \\ p_\downarrow \end{pmatrix} = \begin{pmatrix} -\gamma_2 & \gamma_1 \\ \gamma_2 & -\gamma_1 \end{pmatrix}\begin{pmatrix} p_\uparrow \\ p_\downarrow \end{pmatrix}$
  - 2-spin: $\frac{d}{dt}\begin{pmatrix} p_{\uparrow\uparrow} \\ p_{\uparrow\downarrow} \\ p_{\downarrow\uparrow} \\ p_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -\gamma_2 - \gamma_5 - \gamma_7 & \gamma_4 & \gamma_6 & \gamma_1 \\ \gamma_5 & -\gamma_4 & 0 & 0 \\ \gamma_7 & 0 & -\gamma_6 & 0 \\ \gamma_2 & 0 & 0 & -\gamma_1 \end{pmatrix}\begin{pmatrix} p_{\uparrow\uparrow} \\ p_{\uparrow\downarrow} \\ p_{\downarrow\uparrow} \\ p_{\downarrow\downarrow} \end{pmatrix}$

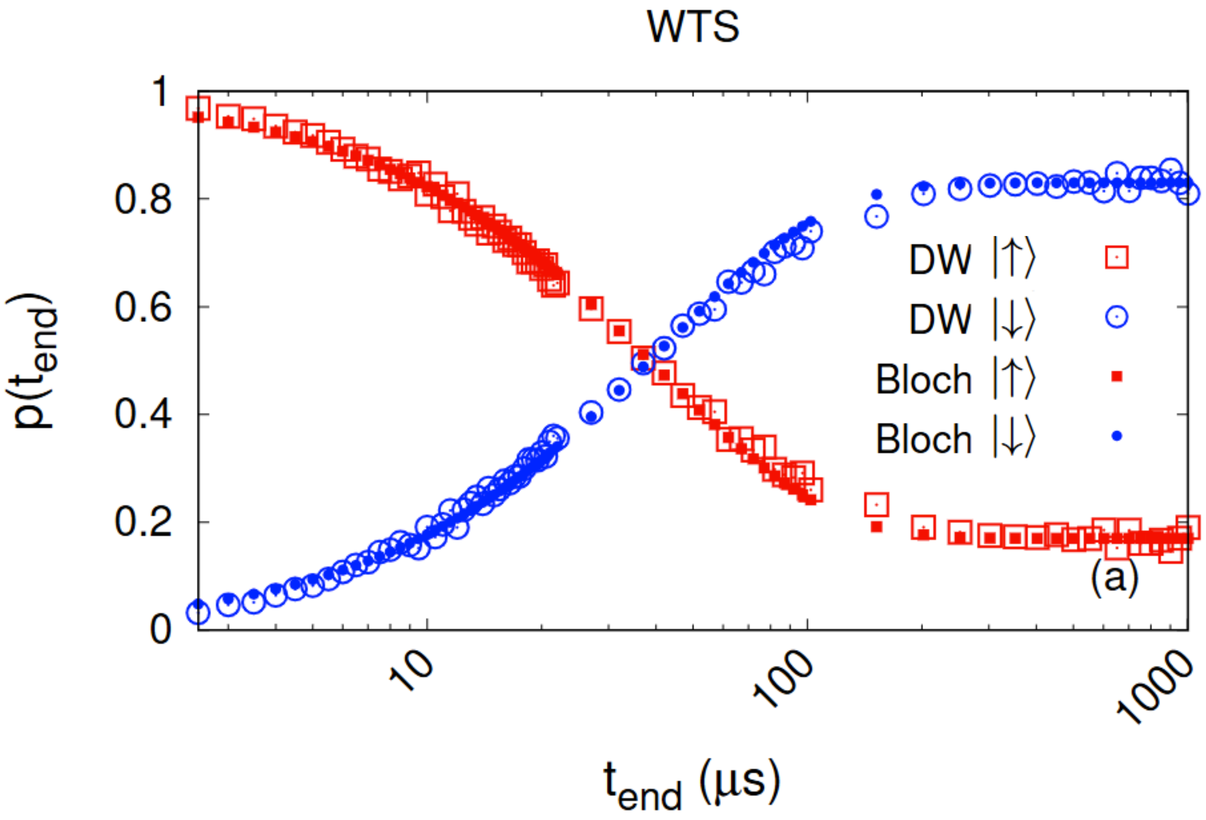
# IV. NUMERICAL RESULTS

# 4.1 1-SPIN RESULTS

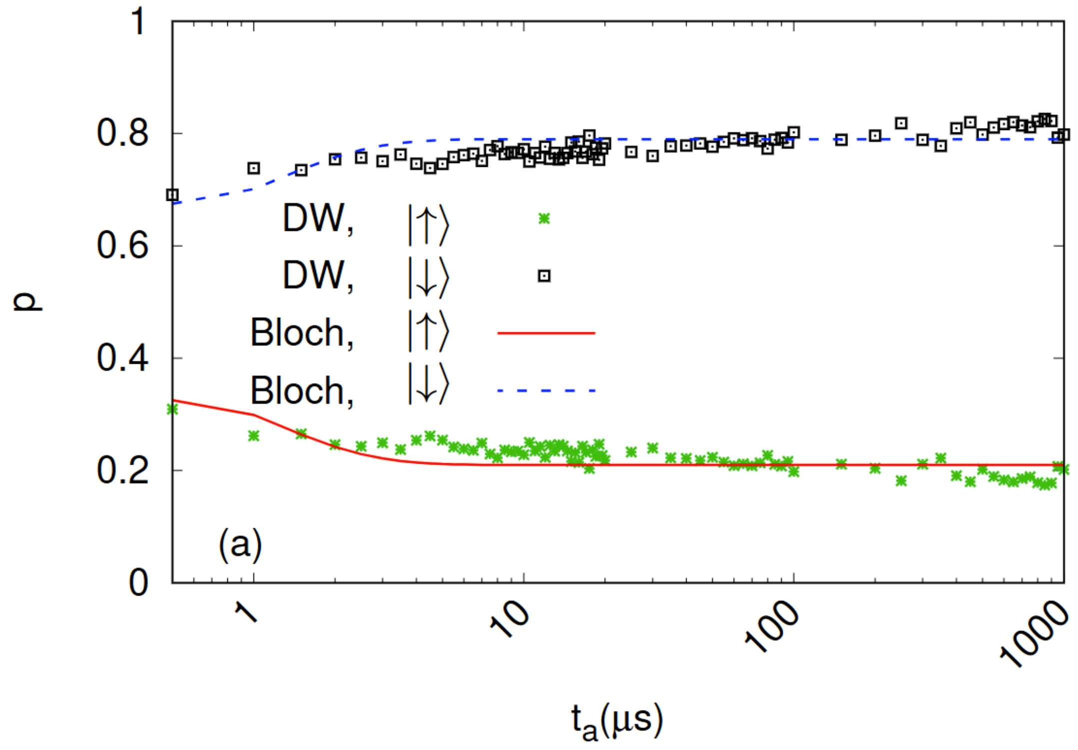
$$H_P = 0.1\,\sigma^z$$

## Bloch equations

**Reverse annealing**




**Standard annealing**





[1] V. Mehta, H. De Raedt, K. Michielsen, F. Jin, arXiv:2502.08575v1 (2025)
[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,



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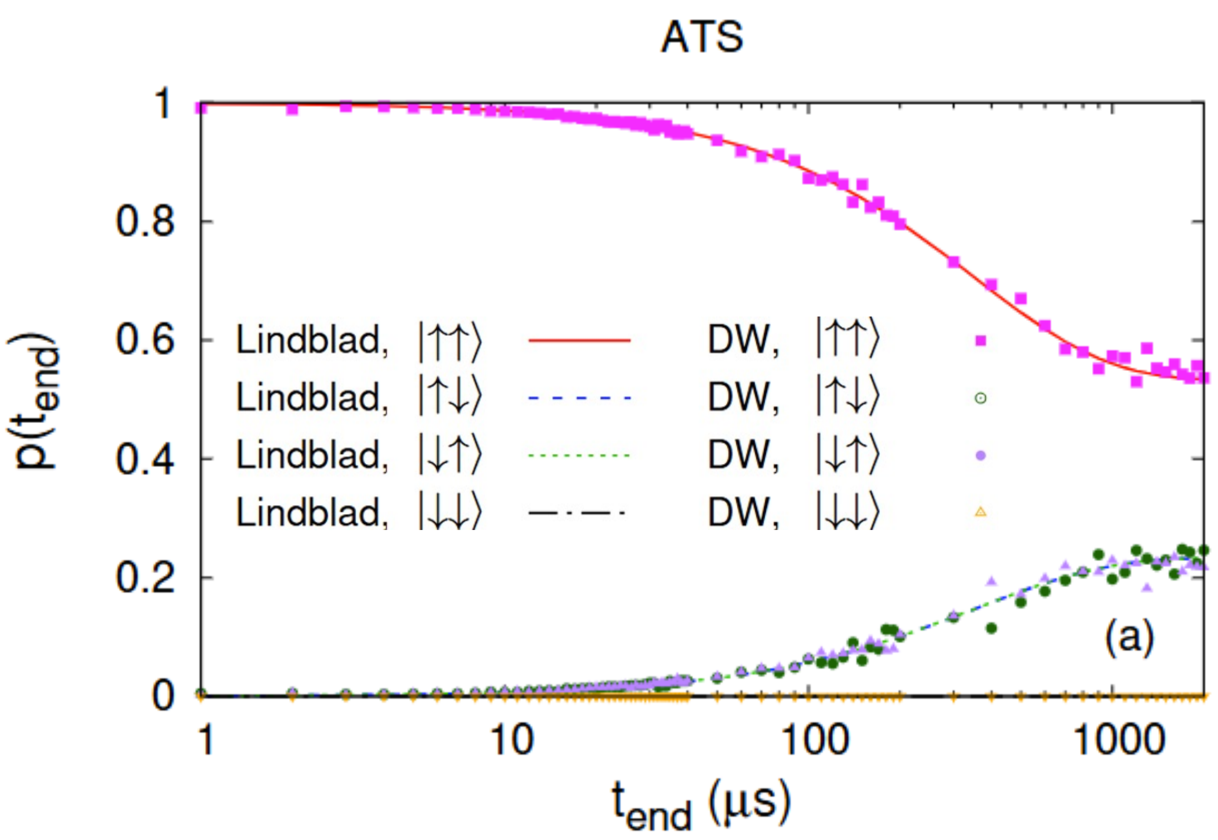
# 4.2 2-SPIN RESULTS

**2S1**

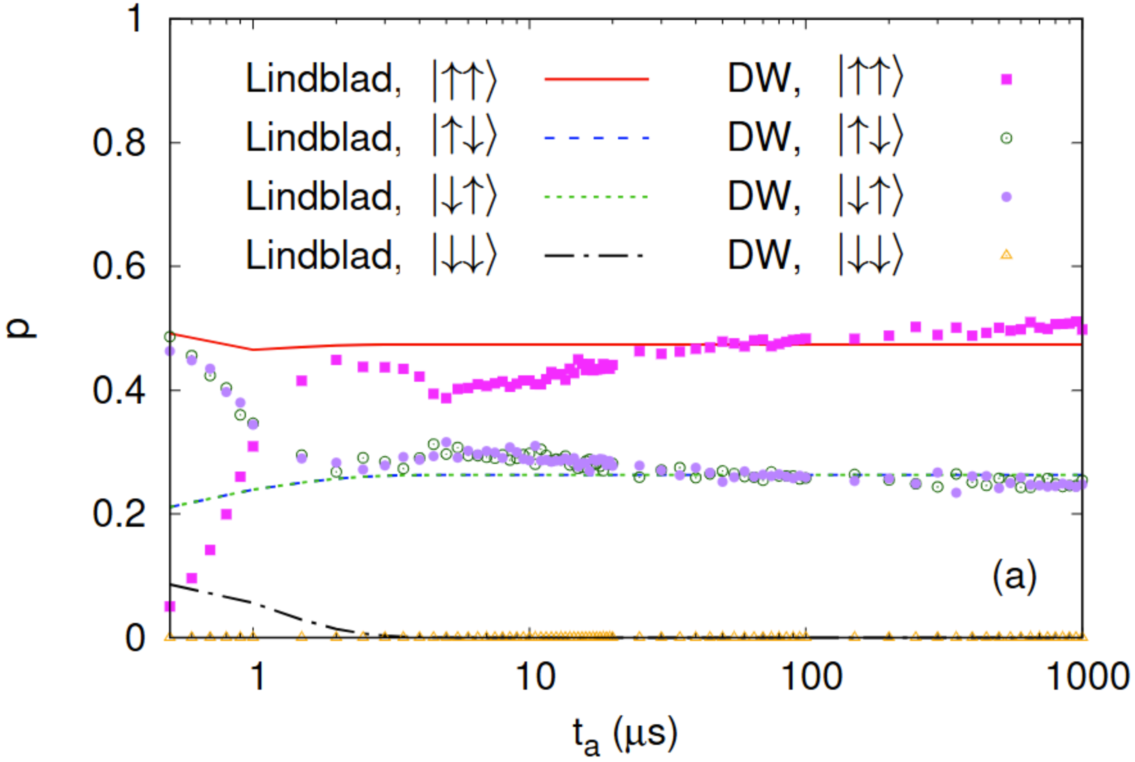
$$H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$$

## 4.2.1 Lindblad master equation

**Reverse annealing**




**Standard annealing**




- Simulations cannot capture the initial dips & bumps

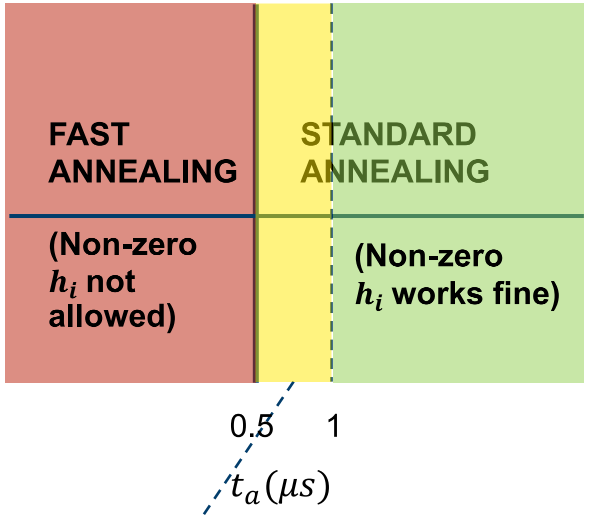
➔ Need time-dependent dissipation rates


[1] V. Mehta, H. De Raedt, K. Michielsen, F. Jin, arXiv:2502.08575v1 (2025)
[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# 4.2 2-SPIN RESULTS

## 4.2.2 2-spin results: Tackling the dips & bumps

$$H = -\frac{1}{2}\sum_i \left[\Delta_q(\Phi_{\mathrm{CCJJ}}(s))\hat{\sigma}_x^{(i)} - 2h_i|I_p(\Phi_{\mathrm{CCJJ}}(s))|\Phi_i^x(s)\hat{\sigma}_z^{(i)}\right] + \sum_{i>j} J_{i,j} M_{\mathrm{AFM}} I_p(\Phi_{\mathrm{CCJJ}}(s))^2 \hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}$$




For implementing B(s) $\Phi_i^x(s) = M_{\mathrm{AFM}}|I_p(s)|$

However, for these potentially very fast ramps in $\Phi_{\mathrm{CCJJ}}(s)$, no attempt is made to keep the relative energy ratio between $h$ and $J$ terms constant (through adjustments of $\Phi_i^x(s)$, as is done by the standard-anneal protocol, where it is set to the linear $M_{\mathrm{AFM}}|I_p(s)|$). Consequently, this protocol is used only for problems with no linear biases ($h = 0$).

Conjecture: For short annealing times (where non-zero allowed), the h and J ratio can no longer be kept constant…

So that instead of $B\left(h_i + J_{i,j}\right)$, we have $B'(h_i) + B(J_{i,j})$?

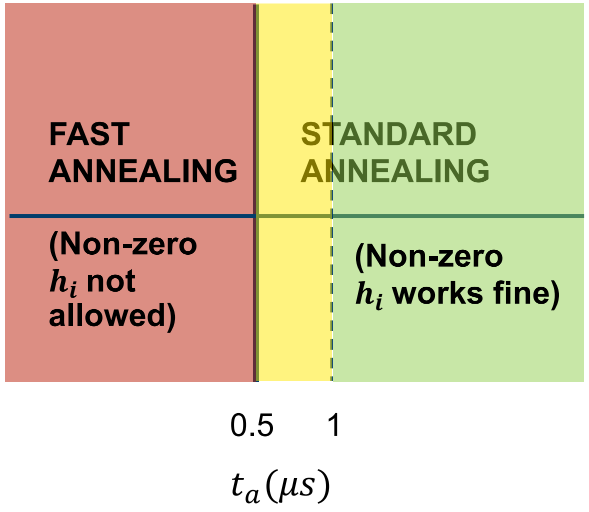


[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# 4.2 2-SPIN RESULTS

## 4.2.2 2-spin results: Tackling the dips & bumps

Conjecture: For short annealing times (where non-zero allowed), the h and J ratio can no longer be kept constant?




So instead of $B(h_i + J_{i,j})$, we have $B'(h_i) + B(J_{i,j})$?

Idea for B'(s): 1) $B'(t) = 0$ for $t < t_{onset}^{start}$

2) $B'(t) = B(t)\sin\left(\frac{\pi}{2}\frac{(t-t_{onset}^{start})}{t_{onset}^{end}-t_{onset}^{start}}\right)$ for $t_{onset}^{start} \leq t \leq t_{onset}^{end}$

3) $B'(t) = B(t)$ for $t > t_{onset}^{end}$



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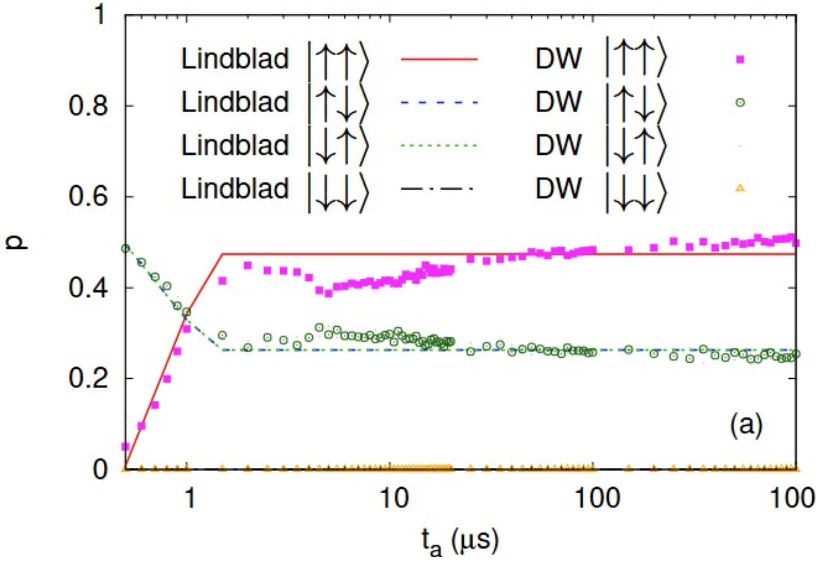
[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# 4.2 2-SPIN RESULTS

## 4.2.2 Tackling the dips & bumps: Numerical results with Lindblad master equation

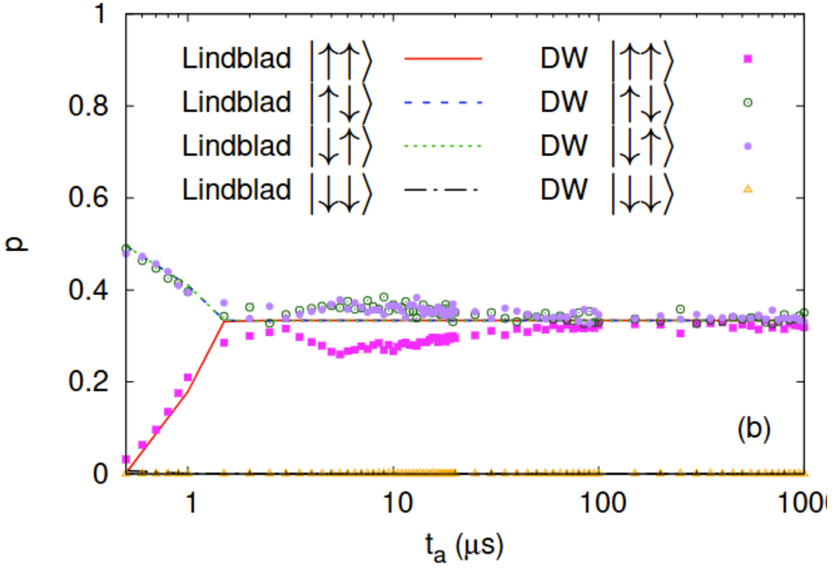
**2S1**

$$H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$$



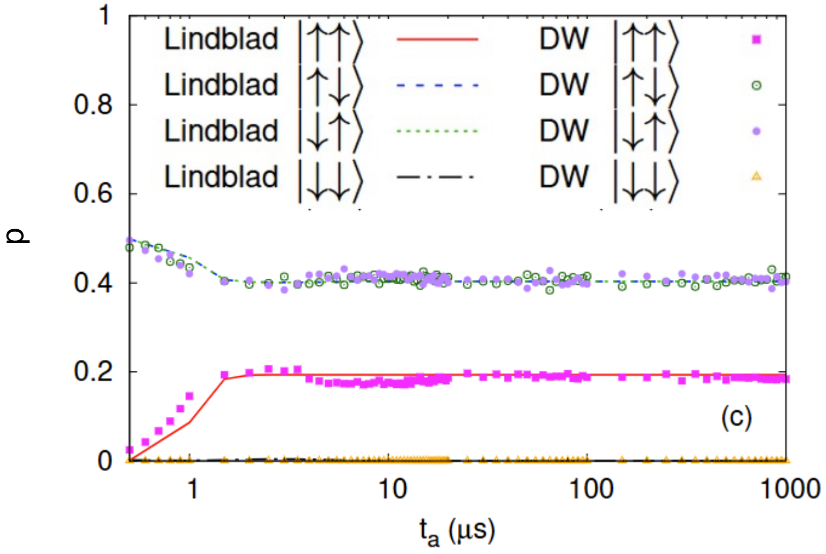

**2S2**

$$H_P = -\sigma_1^z - \sigma_2^z + \sigma_1^z\sigma_2^z$$




**2S3**

$$H_P = -0.95\sigma_1^z - 0.95\sigma_2^z + \sigma_1^z\sigma_2^z$$





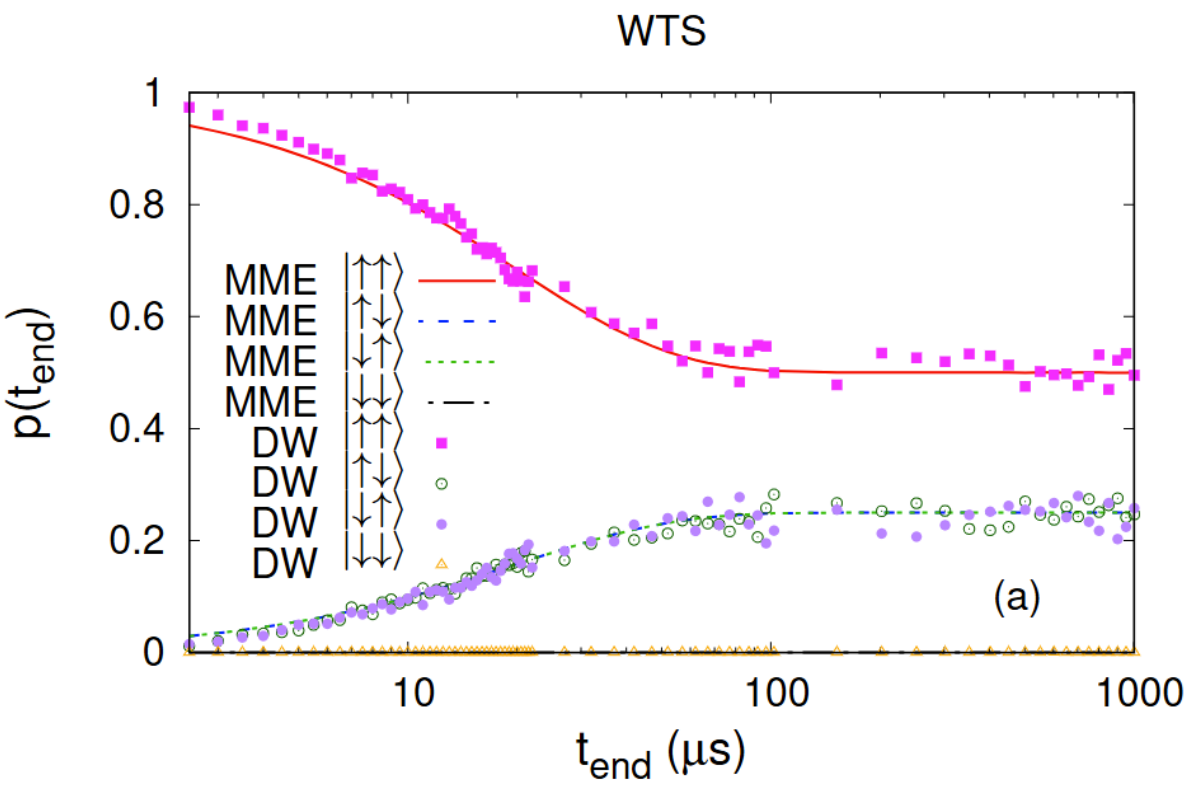

[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# 4.2 2-SPIN RESULTS

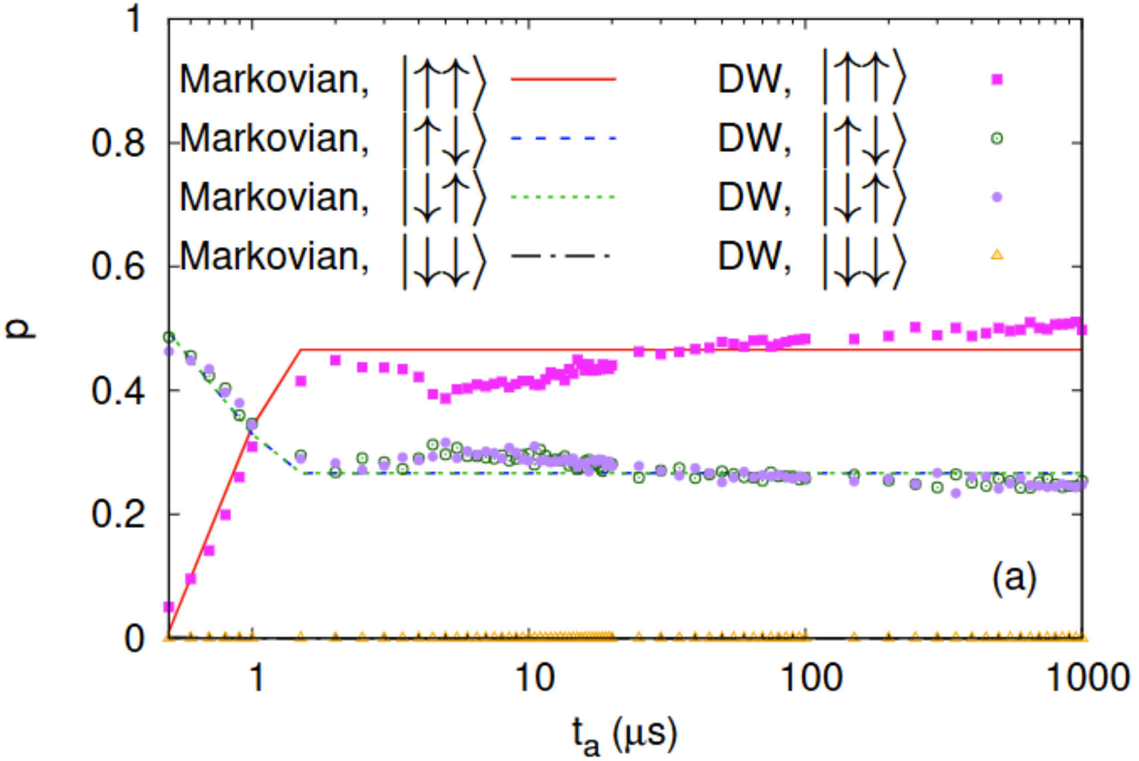
2S1
$$H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$$

## 4.2.3 Markovian master equation

**Reverse annealing**




**Standard annealing**




- B and B' used to reproduce the dips & bumps in forward annealing
- Non-quantum equations can capture the D-Wave behavior equally well
- Can be used to reproduce the results for larger systems

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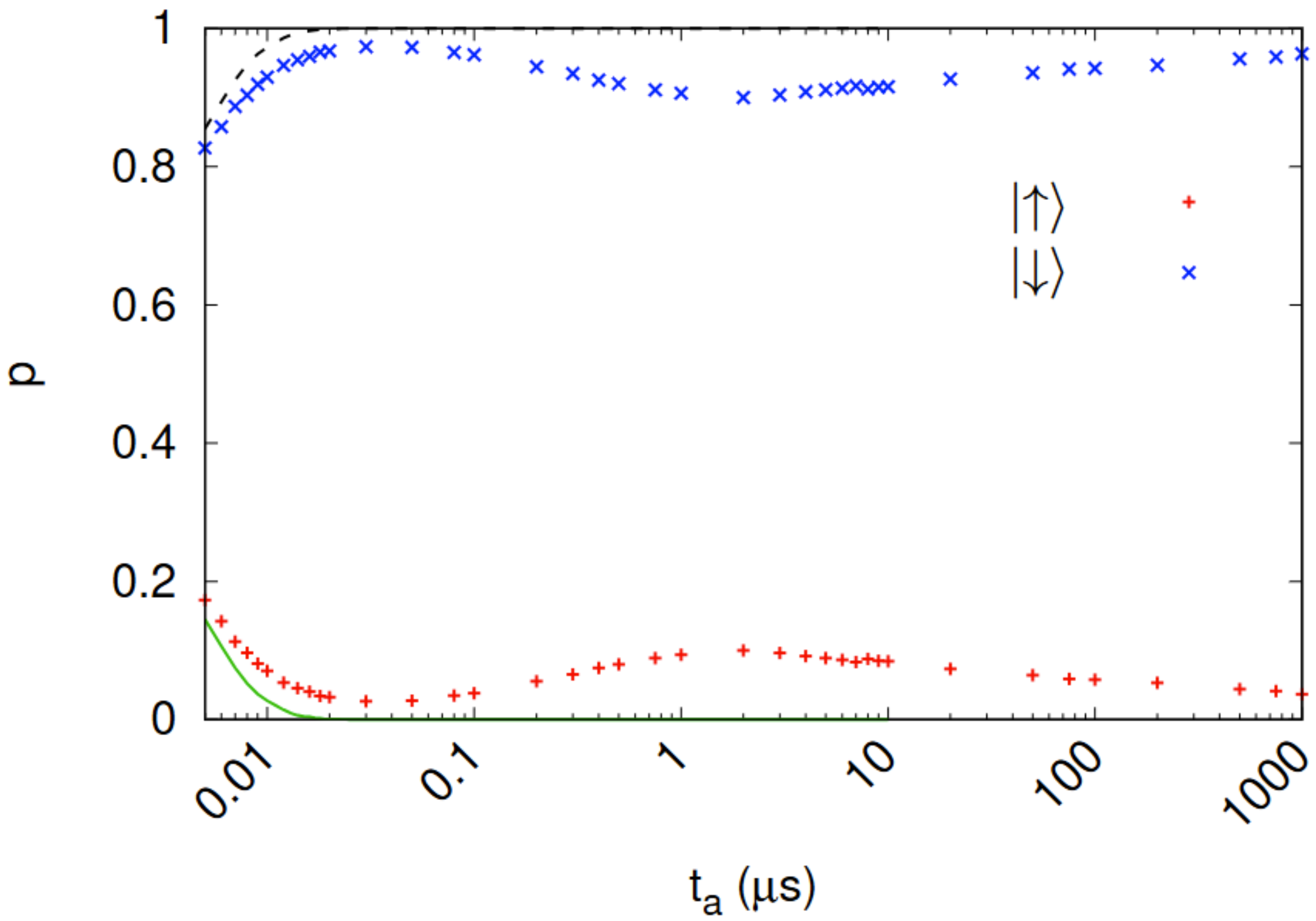
[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# 4.3 FAST ANNEALING

## 4.3.1 1-spin




- Comparison of the D-Wave data with ideal quantum annealing simulations
- Data matches the simulations closely only for $t_a = 5$ ns
- Beyond that data deviates from the coherent simulation results



[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

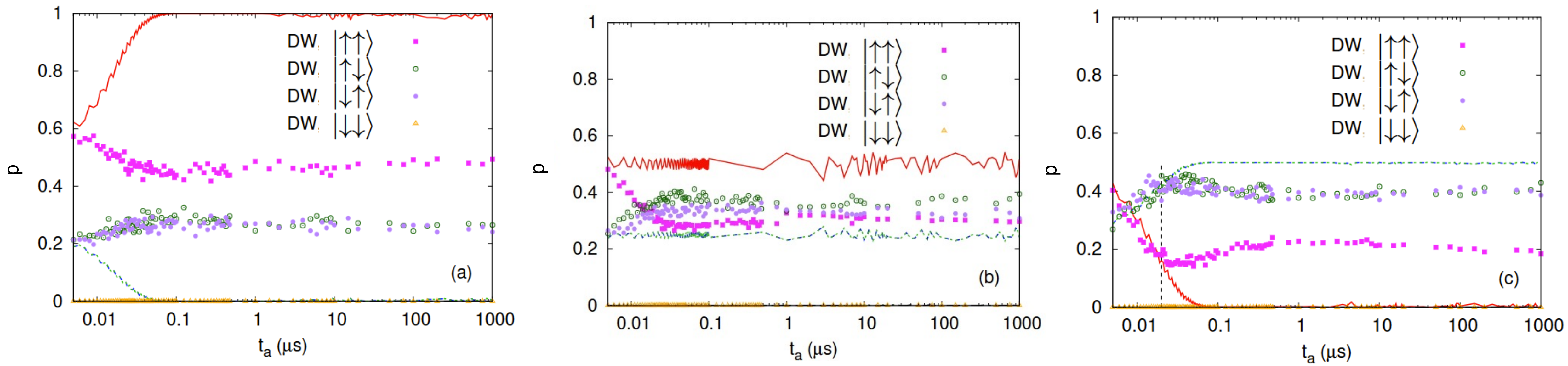
# 4.3 FAST ANNEALING

## 4.3.2 Fast annealing results: 2-spin

**2S1** $H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$

**2S2** $H_P = -\sigma_1^z - \sigma_2^z + \sigma_1^z\sigma_2^z$

**2S3** $H_P = -0.95\sigma_1^z - 0.95\sigma_2^z + \sigma_1^z\sigma_2^z$




- Data deviates from the coherent simulations beyond $t_a = 5$ ns
- Looking at specific cases can be misleading



[2] V. Mehta, H. De Raedt, K. Michielsen, F. Jin,

# V. CONCLUSION

- **Systematic thermalization**: D-Wave results show consistent relaxation towards the thermal equilibrium across standard, reverse, and fast annealing protocols as a function of annealing time

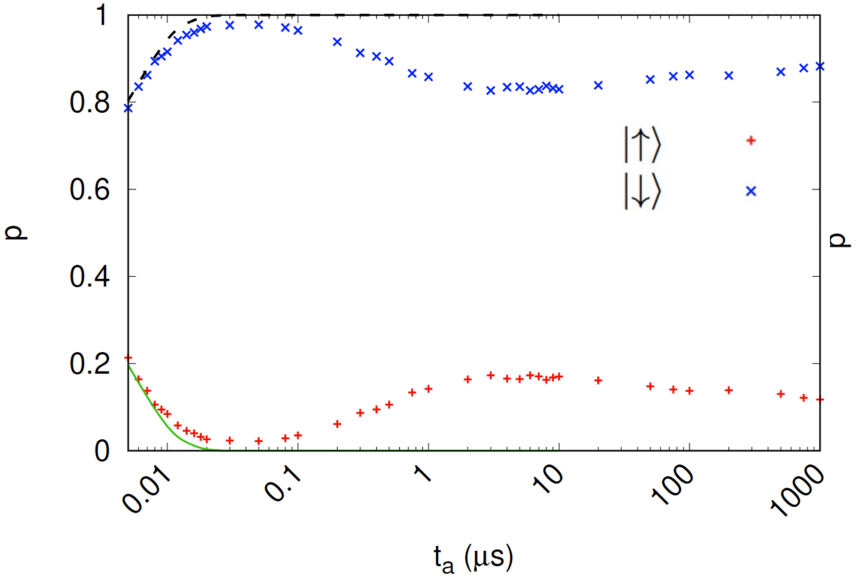
- **Numerical agreement**: Bloch (1-spin) and Lindblad/Markovian master equations (2-spin) reproduce D-Wave data well across all protocols

- **Possible implementation artifact**: Standard annealing shows unexpected features (dips & bumps) at certain annealing times, which can be explained by incorporating a physically motivated modification to the annealing schedule

- **Coherent effects**: Signs of coherence observed but limited to ~5 ns annealing time

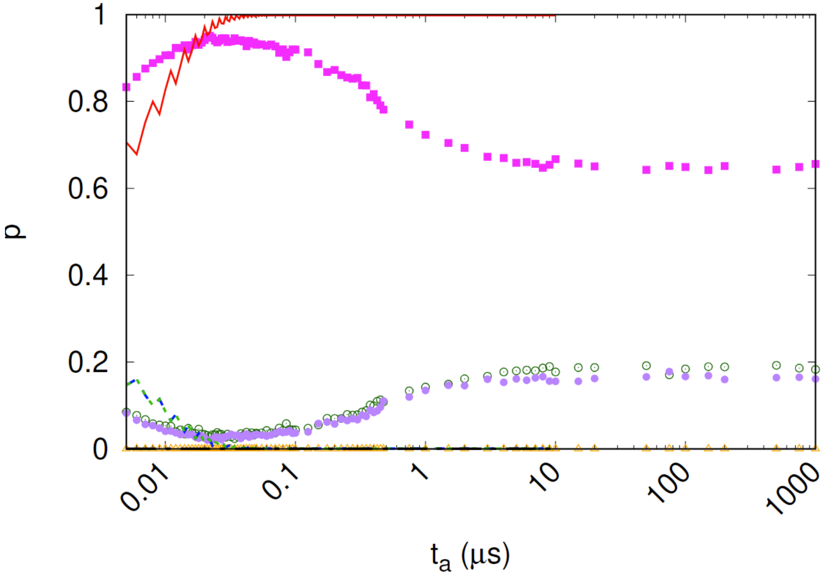
# ADVANTAGE 2 (PROTOTYPE)

$$H_P = 0.1\,\sigma^z$$



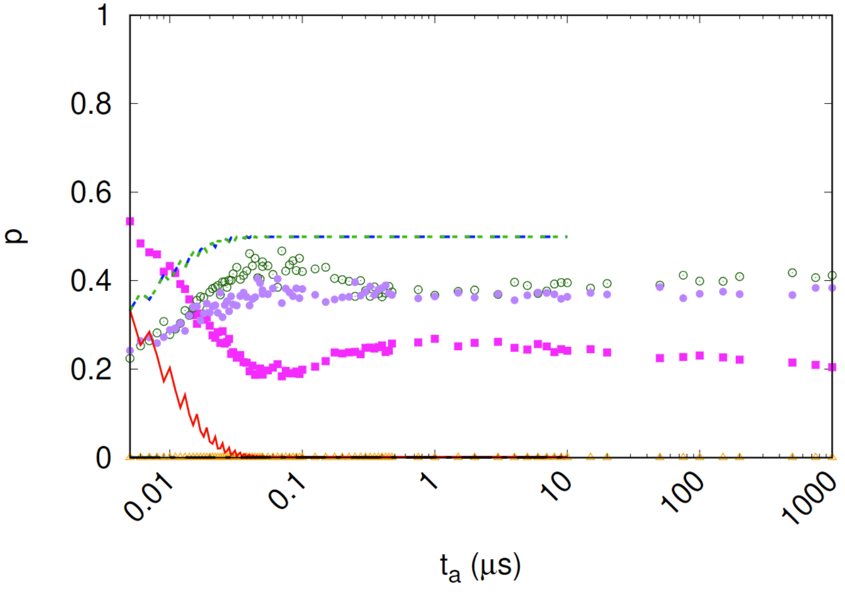

**2S1**

$$H_P = -\sigma_1^z - \sigma_2^z + 0.95\sigma_1^z\sigma_2^z$$




**2S3**

$$H_P = -\sigma_1^z - \sigma_2^z + 1.05\sigma_1^z\sigma_2^z$$



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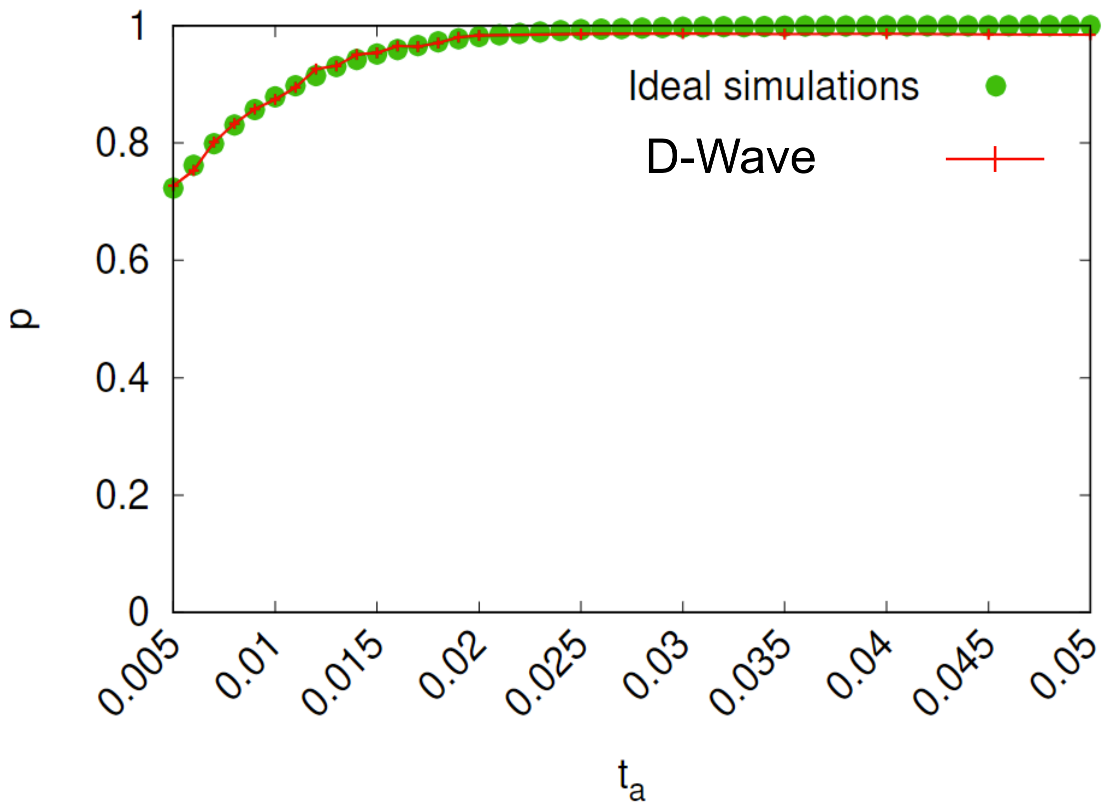
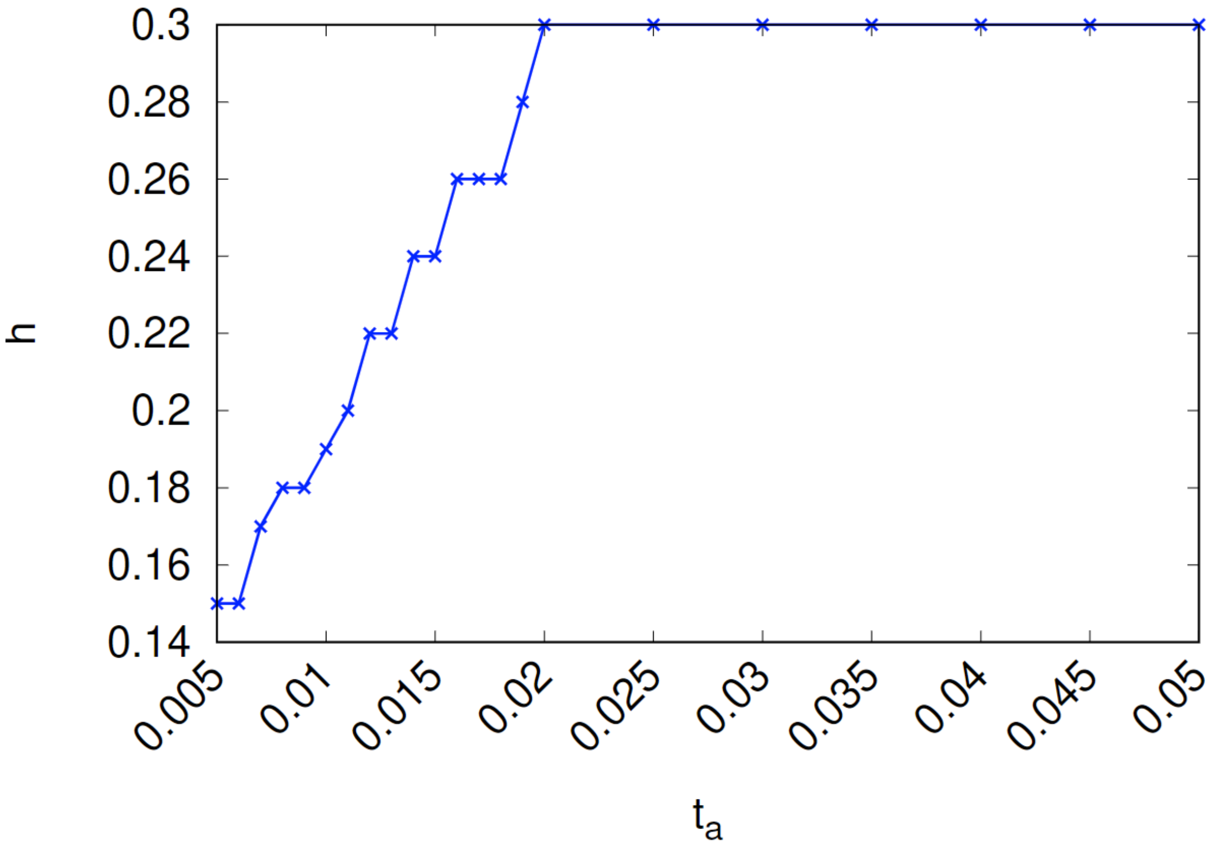
# SHIMMING





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