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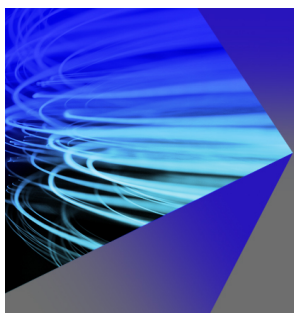
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ABSTRACT

Rubber friction is of major practical importance in applications such as tires, rubber seals, and footwear. This review article focuses on the theory and experimental studies of rubber friction on substrates with random roughness. We examine both steady sliding and accelerated motion, with particular attention to the origins of the breakloose friction force and the influence of pre-slip, elasticity, and flash temperature on friction dynamics. We further discuss rolling friction for cylinders and spheres, as well as sliding friction for triangular sliders on dry and lubricated rubber surfaces. Theoretical predictions are compared with experimental results obtained using different materials, geometries, and environmental conditions, highlighting the importance of accounting for multiscale roughness. Open challenges, such as the role of adhesion enhancement, energy dissipation due to crack opening, and the physical origin of the short-distance roughness cutoff, are discussed.

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I. INTRODUCTION

Rubber friction plays a critical role in many applications, including tires, rubber seals, footwear, and damping systems.^{1–5} In such systems, the frictional behavior is governed by a complex interplay of viscoelastic energy dissipation and adhesion, influenced by factors such as surface roughness, lubrication, temperature, and sliding velocity.

The area of real contact when two elastic solids with random roughness are squeezed together by a normal force F_N is typically proportional to the normal force.^{6–14} This proportionality holds even when adhesion increases the contact area, provided that the adhesion does not result in a (macroscopic) pull-off force, a condition satisfied in most practical applications (e.g., negligible adhesion when lifting a bottle from a table¹⁵). When the real contact area remains proportional to F_N , the sliding friction force is usually also proportional to F_N . In such cases, one defines static and kinetic friction coefficients (μ_s and μ_k), where the force required for initiating sliding (the breakloose friction force) equals $\mu_s F_N$ and the friction force during steady sliding equals $\mu_k F_N$.

Understanding rubber friction has long been a focus of tribological research. Pioneering work by Schallamach,^{16,17} Greenwood and Tabor,¹⁸ and Grosch¹⁹ laid the foundation for interpreting the adhesive contribution (from shearing the real area of contact) to friction as well as the viscoelastic contribution arising from macroscopic deformation.¹⁸ Schallamach proposed a stick-slip mechanism at the interface and discovered detachment waves (Schallamach waves) that characterize the sliding of soft rubber on smooth surfaces. Greenwood and Tabor investigated the role of viscoelasticity in both sliding and rolling configurations at the macroscopic level, while Grosch systematically explored the dependence of friction and wear on sliding velocity and temperature.

Subsequent developments incorporated more detailed descriptions of rubber rheology and surface roughness into theoretical models. For example, Chernyak and Leonov,²⁰ and later Volokitin and Persson,²¹ presented theories that accounted for the adhesive contribution with frequency-dependent viscoelastic properties of rubber. Finally, Persson,⁶ using his multiscale contact mechanics theory, introduced a comprehensive framework that includes the viscoelastic contribution due to roughness over multiple length

scales, as well as the adhesive contribution from the real area of contact. This approach enabled more quantitative predictions of both adhesive and hysteretic friction components, including the effects of surface topography and thermal feedback such as flash temperature.^{22,23} Comparisons with experimental observations under various conditions, including dry and lubricated contacts, have provided strong support for the predictive power of this framework.^{24–27}

This review summarizes theoretical and experimental progress in the study of rubber friction on rigid, randomly rough substrates. We discuss key physical quantities and mechanisms and explore how different geometries and boundary conditions affect the frictional response.

This article is organized as follows: Secs. II and III review two fundamental quantities that govern rubber friction in Persson contact mechanics, the viscoelastic modulus of rubber and the surface roughness power spectrum.

In Sec. IV, we introduce the analytical framework used to calculate the real contact area between a nominally flat elastic block and a rigid rough substrate, based on Persson contact mechanics theory. The theory predicts how the contact area depends on the applied pressure, the effective elastic modulus, and the surface roughness, which enters through the root-mean-square (rms) slope. The extension to viscoelastic sliding conditions is also briefly discussed.

In Sec. V, we present the core of the analytical theory for rubber friction on rough surfaces. We begin with the general concepts, followed by the analytical formulation of rubber friction (Sec. V C), and then comparisons with experimental data (Sec. V D). Additional considerations, such as dynamic effects and their influence on the friction coefficient, are discussed in Sec. V E.

Section VI addresses rolling friction, which can be regarded as a limiting case of sliding friction where the contribution from interfacial shear is negligible and friction arises solely from bulk viscoelastic deformation.

In Sec. VII, we review a special case involving a triangular slider on rubber surfaces under both dry and lubricated conditions. This case effectively summarizes the theoretical framework: under dry conditions, the behavior follows the sliding friction theory described in Sec. V while under lubrication, where interfacial shear is suppressed and the behavior closely resembles that of rolling friction.

In Sec. VIII, we examine the effects of lubrication on rubber friction. This includes the role of fluid-induced separation between the rubber and substrate, the competition between wetting and dewetting mechanisms, and how these influence the transition from boundary to hydrodynamic lubrication. We also analyze how interfacial adhesion and surface roughness affect the wetting dynamics, especially under transient sliding conditions. Furthermore, the contribution of adhesion in lubricated contacts is discussed, with a focus on how lubricant layers modulate the adhesive interaction and the resulting shear stress. These insights are supported by both theoretical analysis and comparisons with experimental findings.

Section IX reviews the similarities and differences between friction on ice and snow. While the mechanisms of friction on ice have been widely studied due to their relevance to winter mobility, rubber–snow interactions remain less well understood. The temperature dependence of friction behavior on ice as well as water content

dependence on snow are discussed. Experimental findings are compared with theoretical predictions, highlighting the role of surface premelting, capillary forces, and viscoelastic dissipation.

Finally, Sec. X provides a summary and conclusion, reflecting on the current understanding of rubber friction and highlighting open questions and future directions.

II. VISCOELASTIC MODULUS

Rubber compounds consist of long molecular chains that are cross-linked, resulting in a viscoelastic solid. When they undergo time-dependent deformation, mechanical energy is converted into thermal motion. Here, we will be sloppy and use the term “energy dissipation” to describe the conversion of mechanical energy into heat. In rubber, energy dissipation arises from internal friction between polymer chains as molecular segments slide past one another.

It is convenient to use complex notation for stress and strain,

$$\sigma(t) = \sigma(\omega)e^{-i\omega t}, \quad \epsilon(t) = \epsilon(\omega)e^{-i\omega t}. \quad (1)$$

The physical stress and strain correspond to the real parts of these expressions. For a linear viscoelastic material, the viscoelastic modulus $E(\omega)$ is defined by

$$\sigma(\omega) = E(\omega)\epsilon(\omega). \quad (2)$$

The modulus $E(\omega)$ is complex,

$$E(\omega) = E_R(\omega) - iE_I(\omega) = |E(\omega)|e^{-i\alpha}, \quad (3)$$

where E_R , E_I , and α are real quantities. Using (1) and (3), we see that if $\epsilon(t) = \epsilon_0 \cos(\omega t)$, then $\sigma(t) = |E(\omega)|\epsilon_0 \cos(\omega t + \alpha)$. This indicates that the stress response lags behind the strain by a phase angle α , a result of energy dissipation.

At low frequencies (or high temperatures), rubber behaves as a soft elastic solid with a typical modulus E_0 on the order of a few MPa. At high frequencies (or low temperatures), it again behaves elastically, but with a much higher modulus, $E_1 \approx 1000E_0$, typically on the order of a few GPa. The low-frequency regime is called the rubbery region, while the high-frequency regime is referred to as the glassy region.^{28,29}

A fundamental physical observation is causality, which states that the response of a system can occur only after the application of the driving force. For the (linear response) viscoelastic modulus, causality results in a relation between the real and imaginary part of $E(\omega)$ given by the Kramers-Kronig relation:

$$\text{Re}E(\omega) = E(\infty) + \frac{2}{\pi} \text{P} \int_0^\infty d\omega' \frac{\omega' \text{Im}E(\omega')}{(\omega' - \omega)(\omega' + \omega)}$$

A more useful consequence of causality is that the complex modulus can be represented using the following spectral representation:

$$\frac{1}{E(\omega)} = \frac{1}{E(\infty)} + \int_0^\infty d\tau \frac{H(\tau)}{1 - i\omega\tau}, \quad (4a)$$

where $H(\omega)$ is real and positive. A simple limit of (4a) is when $H(\tau) = H_0\delta(\tau - \tau_0)$ where

$$\frac{1}{E} = \frac{1}{E_1} + \left(\frac{1}{E_0} - \frac{1}{E_1} \right) \frac{1}{1 - i\omega\tau_0}. \quad (4b)$$

Here, the model is characterized by a single relaxation time τ_0 . In reality, rubber exhibits a broad distribution of relaxation times, and the transition from the rubbery to the glassy region spans several decades of frequency. This behavior is illustrated in Fig. 1 (master curve) and Fig. 2 (loss tangent) for acrylonitrile butadiene rubber (NBR).

Rubber deformation involves stress augmented thermally activated processes, and $E(\omega)$ is strongly temperature-dependent. An important characteristic of rubber is its glass transition temperature T_g . We define T_g as the temperature at which $\tan\delta = \text{Im} E(\omega, T)/\text{Re} E(\omega, T)$ reaches a maximum for a frequency $\omega = 0.01 \text{ s}^{-1}$. This definition gives values of T_g that are close to those obtained using standard viscosity or calorimetric measurements.

When deformed at temperatures well below T_g , rubber is in the glassy state, where molecular motion is limited. At temperatures well above T_g , it is in the rubbery state with high chain mobility and would be a (high-viscosity) liquid if not for the crosslinks.

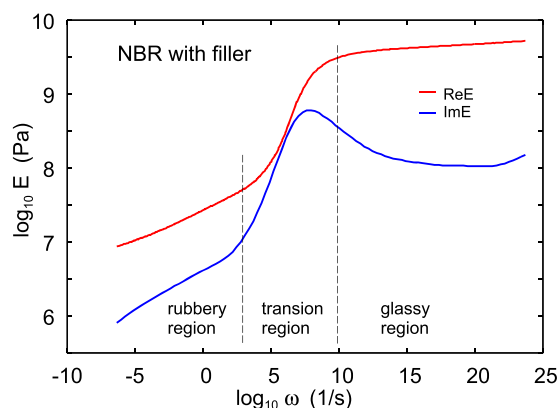


FIG. 1. Real and the imaginary parts of the (linear response) viscoelastic modulus as a function of frequency ω (log-log scale); for a NBR rubber compound with filler particles at $T = 20^\circ\text{C}$.

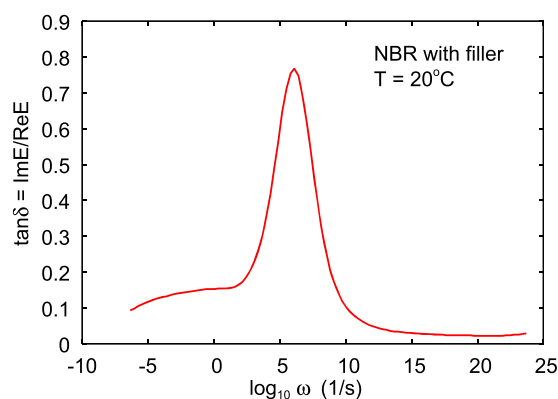


FIG. 2. Loss tangent $\tan\delta = \text{Im} E/\text{Re} E$ as a function of the logarithm of the frequency ω ; for the NBR rubber compound shown in Fig. 1.

The viscoelastic master curve and the temperature dependence of $E(\omega)$ can be determined using a dynamic mechanical analyzer (DMA), which subjects small rubber specimens to elongation or shear at various frequencies and temperatures.

By shifting the frequency segments measured at different temperatures T (typically from -80 to 120°C) and in some frequency range ω (typically from 0.1 to 100 Hz), one can construct a master curve $E(\omega, T) = E(\omega a_T, T_0)$ that spans a wide frequency range. Here, a_T is the shift factor, with $a_T = 1$ at $T = T_0$. For “simple” rubber compounds, if the reference temperature $T_0 = T_g$, a_T is approximately given by the Williams–Landel–Ferry (WLF) equation,

$$\log_{10} a_T = -\frac{C(T - T_g)}{T + T_1 - T_g}, \quad (5)$$

with typical values $C \approx 17.44$ and $T_1 \approx 51.6 \text{ K}$. However, for most rubber compounds, this relation is valid only within a limited temperature range. This is illustrated in Fig. 3, which shows the shift factor a_T for a carbon-filled styrene-butadiene rubber (SBR) compound. It should also be noted that the frequency-temperature shifting procedure used to construct the master curve is strictly valid only at very small strains, where stress and strain are linearly related. In the linear response region, $\text{Re} E$ and $\text{Im} E$ are not independent functions: $\text{Re} E$ can be obtained from $\text{Im} E$ using a Kramers–Kronig relation. Hence, if shifting produces a smooth mastercurve for, e.g., $\text{Im} E$, the same shift function a_T will result in a smooth mastercurve for $\text{Re} E$. This is not true at larger strains, where there is no simple relation between the real and imaginary parts of the effective modulus.

Most rubbers used in tires, seals, and other applications contain filler particles, typically carbon or silica nanoparticles. These fillers form networks within the rubber matrix, making the material stiffer at small deformations, typically when the strain is less than 0.001 , where the filler network remains intact. As the strain increases, the filler network breaks down and the real part of the modulus decreases, and at strain levels relevant to most applications (typically 0.1 – 0.5), the modulus can be reduced by a factor of ~ 5 .

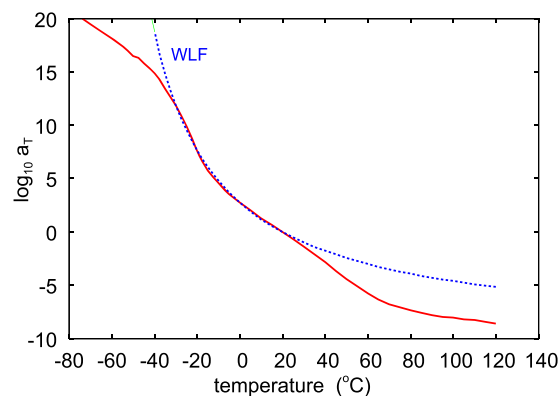


FIG. 3. Horizontal shift factor a_T as a function of temperature for the SBR compound used in Sec. VII. The reference temperature is $T_0 = 20^\circ\text{C}$, with $a_{T_0} = 1$.

To account for this strain-dependent softening, we define an effective modulus E_{eff} , which depends on the strain ϵ and the temperature T ,

$$\text{Re}E_{\text{eff}} = E_R(\omega, T) \eta_R(\epsilon), \quad \text{Im}E_{\text{eff}} = E_I(\omega, T) \eta_I(\epsilon), \quad (6)$$

where E_R and E_I are the small-strain (linear response) modulus. The strain softening functions $\eta_R(\epsilon)$ and $\eta_I(\epsilon)$ can be measured using DMA. They also depend on ω and T , but this dependence is weaker than that of the small-strain viscoelastic modulus. We note that the modulus obtained in this way account for nonlinearity via the secant method, which we consider the most appropriate approach for incorporating nonlinear effects within a linear response framework.

In Fig. 4, we show η_R and η_I for a SBR compound with carbon-filler. The function $\eta_R(\epsilon)$ decreases monotonically with increasing strain. In contrast, $\eta_I(\epsilon)$ first increases and reaches a maximum at around $\epsilon \approx 0.01$ and then decreases continuously over the strain range studied. For very large strain, where the polymer chains are fully stretched, $\eta_R(\epsilon)$ increases rapidly, but this large strain region is not important in most applications.

In many applications, the strain field is complex, involving elongation, compression, and shear components. Experimental studies have shown that rubber nonlinearity is insensitive to the mode of deformation, at least in the static limit. Gregory^{30,31} found that for a natural rubber (NR) compound with carbon black filler, the elastic energy stored in the deformed rubber can be accurately considered as a function of the strain invariant,

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2,$$

where $\lambda_1 = L_x/L_0$ is the extension ratio in the x -direction and similarly for λ_2 and λ_3 .

Figure 5 shows the effective modulus in elongation and compression, $H = \sigma/(\lambda - \lambda^{-2})$ (with $H = E/3$ in the linear response regime), and in shear, $G = \sigma/\gamma$, as a function of $(I_1 - 3)^{1/2}$. Clearly, the effective modulus is nearly independent of the mode of deformation up to $(I_1 - 3)^{1/2} \approx 1$.

Physically, if strain softening results from the breakdown of the filler network as the rubber deforms, this effect should occur

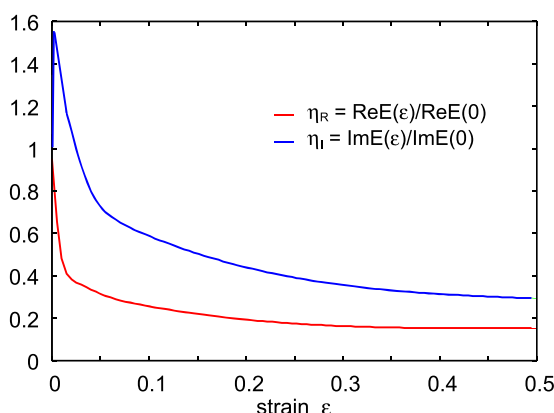


FIG. 4. Viscoelastic modulus at strain ϵ , normalized by the modulus at $\epsilon = 10^{-4}$, measured at a frequency of 1 Hz and at $T = 20^\circ\text{C}$, for the same SBR compound shown in Fig. 3.

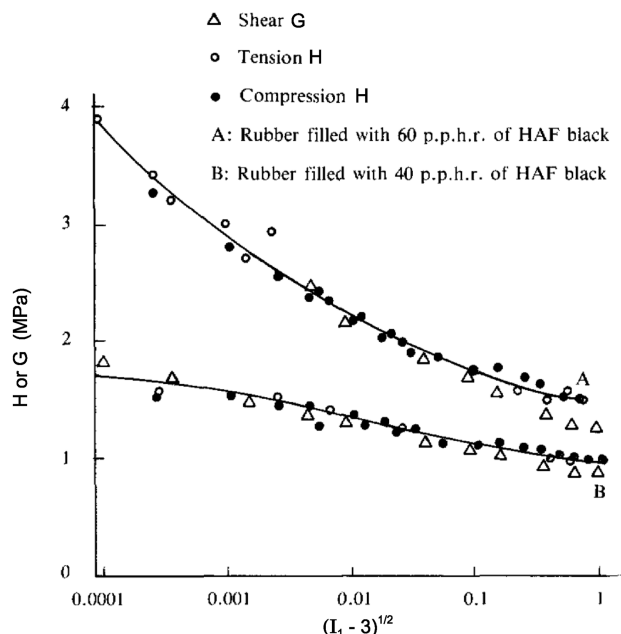


FIG. 5. Dependence of the material parameters H and G on the strain invariant I_1 for natural rubber filled with 60 and 40 p.p.h.r. of HAF (N330) carbon black; adapted from Refs. 30 and 31.

regardless of whether the sample is extended, compressed, or sheared. Hence, one expects strain softening to occur at small strain for all deformation modes, as observed in Fig. 5.

Assuming incompressible rubber, so that $\lambda_1\lambda_2\lambda_3 = 1$, we have for elongation or compression in the x -direction, $\lambda_1 = L/L_0$ and $\lambda_2 = \lambda_3$, leading to

$$I_1 = \lambda_1^2 + \frac{2}{\lambda_1}.$$

Thus, $(I_1 - 3)^{1/2} = 1$ corresponds to $L/L_0 \approx 0.54$ (compression) or $L/L_0 \approx 1.68$ (elongation), which is similar to or larger than the strains expected in most engineering applications.

III. SURFACE ROUGHNESS POWER SPECTRA

There are many ways to study surface roughness,³² but the most accurate methods are engineering stylus and atomic force microscope (AFM) instruments. In such measurements, a sharp tip is moved along a straight line (see Fig. 6), while being pressed lightly against the substrate. The tip follows the surface contour, moving up and down, and records the topography $z = h(x)$.

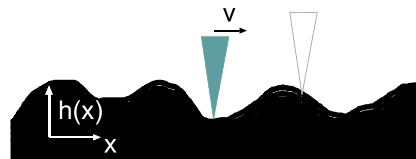


FIG. 6. Surface height profile $h(x)$ along the x axis, as measured using a stylus instrument.

If the surface roughness has isotropic statistical properties, then all statistical properties of the surface can be captured in a sufficiently long line scan of length L . Fourier showed that a (nearly) arbitrary function $h(x)$ can be expressed as a sum or integral of $\cos(2\pi x/\lambda)$ and $\sin(2\pi x/\lambda)$ waves with different amplitudes and wavelengths λ [see Fig. 7(a)]. It is more convenient to use complex notation,

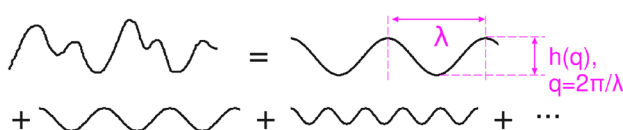
$$h(x) = \int dq h(q_x) e^{-iq_x x}, \quad (7)$$

where the wavevector $q_x = 2\pi/\lambda$. Since $h(x)$ is real, we must have $h^*(q_x) = h(-q_x)$. The one-dimensional (1D) power spectrum is defined as^{33,34}

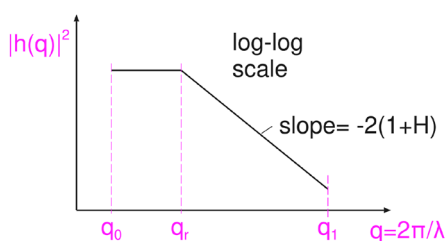
$$C_{1D}(q_x) = \frac{2\pi}{L} |h(q_x)|^2. \quad (8)$$

The two-dimensional (2D) power spectrum C_{2D} , which appears in analytical theories of rubber friction, can be derived from C_{1D} through an integral transform when the surface roughness is isotropic. However, it is more conveniently obtained from measurements of the surface height $h(x, y)$ over a square area $A_0 = L^2$. The 2D power spectrum is given by

(a) Fourier decomposition of surface roughness profile:



(b) Surface roughness power spectrum



(c) Magnification of roughness profile

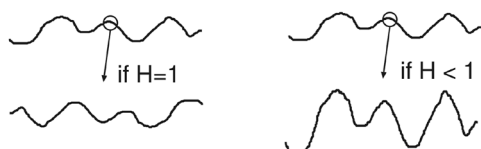


FIG. 7. (a) Surface roughness profile $h(x)$ can be expressed as a sum of periodic functions with different wavelengths and amplitudes. (b) The two-dimensional surface roughness power spectrum is proportional to the square of the amplitude of the Fourier transform of the height $h(x, y)$, measured over a square area. Most engineering surfaces have a power spectrum that appears as a tilted line on a log-log plot, with the slope defined by the Hurst exponent H . A roll-off occurs for wavenumbers $q < q_r$. The fractal dimension of the surface is $D_f = 3 - H$. (c) Surfaces with a fractal dimension $D_f = 2$ or Hurst exponent $H = 1$ are statistically self-similar, meaning that magnified segments resemble the original in a statistical sense. For $D_f > 2$ or $H < 1$, the ratio of roughness amplitude to wavelength increases with magnification. Most engineering surfaces have $0.7 < H < 1$.

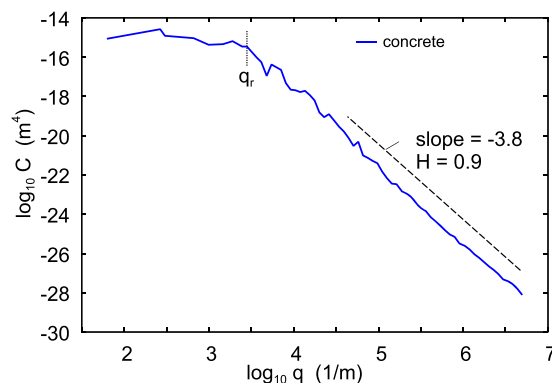


FIG. 8. 2D surface roughness power spectrum of a concrete surface, plotted as a function of the wavenumber on a log-log scale.

$$C_{2D}(\mathbf{q}) = \frac{(2\pi)^2}{A_0} |h(\mathbf{q})|^2, \quad (9)$$

where $h(\mathbf{q})$ is the Fourier transform of the surface height $h(\mathbf{x})$ and $\mathbf{q} = (q_x, q_y)$ is the wavevector. The range of q_x is $\pi/a < q_x < \pi/a$, and similarly for q_y , where a is the lateral spacing between adjacent height data points along the x axis.

In the isotropic case, $C_{2D}(\mathbf{q})$ depends only on the magnitude $q = |\mathbf{q}|$ of the wavevector. For simplicity, we will refer to $C_{2D}(\mathbf{q})$ as $C(q)$ in the following.

Surfaces often exhibit power-law spectra of the form $C(q) \sim q^{-\beta}$, which appear as straight lines on a log-log plot [see Fig. 7(b)], but with a roll-off at long wavelengths (i.e., small wavenumbers). This roll-off arises because most engineering surfaces are designed to be smooth on the scale of the object. For example, for asphalt road surfaces, the roll-off is determined by the largest stone particles in the mixture. In this case, the roll-off wavelength is on the order of a few millimeters, corresponding to a roll-off wavenumber of $q_r = 2\pi/\lambda_r \approx 10^3 \text{ m}^{-1}$.

Self-affine fractal surfaces are characterized by a power-law spectrum $C(q) \sim q^{-\beta}$, where $\beta = 2(1 + H)$ and H is the Hurst exponent. The Hurst exponent is related to the fractal dimension by $D_f = 3 - H$. When $D_f = 2$ or $H = 1$, the surface is self-similar, meaning that a magnified segment statistically “looks the same” as the original surface [see Fig. 7(c)].

For $D_f > 2$ or $H < 1$, the ratio between roughness amplitude and wavelength increases with magnification. Most engineering surfaces have $0.7 < H < 1$ and such surfaces appear rougher at higher magnifications [see Fig. 7(c)].

As an example, Fig. 8 shows the 2D surface roughness power spectrum of a concrete surface. The sloped region is well-approximated by a straight line with a slope of -3.8 , corresponding to a Hurst exponent $H = 0.9$. The roll-off wavenumber is $q_r \approx 3000 \text{ m}^{-1}$, corresponding to a wavelength of $\lambda_r = 2\pi/q_r \approx 2 \text{ mm}$.

IV. ELEMENTS OF CONTACT MECHANICS

Consider a rectangular block squeezed against a substrate with nominal pressure σ_0 . For simplicity, we assume that the substrate is rigid and randomly rough and that the block is elastic with a flat

surface. The normalized area of real contact predicted by Persson contact mechanics theory⁶ is given by

$$\frac{A}{A_0} = \operatorname{erf}\left(\frac{\sqrt{2}\sigma_0}{\xi E^*}\right), \quad (10)$$

where $\operatorname{erf}(x)$ is the error function, A_0 is the nominal contact area (i.e., the area of the bottom surface of the block), and $E^* = E/(1 - \nu^2)$ is the effective elastic modulus (E is the Young's modulus and ν is the Poisson ratio). The rms slope ξ can be obtained directly from the measured height profile $z = h(x, y)$ or calculated from the surface roughness power spectrum using

$$\xi^2 = \int d^2q \, q^2 C_{2D}(\mathbf{q}).$$

The red line in Fig. 9 corresponds to (10).

Using the approximation $\operatorname{erf}(x) \approx 2x/\sqrt{\pi}$ for $x \ll 1$, we obtain, for small arguments of the error function,

$$\frac{A}{A_0} \approx \frac{\kappa\sigma_0}{\xi E^*}, \quad (11)$$

where $\kappa = (8/\pi)^{1/2}$. The linear relation in (11) between A and σ_0 holds approximately for $A/A_0 < 0.3$ (see the dashed line in Fig. 9).

The analytical expression in (10) depends on the surface roughness power spectrum only through the rms slope ξ . Numerical studies^{7,8,35,36} have shown that different power spectra with the same rms slope give nearly the same (universal) relation between A/A_0 and $\sigma_0/\xi E^*$, as predicted by the theory. The blue line in Fig. 9 represents the result of one "exact" numerical study, with the slope parameter $\kappa \approx 2$.

In the case of sliding contact (velocity v) for viscoelastic solids, A/A_0 can be calculated using equations that will be presented later. Here, we note that if the roughness occurs on a single length scale (in the sliding direction) λ , then (10) remains approximately valid if E^* is replaced by $|E(qv)/(1 - \nu^2)|$, where $q = 2\pi/\lambda$, and $E(\omega)$ is the complex viscoelastic modulus. This relation follows from the observation that the viscoelastic solid is deformed at the frequency $\omega = qv$ when sliding on a rigid corrugation with wavelength λ .

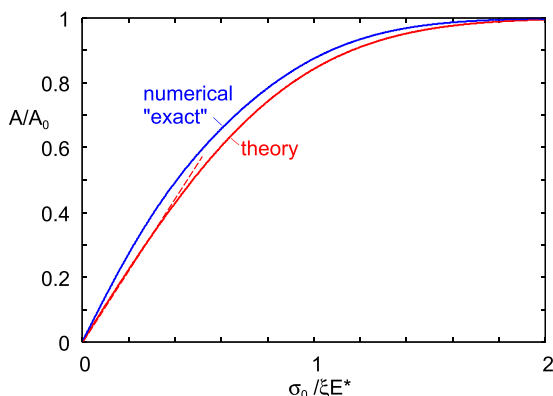


FIG. 9. Normalized area of real contact A/A_0 as a function of $\sigma_0/\xi E^*$, where σ_0 is the nominal contact pressure, ξ is the rms slope, and $E^* = E/(1 - \nu^2)$ is the effective elastic modulus. The blue line represents the result of an "exact" numerical study,^{7,8} and the red line represents the theoretical prediction from (10).⁶

V. RUBBER FRICTION ON ROUGH SURFACES

We consider a rubber block sliding on a hard rough substrate, such as a tire tread block on a road surface. There are two main contributions to the friction force F_f , as illustrated in Fig. 10.

One contribution arises from the viscoelasticity of the rubber. During sliding, the asperities on the road surface induce time-dependent deformations in the rubber, which result in energy dissipation (conversion of mechanical energy into heat). This results in a contact force with a tangential component that opposes the sliding direction [see Fig. 10(a)]. The asperity induced deformations occur mainly within volume elements that extend into the rubber to a distance comparable with the lateral size of the asperity contact regions.

In addition, there is a contribution from the frictional shear stress acting in the area of real contact, often referred to as the adhesive contribution.

Defining the friction coefficient in the usual way as $F_f = \mu F_N$, where F_N is the normal force, we write

$$\mu = \mu_{\text{visc}} + \mu_{\text{cont}}. \quad (12)$$

Experiments have shown that for sliding on randomly rough surfaces, both the viscoelastic contribution μ_{visc} and the contact-area contribution μ_{cont} are independent of F_N , unless the normal force is large enough to approach full contact, the sliding speed is high enough for frictional heating to become significant, or the surfaces are so smooth that adhesion manifest itself on the macroscopic scale as a pull-off force.

A. Viscoelastic contribution

Assume first that adhesion can be neglected. The contribution from rubber viscoelasticity depends on the surface roughness across multiple length scales. This is illustrated in Fig. 11, which shows the case of a rubber block sliding in contact with periodically corrugated substrates having (a) long-wavelength and (b) short-wavelength roughness.

If the ratio h/λ between the amplitude h and the wavelength λ of the roughness is the same, and full contact occurs, then the friction force $F_f(v)$ acting on the block as a function of the logarithm of the

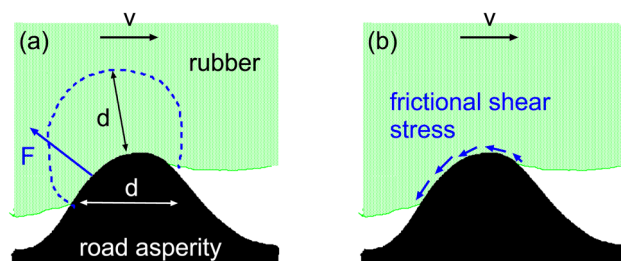


FIG. 10. Several mechanisms contribute to rubber friction. (a) During sliding, surface asperities on the road cause time-dependent deformations in the rubber. These deformations extend into the rubber to a depth comparable with the size of the asperity contact regions. The resulting energy dissipation produces a contact force with a tangential component that opposes the direction of motion. (b) A frictional shear stress acts in the area of real contact. This stress can originate from different mechanisms (see Fig. 12).

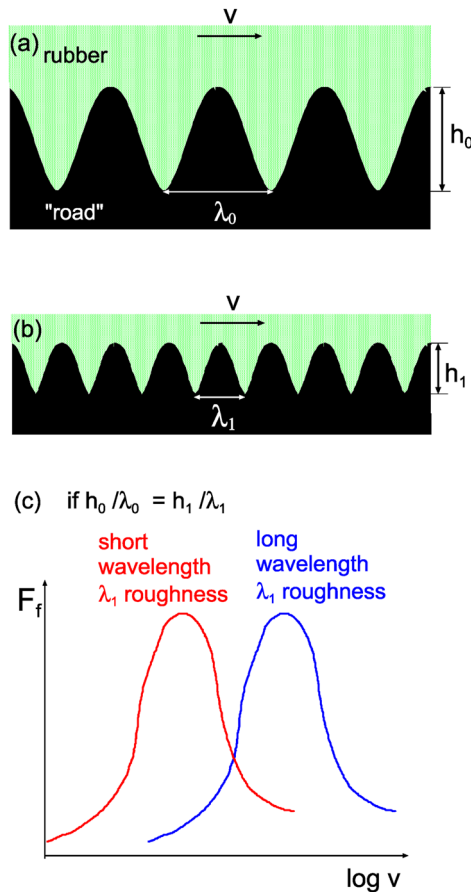


FIG. 11. If rubber slides in complete contact with a sinus corrugated surface, the friction force $F_f(v)$ as a function of the logarithm of the sliding speed appears the same in both cases, except for a shift along the velocity axis. On a $\log v$ -scale, this shift is proportional to $\log(\lambda_1/\lambda_0)$ and results from the different excitation frequencies $\omega = v/\lambda$ generated by the two corrugated profiles.

sliding speed will be the same in both cases, except for a shift along the frequency axis [see Fig. 11(c)]. This shift results from the fact that the deformation frequency induced in the rubber is $\omega \approx v/\lambda$. To achieve the same viscoelastic energy dissipation in both cases, the velocities must be chosen such that the deformation frequency is equal, giving $v_1/v_0 = \lambda_1/\lambda_0$.

In reality, when sliding on a randomly rough surface, the rubber rarely makes full contact with the roughness profile at all length scales. The apparent contact area decreases as shorter-wavelength roughness components are taken into account. For surfaces with a Hurst exponent $H < 1$, the ratio h/λ increases with decreasing wavelength, which would lead to larger contributions from the short-wavelength components if full contact occurred.

In most practical applications, H is close to 1 (typically $H \approx 0.8$), and in this case, the viscoelastic contribution from each decade of roughness length scale is approximately equal. This raises a key question: down to what length scale should roughness be included when calculating the viscoelastic contribution to the friction force?

Analysis of experimental data indicates that, for tire tread rubber sliding on road surfaces, most of the viscoelastic contribution arises from interactions with asperities down to a cutoff length $\lambda_{\min}$ on the order of micrometers.^{16,24,37} However, the origin and precise value of this short-wavelength cutoff are not yet fully understood.

The viscoelastic contribution to the friction from asperities at a single length scale can be estimated as follows: when a rubber block slides in contact with an asperity, as illustrated in Fig. 10, the asperity induces a deformation in the rubber with a typical frequency $\omega \approx v/d$, where d is the characteristic size of the contact region.

The deformation field extends into the rubber to a depth of order d , so the volume of the deformed region scales as d^3 . Over a sliding distance d , the time-dependent stress can be approximated as $\sigma(t) = \sigma_1 \cos(\omega t)$ over a time interval $T = \pi d/v$ (half of one oscillation period). Letting $\dot{\epsilon} = d\epsilon/dt$ denote the strain rate, the energy dissipated during this process is

$$\Delta E \approx d^3 \int_0^T dt \sigma(t) \dot{\epsilon}(t).$$

Using complex notation for harmonic stress and strain, we have

$$\Delta E \approx d^3 \sigma_1^2 \frac{1}{4} \int_0^T dt (e^{i\omega t} + e^{-i\omega t})(i\omega) \left(\frac{e^{i\omega t}}{E^*(\omega)} - \frac{e^{-i\omega t}}{E(\omega)} \right),$$

where we have used $E(-\omega) = E^*(\omega)$. Performing the integral yields

$$\Delta E \approx \frac{\pi}{2} d^3 \sigma_1^2 \operatorname{Im} \frac{1}{E(\omega)}.$$

This energy equals the sliding distance d multiplied by the friction force F_f , so that

$$F_f \approx \frac{\pi}{2} d^2 \sigma_1^2 \operatorname{Im} \frac{1}{E(\omega)}.$$

If there are N macroasperity contact regions, the friction force,

$$F_f \approx N \frac{\pi}{2} d^2 \sigma_1^2 \operatorname{Im} \frac{1}{E(\omega)}.$$

Using the relation $A \approx Nd^2$ for the area of real contact, we can express the total normal force as $F_N = A\sigma_1 = Nd^2\sigma_1$. The friction coefficient is then

$$\mu = \frac{F_f}{F_N} \approx \frac{\pi}{2} \sigma_1 \operatorname{Im} \frac{1}{E(\omega)}.$$

The normal force can also be written as $F_N = \sigma_0 A_0$, where A_0 is the nominal contact area and σ_0 is the nominal contact pressure, which implies $\sigma_1 = \sigma_0 A_0/A$. Using the known result (see Sec. IV) $A/A_0 \approx 2\sigma_0/(\xi|E(\omega)|)$, we obtain $\sigma_1 \approx \xi|E(\omega)|/2$, so that

$$\mu \approx \frac{\xi\pi}{4} |E(\omega)| \operatorname{Im} \frac{1}{E(\omega)} = \frac{\xi\pi}{4} \frac{\operatorname{Im} E(\omega)}{|E(\omega)|}, \quad (13)$$

where ξ is the cumulative rms slope for the considered roughness.

In the transition region between the rubbery and glassy regimes, the ratio $\operatorname{Im} E(\omega)/|E(\omega)| \approx 1$. Therefore, the maximum viscoelastic contribution to the friction coefficient is expected to be of order ξ , which is typically of order 1.

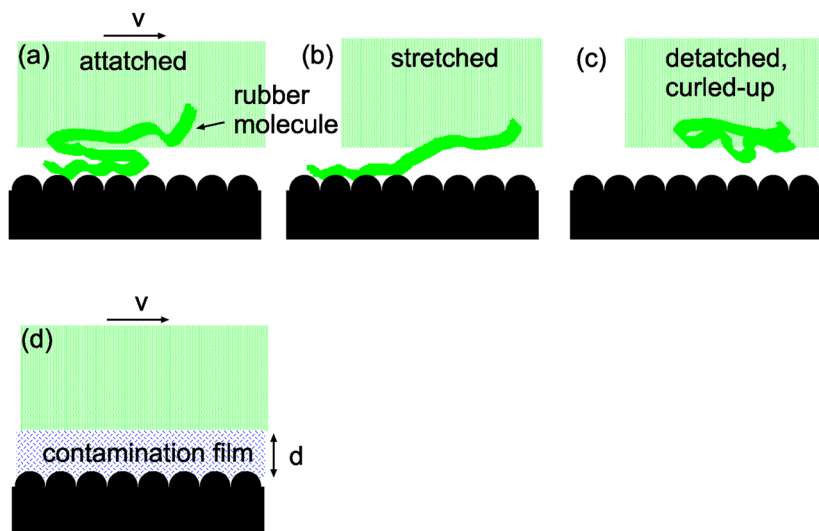


FIG. 12. Frictional shear stress in the area of real contact [see Fig. 10(b)] can arise from (a)–(c) binding-stretching-detachment cycles of rubber molecules at the sliding interface, or (d) from shearing of thin contamination films located between the substrate and the rubber.

B. Area of real contact contribution

We first consider dry and clean surfaces. In sliding contact, rubber molecules and substrate atoms interact, as illustrated in Figs. 12(a)–12(c). In many cases, the interfacial interactions are weak, such as van der Waals forces. During stationary contact, the rubber chains at the interface adjust to the substrate potential in order to minimize the free energy. This bond formation may involve overcoming energy barriers and, therefore, does not occur instantaneously but requires a finite relaxation time.

During sliding at low velocities, thermal fluctuations assist in formation and breaking of the rubber-substrate bonds, resulting in a friction force that approaches zero as the sliding velocity tends to zero. At high velocities, there is not enough time for the rubber molecules to adjust to the substrate potential. As a result, for high sliding speeds, the rubber interface effectively “floats” above the substrate in an incommensurate-like state relative to the corrugated potential. In this regime, the frictional shear stress is also small.

Therefore, the frictional shear stress as a function of sliding speed must exhibit a maximum at some intermediate velocity v^* . This friction mechanism was first studied in a simplified model by Schallamach,¹⁶ and later extended by Chernyak and Leonov,²⁰ and in a more realistic model by Persson and Volokitin.²¹ Because of its conceptual importance, we present in the Appendix a derivation of the Schallamach result using an approach that differs slightly from the one originally used by Schallamach.

The actual elementary stick-slip units are not likely to be individual molecular segments, but rather small patches of molecular segments, which we refer to as stress domains, with a characteristic size D . In Fig. 13, we show the results of calculations (using the model of Persson and Volokitin) for $D = 30$ and 40 nm. Figure 13(a) shows the frictional shear stress, and Fig. 13(b) shows the fraction N/N_0 of the rubber surface in the sliding state, both as functions of the logarithm of the sliding speed.²¹

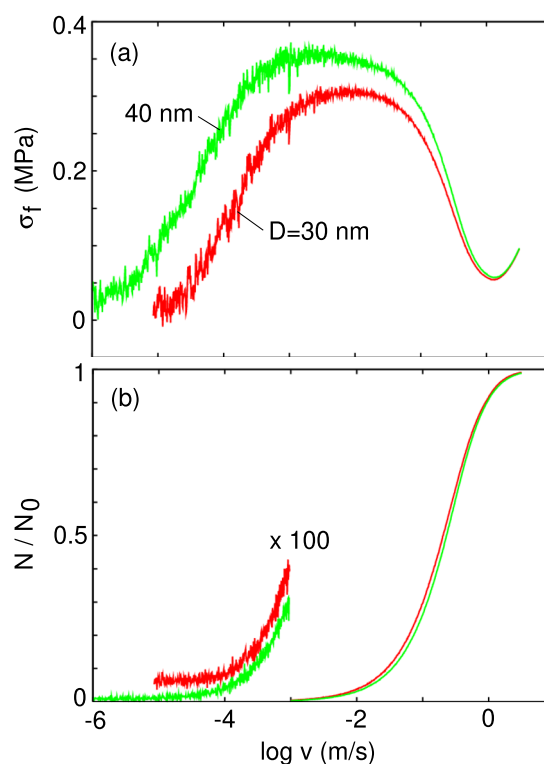


FIG. 13. Calculated frictional shear stress (a) and the fraction N/N_0 of the rubber surface in the sliding state (b), shown as a function of the logarithm of the sliding speed. D is the characteristic size of the interfacial units that undergo stick-slip-stick motion. The results are for styrene-butadiene (SB) rubber; adapted from Ref. 21.

In the simple Schallamach model (see the [Appendix](#)), the frictional shear stress is given by $\sigma_f = \sigma^* f(v/v^*)$, where σ^* is the maximum shear stress (with $D = 2R$),

$$\sigma^* \approx \left(\frac{2Ek_B T}{R^3} \right)^{1/2}, \quad (14)$$

and the reference velocity,

$$v^* = \left(\frac{2k_B T}{ER\tau_0^2} \right)^{1/2}. \quad (15)$$

Here, k_B is the Boltzmann constant; T is the absolute temperature; E is the elastic modulus; and τ_0 is the average time a stress domain spends in the detached state.

The function $f(v/v^*)$ depends on $\Delta E = \epsilon_1 - \epsilon_0$, where ϵ_1 is the energetic barrier to break the rubber–substrate bond in a stress domain and ϵ_0 is the energetic barrier for forming the bond. For $\Delta E/k_B T \approx 3$, $f(v/v^*)$ is approximately a Gaussian function of $\log_{10} v$, with the full width at half maximum of about 2 velocity decades.

The values for σ^* and v^* are consistent with experimental observations (see the [Appendix](#)). Experiments show that the full width at half maximum of $\sigma_f(v)$ is about 4 velocity decades, which is larger than predicted by the Schallamach model, but similar to the prediction by the Persson–Volokitin theory (see [Fig. 13](#)).

Another contribution to the friction force from the area of real contact, which can be significant in particular for smooth surfaces and soft rubber compounds, is illustrated in [Fig. 14](#). During sliding, adhesive interactions between the rubber and the substrate give rise to opening and closing cracks at the edges of asperity contact regions. Due to adhesion hysteresis, more energy is dissipated at the opening crack than is recovered at the closing crack. This asymmetry results in a contribution to the sliding friction force.

The energy dissipation at the crack tip may spatially overlap with bulk viscoelastic dissipation (see [Fig. 14](#)), in which case the total energy loss may not equal the sum of the two mechanisms calculated independently.^{38–42}

In classical theories of interfacial crack propagation, it is typically assumed that the stress field extends far from the crack tip into the contact region and where it remains in a relaxed state. However, this assumption does not hold in situations where an indenter,

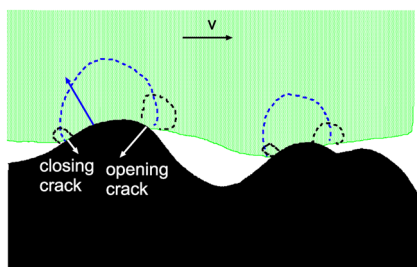


FIG. 14. Adhesive interaction between rubber and substrate leads to the formation of opening and closing cracks at the edges of asperity contact regions during sliding. Due to adhesion hysteresis, more energy is dissipated at the opening crack than is recovered at the closing crack, resulting in a net contribution to the sliding friction force.

such as a rigid sphere, is rapidly brought into and lifted out of contact with a viscoelastic solid. A similar condition applies to most sliding friction scenarios, where the rubber at a given surface point is initially nearly undeformed but undergoes rapid deformation as it enters and exits contact with substrate asperities, particularly at high sliding speeds. In such cases, the stress remains confined to the immediate vicinity of the contact region and does not penetrate deeply into the bulk. The influence of time-dependent deformation on interfacial crack propagation has been discussed qualitatively in [Ref. 43](#) and quantitatively in [Refs. 93 and 44](#).

Opening and closing crack propagations govern the friction behavior when soft rubber slides on a substrate, where both the rubber and the substrate surface are very smooth. This is illustrated in [Fig. 15](#), where a smooth rubber block slides on a flat substrate with negligible roughness. In this case, the rubber adheres to the substrate and no direct slip occurs at the interface. Instead, localized detachment zones form at the leading edge and propagate opposite to the sliding direction. These are known as Schallamach waves.

Schallamach wave motion can be understood by analogy with the motion of a rug on a carpet: one can either pull the entire carpet or create a local deformation and move it forward. Due to adhesion hysteresis, more energy is required to break the bonds at the opening crack than is recovered at the closing crack. This results in a friction force that can be estimated as follows.

Assume a rectangular rubber block of length L_x in the sliding direction and width L_y in the transverse direction. Let the Schallamach waves move with speed v and assume that their spacing is λ . If v_0 is the sliding speed of the block, then the frictional work over a time interval Δt is $F_f v_0 \Delta t$. This must equal the energy dissipated by peeling during that time.

There are $N = L_x/\lambda$ waves, each of length L_y , and the energy dissipated is $N L_y \gamma_{\text{eff}} v \Delta t$, where $\gamma_{\text{eff}} = \gamma_{\text{open}} - \gamma_{\text{close}} \approx \gamma_{\text{open}}$ is the effective adhesion energy. Equating this expression for the energy dissipation with $F_f v_0 \Delta t$ gives

$$F_f = A_0 \gamma_{\text{eff}} \frac{v}{v_0 \lambda}, \quad (16)$$

where $A_0 = L_x L_y$ is the surface area. Roberts and Thomas⁴⁵ have shown that [\(16\)](#) is in good agreement with experimental results.

In many practical cases, surfaces are not perfectly clean, and thin, liquid-like contamination films may exist within the area of real contact. In such cases, there is an additional contribution to the friction force from shearing of the contamination film. These confined

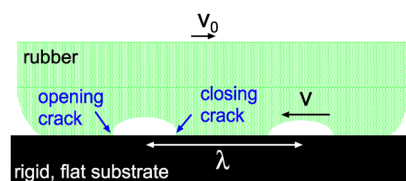


FIG. 15. When a block of soft rubber slides on a flat, smooth substrate (with negligible roughness), Schallamach waves are observed above a critical sliding speed. The rubber adheres to the substrate, and no direct slip occurs at the interface. Instead, localized detachment zones form at the leading edge and propagate opposite to the sliding direction.

films generally exhibit velocity-dependent viscosity, so the resulting shear stress is not linearly proportional to the strain rate.

C. Analytical theory of rubber friction on rough surfaces

As mentioned above, there are two main contributions to the rubber friction force F_f on rough surfaces: one arising from the area of real contact and the other from viscoelastic deformations induced by asperities.

We express the total friction force as

$$F_f = F_{\text{visc}} + F_{\text{cont.}}$$

If the normal force is given by $F_N = A_0 \sigma_0$, where A_0 is the nominal contact area and σ_0 the nominal contact pressure, the friction coefficient becomes

$$\mu = \mu_{\text{visc}} + \frac{\sigma_f A}{\sigma_0 A_0},$$

where A is the area of real contact and σ_f is the frictional shear stress acting within the contact area.

The Persson contact mechanics theory predicts the viscoelastic contribution μ_{visc} and the area of real contact A . However, the dependence of the frictional shear stress σ_f on sliding velocity and temperature must be determined using other theoretical approaches or extracted from experimental data.

1. Viscoelastic contribution

We first consider sliding at constant speed and neglect frictional heating, assuming the temperature is uniform and equal to $T = T_0$. In Persson theory, the viscoelastic friction force acting on a rubber block pressed with nominal stress σ_0 against a hard, randomly rough surface is given by^{6,46}

$$\mu_{\text{visc}} \approx \frac{1}{2} \int_{q_0}^{q_1} dq q^3 C(q) S(q) P(q) \times \int_0^{2\pi} d\phi \cos \phi \operatorname{Im} \frac{E(qv(t) \cos \phi, T_0)}{(1 - v^2) \sigma_0}. \quad (17)$$

Here, $P(q) = A(q)/A_0$ is the normalized contact area observed at magnification $\zeta = q/q_0$,

$$P(q) = \frac{1}{\sqrt{\pi}} \int_0^{\sqrt{G(q)}} dx e^{-x^2/4} = \operatorname{erf} \left(\frac{1}{2\sqrt{G(q)}} \right). \quad (18)$$

The function $G(q)$ is defined as

$$G(q) = \frac{1}{8} \int_{q_0}^q dq q^3 C(q) \int_0^{2\pi} d\phi \left| \frac{E(qv \cos \phi, T_0)}{(1 - v^2) \sigma_0} \right|^2. \quad (19)$$

The correction factor $S(q)$ accounts for the incomplete contact and is given by

$$S(q) = \gamma + (1 - \gamma) P^2(q), \quad (20)$$

with $\gamma \approx 1/2$. Note that $S \rightarrow 1$ as $P \rightarrow 1$, which recovers the exact result for full contact. In fact, (17) becomes exact in the limit of complete contact.

The general theory also includes temperature effects, both the flash temperature and the gradual heating of the sliding rubber block with increasing sliding speed.²² The theory can also be applied to layered materials, e.g., for a rubber film on another solid. For a layered material consisting of two materials 0 and 1, with 0 forming a film of thickness d on top of a semi-infinite 1, one needs to replace $E^*(\omega)$ in (17)–(19) with $E_0^*(\omega)/K(\omega)$, where

$$K = \frac{1 + 4mqde^{-2qd} - mne^{-4qd}}{1 - (m + n + 4mq^2 d^2)e^{-2qd} + mne^{-4qd}}$$

and

$$m = \frac{\mu_0/\mu_1 - 1}{\mu_0/\mu_1 + 3 - 4v_0}, \quad n = 1 - \frac{4(1 - v_0)}{1 + (\mu_0/\mu_1)(3 - 4v_1)}.$$

The shear modulus $\mu_0 = E_0/2(1 + v_0)$ and similarly for μ_1 . Note that K is dimensionless and depends only on qd , v_0 , v_1 , and E_0/E_1 . Note also that $K = 1$ for $d = \infty$, and $K = E_0^*/E_1^*$ for $d = 0$.

As an example, consider a thin rubber film (thickness d) on a rigid substrate sliding on a concrete surface. Figure 16 shows the viscoelastic contribution μ_{visc} to the sliding friction coefficient, and the relative contact area A/A_0 as a function of the sliding speed for the film thicknesses $d = 0.1, 0.3$, and 1 mm and for $d = \infty$. The concrete

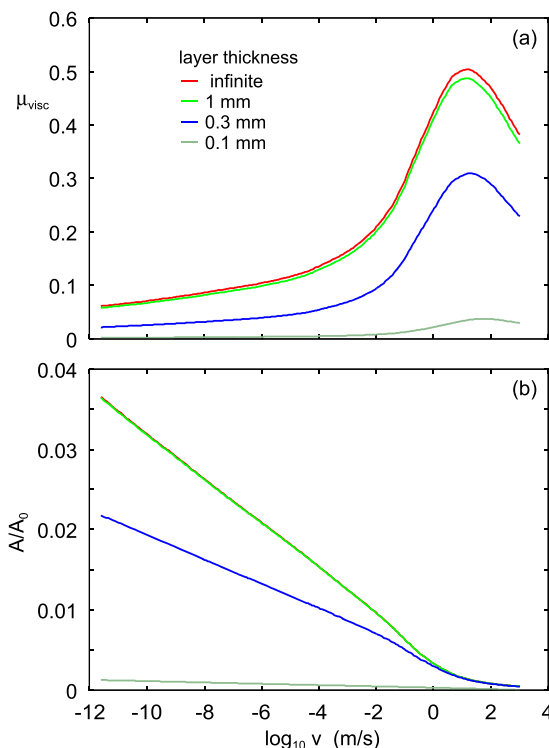


FIG. 16. Viscoelastic contribution μ_{visc} to the sliding friction coefficient and the relative contact area A/A_0 , as functions of the sliding speed for a flat rigid solid covered with a tread rubber film of thickness d sliding on a concrete surface. The concrete surface is relatively smooth, with an rms roughness of 0.05 mm, and the calculation includes all roughness components down to a cutoff wavenumber of $\approx 10^6 \text{ m}^{-1}$, where the rms slope is ≈ 1 . Frictional heating is not included in the calculation. The nominal contact pressure is $\sigma_0 = 0.2 \text{ MPa}$.

surface is relatively smooth, with an rms roughness $h_{\text{rms}} \approx 0.05$ mm, which is considerably smaller than all the film thicknesses used. However, h_{rms} does not determine the depth of the deformation field into the rubber; this is determined by the width of the asperity contact regions. The surface used has a small-wavenumber cutoff $q_0 \approx 615 \text{ m}^{-1}$, corresponding to a wavelength $\lambda_0 = 2\pi/q_0 \approx 10$ mm, and the width of the largest asperity contact regions will be some fraction of this. This explains why the friction drops already for $d < 1$ mm.

2. Adhesive contribution

We now consider the contribution to the friction force arising from the area of real contact A . In sliding contact, rubber molecules interact with substrate atoms, as illustrated in Figs. 10(a)–10(c).

For this case, the theory in Ref. 21 (also see the Appendix) predicts that the frictional shear stress exhibits a Gaussian-like dependence on the logarithm of the sliding speed. The curve has a full width at half maximum of ~ 4 (or more) frequency decades, and, at room temperature, is centered around a characteristic sliding speed typically on the order of $v^* \approx 1$ cm/s.

In Ref. 27, we found that using the following empirical shear stress law $\sigma_f(v, T)$, which is similar to the theoretical prediction,²¹ resulted in good agreement between theoretical results and experimental measurements:

$$\sigma_f(v) \approx \sigma_{f0} \exp\left(-c\left[\log_{10}\left(\frac{v}{v^*}\right)\right]^2\right), \quad (21)$$

where for passenger car tread rubber, typically $c \approx 0.17$, $\sigma_{f0} \approx 4$ –8 MPa, and the reference sliding speed is $v^* \approx 1$ cm/s at $T = 20^\circ\text{C}$.

The full width at half maximum of $\sigma_f(v)$ as a function of $\log_{10} v$ is given by $\Gamma = 2(\ln 2/c)^{1/2} \approx 4$.

The master curve described by (21) is valid at the reference temperature $T_{\text{ref}} = 20^\circ\text{C}$. The frictional shear stress at other temperatures can be obtained by replacing v with va'_T , where a'_T is a shift factor derived during the construction of the friction master curve. This shift factor is often close to the bulk viscoelastic shift factor a_T , but in some cases, we have found that an Arrhenius-type expression provides a more accurate description,

$$\ln a'_T \approx \frac{\epsilon}{k_B} \left(\frac{1}{T} - \frac{1}{T_{\text{ref}}} \right), \quad (22)$$

where $\epsilon \approx 1$ eV is an activation energy and k_B is the Boltzmann constant. This temperature dependence is slightly weaker than that predicted by the bulk viscoelastic shift factor and may reflect a higher mobility of the polymer chains at the rubber surface compared with the bulk.

The contribution of the area of real contact to rubber friction depends sensitively on the presence of contamination particles and fluids. On a wet road surface, at sufficiently high sliding or rolling speeds, the surfaces in the apparent contact regions can become separated by a thin fluid film. In such cases, viscoelastic deformation of the rubber becomes the dominant contribution to friction.

3. Role of frictional heating

Temperature has a strong influence on rubber sliding friction, especially when the sliding speed varies with time (see Sec. V C). The

temperature distribution within a rubber block sliding on a rough surface is highly inhomogeneous.

This distribution can be decomposed into two components: the *background* temperature $T_0(\mathbf{x}, t)$, which varies slowly in space and time, and the *flash* temperature $\Delta T(\mathbf{x}, t)$, which is nonzero only in the vicinity of asperity contact regions, where frictional energy dissipation occurs.

The background temperature $T_0(\mathbf{x}, t)$ depends on the *sliding history* and on external conditions. For example, in the case of a tire, $T_0(\mathbf{x}, t)$ is dependent on the ambient air and road temperatures, whether the road is dry or wet, the rolling resistance of the tire, and past driving conditions such as braking, cornering, and speed.

In contrast, the flash temperature ΔT is largely independent of external conditions and sliding history. It arises from the energy dissipation in the asperity contact regions.

At low sliding speeds v (typically less than 1–10 mm/s), thermal diffusion significantly reduces the flash temperature. However, at higher sliding speeds, the temperature increase ΔT is large and shifts both the viscoelastic and adhesive friction contributions toward higher sliding speeds.

As with friction force, both the frictional shear stress acting in the area of real contact (adhesive contribution) and the viscoelastic deformations contribute to frictional heating. We denote the corresponding friction coefficients as μ_{cont} and μ_{visc} , respectively. The adhesive contribution results in energy dissipation within a nanometer-thick surface layer.

To estimate the flash temperature, one must determine the size of the contact regions, which depends on the magnification. At low magnification, only the contact with the largest road asperities is visible, as illustrated in Fig. 17(a). These are referred to as the macroasperity contact regions. As the magnification increases, smaller asperities located on top of the larger ones become visible and contact is observed in these regions as well [Fig. 17(b)]. The true area of real contact is revealed only at the highest magnification, as shown in Fig. 17(c).

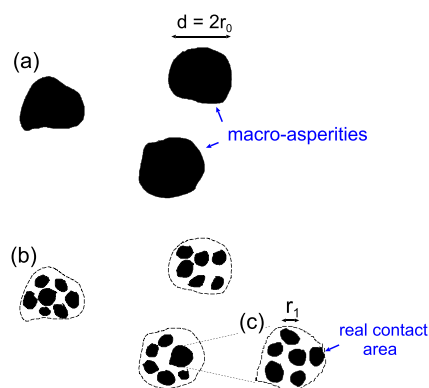


FIG. 17. Multiscale nature of contact between two solids as a function of magnification. (a) At low magnification, contact appears to occur uniformly across macroasperities. (b) At intermediate magnification, smaller non-contact regions are revealed within the nominal contact patches. (c) At high magnification, even the apparent contact areas are found to consist of discrete microcontacts separated by non-contact gaps, illustrating the true real contact area.

In Ref. 25, a theory was developed that predicts the size of contact regions as a function of magnification. However, unless the sliding speed is very high, a simpler approach can be used. In this approach, the heat generated in the contact regions within the macroasperity contacts is assumed to spread laterally, and the resulting energy dissipation is expressed as a function of depth into the rubber. Using this heat distribution as a source term, the temperature increase in the rubber at the asperity contacts can be calculated by solving the heat diffusion equation, which includes heat exchange with the road surface. This method was introduced in Ref. 22, and here, we only provide a qualitative overview.

In the following analysis, we assume that the macroasperities have a diameter of $d \approx 1$ mm, which is typical for rubber in contact with rough road surfaces.

First, consider the adhesive contribution. The frictional energy dissipated per unit area and unit time in the asperity contact regions is given by $\dot{q} = \mu_{\text{cont}} \sigma v$, where $\sigma = F_N/A$ is the normal contact stress. The asperity contact time is $\tau = d/v$, and the energy dissipated per unit volume is $Q = \dot{q}\tau/h = \dot{q}d/hv = \mu_{\text{cont}} \sigma d/h$, where $h \approx 1$ nm is the thickness of the region at the surface where the adhesive frictional energy is deposited.

At very high sliding speeds, thermal diffusion is negligible, and the temperature increase in the thin surface layer of thickness h is given by

$$\Delta T = \frac{Q}{\rho C_V} = \frac{\dot{q}d}{h v \rho C_V} = \frac{\dot{q}d}{\kappa_{\text{th}} v h}, \quad (23)$$

where $D = \kappa_{\text{th}}/\rho C_V$ is the thermal diffusivity and ρ , C_V , and κ_{th} are the rubber mass density, heat capacity, and thermal conductivity, respectively. For rubber with carbon fillers, typical values are $\rho \approx 1500$ kg/m³, $C_V \approx 1500$ J/(kg K), and $\kappa \approx 0.3$ W/(m K), giving a thermal diffusivity of $D \approx 10^{-7}$ m²/s.

For this equation to be valid, the heat must spread only over a distance of order h during the contact time $\tau = d/v$. If the phonon mean free path is of the order of h or smaller, then the thickness of the affected layer can be estimated from the heat diffusion equation and is given by $\approx D/v$. Introducing the dimensionless parameter $J_h = v h/D$, the condition becomes $J_h > 1$, and (23) can be written as

$$\Delta T = \frac{\dot{q}d}{\kappa_{\text{th}} J_h}. \quad (24)$$

In the opposite limit of very small sliding speeds, the flash temperature corresponds to that of a stationary heat source. In this case, for a circular contact region with diameter d , the maximum temperature increases, which occurs at the surface in the center of the contact region, given by:^{47–49}

$$\Delta T = \frac{\dot{q}d}{2\kappa_{\text{th}}}. \quad (25)$$

This result is valid only when the sliding speed is sufficiently low such that the heat diffusion length, approximately D/v , is larger than the lateral size of the contact region, d . Introducing the dimensionless parameter $J_d = v d/D$, the condition becomes $J_d < 1$.

An interpolation formula that is approximately valid across all sliding speeds was presented by Greenwood in Ref. 50 and is extended here to include a J_h^2 term as follows:

$$\Delta T \approx \frac{\dot{q}d}{2\kappa_{\text{th}}} \left(\frac{1}{1 + (\pi/16)J_d + J_h^2/4} \right)^{1/2}. \quad (26)$$

Note that $J_d = J_h^2$ when $v = Dd/h^2$, which, for the adhesive contribution, where $h \approx 1$ nm, corresponds to an extremely high velocity in most cases. For example, if $d = 1$ mm, then $v \approx 10^8$ m/s. Hence, for the adhesive contribution, the J_h^2 term can always be neglected.

In contrast, for the viscoelastic contribution, energy is dissipated in a volume element extending a distance $h \approx d$ into the rubber, so that $v = Dd/h^2 \approx D/d \approx 0.1$ mm/s for $d = 1$ mm. Thus, for the viscoelastic case, the J_h^2 term cannot be neglected. However, in this case, more detailed analysis (as presented in Ref. 22) is required since viscoelastic energy dissipation decays rapidly with depth into the rubber.

Since $J_d = 1$ corresponds to the sliding speed $v = D/d \approx 10^{-4}$ m/s (using $d = 1$ mm), and since for this velocity $\Delta T \approx \dot{q}d/2\kappa_{\text{th}} = \mu\sigma v d/2\kappa_{\text{th}} \approx 0.1$ –1 K for $\sigma \approx 1$ –10 MPa (see in the following), we conclude that frictional heating is negligible at sliding speeds below $v \approx 10^{-4}$ m/s.

For higher sliding speeds, the maximum temperature increase due to adhesive energy dissipation is approximately given by

$$\Delta T \approx \frac{\dot{q}d}{2\kappa_{\text{th}}} (16/\pi J_d)^{1/2} \approx 1.13 \frac{\mu_{\text{cont}} \sigma}{\kappa_{\text{th}}} (v D d)^{1/2}. \quad (27)$$

To estimate the temperature increase, we use the contact mechanics result [see (11)] $\sigma \approx F_N/A \approx \xi |E(\omega)|/2$, where $\omega \approx v/d$. For typical sliding speeds (including strain softening), $|E(\omega)| \approx 2 \times 10^7$ Pa and using the typical rms-slope of $\xi \approx 1$ gives $\sigma \approx 10$ MPa. Using $\sigma \approx 10$ MPa in Fig. 18 shows the flash temperature predicted from (27), assuming typical thermal parameters. The calculation neglects heat exchange with the substrate and the contribution from viscoelastic energy dissipation. It also assumes that the contact area and the friction coefficient remain constant, although in reality both depend on the sliding speed and temperature. The predicted temperature rise can be significant, e.g., $\approx 40^\circ\text{C}$ at $v \approx 1$ m/s, which would shift the viscoelastic mastercurve toward higher frequencies (and thus higher velocities). This is an important effect in rubber sliding friction and is included in the full theory.²²

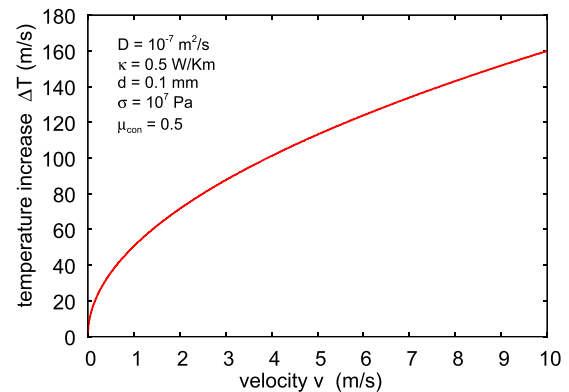


FIG. 18. Flash temperature as a function of sliding speed, estimated using (27) with typical parameter values: $D = 10^{-7}$ m²/s, $\kappa_{\text{th}} = 0.5$ W/(K m), $d = 0.1$ mm, $\sigma = 10^7$ Pa, and $\mu_{\text{cont}} = 0.5$.

Figure 19 shows the influence of the flash temperature on the sliding friction coefficient and the area of real contact. The green and red lines in panel (a) represent the calculated viscoelastic contribution to the friction, μ_{visc} , and the total friction coefficient, $\mu = \mu_{\text{visc}} + \mu_{\text{cont}}$. The solid lines correspond to calculations with the flash temperature included and the dashed lines correspond to those without it. Figure 19(b) shows the relative contact area A/A_0 , with and without the flash temperature. Note that the flash temperature makes the rubber elastically softer, which results in an increase in the contact area with increasing sliding speed for $v > 1$ m/s.

The equations for the sliding friction force and frictional heating, incorporating both viscoelastic and adhesive energy dissipation, were presented in Ref. 22. The theory in Ref. 22 predicts the non-uniform temperature distribution and the corresponding friction force during accelerated motion. As an example, Fig. 20 shows the temperature profile as a function of depth into the rubber within a macroasperity contact region for several sliding speeds. The results correspond to a total sliding distance of 3 cm on an asphalt road surface.

In the simulation, the rubber block has a length $L = 5$ cm in the sliding direction, and the macroasperity contact region is located 3 cm from the leading edge of the block. The initial temperatures of both the rubber and the road are 20 °C. Note that for the two

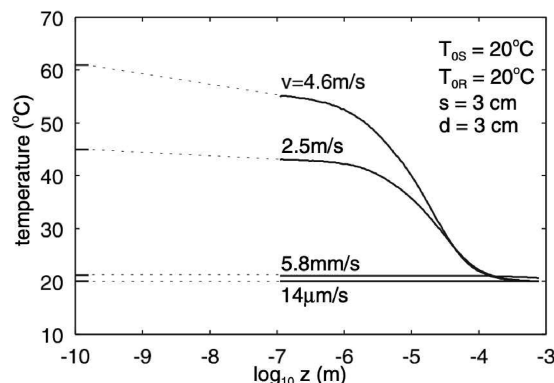


FIG. 20. Temperature as a function of the distance into the rubber within a macroasperity contact region, for different sliding speeds. The results are for a rubber block sliding on an asphalt road surface; the figure is adapted from Ref. 22.

higher sliding speeds ($v = 4.6$ and 2.5 m/s), the maximum temperature occurs at the rubber surface, while for the lower sliding speeds, the temperature remains nearly constant throughout the depth.

4. Open problems

Analysis of experimental data indicates that, for tire tread rubber sliding on road surfaces, most of the viscoelastic contribution to friction arises from interactions with asperities down to a cutoff length $\lambda_{\text{min}} = 2\pi/q_1$, which is typically on the order of micrometers.^{16,24,37} However, the origin and precise value of this short-wavelength cutoff remain poorly understood.

Understanding of this viscoelastic cutoff length is essential for any theory of rubber friction, whether analytical or numerical. This requirement is not only limited to Persson's contact mechanics theory⁶ but also applies to numerical methods such as the finite element method (FEM)^{51,52} or boundary element (BE) methods.^{46,53}

The physical origin of λ_{min} depends on the nature of the surface roughness. For relatively smooth surfaces, viscoelastic contributions may arise from roughness features down to atomic dimensions. On such surfaces, adhesion also becomes significant and influences the viscoelastic component of friction.^{54,55}

Real rubber materials often contain mobile molecules, such as waxes, which diffuse to the surface and form a thin film. This film thickness may act as an effective cutoff length, at least for short sliding distances before being sheared off.^{24,56} On very rough surfaces, the high stresses in the asperity contact regions can lead to the formation of a thin surface layer with properties different from those of the bulk rubber, effectively defining the cutoff length. For instance, formation of micrometer-scale cracks,²⁵ or breaking of molecular chains,⁵⁷ may occur in a thin layer at the rubber surface during the run-in process.

In our analysis of rubber friction on road surfaces, we have found that including surface roughness down to length scales where the cumulative rms slope ξ is of order 1 yields results in good agreement with experimental data. This typically corresponds to a cutoff wavenumber of $q_1 = 2\pi/\lambda_{\text{min}} \approx 10^6 \text{ m}^{-1}$. For a further discussion of the large-wavenumber cutoff, see Ref. 58.

It is important to note that all analytical theories of rubber friction are based on the small-slope approximation. Therefore,

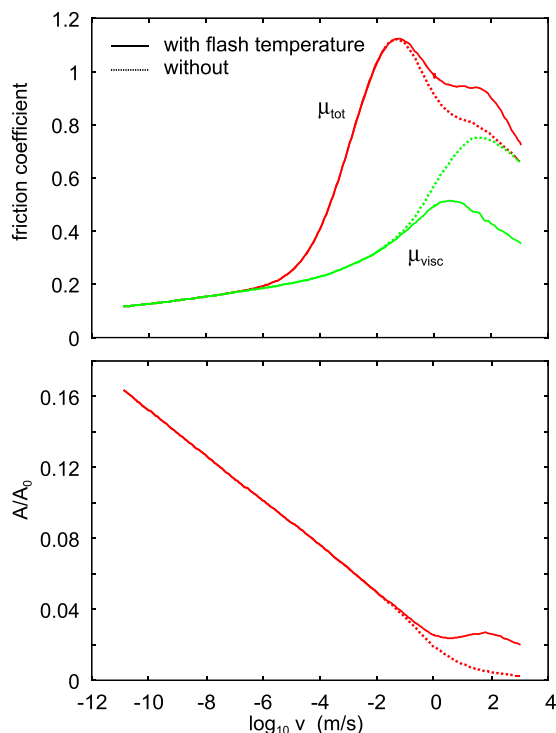


FIG. 19. Influence of the flash temperature on (a) the sliding friction coefficient μ and (b) the relative contact area A/A_0 . The green and red lines in panel (a) represent the viscoelastic contribution to the friction, μ_{visc} , and the total friction coefficient, $\mu = \mu_{\text{visc}} + \mu_{\text{cont}}$. The solid lines correspond to calculations with the flash temperature included and the dashed lines correspond to those without it. The results are for a tire tread compound on a concrete surface at a background temperature of 30 °C and a nominal contact pressure of 0.3 MPa.

including roughness beyond the $\xi \approx 1$ threshold lacks theoretical justification. Moreover, there is currently no surface topography measurement technique that can reliably capture height profiles at resolutions where $\xi > 1$.

Even if the exact value of $\lambda_{\min}$ is not known, the total friction coefficient can still be expressed as in (12), where the contact area contribution is written as $\mu_{\text{cont}} = \sigma_{\text{eff}} A / \sigma_0 A_0$. Here, A is the real contact area, calculated by including all surface roughness down to the assumed cutoff length. In this case, the shear stress is an effective shear stress, which could have contributions both from viscoelasticity and from the adhesive interaction in the area of real contact.

Thus, in some studies, A has been chosen as the nominal contact area A_0 , in which case μ_{visc} arises solely from macroscopic deformations of the contact region. This approach was employed in Ref. 59, where a rigid sphere was slid on human skin, a viscoelastic material. In that case, the nominal contact was Hertzian-like and the macroscopic contact area had a distorted circular shape. Under such conditions, all asperity-induced viscoelastic energy dissipation is effectively captured within the definition of σ_{eff} .

Another important issue concerns the role of adhesion. Here, we are not referring to the adhesive interaction between rubber molecules and the substrate within the real contact area, which gives rise to the frictional shear stress σ_f . Instead, we refer to two distinct adhesion-related mechanisms: (i) the increase in real contact area due to adhesion and (ii) the energy dissipation from opening cracks at the edges of asperity contacts, occurring at length scales where adhesion is relevant.

The contribution of adhesion to the enhancement of contact area was investigated by Plagge and Hentschke,⁵⁴ while the contribution from opening cracks was studied by Le Gal *et al.*⁶⁰ and by Carbone *et al.*³⁸ In Refs. 38 and 55, Carbone *et al.* also showed that adhesion increases the contact area. These mechanisms may significantly affect friction in very soft rubber compounds sliding on smooth surfaces. However, for rough substrate surfaces with roughness on many length scales, such as road surfaces, experimental evidence suggests that this form of adhesion is often negligible.

For the relatively stiff rubber compounds used in practical applications such as tires and seals, there is consistent experimental evidence that crack-opening mechanisms contribute little to the overall sliding or rolling friction. This is primarily due to the multi-scale roughness of the surface, which strongly suppresses adhesion.⁶¹ In Sec. VIII, it is shown that for rubber compounds exhibiting strong adhesion on smooth surfaces under dry conditions, but no adhesion in water, display nearly identical sliding friction on rough surfaces under both dry and wet conditions.²⁶ Similar observations have been made by Greenwood and Tabor.¹⁸

The influence of adhesion on the sliding friction for a viscoelastic solid with the simple modulus given by (4b), with $E_0 = 10$ MPa and $E_1/E_0 = 10$, was studied theoretically in Ref. 62. The sliding friction was calculated for a surface with the height profile $h_0 \sin(q_0 x)$, and for a surface with one-dimensional (1D) self-affine roughness, with the ratio between the large and small wavenumber cutoffs $q_1/q_0 = 60$ and the Hurst exponent $H = 0.8$. The interfacial work of adhesion was $\gamma = 0.01$ J/m², as is typical for rubber in contact with many solids (smooth surfaces).

Figure 21 shows the friction coefficient μ divided by the surface rms slope ξ as a function of the dimensionless sliding velocity

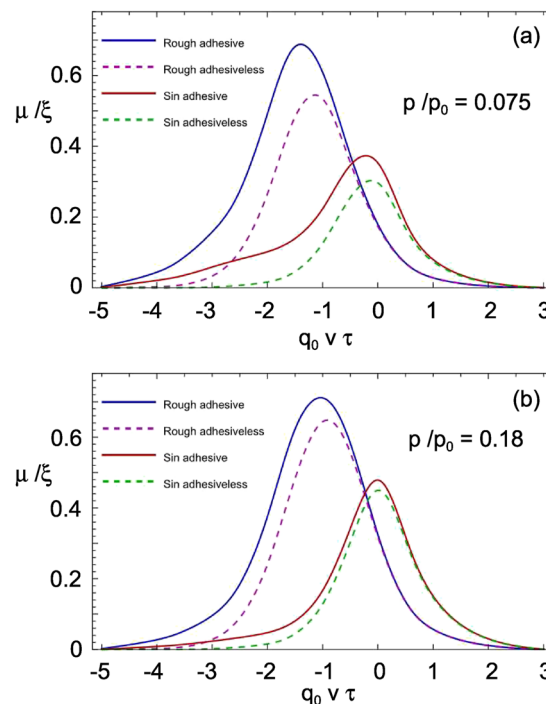


FIG. 21. Friction coefficient, divided by the surface rms slope ξ , as a function of the dimensionless sliding speed $q_0 v \tau$, for a surface with the height profile $h_0 \sin(q_0 x)$ (dashed lines), and for a surface with one-dimensional self-affine roughness, with the ratio between the large and small wavenumber cutoffs $q_1/q_0 = 60$ and the Hurst exponent $H = 0.8$ (solid lines). The viscoelastic modulus is of the form given in (4b), with $E_1/E_0 = 10$ and relaxation time τ . The applied pressure is $p = 0.075p_0$ in panel (a) and $0.18p_0$ in panel (b), where the reference pressure is $p_0 = E_0^* q_0 h_{\max}/2$, and $h_{\max}$ is the height of the highest asperity relative to the average surface plane; the figure is adapted from Ref. 62.

v/v_0 for two different values of the dimensionless pressure p/p_0 . The reference velocity is $v_0 = 1/(q_0 \tau)$, and the reference pressure is $p_0 = E_0^* q_0 h_{\max}/2$, where $h_{\max}$ is the height of the highest asperity relative to the average surface plane.

The results are shown both with and without adhesion. As the pressure increases, the influence of adhesion decreases. It is not clear how these results would be modified for real surfaces with 2D roughness and with larger values of q_1/q_0 .

Another important challenge is how to incorporate the multiscale nature of frictional heating. In the treatment presented in Ref. 22, the frictional energy dissipation occurring in the (small) contact regions within the macroasperity contacts was approximated as a laterally uniform heat source within the macroasperity contact regions. However, this simplification may not always be valid. The problem is inherently complex because, during sliding, a given contact region may be influenced by the thermal “hot tracks” generated by asperity contact regions located ahead of it in the sliding direction (see Fig. 22). In Ref. 22, this effect was accounted for at the level of macroasperity contacts, but a more comprehensive theory should also include such interactions between the smaller contact regions nested within the macroasperity zones.

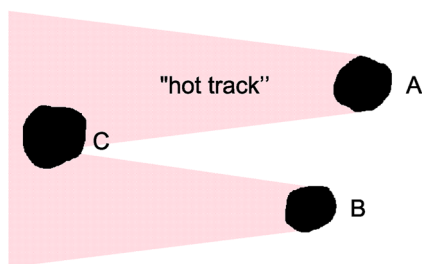


FIG. 22. Asperity contact region C experiences a temperature increase due to the frictional heating generated by asperity contacts A and B located ahead of it in the sliding direction. The "hot tracks" originating from asperity contacts spread laterally with increasing distance from the contact due to thermal diffusion.

D. Comparing the theory with experimental results

Figure 23 shows the measured friction coefficient as a function of the logarithm of sliding speed for three different rubber compounds sliding on a concrete surface. The square symbols represent measurements under dry conditions, while the star symbols correspond to measurements in water. The dashed green lines represent the theoretical viscoelastic contributions, and the solid lines represent the sum of the viscoelastic contribution and the contribution from the area of real contact.

The results in Fig. 23 demonstrate that, in most cases, at low sliding speeds, water is almost completely expelled from the asperity contact regions, resulting in a friction coefficient similar to that observed under dry conditions. However, at much higher sliding speeds, there is insufficient time for the water film to be fully squeezed out, leading to a significant reduction in friction. At very high speeds, hydroplaning may occur and the friction becomes negligible.

While water is generally a poor lubricant on most surfaces, the addition of soap can produce effective lubrication, particularly when the contact pressure is not too high. On hydrophilic surfaces, lubrication may arise from osmotic pressure⁶³ or hydration effects.⁶⁴ To illustrate this, Fig. 24 shows the measured friction coefficient as a function of the logarithm of sliding speed for a rubber compound sliding on an asphalt road surface composed of large stones with relatively flat and smooth upper surfaces. The filled square symbols represent dry conditions; the stars indicate in-water conditions; and the open squares correspond to lubrication by soapy water. The lower green line indicates the calculated viscoelastic contribution, and the upper green line represents the total friction coefficient. Notably, the friction coefficient in soapy water is in close agreement with the theoretical prediction based solely on the viscoelastic contribution.

One important quantity that characterizes rubber is its glass transition temperature, T_g , defined as the temperature at which $\tan \delta$ reaches its maximum for a given reference frequency (we use $\omega = 0.01 \text{ s}^{-1}$). Due to the WLF temperature-frequency relationship, T_g also provides information about the frequency at which $\tan \delta$ is maximal.

To get a large viscoelastic contribution to friction, the perturbation frequencies generated by the road asperities should lie within the transition region between the rubbery and glassy states, i.e., the region where $\tan \delta$ is large. Therefore, T_g is a critical parameter in

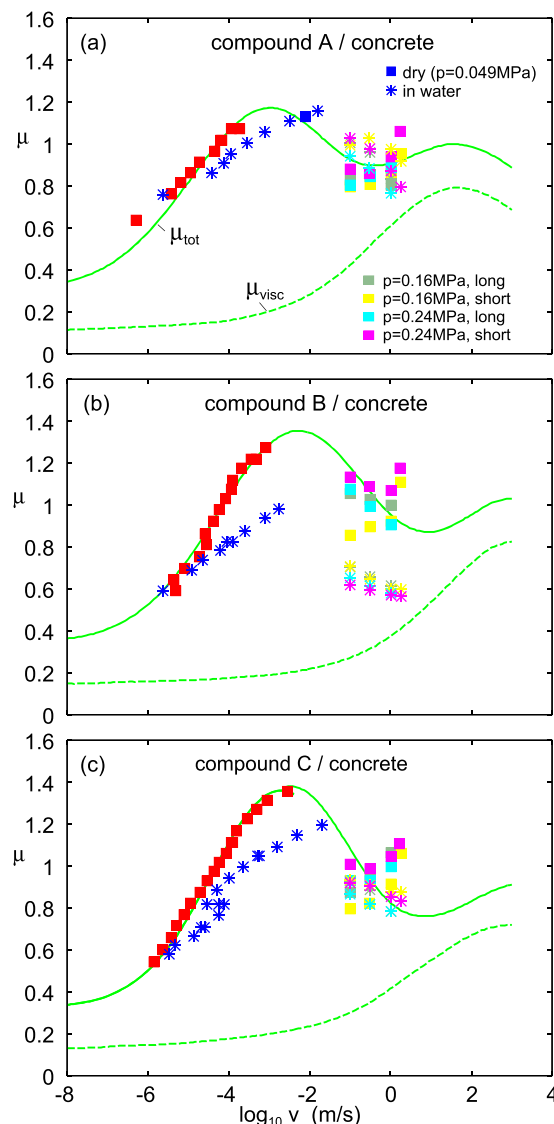


FIG. 23. Measured friction coefficients as a function of the logarithm of sliding speed for three different rubber compounds sliding on a concrete surface. The square symbols correspond to dry conditions and the star symbols to wet conditions (in water). The dashed green lines represent the predicted viscoelastic contribution, while the solid lines represent the sum of the viscoelastic contribution and the contribution from the area of real contact.

the design of tread compounds for tires. Since different types of tires operate at different working temperatures, rubber compounds with different T_g values are used for winter, summer, and racing tires. As a result, the maximum of the friction coefficient as a function of sliding speed at a fixed temperature differs significantly among tire types. The friction master curve typically reaches its maximum at sliding speeds on the order of a few meters per second when the tire is used at its designed operating temperature.

The friction curves shown in Figs. 23 and 24 correspond to summer tires for passenger cars at room temperature, where the

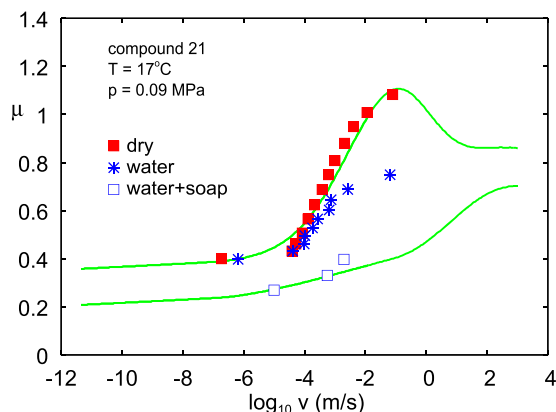


FIG. 24. Measured friction coefficients as a function of the logarithm of sliding speed for a rubber compound sliding on an asphalt road surface composed of large stones with relatively flat and smooth upper surfaces. The filled square symbols represent dry conditions, the stars represent wet conditions (in water), and the open squares correspond to lubrication by soapy water. The lower green line shows the calculated viscoelastic contribution, and the upper green line represents the total friction coefficient.

maximum occurs at ~ 1 cm/s. The typical operating temperature for summer tires is around 50°C , and at such temperatures, the maximum shifts to a few meters per second. In contrast, the operating temperature for racing tires is $\sim 100^\circ\text{C}$ or higher. To ensure the maximum in the friction coefficient falls within a relevant velocity range (~ 1 – 10 m/s), racing tires are formulated with significantly higher T_g than passenger car tires. This is illustrated in Fig. 25 where, for Formula One (F1) racing compound, the maximum friction occurs at extremely low speed at room temperature. Figure 26 schematically illustrates the differences in friction behavior between summer and winter compounds for passenger car tires and a racing compound.

Adhesion and viscoelasticity can have a strong influence on the area of real contact.^{65,66} The stiffening of the elastic properties

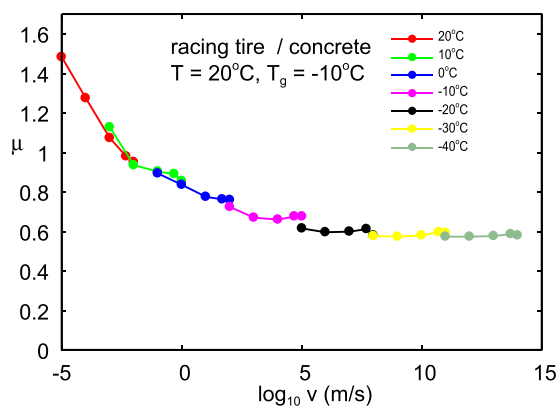


FIG. 25. Sliding friction master curve for the racing compound. The velocity-dependent segments of the friction data were measured at temperatures between -40 and 20°C and were shifted along the velocity axis using the viscoelastic shift factor a_T , obtained from the construction of the viscoelastic master curve $E(\omega, T_0)$. The reference temperature is $T_0 = 20^\circ\text{C}$.

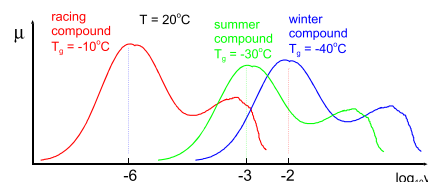


FIG. 26. Friction coefficient at room temperature as a function of the logarithm of the sliding speed for a racing compound (red line), a passenger summer compound (green line), and a winter compound (blue line) (schematic).

of rubber when perturbed at high frequencies results in a drastic decrease in the contact area with increasing sliding speed in a region of intermediate velocities [see Fig. 19(b)]. At very high sliding speeds, the contact area can increase due to frictional heating, which reduces the elastic modulus of the rubber.

At low sliding speeds on smooth surfaces, experimental observations also indicate that adhesion can increase the contact area with increasing sliding speed⁶⁵ (see Fig. 27). This may be due to the increased energy required to propagate opening cracks at higher sliding speeds.

1. Sliding friction on smooth surfaces

Rubber friction for tire tread compounds on smooth surfaces is a complicated topic because mobile components (waxes), which are added to the rubber compounds, diffuse to the rubber surface and are slowly sheared off during sliding, resulting in a friction coefficient that depends sensitively on the sliding distance. Experiments on “simpler” rubber compounds such as poly(dimethylsiloxane) (PDMS) elastomers give more reproducible results.

Grosch¹⁹ studied the friction of acrylonitrile-butadiene rubber on smooth but macroscopically wavy glass. Figure 28 shows the master curve of the friction coefficient at the reference temperature $T_0 = 20^\circ\text{C}$. The master curve was obtained by shifting velocity segments measured at temperatures between -15 and 85°C , using the bulk viscoelastic shift factor a_T . The friction master curve has the general form expected from the Schallamach model¹⁶ of the adhesive contribution to friction, but the width of the curve is larger than that predicted by the Schallamach model and instead agrees better with the prediction of the Persson–Volokitin model.²¹

Vorvolakos and Chaudhury⁶⁷ performed experiments in which spherical cups of PDMS were slid on Si wafers coated with a

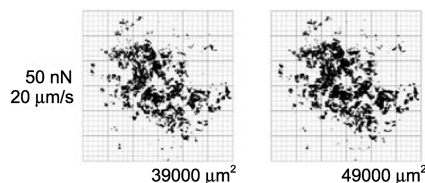


FIG. 27. Two contact footprints between a buna-N nitrile rubber sphere (diameter 4.8 mm, with rms roughness $h_{\text{rms}} = 5.2 \mu\text{m}$) and a borosilicate float glass optical window. Panel (a) shows the contact area before sliding and panel (b) shows that at a sliding speed of $20 \mu\text{m/s}$. The contact area during sliding is larger by a factor of $49/39 \approx 1.26$.

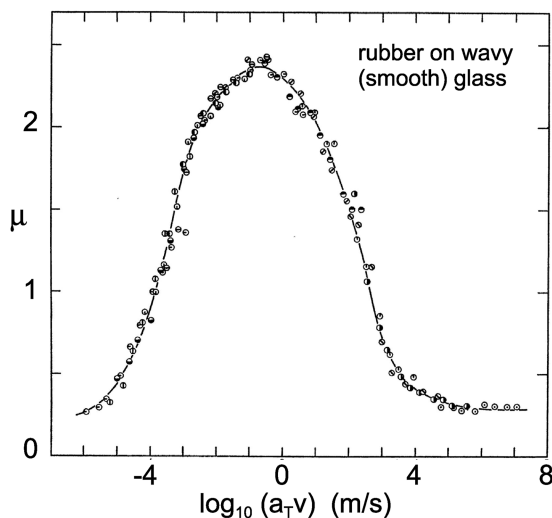


FIG. 28. Master curve of the friction coefficient for acrylonitrile-butadiene rubber on smooth but macroscopically wavy glass. The reference temperature is $T_0 = 20^\circ\text{C}$. The master curve was obtained by shifting velocity segments (velocity range approximately from $1\ \mu\text{m/s}$ to $1\ \text{cm/s}$), measured at temperatures between -15 and 85°C , using the bulk viscoelastic shift factor a_T . When the friction coefficient decreases with increasing sliding speed, stick-slip occurs. For these velocities, the reported friction coefficient corresponds to the peak friction force during the stick-slip oscillations; the figure is adapted from Ref. 19.

hexadecylsiloxane monolayer or on thin films of polystyrene. Elastomers with various molecular weights were used on both surfaces. Figure 29 shows the shear stress as a function of velocity and temperature between PDMS and a Si wafer with a self-assembled monolayer of hexadecylsiloxane. From the temperature-dependent studies, the activation energy of friction on both surfaces was estimated to be $25\ \text{kJ/mol}$, which compares well with values obtained from studies of melt dynamics.

The friction first increases with sliding velocity, reaches a maximum, and then decreases, as expected from the Schallamach model. The maximum of the frictional shear stress σ_f^* was found to be roughly proportional to the square root of the elastic modulus, as expected from the Schallamach model if the size of the stress domains is independent of the modulus [see (A22)]. The sliding velocity v^* at which σ_f is maximum differs for the two substrates, which may reflect different relaxation times τ_0 , as expected since the free energy barrier for attachment depends on the substrate.

Maksuta *et al.*⁶⁸ measured the frictional shear stress for PDMS rubbers with different elastic moduli sliding on three surfaces with similar interfacial chemistry but different roughness. Analysis of the experiments showed that friction on all the surfaces was dominated by adhesive contributions rather than viscoelastic dissipation and that in all cases the rubber made complete contact with the substrate.

Contrary to the expectation that roughness only influences the available contact area for adhesive friction, it was found that roughness caused a shift in the velocity at which the adhesive friction is maximum. The authors suggested that this resulted from roughness-induced vertical oscillations of the rubber surface during sliding. The frictional shear stress scaled with the elastic modulus as

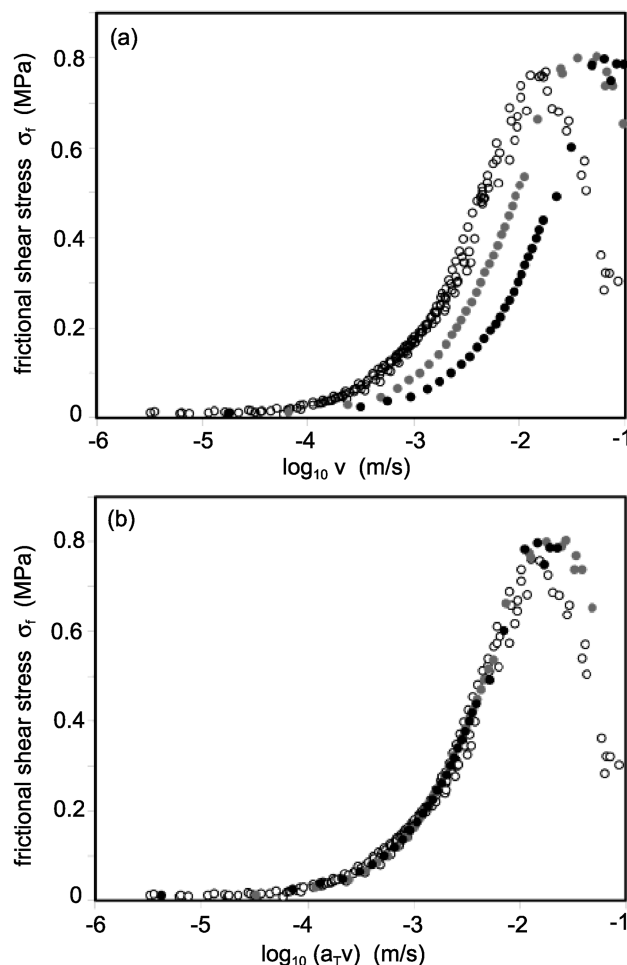


FIG. 29. (a) Shear stress as a function of velocity and temperature for sliding between PDMS ($M = 2.7\ \text{kg/mol}$) and a silicon wafer coated with a hexadecylsiloxane self-assembled monolayer (SAM). The open, gray, and black circles represent data measured at 298, 318, and 348 K, respectively. (b) The temperature-dependent shear stress data shifted to room temperature using an Arrhenius shift factor. The activation energy for sliding between PDMS and the SAM is estimated to be $25\ \text{kJ/mol}$. The open, gray, and black circles again represent data measured at 298, 318, and 348 K, respectively.

$E^{1/2}$ (see Fig. 30), as expected from the Schallamach model for adhesive friction on smooth surfaces.

2. Dependency of the kinetic friction on the shape of the rubber block

For contact between most solids with surface roughness, the area of real contact is proportional to the normal force and independent of the size or shape of the nominal contact area. However, experiments have shown that for rubber, the friction force may depend on the shape of the nominal contact area.⁵ In one study, Hale and Lewis slid rectangular blocks of different shapes [see Fig. 31(a)] on rough surfaces. Figure 31(b) shows that the friction coefficient decreased with increasing length of the blocks in the sliding direction. One mechanism for this is frictional heating. Rubber friction

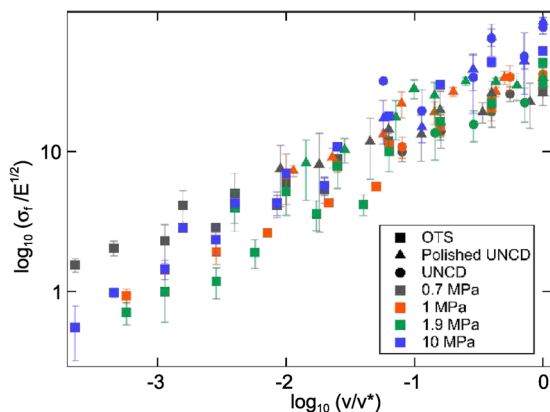


FIG. 30. Frictional shear stress divided by the elastic modulus raised to the $1/2$ power as a function of the normalized sliding speed v/v^* (log-log scale). The data points are for PDMS rubber with different moduli sliding on three different surfaces (see inset); the figure is adapted from Ref. 68.

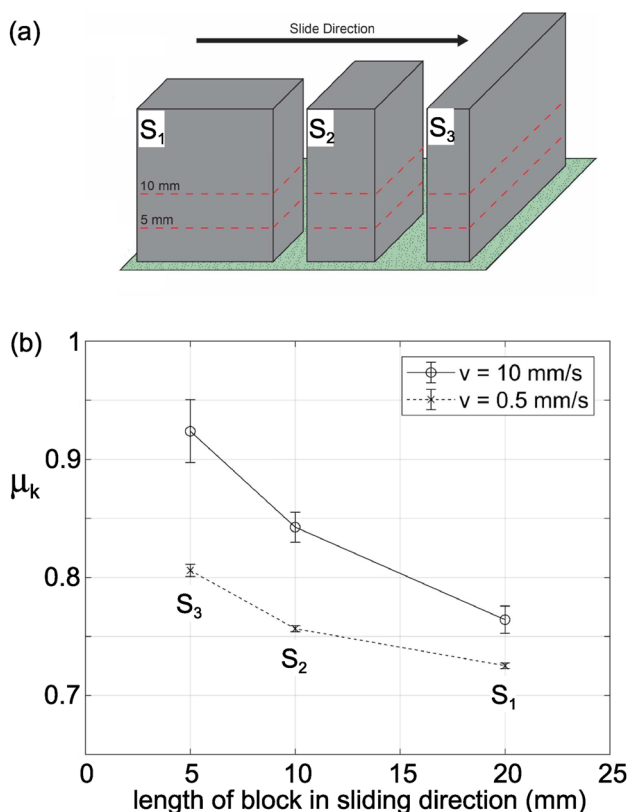


FIG. 31. (a) Three rectangular tread rubber blocks sliding on a rough substrate surface. The rubber blocks have the same bottom surface area. They were clamped at different distances from the bottom surface, but no significant dependence on the clamping position was observed. (b) The kinetic friction coefficient decreases with increasing length of the rubber block in the sliding direction. At the same time, the rubber wear rate decreased from 4.6 to 1.9 to 1.6 mg/Nm for the blocks with the shortest to the longest length in the sliding direction. This strong dependence of wear on the friction coefficient (roughly $\sim \mu_k^{11}$) is consistent with the rubber wear theory presented in Ref. 25; the figure is adapted from Ref. 5.

is highly temperature dependent, and the longer a rubber surface remains in contact in the sliding direction with road asperities, the warmer the asperities become. This, in turn, increases the temperature of the rubber in the asperity contact regions. For rubber sliding on smooth surfaces, frictional heating in the rubber asperity contact regions produces a similar effect and becomes more significant as the duration of contact with the smooth counter surface increases.

At velocities around 10 mm/s, frictional heating is expected to occur and may explain why longer shapes (oriented in the sliding direction) result in lower friction coefficients. However, even at the lower sliding speed $v = 0.5$ mm/s, where frictional heating is negligible, the friction force still decreases with increasing block length along the sliding direction [see Fig. 31(b)]. This cannot be due to frictional heating but may be an “edge” effect. Thus, an investigation of the wear rate and its spatial distribution showed that the length of the leading edge also influences wear, with longer front-edge geometries oriented perpendicular to the sliding direction producing the highest wear mass. This increased wear for wider rubber blocks can be explained by shear deformation of the block, which leads to the highest contact pressure at the leading edge where most of the wear was observed⁵ (also see Fig. 32). The substrate used in the friction study reported in Ref. 5 had very sharp roughness, consisting of a sand and acrylic paint mixture on a wood plate. This configuration resulted in very high wear rates, ranging from 1.6 to 4.6 mg/N m, compared to the much lower wear rates observed in Ref. 25 for SB rubber sliding on a concrete surface, which were between 0.02 and 0.04 mg/N m. These values were ~ 100 times smaller. In most cases, wear contributes negligibly to the friction force. However, in the case of the large wear rates observed in Ref. 5, the wear contribution to friction may no longer be negligible.

E. Dynamical friction

Up to now, we have assumed stationary conditions under which the sliding speed was constant. Here, we describe two dynamical effects that become crucial when the sliding speed changes, especially when slip initiates after some time of stationary contact ($v = 0$). This situation arises in many practical applications. For example, when a tread block of a tire comes into contact with the road surface, it initially does not slip but only deforms elastically. Slip begins when the shear stress becomes higher the static frictional shear stress. Hence, in general, for small tire slip [e.g., during ABS (Anti-lock Braking System) braking], the tread blocks will begin to slip near the exit of the tire-road footprint (see Fig. 33).

The breakloose friction force is defined as the maximum friction force before the onset of steady sliding and is often referred to as the static friction force. It depends on the history of contact between two solids. For instance, if a block slides on a substrate, the resulting temperature increase in the block, particularly near

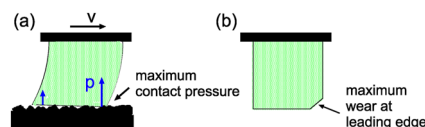


FIG. 32. During sliding, the rubber block deforms such that the maximum contact pressure occurs at the leading edge. Visual inspection after sliding shows that most of the wear has occurred at the leading edge.⁵

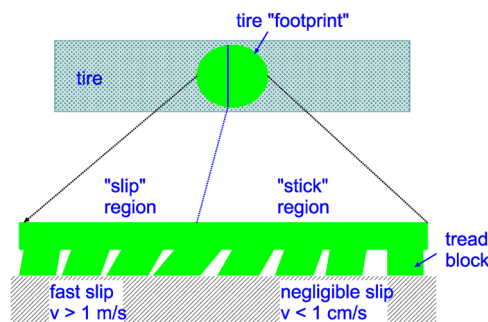


FIG. 33. Deformation of the tire tread blocks as they pass through the tire footprint under finite tire slip, defined as $s = (v_c - v_r)/v_c$, where v_c is the velocity of the car and v_r is the tire rolling velocity. The blocks initially deform elastically without slip, but begin to slip near the exit of the tire-road footprint.

asperity contact regions, may affect the friction force. If the block motion stops, the temperature distribution in the block and in the asperity contact regions of the substrate evolves over time, leading to a breakloose friction force that depends on the contact duration. This effect is particularly significant for rubber materials, where rapid flash temperature changes can cause substantial variations in friction during non-stationary sliding.

Other processes that can lower the interfacial free energy and increase the breakloose friction force include a slow increase in contact area, thermally activated (and hence contact-time-dependent) bond formation at the interface, or structural relaxation of the rubber chain molecules at the contact interface.

1. Elasticity effects and pre-slip

Strengthening of the contact during stationary contact does not always result in a breakloose friction force exceeding the kinetic

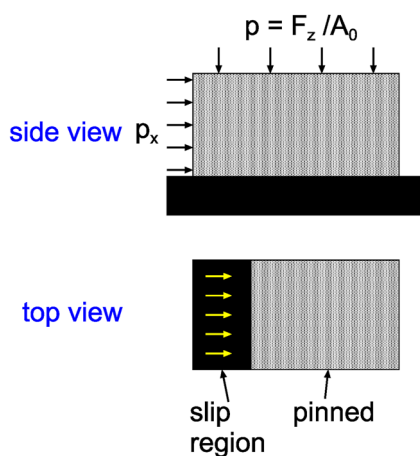


FIG. 34. Slip at the onset of sliding when the driving force is applied on one side of the rubber block. The region where slip occurs gradually expands as the system approaches the onset of global slip, at which point the entire contact area is sliding. The frictional shear stress in regions where the slip displacement $u > \lambda_{asp}$ is equal to the kinetic frictional shear stress σ_k . Consequently, the breakloose friction force is smaller than $F_s = A\sigma_s$ and approaches $F_k = A\sigma_k$ as the elastic modulus decreases.

friction force, especially for elastically soft materials. We now analyze this phenomenon with a particular focus on rubber friction.

Rubber is an elastically soft material, and as a result, the interfacial slip at the onset of sliding between a rubber block and a substrate is not uniform, as illustrated in Fig. 34 for an elastic block where the driving force is applied on the left-hand side. We first show that pre-slip within the contact region causes the breakloose friction force to be lower than the product of the true contact area and the static shear stress. For elastically soft materials, in the absence of the flash temperature effect described above, this can result in a breakloose friction force that is equal to the kinetic friction force.

Consider a “kinetic” frictional shear stress σ_k acting on the contact area A between a block and substrate during sliding. After a stationary contact duration t_s , the shear stress at the new onset of sliding becomes σ_s , which we assume to be larger than σ_k , e.g., due to thermally activated interfacial bond formation. To reduce the shear stress to its kinetic value σ_k , the slip distance must be of the order of the size D of the macroasperity contact regions. If, due to elasticity, different regions at the interface have already slipped a distance larger than $\lambda_c \approx D$ when full slip occurs, then the breakloose friction will be smaller than $\sigma_s A$, which would be the friction force if all contact regions slipped simultaneously, as in the case of a rigid solid.

For the situation in Fig. 34, one can show that the elastic deformations are characterized by a length scale $\lambda_{el} \approx (\sigma_s - \sigma_k)L^2/Eh$, where E is Young’s modulus, h is the block height, and L is its length in the sliding direction. If $\lambda_{el} > \lambda_c$, the breakloose friction force is smaller than $\sigma_s A$, and if $\lambda_{el} \gg \lambda_c$, it may vanish.

To illustrate this, Fig. 35 shows the ratio between the tangential (friction) force F_x and the normal force F_N as a function of sliding distance during oscillatory motion of rubber blocks made from a passenger car compound sliding on concrete at $T = 20^\circ\text{C}$. The blocks first move forward at $v = 3\text{ mm/s}$ for 20 cm and then backward at the same velocity, returning to the starting point. This motion is repeated four times. Note that the breakloose friction force

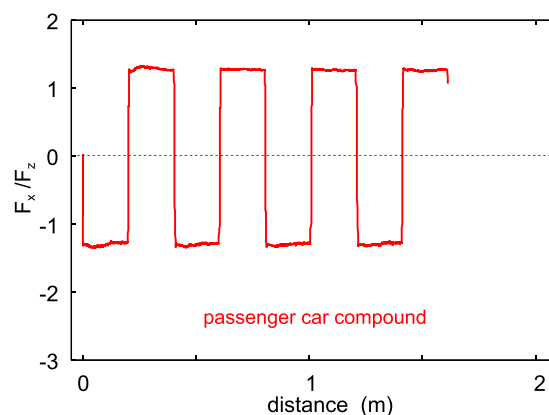


FIG. 35. Ratio of tangential (friction) force F_x to normal force F_N as a function of sliding distance during oscillatory movement of rubber blocks. Data correspond to a passenger car tread compound sliding on a concrete surface at $T = 20^\circ\text{C}$. The rubber blocks first move forward at $v = 3\text{ mm/s}$ for 20 cm, and then backward at the same velocity, returning to the starting point. This motion is repeated four times. The rubber blocks are 0.5 cm high with lateral dimensions of $4 \times 4\text{ cm}^2$.

equals the kinetic friction force at the start of sliding and also at the turning points where the slip velocity passes through $v = 0$.

2. Pre-slip on smooth surfaces

The study presented above involved rubber sliding on very rough substrate surfaces, but similar results have been observed for rubber in contact with smooth glass plates. In a very interesting study by Tuononen,⁶⁹ a rectangular block of a tire tread compound was glued to a rigid plate and pressed against a smooth glass surface, both in the dry state and under water. In the dry state, when a tangential force was applied, slip initiated at the trailing-edge corners and propagated forward toward the leading edge of the block [see Fig. 36(a), where the green area indicates the no-slip region]. Slip began at the trailing edge because the tangential loading introduces a moment that reduces the contact pressure at the trailing edge. As a result, at the onset of global slip, where the entire interface begins to slip, the slip distance is almost everywhere longer than λ_c , and the breakloose friction force is nearly equal to the kinetic friction force.

On the wet glass surface, the friction is much smaller. At the onset of sliding, the motion occurs nearly simultaneously across the entire interface. As a result, slip at the interface is more uniform [see Fig. 36(b)] and the breakloose friction force becomes larger than the kinetic friction force.

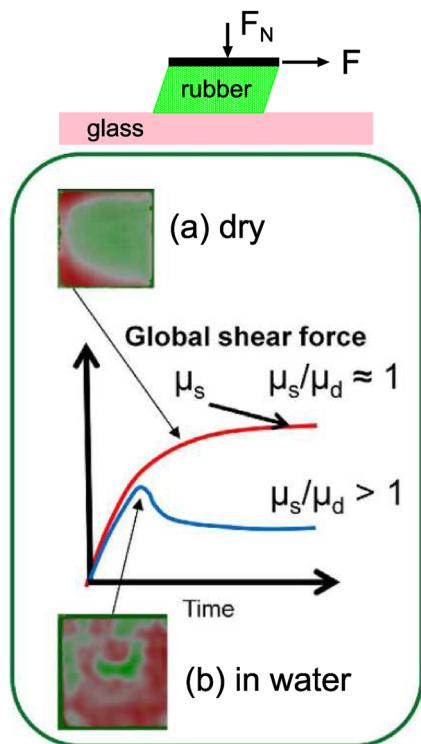


FIG. 36. (a) In the dry state, slip initiates at the trailing edge and propagates forward, resulting in a breakloose friction force that is approximately equal to the kinetic friction force. (b) In the presence of water, slip initiates more uniformly along the interface, leading to a breakloose friction force that is larger than the kinetic friction force. In the images, the green areas indicate stick regions and the red areas indicate slip regions; the figure is adapted from Ref. 69.

In Ref. 69, Schallamach waves were not observed for rectangular rubber blocks. These waves are usually studied using hemispherical sliders, which may be necessary to create suitable conditions for buckling. In addition, tread compounds (which contain fillers) are elastically stiffer than the rubber types typically used in studies of Schallamach waves, which may further suppress buckling.

3. Effective breakloose force

Even when $\lambda_{el} \gg \lambda_c$, the breakloose friction may exceed the kinetic friction. We will now consider two *kinetic effects*, which result in effective breakloose friction forces larger than the kinetic friction force.

The first kinetic effect becomes important if the kinetic friction force $F_k(v)$ has a maximum at a low sliding speed, say at $v = v_c$. If the driving speed v is larger than v_c , then at the onset of slip, the friction force will first increase rapidly to approximately $F_k(v_c)$, and then drop quickly to $F_k(v)$, resulting in an effective breakloose friction force greater than the kinetic friction force.

The second kinetic effect is caused by the flash temperature, which requires a finite slip distance (typically ~ 1 mm) to fully develop. Since the friction force tends to increase with increasing sliding speed at low slip velocities and decrease with increasing temperature, the friction rises to a maximum before the flash temperature is fully developed.

Figure 37(a) shows the friction coefficient as a function of sliding speed acting on an elastic block, assuming that the $\mu_k(v)$ curve has a maximum at $v = v_c$, with $\mu_k(v_c) = \mu_c$. If the block is stationary for $t < 0$ and the upper surface of the block moves at velocity v for $t > 0$, then the friction force F_x will rapidly and monotonically increase to $\mu_k(v)F_N$ if $v < v_c$ [case (b)]. However, if $v > v_c$, the friction force will initially rise rapidly to approximately $\mu_c F_N$ and

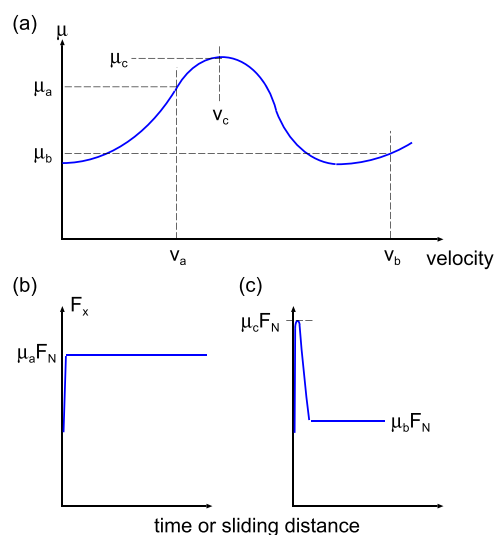


FIG. 37. (a) Friction coefficient as a function of sliding speed. The $\mu_k(v)$ curve reaches a maximum at $v = v_c$, where $\mu_k(v_c) = \mu_c$. (b) If a rubber block is initially stationary for $t < 0$ and its upper surface begins to move at velocity v for $t > 0$, then the friction force F_x increases rapidly and monotonically to $\mu_k(v)F_N$ if $v < v_c$. (c) If $v > v_c$, the friction force first increases rapidly to approximately $\mu_c F_N$ and then decreases quickly to $F_k = \mu_k(v)F_N$.

then decrease rapidly to $F_k = \mu_k(v)F_N$ [case (c)]. For a rigid block, these transitions would occur instantaneously with negligible displacement. For an elastic block, the transitions are continuous, but fast enough that the peak force appears as an effective breakloose (or static) friction force. In the first case, the effective breakloose friction force equals the kinetic friction force, while in the second case, it approximately equals $\mu_c F_N$.

As an illustration of this kinetic origin of breakloose friction, we consider the sliding of a racing tire compound under the same test conditions as in Fig. 35. Note that for rubber friction on rough surfaces at room temperature, the friction force for passenger car tires typically reaches a maximum at a sliding speed of ~ 10 mm/s (higher than the 3 mm/s used in the experiment), while for racing tires, the maximum occurs at much lower speeds, typically around 1 $\mu\text{m/s}$ (see Fig. 25).

Figure 38 shows the ratio F_x/F_N at $T = 20^\circ\text{C}$ as a function of sliding distance for a racing compound with a glass transition temperature $T_g = -12^\circ\text{C}$. The rubber block first moves forward at $v = 3$ mm/s for 20 cm, then reverses direction at the same velocity, repeating this cycle five times. Notably, large friction peaks occur when the velocity changes sign, as the bottom surface of the block undergoes a sharp (but continuous) reversal from $v = 3$ to -3 mm/s. This is expected, since for the racing compound, the maximum friction occurs at $v \approx 1$ $\mu\text{m/s}$.

For the racing compound, the breakloose friction is larger than the friction peak observed during velocity reversal. This may be attributed to a larger real contact area at the onset of sliding compared to that during velocity reversal, where the system only briefly remains near $v \approx 0$.

Note that the breakloose friction equals exactly $\mu_c F_N$ only if the slip velocity at the interface is uniform when the average velocity reaches v_c . This is not always the case, as different surface regions may begin to slip at different times. In particular, if v_c is very small, some parts of the interface may move faster than v_c when the average interface speed equals v_c . In this case, the effective breakloose

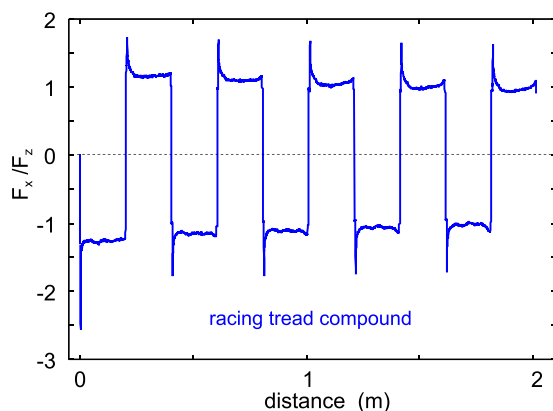


FIG. 38. Ratio between the tangential (friction) force F_x and the normal force F_N as a function of sliding distance during oscillatory motion of rubber blocks made from a racing compound, sliding on concrete at $T = 20^\circ\text{C}$. The rubber blocks first move forward at $v = 3$ mm/s for 20 cm, then backward at the same velocity, returning to the starting point. This motion is repeated five times. The blocks are 0.5 cm high, with lateral dimensions of 4×4 cm².

friction force will be less than $\mu_c F_N$ but still greater than the kinetic friction force F_k .

An increase in the contact area with stationary contact duration is also expected for the passenger car compound. However, this effect is not evident in the friction curve in Fig. 35, for the following reason.

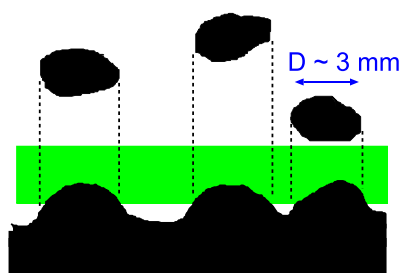
For the passenger car rubber compound, the frictional shear stress increases with sliding speed. When the velocity reaches the driving speed of 3 mm/s, the bottom surface of the block has already slipped far enough to renew the asperity contact regions, achieving the steady-state sliding contact area. At this speed, the flash temperature effect is negligible, and the rubber temperature remains constant.

Many other experimental and theoretical studies have shown that the breakloose (or static) friction force depends on the geometry of the contacting bodies and on how the driving force is applied. As an example, Zhang and Ciavarella⁷⁰ (see also Refs. 71 and 72) used the Cattaneo–Mindlin friction model with Griffith friction (where the effect of roughness is included in a fracture energy linearly dependent on the local pressure), and predicted that at low normal loads, the breakloose friction coefficient depends on the geometry of the contact.

4. Flash temperature effect

A second effect that can generate an effective breakloose friction force is the flash temperature effect.⁷³ A tire tread block makes contact with road asperities at many length scales. The macroasperity contact regions are those observed at low magnification, where the surface roughness on top of the large stone particles cannot be observed (black area in Fig. 39). The diameter D of the macroasperity contact regions is typically on the order of 0.1–3 mm. To fully develop the flash temperature in the macroasperity contact regions,

rubber in contact with the
biggest road asperities:
macro asperity contact regions



to fully develop the flash
temperature the sliding
distance must be $> D$

FIG. 39. Macroasperity contact region (black area) refers to the contact regions observed at low magnification, where the surface roughness of the large stone particles is not visible. To fully develop the flash temperature in the asperity contact regions, the slip distance must be at least as large as the macroasperity contact region.

the tread blocks must slip at least a distance comparable with the size of these regions.

For slip distances much smaller than the size of the macroasperity contact regions, the friction coefficient is determined by the theory without considering the flash temperature. We denote this branch of the $\mu(v)$ relation as the *cold* branch (blue line in Fig. 40). As the slip distance increases, the flash temperature begins to develop, and for large slip, the friction follows the *hot* branch of the $\mu(v)$ relation (red line in Fig. 40). The tread blocks must slip a distance on the order of the macroasperity contact region before the flash temperature is fully established. The black lines represent the tread block velocities as they pass through the tire footprint at different tire slip levels, as obtained from numerical simulations of tire dynamics.

Figure 41 shows the effective friction coefficient experienced by a tread block as it passes through the tire-road footprint during cornering (cornering angle 20°). The result was obtained from a full tire dynamics simulation. It was found that using the following relation for the friction coefficient in the tire model gave nearly the same tread block motion as observed when the full dynamical friction coefficient was used:

$$\mu(t) = \mu_{\text{cold}}(v(t), T_0)e^{-s(t)/s_0} + \mu_{\text{hot}}(v(t), T_0)(1 - e^{-s(t)/s_0}), \quad (28)$$

where $s_0 \approx 0.2D$ and μ_{cold} and μ_{hot} are the upper and lower (steady-state) friction branches, respectively, in Fig. 40.

Including the flash temperature effect described above is crucial in tire dynamics and effectively determines the boundary in the tire footprint between tread blocks that have undergone significant slip and those that have experienced negligible slip.

VI. ROLLING FRICTION FOR A HARD CYLINDER AND SPHERE ON A VISCOELASTIC SOLID

We investigate the friction force acting on a hard cylinder or spherical ball rolling on the flat surface of a viscoelastic solid. The

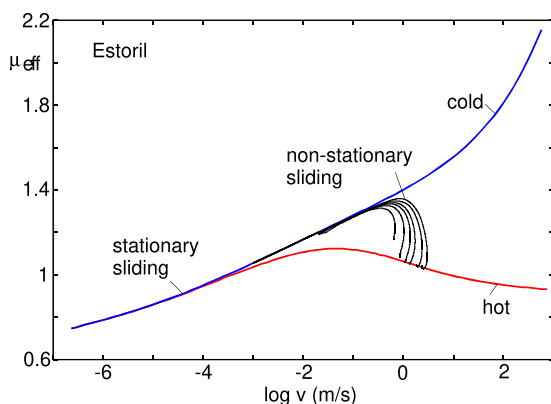


FIG. 40. Friction coefficient as a function of sliding velocity during steady sliding, neglecting the flash temperature (the *cold* branch, blue line) and including the flash temperature (the *hot* branch, red line). The tread blocks must slip a distance comparable with the size of the macroasperity contact region before the flash temperature is fully developed. The black lines represent the friction coefficients obtained from tire dynamics calculations experienced by the tread blocks as they pass through the tire footprint at different tire slip values.

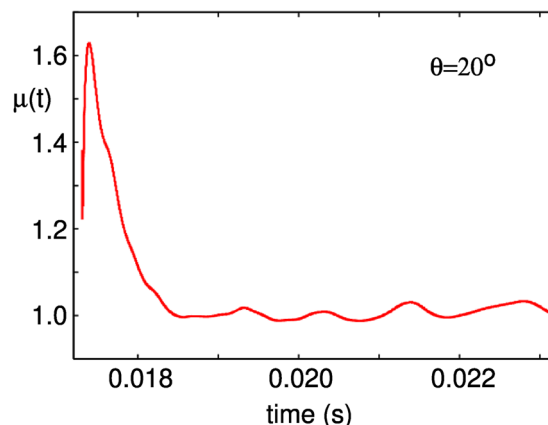


FIG. 41. Effective (time-dependent) friction coefficient experienced by a tread block as it passes through the tire footprint during cornering.

rolling friction coefficient depends nonlinearly on both the normal load and the rolling velocity. The results of rolling friction experiments have often been analyzed using a simple model proposed by Greenwood and Tabor.¹⁸ However, this model includes an unknown factor α , representing the fraction of the input elastic energy dissipated due to the internal friction of the rubber. Here, we review a simple theory for the friction force acting on a hard cylinder or spherical ball rolling on a flat rubber surface.⁷⁴

Greenwood and Tabor¹⁸ have studied the sliding and rolling friction of a hard sphere on a well-lubricated rubber surface. They found that nearly the same friction force is observed during sliding as during rolling, under the condition that the interface is lubricated and the sliding velocity and fluid viscosity allow for the formation of a thin lubrication film. This film must be much thinner than the indentation depth of the sphere but thicker than the amplitude of the surface roughness.¹⁸

Consider a sphere or cylinder rolling with a constant velocity $\mathbf{v}$ on the surface of a viscoelastic solid and assume that adhesion can be neglected. The normal stress acting on the rubber will depend on time,

$$\sigma_z(\mathbf{x}, t) = \sigma_z(\mathbf{x} - \mathbf{v}t).$$

This stress results in time-dependent deformations of the rubber and leads to viscoelastic energy dissipation, which gives rise to the rolling friction force,⁷⁴

$$F_f = \frac{2(2\pi)^2}{v} \int d^2q \frac{\omega}{q} \text{Im} \frac{1}{E_{\text{eff}}(\omega)} |\sigma_z(\mathbf{q})|^2. \quad (29)$$

Here,

$$\sigma_z(\mathbf{q}) = \frac{1}{(2\pi)^2} \int d^2x \sigma_z(\mathbf{x}) e^{-i\mathbf{q} \cdot \mathbf{x}}$$

is the Fourier transform of the stress. The effective viscoelastic modulus of the rubber is defined as $E_{\text{eff}} = E/(1 - \nu^2)$.

Note that (29) is also valid for a sliding object of arbitrary shape, if the frictional shear stress in the area of real contact can be neglected. We will apply (29) in Sec. VII to a sliding triangular

slider. The theory that leads to (29) is highly versatile and can also be applied to layered viscoelastic materials. Here, we first apply it to rolling cylinders and spheres.

A. Cylinder

Consider a hard cylinder (radius R and length $L_y \gg R$) rolling on a viscoelastic solid. The same result is obtained during sliding if one assumes lubricated contact and if the viscous energy dissipation in the lubrication film can be neglected. Assuming that the stress at the interface can be approximated by the Hertz stress field for the cylinder-flat contact, one obtains from (29) the friction coefficient,

$$\mu = \frac{8f_N}{\pi} \int_0^\infty dq_x \operatorname{Im} \frac{1}{E_{\text{eff}}(q_x v)} \frac{1}{(aq_x)^2} J_1^2(q_x a). \quad (30)$$

The half-width of the contact area in Hertz contact theory is

$$a = \left(\frac{4f_N R}{\pi |E_{\text{eff}}|} \right)^{1/2}, \quad (31)$$

where we take E_{eff} to be $|E_{\text{eff}}(\omega)|$ with $\omega = \mathbf{q} \cdot \mathbf{v}$.

Let us now assume the simplest possible viscoelastic modulus characterized by a single relaxation time τ Eq. (4b),

$$\frac{1}{E} = \frac{1}{E_1} + \left(\frac{1}{E_0} - \frac{1}{E_1} \right) \frac{1}{1 - i\omega\tau},$$

where E_1/E_0 is the ratio between the high-frequency and low-frequency modulus.

In Fig. 42, we show the friction coefficient (multiplied by the radius R of the cylinder and divided by the half-width $a_0 = a(v=0)$ of the static contact region) as a function of $v\tau/a_0$, where v is the rolling velocity. We have assumed $E_1/E_0 = 10$. We compare the exact result (blue curve) of Hunter⁷⁵ with the prediction of (30) (green curve). Note that some distance away from the maximum, the agreement between the two curves is nearly perfect. This is expected because these regions correspond to small $\tan \delta$, where (13) should be essentially exact. Near the maximum, a small difference occurs

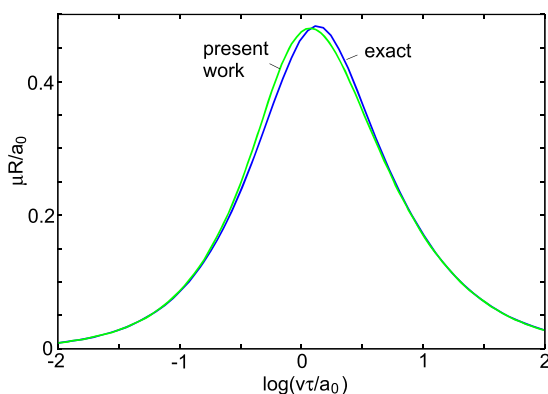


FIG. 42. Friction coefficient (multiplied by the radius R of the cylinder and divided by the half-width a_0 of the static contact region) as a function of $v\tau/a_0$, where v is the rolling velocity and τ is the rubber viscoelastic relaxation time (for $E_1/E_0 = 10$). We compare the exact result (blue curve) of Hunter⁷⁵ with the prediction of (30) (green curve).

between the two curves, but from a practical point of view, this is not important since real rubber exhibits some nonlinearity, making any linear viscoelastic theory only approximately valid.

B. Sphere

Consider now a hard spherical ball (radius R) rolling on a viscoelastic solid. Assuming that the stress at the interface can be approximated by the Hertz stress field for the sphere-flat contact, one obtains from (29) the friction coefficient,

$$\mu = \frac{9F_N}{2\pi^2} \int_0^\infty dq q \int_0^{2\pi} d\phi \cos\phi \operatorname{Im} \frac{1}{E_{\text{eff}}(qv \cos\phi)} \times \frac{1}{(qr_c)^6} [\sin(qr_c) - qr_c \cos(qr_c)]^2. \quad (32)$$

The radius of the contact area in the Hertz contact theory is given by

$$r_c = \left(\frac{3F_N R}{4|E_{\text{eff}}|} \right)^{1/3}, \quad (33)$$

where we take $|E_{\text{eff}}|$ to be $|E_{\text{eff}}(\omega)|$ with $\omega = q_x v$, as the velocity is along the x direction.

Equations (30) and (32) are very general and impose no restrictions on the viscoelastic properties of the rubber or on the sliding velocity v (assuming that v is small enough for frictional heating to be negligible and that $v \ll c_T$).

In Fig. 43, we show the rolling friction coefficient as a function of $v\tau/r_0$ for a sphere and as a function of $v\tau/a_0$ for a cylinder. Here, we use the simple rheological model described in (4b), and a_0 is the half-width of the cylinder-substrate contact area and r_0 is the radius of the contact region for the sphere, both defined in the limit of vanishing rolling velocity. In the calculations, the radius of the sphere and the cylinder are equal, and the applied loads were chosen such that the average contact pressure under stationary conditions was the same.

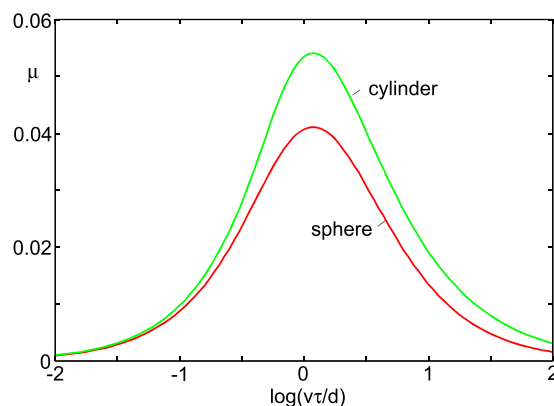


FIG. 43. Friction coefficient as a function of $v\tau/d$, where v is the rolling velocity, τ is the rubber viscoelastic relaxation time, and $d = a_0$ for the cylinder (upper curve) or $d = r_0$ for the sphere (lower curve), rolling on a rubber substrate described by a simple viscoelastic model [see Eq. (4b)].

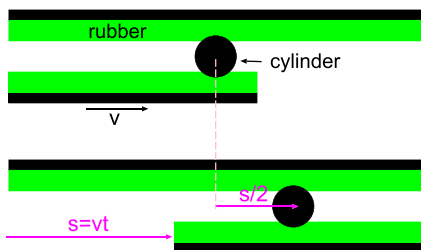


FIG. 44. Rigid cylinder squeezed between two rubber slabs glued to flat steel surfaces. The lower plate moves with velocity v , and the roller exerts a force F on the upper plate. The rolling friction force is $F = F_R$, and the rolling velocity is $v_R = v/2$.

C. Comparison with experiment

Rolling friction experiments⁷⁶ were performed using the setup shown in Fig. 44. A rigid cylinder was squeezed between two rubber slabs glued to flat steel surfaces. The lower plate moves with velocity v , and the roller exerts a force F on the upper plate. The rolling friction force is $F = F_R$, and the rolling velocity is $v_R = v/2$. To vary the temperature, the entire setup was placed inside a deep freezer capable of cooling down to -40°C .

Figure 45 shows the rolling friction master curve for a rubber compound (denoted as compound B). The squares represent the measured rolling friction values at the indicated temperatures, shifted using the bulk viscoelastic shift factor a_T . The solid and dashed green lines are the calculated rolling friction coefficients with and without accounting for strain softening, respectively. The difference between the measured data (square symbols) and the theoretical prediction (solid green line) is likely due mainly to uncertainties in the strain softening measurements and the simplified way strain softening is incorporated into the theory (see Sec. II). Note the significant influence of strain softening on the magnitude of the rolling friction.

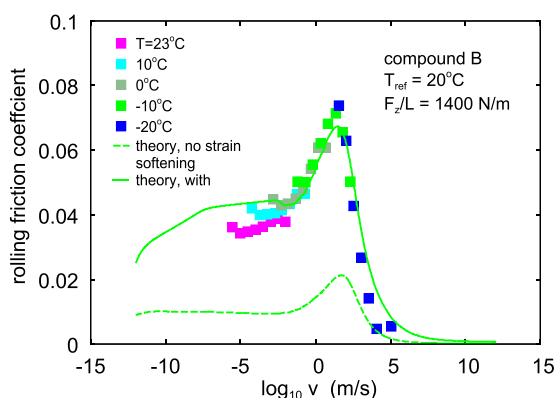


FIG. 45. Rolling friction master curve for a tire tread compound. Two Plexiglas cylinder rollers (each with 10 cm in contact with the rubber) have a diameter of 1 cm. The squares represent measured rolling friction values at the indicated temperatures, shifted using the bulk viscoelastic shift factor a_T . The solid and dashed green lines represent the calculated rolling friction coefficient with and without including strain softening, respectively.

D. Adhesive contribution to rolling friction

The theory presented above for rolling friction assumes negligible adhesion. However, for very smooth and clean surfaces, adhesion is important and for small normal forces it can give the dominant contribution to the rolling friction force.

Consider a rigid sphere or cylinder in contact with a viscoelastic solid with a flat surface. When adhesion occurs, the boundary line between the contact and non-contact surface area can be considered as an adhesive crack tip. When a cylinder (or sphere) rolls on a rubber surface, the rubber comes into contact with the cylinder on one side (closing crack) and moves out of contact on the other side (opening crack). If a crack moves with a finite speed v , the energy required to break the bonds at the crack tip is in general larger at the opening crack than the energy gained by bond formation at the closing crack. This adhesion hysteresis can be understood by a simple example.

Consider a flat surface approaching a solid consisting of long-chain molecules; see Fig. 46. At a finite approach speed v , the solids make contact only when they are within the range of the interaction potential between the solids (usually of order 1 nm or less). This bond formation gives an energy gain per unit surface area denoted by γ_0 . Next, we separate the solids. If the separation velocity is high, the bond between the chain molecules and the flat surface breaks only when the chain molecules are nearly fully stretched. Hence, the work per unit surface area γ_1 required to separate the surfaces may be much larger than γ_0 . In general, $\gamma_0(v)$ and $\gamma_1(v)$ depend on the speed v . For infinitesimally slow motion ($v \rightarrow 0$), thermal equilibrium occurs at all times and $\gamma_0(0) = \gamma_1(0)$.

For viscoelastic solids, the opening crack propagation energy $\gamma_{\text{open}}(v)$ usually increases rapidly with increasing crack-tip velocity due to viscoelastic energy dissipation close to the crack tip [see Fig. 47(a)].⁷⁷ Hence, $\gamma_{\text{open}}(v)$ can be much larger than $\gamma_1(v)$. Similarly, viscoelastic energy dissipation close to the closing crack tip reduces the energy gain from bond formation, so that $\gamma_{\text{close}}(v)$ may be much smaller than $\gamma_0(v)$. Rolling friction depends on the difference $\gamma_{\text{open}}(v) - \gamma_{\text{close}}(v) \approx \gamma_{\text{open}}(v)$ (see the following), so no direct information about $\gamma_{\text{close}}(v)$ can be obtained from rolling friction experiments. However, both $\gamma_{\text{open}}(v)$ and $\gamma_{\text{close}}(v)$ can be deduced from adhesion experiments in which a spherical ball is moved into and out of contact with the viscoelastic solid. One example is shown in Fig. 48, where negligible adhesion occurs on approach (closing crack), while strong adhesion is observed on retraction.

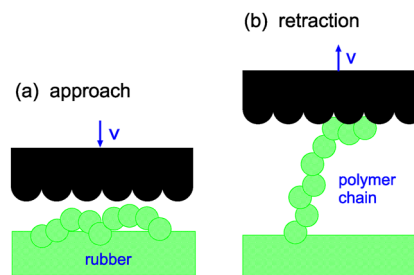


FIG. 46. (a) Solid with a flat surface (black) approaching a solid consisting of long-chain molecules. The solids make contact only when they are at atomic separation. (b) During separation, a chain molecule is bonded to the upper solid, and the bond breaks only when the chain is nearly fully stretched.

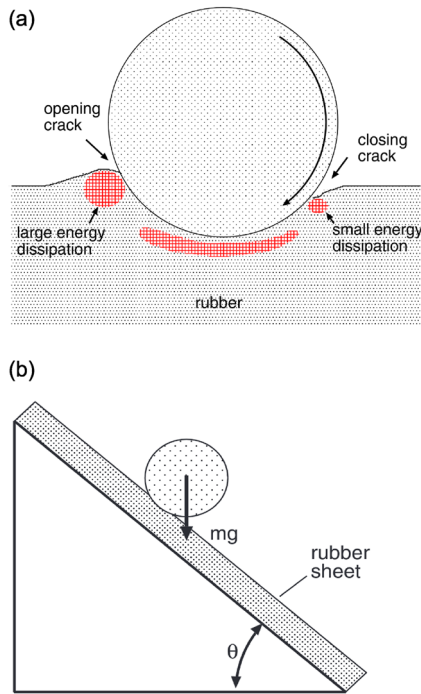


FIG. 47. (a) Hard cylinder or ball rolling on a flat rubber track. The regions where most of the viscoelastic energy dissipation occurs are indicated by the red dashed areas. (b) A rigid cylinder rolling down a rubber track. From the tilt angle θ , one can calculate the effective interfacial energy γ_{eff} .

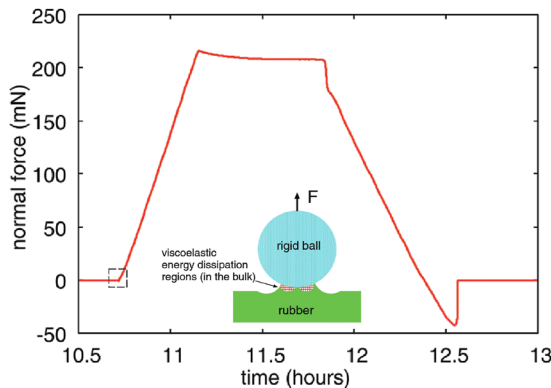


FIG. 48. Glass sphere (radius $R = 2$ cm) brought into contact with a weakly cross-linked PDMS rubber surface, kept in contact for about 1 h, and then removed. The approach and retraction speed is $0.87 \mu\text{m/s}$. Note that no adhesion force can be observed during contact formation (closing crack) (dashed rectangle), while a large attractive force is observed during retraction (opening crack). The pull-off force is related to γ_{open} via the JKR formula $F_{\text{pull-off}} = (3\pi/2)\gamma_{\text{open}}$; the figure is adapted from Ref. 78.

The adhesive contribution to the rolling friction of a cylinder on a flat viscoelastic solid can be obtained as follows: if the cylinder has length L and rolls a distance Δx , the opening and closing crack tips move over the surface area $A = L\Delta x$, so the energy dissipated at

the opening crack tip is $\gamma_{\text{open}}(v)L\Delta x$ and the energy gained at the closing crack tip is $\gamma_{\text{close}}(v)L\Delta x$, giving the net energy loss,

$$\Delta E = [\gamma_{\text{open}}(v) - \gamma_{\text{close}}(v)]L\Delta x.$$

If f_R is the rolling force per unit length, the rolling friction energy is $f_R L\Delta x$, giving

$$f_R = \gamma_{\text{open}}(v) - \gamma_{\text{close}}(v) = \gamma_{\text{eff}}(v).$$

Note that f_R is independent of the normal force, so the rolling friction coefficient $\mu = f_R/f_N$ increases without limit as $f_N \rightarrow 0$. This differs from the viscoelastic contribution studied before [see (30)], where f_R depends nonlinearly on f_N (typically $f_R \sim f_N^{1/2}$ for a cylinder and $F_R \sim F_N^{1/3}$ for a sphere) and in particular $f_R \rightarrow 0$ as $f_N \rightarrow 0$. Hence, the adhesive contribution dominates the rolling friction for sufficiently small normal loads. In Ref. 79, an equation was derived for the load where the rolling friction crosses over from the adhesive to the viscous limiting case,

$$\gamma_{\text{eff}} \approx \left(\frac{\sigma_0}{E_0}\right)^3 R \text{Im}E(\omega_0), \quad (34)$$

where σ_0 is the average contact pressure and $\omega_0 = r/d$, where d is the width of the contact regions. σ_0 and d can be estimated using Hertz contact theory. Equation (34) shows that for large enough contact pressure, the viscoelastic bulk contribution will dominate.

The rolling friction can be measured using a tilted substrate, as indicated in Fig. 47(b). The tangential component of the gravitational force acting on the cylinder is $F = mg \sin \theta$, which must equal the friction force Lf_R during rolling at constant speed. This gives $\gamma_{\text{eff}}(v) = (mg/L) \sin \theta$. We note that steady rolling occurs only if $\gamma_{\text{eff}}(v)$ increases with the crack-tip velocity v , which is almost always the case for rubber due to the strong increase in viscoelastic energy dissipation at the crack tip with increasing velocity.⁸⁰

Roberts and Thomas⁸⁰ performed rolling friction experiments with glass cylinders on smooth plane rubber tracks. Figure 49 shows $\gamma_{\text{eff}}(v)$ for natural rubber and polychloroprene as a function of the

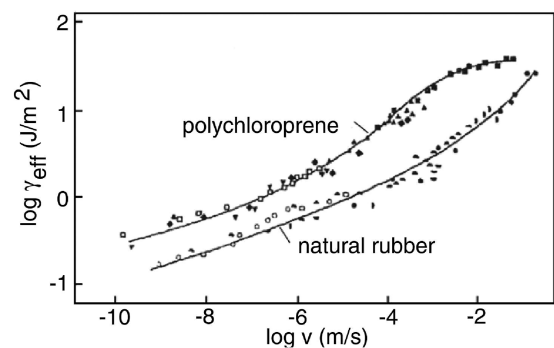


FIG. 49. Logarithm (base 10) of the effective interfacial energy $\gamma_{\text{eff}}(v)$ as a function of the logarithm of the crack-tip velocity v , deduced from rolling experiments with glass cylinders on smooth plane rubber tracks. The different symbols correspond to different glass cylinder loads (0, 6, 18, 28, and 102 g), where increasing velocity corresponds on average to increasing load. The results are shown for natural rubber and polychloroprene rubber tracks; the figure is adapted from Ref. 80.

crack-tip velocity v (log-log scale). The different symbols correspond to different glass cylinder loads (0, 6, 18, 28, and 102 g). As the cylinder mass increases, the rolling velocity increases, and the fact that all data points collapse onto a common master curve shows that the rolling friction is independent of the normal force and depends only on the rolling speed.

VII. TRIANGULAR SLIDER ON RUBBER SURFACES

Here, we investigate the sliding friction of a rigid triangular steel slider (wedge) on a rubber surface under both dry and lubricated conditions.⁸¹ The substrate is a carbon-filled styrene-butadiene rubber (SBR) compound with a glass transition temperature of $T_g \approx -50^\circ\text{C}$. We first consider lubricated friction at low sliding speeds, assuming that the frictional shear stress in the area of real contact is negligible. The experimental results are compared to the theory developed in Ref. 74 for rolling friction, which is also valid for lubricated sliding at low speeds if the frictional shear stress can be neglected.

We use the symmetric slider shown in Fig. 50. The profile has a half-opening angle $\alpha = 65^\circ$ and is pressed against a flat rubber sheet with a normal force per unit length $f_N = F_N/L_y$. The pressure distribution between the slider and the rubber is given by⁸²

$$p(x) = p_0 \cosh^{-1}\left(\frac{a}{x}\right),$$

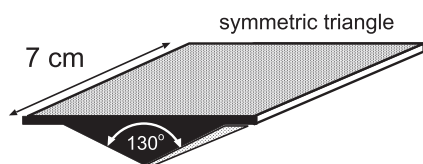


FIG. 50. Triangular steel slider (wedge) used in the rubber friction experiments.

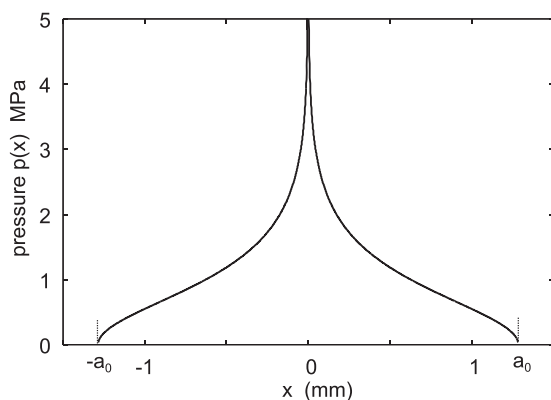


FIG. 51. Pressure distribution $p(x)$ as a function of x for the symmetric indenter pressed against the rubber with a normal force per unit length $F_N/L_y = 3000\text{ N/m}$. The effective elastic modulus, including a strain softening factor of 0.2, is taken as $E_{\text{eff}}(1/t_0, \epsilon) = 25 \times 0.2 = 5\text{ MPa}$, corresponding to a contact time of approximately $t_0 \approx 1000\text{ s}$. The indenter tip is assumed to be perfectly sharp, i.e., $r_{\text{tip}} = 0$. The width of the contact region is $w = 2a_0 \approx 2.5\text{ mm}$.

where $p_0 = f_N/\pi a$. Figure 51 shows the pressure distribution $p(x)$ taking into account the influence of strain softening on the elastic modulus. The half-width of the contact region is

$$a = \frac{f_N}{E^* \cot \alpha},$$

where $E^* = E/(1 - \nu^2)$. When used in (29), E^* is replaced with $|E_{\text{eff}}(\omega, \epsilon)|$, with $\omega = \mathbf{q} \cdot \mathbf{v}$, so the contact width $w = 2a$ depends on the sliding speed v .

Figure 52 shows two pressure footprints of the contact between the slider and the rubber substrate obtained using a pressure-sensitive film. The slider was loaded against the rubber surface with $f_N \approx 3000\text{ N/m}$. The two footprints correspond to contact times of 6 and 600 s, with widths (indicated by the blue bars) of 2 and 2.5 mm, respectively.

The strain in the region where viscoelastic deformations occur varies spatially, but it can be shown to be of the order $\epsilon \approx 1/(2 \tan \alpha)$, which in our case gives $\epsilon \approx 0.25$. In the following analysis, we present theoretical results that include strain softening, using the effective modulus (see Sec. II) $E_{\text{eff}}(\omega, \epsilon)$ for $\epsilon \approx 0.25$. For this relatively large strain, the nonlinear (strain softening) correction is significant, increasing the predicted sliding friction by approximately a factor of 5.

Figure 53 shows the measured sliding friction for the wedge on dry (blue squares) and lubricated (green squares) rubber. The lubricated surface was prepared by spreading a teaspoon of silicone oil with a viscosity of 1 Pa s over the rubber. Due to the low sliding speed and high contact pressure, the silicone oil acts as a boundary lubricant (without hydrodynamic effects).

Figure 54 shows the measured sliding friction on the lubricated rubber substrate at different temperatures: $T = 20^\circ\text{C}$ (green),

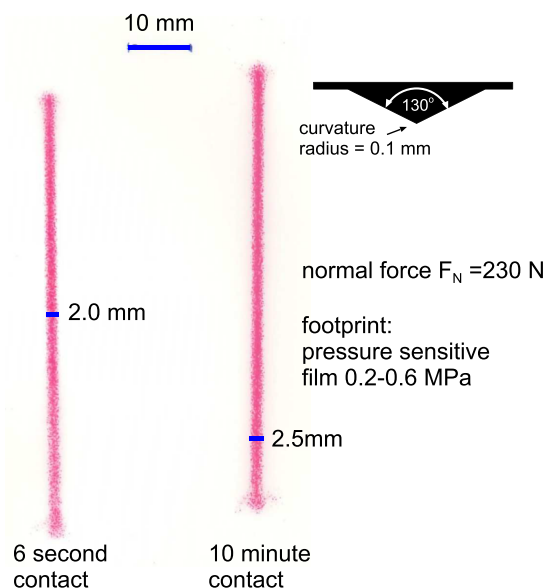


FIG. 52. Wedge loaded against the rubber surface with a pressure-sensitive film placed in between. The two pressure footprints correspond to contact times of 6 and 600 s, respectively. The short blue bars indicate contact widths of 2 and 2.5 mm.

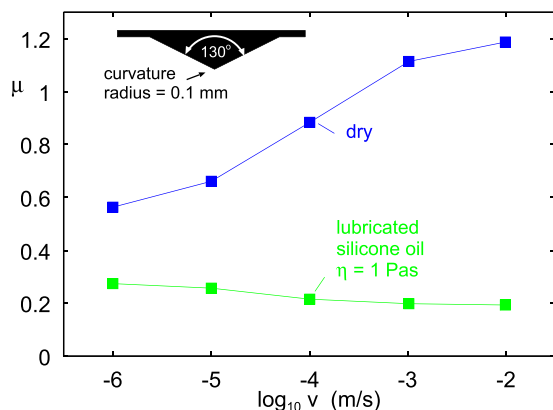


FIG. 53. Sliding friction for the wedge on rubber under dry (blue squares) and lubricated (green squares) conditions.

−20 °C (pink), −40 °C (blue), and a repeated measurement at 20 °C (dark green). The red line represents the theoretically predicted viscoelastic contribution to the friction. The measured data at −20 and −40 °C are shifted horizontally using the viscoelastic bulk shift factor, so that they correspond to equivalent friction values at 20 °C but at higher sliding speeds. However, these measured friction values lie significantly above the theoretical prediction (red curve).

We attribute the enhanced friction at −20 and −40 °C to the high contact pressure resulting from the increased elastic modulus of the rubber at low temperatures, which may lead to penetration of the lubricant film by surface asperities. Nevertheless, the sliding friction at −20 and −40 °C remains lower than expected for dry surfaces, indicating that the lubricant film continues to reduce friction even under these colder conditions.

Assuming, as supported by the theory, that the sliding friction on the lubricated surface at room temperature is dominated by viscoelastic effects, we can estimate the shear stress acting in the area of real contact under dry conditions using

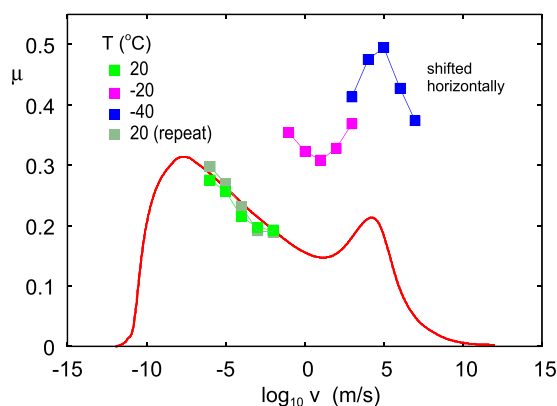


FIG. 54. Sliding friction for the wedge on lubricated rubber at $T = 20$ °C (green), −20 °C (pink), and −40 °C (blue), along with a repeated measurement at 20 °C (dark green). The data at −20 °C and −40 °C are shifted horizontally using the bulk viscoelastic shift factor. The red line represents the theoretically predicted viscoelastic contribution, assuming a tip radius of $r = 0.1$ mm.

$$\sigma_f(v) = [\mu(v) - \mu_{\text{visc}}(v)] \frac{F_N}{w(v)L_y}. \quad (35)$$

Here, $w(v)$ is the width of the contact region, $\mu(v)$ is the total friction coefficient, and μ_{visc} is the viscoelastic contribution, as indicated by the green and blue curves in Fig. 53. Using this equation gives a frictional shear stress $\sigma_f(v)$ that increases nearly linearly with the logarithm of the sliding speed, as expected for thermally activated processes in the low-velocity regime.

Rubber filled with reinforcing particles exhibits strong strain softening, as shown in Fig. 4 for the compound used in this study. It is crucial in studies of rolling and sliding friction to account for the rapid decrease in effective modulus with increasing strain. This has been emphasized previously in the context of rolling friction and sliding friction. In the present study, the strain in the contact region is ~ 0.25 , which is similar to the strain encountered in asperity contacts when a tire is in contact with a road surface. Figure 55 compares the calculated friction using the linear response modulus $E(\omega)$ with the result obtained using the effective modulus $E_{\text{eff}}(\omega, \epsilon)$ for the strain $\epsilon = 0.25$. The calculated result that includes strain softening yields a friction value approximately five times higher than the result without softening. A similarly strong effect of strain softening is observed in rolling friction on rubber with filler particles. Strain softening is equally important for static contacts, and the width of the wedge footprints in Fig. 52 can only be reproduced by the theory if both strain softening and the frequency dependence of the effective viscoelastic modulus are taken into account.

In Ref. 83, the authors studied the sliding of non-cylindrical wedges on a viscoelastic half-space. Using the boundary element method, they found that for wedges with the height profile $h(x) \sim |x|^k$, with $k = 4, 6$, and 8 , the viscoelastic contribution to the friction force differs from the prediction of (30). However, if the friction coefficients are scaled by E_0/p_0 , where E_0 and p_0 are the modulus and mean pressure at zero speed, nearly identical results are obtained for all wedges, including the cylindrical wedge [where $h(x) \sim x^2$]. Hence, the friction coefficients as a function of sliding speed can be obtained from the cylindrical case, where (30) is accurate, by scaling.

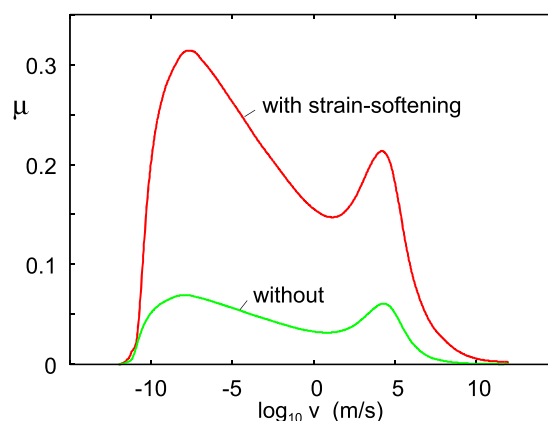


FIG. 55. Calculated sliding friction coefficient for the wedge, using the measured tip radius $r = 0.1$ mm. The red curve includes the effect of strain softening (nonlinear modulus), while the green curve uses the linear viscoelastic modulus (low-strain response).

VIII. LUBRICATED RUBBER FRICTION

This review mainly focuses on rubber friction on dry surfaces. However, rubber friction on wet surfaces, or on surfaces covered by lubrication or contamination films, is equally important. Applications include tires operating on wet or contaminated road surfaces and dynamic seals in which the sliding surfaces are covered by oil films. Other examples include the friction between shoes and contaminated ground surfaces, and syringes, where the interface between the rubber stopper and the glass or polymer barrel is often lubricated with silicone oil.

Here, we do not present a comprehensive study of rubber friction on lubricated surfaces, but we highlight a few basic facts.

When an elastic body slides on a substrate in the presence of a fluid, one can distinguish three regimes, as illustrated in Fig. 56, which schematically shows the friction coefficient as a function of sliding speed. Here, we focus on the behavior of thin fluid films that may exist in the boundary and early mixed lubrication regions of the Stribeck curve.

The hydrodynamic region prevails when there is no direct contact between the solids. For randomly rough surfaces with a roll-off, the highest asperities are typically on the order of a few times h_{rms} . However, if the surfaces have already been in contact in the boundary lubrication region, the highest asperities may be significantly reduced due to plastic deformation or wear.

In the boundary lubrication region, most of the load is carried by the area of real contact. Nevertheless, a small fraction of the load—typically around 10%—may be supported by pressurized fluid trapped in sealed-off regions.

Thin films between solid surfaces may be stable or unstable depending on interfacial energies and the nominal contact pressure. We first consider smooth surfaces as studied in Refs. 84–88.

Interfacial dewetting in water is observed for surfaces of hydrophobic solids. A fluid film between two solids with flat

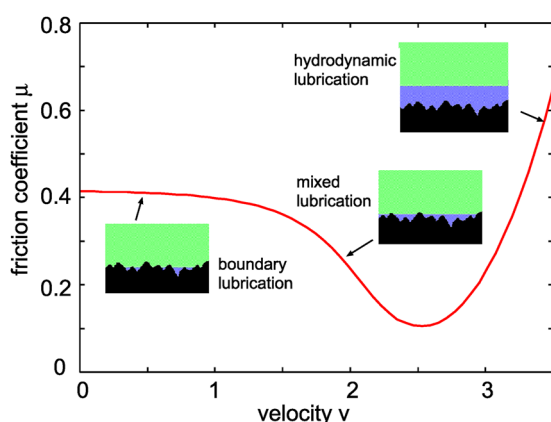


FIG. 56. Stribeck curve: the typical relation between the friction coefficient and the sliding speed. In the boundary lubrication regime, the rubber (green) is in contact with the substrate asperities, and there is no hydrodynamic lift-off pressure from the fluid, except possibly from some trapped (sealed-off) fluid islands with pressurized fluid. In the mixed lubrication regime, the fluid pressure increases the separation between the surfaces, although some asperity contacts still occur. In the (elasto)hydrodynamic regime, the fluid pressure is sufficient to fully separate the surfaces so that no asperity contact remains.

surfaces is unstable and will be removed by dewetting if this results in a reduction of the free energy. If the interfacial energies (per unit surface area) between the solids and the fluid are denoted by γ_{01} (fluid 0, solid 1) and γ_{02} (fluid 0, solid 2), and the interfacial energy for the dry solid–solid contact by γ_{12} , then the condition for dewetting is that the interfacial (one-dimensional or line) spreading pressure,

$$S = \gamma_{12} - \gamma_{01} - \gamma_{02}, \quad (36)$$

is negative. Note that S represents the change in free energy per unit surface area as the two solids come into contact in the fluid. If θ_1 and θ_2 are the fluid (thermal equilibrium) contact angles on the surface of solids 1 and 2, respectively, and γ_1 and γ_2 the corresponding solid–vapor surface energies, then by Young's equations,

$$\gamma_1 = \gamma_{01} + \gamma_0 \cos \theta_1, \quad \gamma_2 = \gamma_{02} + \gamma_0 \cos \theta_2,$$

where γ_0 is the surface tension of the fluid. Hence, we can also write (36) as

$$S = \gamma_{12} - \gamma_1 - \gamma_2 + \gamma_0 (\cos \theta_1 + \cos \theta_2). \quad (37)$$

The term,

$$w = \gamma_1 + \gamma_2 - \gamma_{12}$$

is the work of adhesion, that is, the energy per unit surface area required to separate the solids in dry conditions (no fluid). Thus,

$$S = -w + \gamma_0 (\cos \theta_1 + \cos \theta_2).$$

If θ_1 and θ_2 are larger than 90° , then S will be negative regardless of the work of adhesion, which is always positive for neutral solids.

If S is positive, then in the absence of external forces, a thin fluid film will separate the surfaces of the solids. If S is negative, the fluid film is unstable and dewetting will occur. For soft solids such as rubber, this may take place by the fluid accumulating in a rim at the front of the expanding dry surface area, as illustrated in Fig. 57.

Here, we are interested in rubber friction on solids in a fluid environment, so that $\gamma_{12} = \gamma_{rs}$ (rubber–solid), $\gamma_{01} = \gamma_{rf}$ (rubber–fluid), and $\gamma_{02} = \gamma_{sf}$ (solid–fluid).

A. Wetting and dewetting experiments

In a series of pioneering studies, Roberts and Tabor^{89,90} investigated the squeezing of liquid films between rubber balls and flat hard substrates. The thickness of the liquid layer was deduced by analyzing the optical interference pattern in the junction. During squeezing of the rubber ball against the substrate in a liquid, the profile of the rubber surface in the contact zone bends upward, and the rubber–substrate separation becomes smallest at the periphery of the contact area, as expected from fluid squeeze-out theory. When the thickness

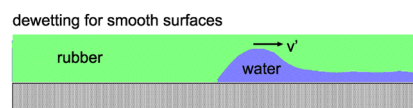


FIG. 57. Dewetting for a soft solid (rubber) initially separated from the substrate by a thin fluid film. During dewetting, the fluid accumulates in a rim at the front of the expanding dry surface area.

of the film falls below 40 nm, the surfaces suddenly spring together at various points, leading to adhesive contact over a major part of the contact region. The contact first occurs at some protrusion (defect) near the edge of the contact zone, where the film thickness is smallest. Once a point of contact is established, it pulls the rest of the rubber into contact. This is referred to as a dewetting transition.

With a low-viscosity fluid such as water as the lubricant, at the stage of film collapse, the surfaces seal together exceedingly quickly and trap small islands of fluid. After a few hours, these islands disappear, possibly due to a slow drift toward the periphery of the contact area, driven by the spatial (Hertzian) pressure distribution in the contact zone. If instead of distilled water, a dilute solution of a negatively charged soap [sodium dodecyl sulfate (SDS)] is used, a drastically different result is observed; see Fig. 58(a) (right). The surfaces no longer spring together, trapping islands of liquid, but instead remain separated by an almost uniform distance of about 20 nm. The film does not collapse over time. In this case, the film is stabilized by electrical double-layer repulsive forces, which support the normal load even when the squeezing pressure in the contact region reaches 0.1 MPa. The SDS is adsorbed onto the rubber surface [see Fig. 58(b)] with its negatively charged polar end-groups immersed in the water. The glass surface most likely acquires a negative charge due to the reaction of water molecules with Si=O groups on the surface, forming HOSi. The two negatively charged surfaces attract positive ions from the solution, establishing a double layer of charge, which results in repulsive forces between the surfaces at small wall-to-wall separations. One important manifestation of this thin liquid layer is its lubricating effect: sliding at 1 m/s results in a friction coefficient of 0.001, compared to 10 under dry conditions or when using distilled water.

Roberts⁹⁰ also performed experiments in which positively charged soap was adsorbed onto the rubber surface, while the glass surface remained negatively charged. In this case, the rubber and the glass surfaces snapped together (dewetting transition), and the friction was much higher. The ability to affect adhesion and sliding friction of solids in liquids by adsorbing molecules with ionic groups on solid walls has great practical relevance, for example, in conditioners for hair-care applications. The ability to modulate the contact between two soft solid bodies by changing the ionic composition of the surrounding fluid may also be important in many biological processes.

The discussion above has not considered the influence of external pressures or shear stresses on the lubrication film thickness. However, if the pressure due to an external load is sufficiently large, a fluid film can be removed from the interface even if this increases the interfacial energy, that is, even if the spreading pressure $S > 0$. We refer to this as forced dewetting. Conversely, even if interfacial energies favor removing a fluid film from the interface (i.e., $S < 0$), sliding motion may force a fluid film to enter the interface. We refer to this as forced wetting.

B. Forced wetting

In a pioneering work, Martin *et al.*⁸⁴ studied sliding-induced (forced) wetting of a contact that was initially dry. They observed (using optical interferometry) the contact of a rubber cap squeezing a nonwetting liquid against a plate moving at velocity v , as shown in Fig. 59. At low velocities, the contact remains dry [Fig. 59(a)].

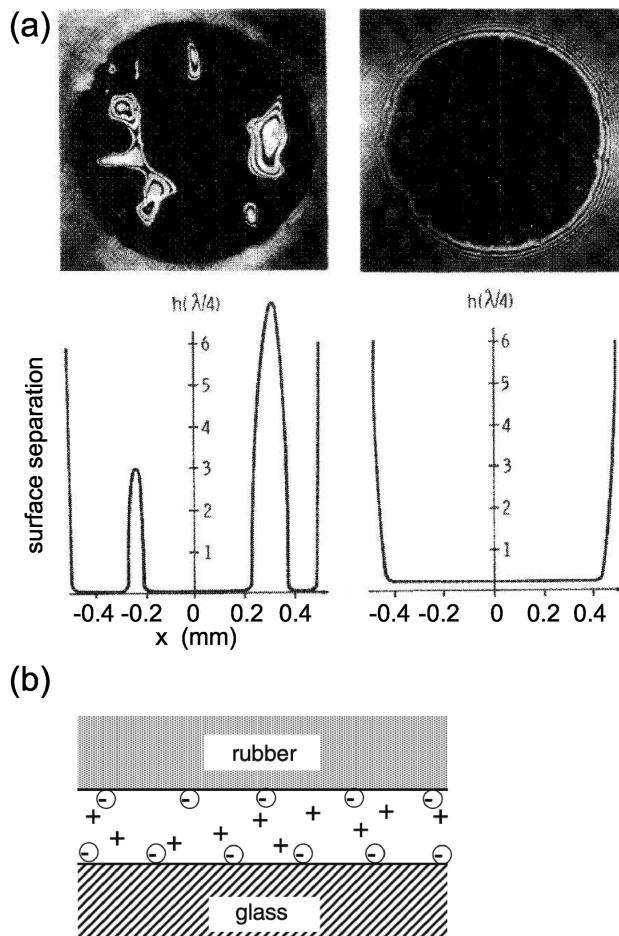


FIG. 58. (a) Interferograms (top) and deduced surface separation profiles (bottom) showing the marked difference between distilled water (left) and water containing 0.01 mol/l SDS. Distilled water becomes trapped in pockets, while the SDS solution forms a thin equilibrium film of ~ 20 nm uniform thickness. (b) Water with positive ions (hydrated protons) confined between a rubber surface and a glass surface with negatively charged atomic groups. As the water is squeezed out, the concentration of positive charges between the surfaces increases (the total number of positive and negative charges remains constant), resulting in an increase in entropy and a repulsive force between the solid walls; the figure is adapted from Ref. 88.

It becomes partially wetted above a threshold velocity $v = v_{c1}$, with two symmetrical dry patches remaining at the rear of the contact. At a second critical velocity $v = v_{c2}$, the contact becomes fully wet. The regime $v > v_{c2}$ corresponds to viscous hydroplaning. The transitions at v_{c1} and v_{c2} result from the competition between liquid invasion induced by shear and spontaneous dewetting of the liquid (between nonwetting surfaces).

For a smooth spherical lens (with radius R) sliding on a smooth rubber substrate, the critical velocity v_c was found to scale with the parameter,⁸⁴

$$v_c \sim \frac{|S|}{\eta} \left(\frac{|S|}{ER} \right)^{1/3}, \quad (38)$$

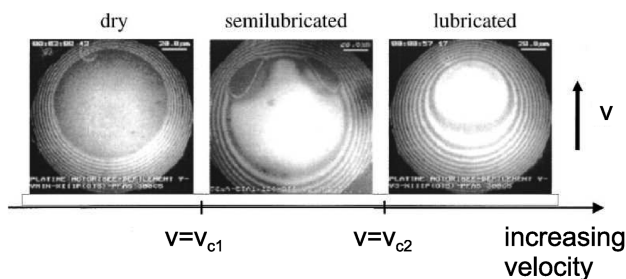


FIG. 59. Regimes of forced wetting observed in the sliding rubber/liquid/glass contact as a function of increasing sliding speed v . The arrow indicates the direction of motion of the glass plate.

where $S < 0$. This relation was derived using simple and approximate arguments, and the experiments performed in Ref. 84 showed a slightly weaker dependence on the fluid viscosity, namely, $v_c \sim \eta^{-3/4}$.

C. Forced dewetting

If S is positive, a thin fluid film will separate the surfaces of the solids in the absence of external forces. This remains true even under an applied squeezing pressure p_0 , provided it is not too high. However, if S is small and the squeezing pressure is large, the fluid film may be removed by what we refer to as *forced interfacial dewetting*. If p^* denotes the minimum pressure needed to squeeze out the fluid film, then $p^*d^* = S$, where d^* is expected to be a molecular-scale distance, ~ 1 nm. We define the effective spreading pressure as $S^* = S - pd^*$, where p is the pressure in the asperity contact region. If $S^* < 0$, then the fluid is removed from the contact regions even when $S > 0$.

We now present a case where forced dewetting may occur at low sliding speeds. Consider rubber sliding on a concrete surface lubricated with water–glycerol mixtures. Figure 60 shows the sliding friction master curve for an SBR rubber with carbon filler sliding

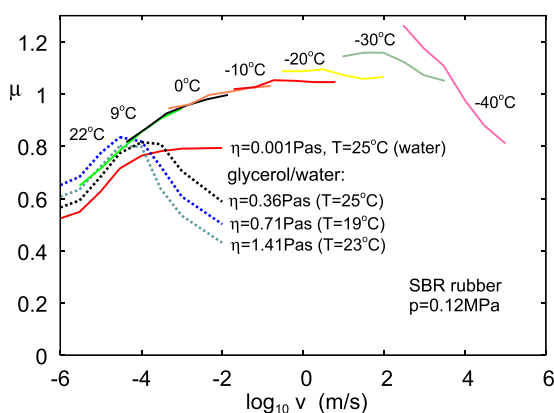


FIG. 60. Sliding friction master curve for a SBR rubber with carbon filler sliding on a dry concrete surface, obtained by shifting velocity segments acquired at the indicated temperatures using the bulk viscoelastic shift factor. The friction coefficients at room temperature for the concrete surface lubricated by glycerol–water mixtures are also shown; the figure is adapted from Ref. 91.

on a dry concrete surface, obtained by shifting velocity segments at the indicated temperatures using the bulk viscoelastic shift factor. The friction coefficients at room temperature for the concrete surface lubricated by glycerol–water mixtures are also shown. For $v < 0.1$ mm/s, the friction force is nearly the same in the dry state and in the presence of the water–glycerol mixtures.

The surface tension of water (and glycerol) is ~ 0.07 J/m². If S is positive, its magnitude is expected to be at most a few times 0.01 J/m². Using $S = 0.01$ J/m² and $d^* = 1$ nm gives $p^* = 10$ MPa, which is also the typical pressure in the rubber–concrete real contact areas. Thus, assuming that the relative asperity contact area A/A_0 is ~ 0.01 (as estimated in Ref. 91), and taking $p_0 = 0.12$ MPa, we find that $p = p_0 A_0/A = 12$ MPa. Hence, the fluid film may be removed from the asperity contact regions even if $S > 0$, provided that S is small enough.

For sliding speeds $v > 0.1$ mm/s, the friction in the fluids is smaller than in the dry state. However, a very thin fluid film is sufficient to explain the observed drop in the friction coefficient. For example, if the fluid film is ~ 3 nm thick, then for pure glycerol ($\eta \approx 1.4$ Pa s, not accounting for shear thinning) at the sliding speed $v = 1$ cm/s, the shear stress is expected to be $\sigma_f \approx \eta v/d \approx 5$ MPa. This gives a friction coefficient of approximately $\sigma_f A/p_0 A_0 \approx 0.2$, which is consistent with experimental observations.

The drop in the friction coefficient in glycerol, starting at the sliding speed $v \approx 0.03$ mm/s, is likely due to forced wetting. Assuming $|S^*| \approx 0.01$ J/m², a fluid viscosity of $\eta \approx 1$ Pa s, an elastic modulus $E = 10^7$ Pa, and using a macroasperity radius of curvature $R = 26$ mm—as obtained from the concrete surface roughness power spectrum (see Ref. 91)—we obtain from (38) a critical velocity $v_c = 0.03$ m/s, which is consistent with the results shown in Fig. 60.

D. On the adhesive contribution to sliding friction

As discussed in Sec. V, there are several ways in which adhesion can affect the sliding friction force. First, it can increase the area of real contact A , which would influence both the viscoelastic contribution and the real contact area contribution to friction.⁵⁴ There may also be a contribution from the edges of the contact regions due to opening cracks.^{38,78,79,92,93} Finally, and most importantly, rubber molecules at the sliding interface may undergo stick–slip motion [see Figs. 12(a)–12(c)], as first suggested by Schallamach. This results in a friction force contribution proportional to the real contact area A .

Here, we compare adhesion and friction for three types of rubbers in the dry state and in water. Experimental observations indicate that, for rough surfaces, the increase in contact area due to adhesion and the crack-opening mechanism appear to be unimportant for rubber friction on rough surfaces.²⁶

We present results for three types of rubber: NBR, GECO (Polyepichlorohydrin), and EPDM (ethylene propylene diene monomer). The compounds were produced without oil and filler particles and are, therefore, “clean” compounds, suitable for fundamental studies. The GECO compound is strongly hydrophilic and absorbs water (swelling) in the wet state. All rubbers were produced in the same mold and have the same (small) roughness.

Figure 61 shows the logarithm of the work of adhesion w during retraction, measured during repeated contact (drive speed

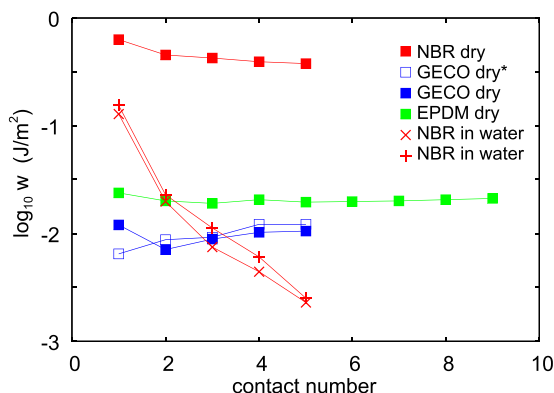


FIG. 61. Logarithm of the work of adhesion w during retraction measured during repeated contact (drive speed $\approx 1 \mu\text{m/s}$) between a glass ball and three types of rubbers (without filler): NBR, GECO, and EPDM. In water, no adhesion is observed for GECO and EPDM.

$\approx 1 \mu\text{m/s}$) between a glass ball and the rubber samples. In water, no adhesion is observed for GECO and EPDM, while for NBR, the adhesion in water is (depending on the number of contacts) ~ 5 – 100 times weaker than in the dry state. Thus, water either completely eliminates adhesion (for GECO and EPDM) or strongly reduces it.

For the same three rubbers, Fig. 62 shows the friction coefficients as a function of the logarithm of the sliding speed. The green lines indicate the calculated viscoelastic contributions. Note that for the EPDM rubber, the friction in the dry state and in water is nearly the same up to a sliding speed of $\sim 1 \text{ mm/s}$, despite the absence of adhesion in water. Therefore, for this compound, neither crack opening nor increased contact area in water contributes to the sliding friction. We conclude that these effects make a negligible contribution to the sliding friction of EPDM.

The figure also suggests that the pressure in the asperity contact regions is high enough to squeeze out the fluid film. Since no adhesion is observed in water, the spreading pressure S is positive, but clearly, at low sliding speed, forced dewetting must occur such that $S^* < 0$. For sliding speeds higher than $\sim 1 \text{ mm/s}$, forced wetting takes place and the friction decreases, but this cannot be measured using the Leonardo da Vinci setup employed in this study, as it results in run-away motion. With the Leonardo da Vinci setup, the driving force is constant, so if the friction decreases with increasing sliding speed, accelerated motion occurs.

For the NBR rubber, the friction in water is nearly the same or even higher than in the dry state, again indicating that the increase in contact area and crack-opening effects are negligible. For GECO, some change in the friction between the dry state and the wet state is observed. This can be attributed to the fact that GECO is strongly hydrophilic and absorbs water, resulting in swelling and modification of the bulk viscoelastic and other material properties.

IX. RUBBER FRICTION ON ICE AND SNOW

Rubber friction on ice and snow has attracted long-standing interest due to its critical relevance to mobility and safety in cold environments.^{94–100}

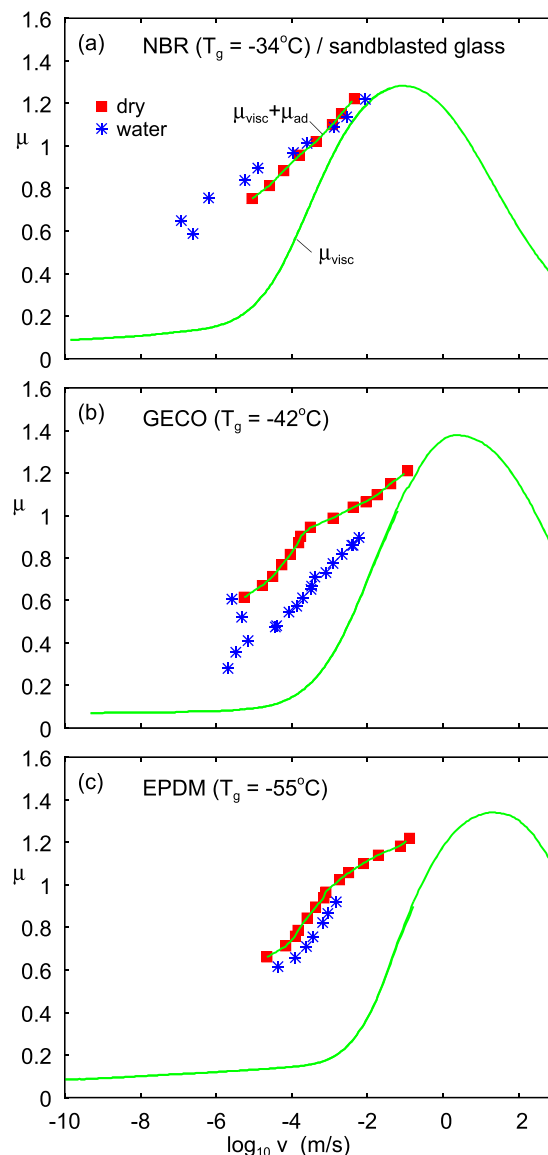


FIG. 62. Friction coefficients as a function of the logarithm of the sliding speed for the NBR, GECO, and EPDM rubbers on dry and wet sandblasted glass. The nominal contact pressure $p \approx 0.09 \text{ MPa}$. The green lines represent the calculated viscoelastic contributions.

A. Friction on ice

Friction on ice is a fascinating and long-studied subject,^{101–105} with pioneering work by Faraday¹⁰⁶ over 150 years ago, who proposed that the surface of ice is covered by a liquid-like film of water. This phenomenon is now known as *premelting*. The presence of a premelted layer depends on both temperature and the hydrophobic or hydrophilic nature of the counter surface.^{107,108} For inert hydrophobic materials, such as polytetrafluoroethylene (Teflon), such a layer may form well below the melting point of ice, while for hydrophilic surfaces, such as silica, it may not occur at all, or only very close to the melting point.

The origin of this (liquid-like) water film has long been debated. One explanation, proposed by Thomson,¹¹⁰ is pressure-induced melting, but this mechanism is unlikely since plastic deformation of ice usually occurs before the pressure is sufficient to induce melting. A more widely accepted explanation is frictional heating, suggested by Bowden and Hughes,¹¹¹ who argued that sliding at sufficiently high velocity produces local melting of the ice surface. This view has been supported by experimental studies.¹¹² An important observation for friction on ice is that when the sliding speed increases, the friction drops, which implies reduced frictional heating. This shifts the onset of ice melting to higher sliding speeds and can explain the velocity dependence of friction on ice.¹¹³

More recent work has challenged the idea that interfacial water films alone are responsible for the low friction of ice. Molecular dynamics simulations by Atila *et al.*,¹⁰⁹ inspired by the hypothesis of Moras *et al.*¹¹⁴ on amorphization in incommensurate contacts showed that ice surfaces form local “cold-welded” sites during sliding. These sites rapidly transform into amorphous layers under lateral displacement, without requiring significant heating or reaching melting conditions. The structure of the amorphous layer resembles supercooled water, providing a lubricating film that reduces friction. Its thickness scales with the square root of the sliding distance, indicating a displacement-driven process, while local pressure gradients further promote amorphization. In this framework, frictional heating plays a secondary role by lowering the effective viscosity of the amorphous layer, thereby reducing shear stress, while amorphization itself remains the dominant mechanism. The simulations further reveal that the mechanism operates similarly at hydrophilic and hydrophobic interfaces, although capillary and adhesion-enhanced dissipation increase friction for the former. This new perspective helps explain observations that traditional theories could not account for, such as the persistence of liquid-like layer well below 0 °C and the pronounced frictional contrast between hydrophilic and hydrophobic counterfaces. It also clarifies the dual temperature effect: at lower temperatures, amorphous layers form more rapidly but exhibit much higher viscosity, resulting in higher macroscopic friction.

Rubber friction on ice can be categorized according to temperature and sliding speed.

- At temperatures close to the bulk melting point of ice, or at high sliding speeds, a water film or a low-viscosity amorphous layer is present at the interface. In this regime, the friction is dominated by the viscoelastic deformation of the rubber induced by the ice roughness.^{98,107,109,115}
- At lower temperatures and lower sliding speeds, the interfacial water film is absent or the amorphous layer exhibits a much higher effective viscosity. In this case, the friction arises from both the viscoelastic response of the rubber and the shear in the rubber–ice contact regions, or between ice fragments attached to the rubber surface and the underlying ice, which corresponds to an ice–ice interaction.^{107,109}
- At even lower temperatures (below T_g), the viscoelastic contribution becomes negligible. As the rubber stiffens, it can cause wear of the ice asperities upon contact, leading to increased friction through a plowing-type process.^{115,116}

In Ref. 98, the Persson rubber friction theory was applied to rubber sliding on ice. The study incorporates both the adhesive contribution from the real area of contact and the viscoelastic contribution from the rubber. A temperature-dependent large wavenumber cutoff, $q_1(T)$, was introduced to account for the smoothing of short-wavelength roughness by plastic deformation of the ice. Since the ice penetration hardness depends on the temperature and indentation speed dependent, the cutoff depends on the ice surface temperature and on the rubber sliding speeds. The model also includes the effect of frictional heating in the rubber, which leads to a shift in the viscoelastic modulus to higher frequencies. The model offers a consistent explanation of the measured friction behavior for three very different tread compounds.

Figure 63(a) shows the rubber friction on ice as a function of the sliding speed for several different temperatures. In Fig. 63(b), the experimental measurements are compared to theoretical predictions based on the viscoelastic contact mechanics theory developed

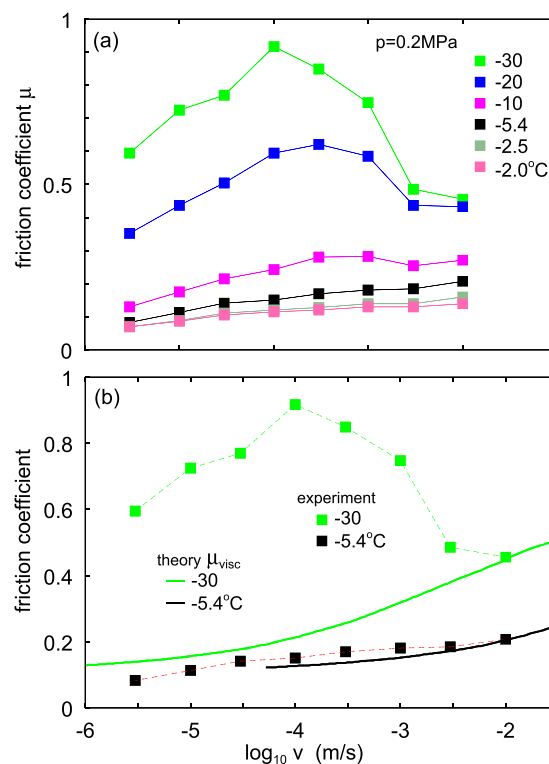


FIG. 63. (a) Measured rubber–ice friction coefficient as a function of the logarithm of the sliding speed for several indicated temperatures. At low temperatures, the friction is very similar to that of ice sliding on ice, suggesting that the friction originates from ice wear particles on the rubber surface sliding against the ice surface. (b) Comparison between experimentally measured friction coefficients (symbols) and theoretical predictions of the viscoelastic contribution (solid lines), based on the theory developed in Ref. 98. At the higher temperature (black), the measured friction closely matches the viscoelastic prediction, indicating that a thin premelted water film forms at the rubber–ice interface and that friction is dominated by viscoelastic deformation. At the lower temperature (green), the measured friction is significantly higher than the theoretical prediction, suggesting additional contributions from interfacial shear; the figure is adapted from Ref. 107.

in Ref. 98. At the higher temperature ($T = -5.4^\circ\text{C}$), the experimental data nearly coincide with the predicted viscoelastic contribution, indicating that a thin interfacial film is present, which may consist of a premelted water layer or a low-viscosity amorphous layer. In this regime, the friction is dominated by viscoelastic deformation of the rubber induced by the ice roughness. In contrast, at the lower temperature ($T = -30^\circ\text{C}$), the measured friction coefficient significantly exceeds the viscoelastic prediction. This suggests the absence of a liquid-like film or the presence of a highly viscous amorphous layer, so that friction arises not only from the viscoelastic response but also from interfacial shear in the rubber–ice contact regions. For $T < -20^\circ\text{C}$, the friction of rubber on ice approaches that of ice on ice (see Fig. 64), indicating a substantial contribution from shear between ice fragments and the underlying ice surface.

B. Friction on ice with sand

In many practical situations, ice surfaces are not clean but are covered with contamination. For example, sand is often spread on ice surfaces to increase friction. The rubber bends around the sand particles and most of the slip occurs between the sand particles and the ice, resulting in a plowing contribution to the friction force (see Fig. 65), which can be estimated as follows:

Suppose there are N sand particles in the tire-road footprint. If the concentration of sand particles is high enough, the full normal load of the tire will act on the sand particles, giving the force $f_N = F_N/N$ that squeezes each sand particle against the ice surface. If the sand particles are spheres with radius R , and if σ_P is the ice penetration hardness, then the sand–ice contact area projected onto the horizontal xy -plane is $A = f_N/\sigma_P$.

The maximum friction occurs when the force f_N is large enough to squeeze half of the particle into the ice. If f_N exceeds the value required to embed half of the sand particle, the entire particle may be pressed into the ice, and the contact would become rubber–ice contact. For the stationary case, the contact area projected onto the horizontal xy -plane would be πR^2 , and the maximum penetration resistance is $\pi R^2 \sigma_P$. However, for the sliding (plowing) case, we assume the contact area is half of this, $A = \pi R^2/2$, since the sand particle only contacts the ice on one side. In this case, the ice wear track

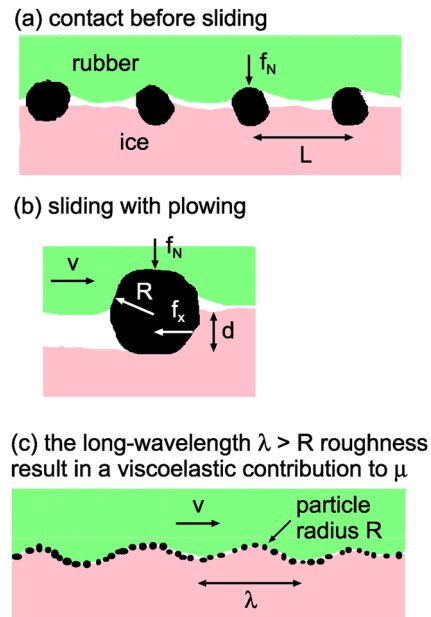


FIG. 65. (a) Rubber block squeezed in contact with an ice surface covered by stone particles. (b) During sliding, ice plowing occurs, resulting in a friction force that is maximal when the projected area of the particles occupies the fraction $\psi \approx 2\sigma_0/\sigma_P$ of the nominal surface area, where σ_0 is the nominal contact pressure squeezing the rubber block against the substrate, and σ_P is the ice penetration hardness (temperature dependent). The maximum plowing friction coefficient is $\mu \approx 1$. (c) The ice roughness with wavelengths larger than the sand particles contributes a viscoelastic component to the friction force.

will have a width of $2R$, and the vertical cross-sectional area of the plowing track will be $A_{\text{plow}} = \pi R^2/2$.

If the plowing stress (the stress acting on the sand particle from the ice it displaces during sliding) is similar to the ice penetration hardness σ_P , we obtain the friction force $F_f = NA_{\text{plow}}\sigma_P$ and the normal force $F_N = NA\sigma_P$, leading to the friction coefficient,

$$\mu_{\text{max}} = \frac{F_f}{F_N} = \frac{A_{\text{plow}}}{A} = 1. \quad (39)$$

This equation is only valid if the force squeezing a particle equals $f_N = (\pi R^2/2)\sigma_P$.

We now apply (39) to tires and ask for which particle concentration this relation holds. Assume the tire footprint area is A_0 , and that the projected area of the particles occupies a fraction $\psi < 1$ of the footprint area. The number of particles in the tire footprint is $N = \psi A_0/\pi R^2$, and the normal force acting on a particle is

$$f_N = \frac{F_N}{N} = \frac{\pi R^2 \sigma_0}{\psi}, \quad (40)$$

where $\sigma_0 = F_N/A_0$ is the average tire-road footprint pressure.

Using (40) and $f_N = \sigma_P \pi R^2/2$ gives $\psi = \psi_0$, where

$$\psi_0 = 2 \frac{\sigma_0}{\sigma_P}. \quad (41)$$

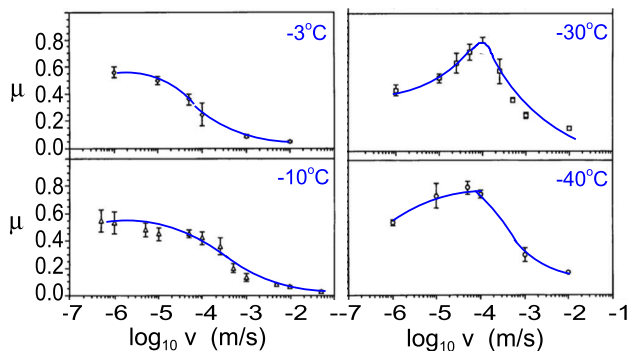


FIG. 64. Friction coefficient as a function of the logarithm of the sliding speed for ice sliding on ice at different temperatures; the figure is adapted from Ref. 117.

Only for this particle concentration will the friction coefficient equal (39). We also note that (41) is the minimum possible particle concentration for which plowing occurs since if $\psi < \psi_0$, then the normal force will be larger than $(\pi R^2/2)\sigma_p$, and the particles will be fully squeezed into the ice surface during plowing.

The penetration hardness depends on temperature (see the following), but since rubber friction on ice is lowest near the ice melting temperature, we estimate the particle concentration for this case. Close to the ice melting temperature,^{116,118,119} $\sigma_p \approx 10$ MPa, and using the average tire-road footprint pressure $\sigma_0 \approx 0.2$ MPa, we obtain $\psi \approx 0.04$.

If L is the average distance between the centers of the particles, using $\psi \approx 2\pi R^2/(\sqrt{3}L^2)$ gives $L/R \approx 10$. For example, if $R = 1$ mm, this gives $L \approx 1$ cm. At lower temperatures, σ_p increases; for example, $\sigma_p \approx 30$ MPa at $T = -20^\circ\text{C}$, and the optimum concentration of sand particles decreases to $\psi \approx 0.01$. However, at low temperatures, rubber friction on clean ice is not so low unless the sliding speed is very high. Very close to the ice melting temperature, a thin water film may form on the ice surface, and in this case, without sand particles, viscous hydroplaning effects could result in very low sliding friction.

There is a second condition that must also be satisfied for (39) to be valid. If a single spherical particle is squeezed against a rubber surface, it will penetrate into the rubber by a distance (Hertz's theory),

$$\delta = \left(\frac{9f_N^2}{16E_*^2 R} \right)^{1/3},$$

where $E_* = E/(1 - \nu^2)$. If this distance is larger than R , the rubber may contact the ice, and part of the normal force (the load) would then be carried by direct rubber-ice contact. As a result, the force f_N acting on the sand particles would be reduced. Hence, for (39) to be valid, we must have

$$\delta = \left(\frac{9f_N^2}{16E_*^2 R} \right)^{1/3} < R$$

or

$$f_N < \frac{4}{3} E_* R^2. \quad (42)$$

This is a conservative estimate of the maximum force, as the result is obtained in the limit of a low concentration of particles. For a finite concentration of particles, the upward displacement of the rubber at a given point between the particles will have contributions from all nearby particles. In general, a weaker condition than (42) (i.e., a larger f_N) will prevail.

Using (40) and (42), we conclude that the rubber will not make direct contact with the ice if $\psi > \psi_1$, where

$$\psi_1 = \frac{3\pi}{4} \frac{\sigma_0}{E_*} \approx 2 \frac{\sigma_0}{E_*}. \quad (43)$$

Similar to the viscoelastic modulus E_* , the penetration hardness σ_p is determined by stress-aided, thermally activated processes and depends strongly on temperature and deformation rate. The penetration hardness of ice was studied by Barnes and Tabor^{118,120}

and other groups.^{116,119} In particular, both σ_p and E_* increase as the temperature decreases and are of similar magnitude for most relevant temperatures and deformation frequencies.

Let us now consider the limit $d \ll R$. In this case, if the width of the plowing track is $2r_0$, the projected contact area is $A = \pi r_0^2/2$. The plowing area A_{plow} during slip is the area of the segment of the circle penetrating below the ice surface and is given approximately by $A_{\text{plow}} \approx 4r_0 d/3$. Thus, the friction coefficient becomes

$$\mu = \frac{A_{\text{plow}}}{A} = \frac{4r_0 d/3}{\pi r_0^2/2} = \frac{8}{3\pi} \frac{d}{r_0}.$$

For $d \ll R$, the penetration $d \approx r_0^2/2R$, so that

$$\mu = \frac{4}{3\pi} \frac{r_0}{R}.$$

The normal force is $f_N = (\pi r_0^2/2)\sigma_p$, and using this in (39) gives $r_0/R = (2\sigma_0/\psi\sigma_p)^{1/2}$, resulting in

$$\mu = \frac{4\sqrt{2}}{3\pi\sqrt{\psi}} \left(\frac{\sigma_0}{\sigma_p} \right)^{1/2} \approx \frac{0.6}{\sqrt{\psi}} \left(\frac{\sigma_0}{\sigma_p} \right)^{1/2}. \quad (44)$$

Note that the friction coefficient is independent of the particle radius and decreases with increasing particle concentration as $1/\sqrt{\psi}$. This result is only valid as long as $\psi \gg \psi_0$.

Sanding of road surfaces typically involves adding 150 g/m² of sand particles with a typical particle radius of 1 mm. Using the sand mass density 2.6 g/cm³, this corresponds to a volume per surface area $\rho \approx 58$ cm³/m². Using $\psi = 3\rho/4R$, this gives $\psi \approx 0.043$. If $\sigma_0 = 0.2$ MPa and $\sigma_p = 10$ MPa, this is close to the optimum particle coverage where the friction $\mu \approx 1$.

However, if the friction were this large, it cannot be excluded that the rubber would slip relative to the sand particles instead of maintaining the stick condition assumed in deriving (39). In any case, the exact conditions in terms of particle concentration required for (39) to hold will never be perfectly satisfied in practical applications, where the particle concentration varies along the sliding track. In particular, if ψ is close to ψ_0 , stochastic fluctuations in the number of particles in the tire-road footprint may result in some particles being exposed to squeezing forces f_N large enough to be pressed fully into the ice surface rather than contributing to plowing. This can explain why the friction in most cases is lower than unity, e.g., ≈ 0.3 in the study reported in Fig. 66.

In addition to the plowing contribution to the friction force, there will be a viscoelastic contribution from ice height fluctuations on length scales longer than the size of the sand particles [see Fig. 65(c)]. This arises from the pulsating fluctuations that such long-wavelength roughness exerts on the rubber surface. This contribution to the friction force may be rather small, particularly if the sand particles are large.

Sanding with a high concentration of sand particles, $\psi > 0.2$, may result in friction smaller than that on a clean surface, unless the temperature is close to the ice melting temperature (see Fig. 66). This is consistent with some experimental observations, where the friction level of rubber on sanded ice was found to be lower than that on clean ice.^{122,123} Norwegian airport maintenance personnel have also pointed out that, in some cases, sanding has a negative

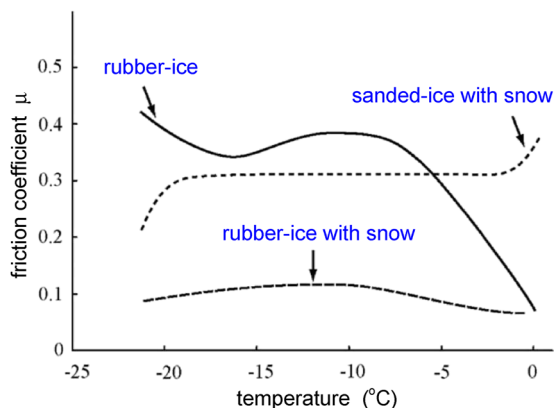


FIG. 66. Measured rubber friction coefficient as a function of temperature for rubber sliding on clean ice, on ice with a thin snow film, and on ice covered by a thin snow film and sand. The friction was measured using the British Pendulum Tester (sliding speed on the order of ~ 1 m/s); the figure is adapted from Ref. 121.

effect on the friction level, and they commonly refer to it as a “ball-bearing” effect.^{121,124} (However, there is no general reason for the sand particles to roll on the ice surface.)

This behavior is expected when the sand particle concentration is high because in this case, (44) predicts $\mu < 0.2$ if $\psi > 9(\sigma_0/\sigma_P)$, and using $\sigma_0 = 0.2$ MPa and $\sigma_P = 10$ MPa gives $\psi > 0.18$. Since sanding removes the adhesive contribution to friction and the viscoelastic contribution from roughness with wavelengths shorter than the sand particles, this may result in a friction coefficient smaller than that observed on clean ice for $T < -5^\circ\text{C}$.

Finally, we note that ice wear tracks have been observed even when rubber slides on clean ice surfaces.^{125,126} In Ref. 125, it was suggested that the narrow wear tracks observed may have resulted from contamination particles (dust), which cannot be avoided when performing experiments in the normal atmosphere. However, in Ref. 126, it was suggested that hard filler particles on the rubber surface scratch the ice surface and result in the strongly modified (smoothed) ice surface observed after repeated sliding on the same area. In addition, the stress in rubber asperities that are strongly deformed (strain of order unity) is on the order of the modulus E^* ,

which can be similar to the yield stress of ice. Hence, wear tracks may arise if large enough rubber asperities occur, even without considering the fillers.

C. Friction on snow

The friction of rubber on snow shares many similarities with that on ice, as snow can be regarded as loosely packed ice particles. However, snowflakes can have a complex dendritic structure or consist of small, nearly spherical ice grains, and these two types of snow may exhibit different frictional properties against rubber or other solids. Although dedicated studies on the frictional behavior of rubber on snow surfaces remain limited (mostly for tires),^{127–130} there are many studies of the friction of polymer sliders (skis) on snow.^{131–133} This field was pioneered by Bowden and Hughes,¹³⁴ who used skis to investigate friction on snow and found characteristics similar to friction on ice, although they did not propose a specific model for snow friction. For stiff solid sliders such as skis, Colbeck^{135–139} identified the key components of snow friction as plowing, direct solid–solid contact (i.e., adhesive friction), meltwater lubrication due to frictional heating, and capillary forces from liquid bridges in partially melted snow.

Some of these mechanisms for friction between snow and hard polymers may also apply to rubber materials, but additional considerations are required due to their viscoelastic nature (see Fig. 67). Any fluctuation in the height of the compacted snow layer that is not flattened in the tire footprint could generate a viscoelastic contribution to the friction during slip. However, if the frictional shear stress exceeds the shear yield strength of ice, the compacted snow may instead undergo internal shear deformations, which would also contribute to the friction force.

The shear strength of snow compressed by a normal stress σ has been studied experimentally. Figure 68 shows results for the shear stress immediately after compression (without aging), as relevant for tire applications. The shear stress increases nearly linearly with the applied pressure and is of similar magnitude to the normal stress. This implies that if the sheared area were similar in size to the nominal tire-road contact area, a friction coefficient of order unity could be expected. In reality, the area where the shear plane lies inside the compacted snow rather than at the rubber-snow interface may be much smaller than the nominal contact area. However, if the tire

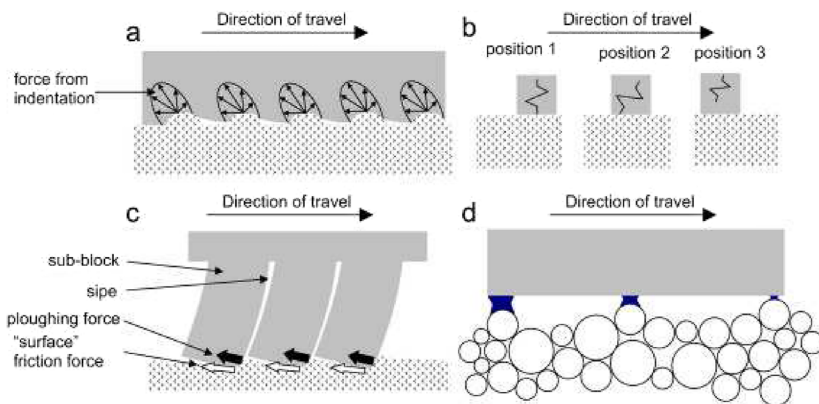


FIG. 67. Schematic illustration of the primary friction mechanisms of rubber on snow: (a) indentation-induced resistance due to the normal force from local deformation, (b) molecular adhesion through the formation and rupture of van der Waals bonds at the interface, (c) plowing force and interfacial (surface) friction generated by tread blocks and sipes interacting with snow, and (d) lubricating effect of a meltwater film, which increases in thickness with the residence time of snow grains in contact with the rubber; the figure is adapted from Ref. 131.

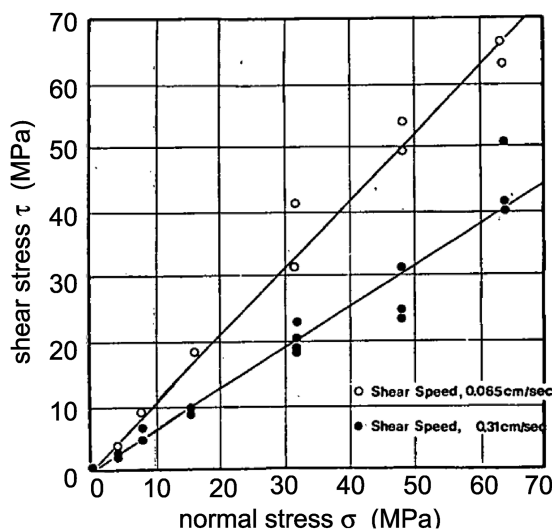


FIG. 68. Shear stress of snow as a function of the normal stress for the sliding speeds 0.65 and 3.1 mm/s; the figure is adapted from Ref. 140.

had a large concentration of cavities as in Fig. 69, and if the pressure between the snow and the rubber in the cavities were comparable with that in the nominal tire-road footprint elsewhere, then a large friction coefficient could be expected.

Overall, the friction of rubber on snow depends not only on the material properties but also on the snow morphology and water content (which is temperature-dependent).

- Wet snow or slush (snow mixed with water, salt, mud, etc.) can act as a high-viscosity fluid that cannot be squeezed out of the tire-road footprint even at relatively low speeds. Hence, it may act as a high-viscosity lubricant, which could result in viscous hydroplaning and low friction.
- On dry snow, friction arises in part from shearing snow contacts and in part from viscoelastic deformation of the rubber by inhomogeneities in the height of the compressed snow film. There is also a contribution from shearing the rubber-snow interface if slip occurs at this interface.

For rubber tires, tread design and sipes play a critical role. Compared to smooth surfaces, treads with sipes generate additional

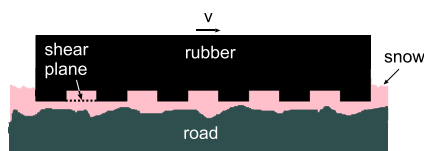


FIG. 69. If the normal pressure acting on the snow in the rectangular cavities of the rubber surface is similar to the nominal pressure acting in the tire-road footprint, then shearing the snow-snow interface as indicated by the dashed line could contribute to the friction coefficient. Depending on the slip velocity, this contribution could be of the order of the fraction of the nominal contact area covered by the cavities, e.g., 0.5 if the cavities cover 50% of the total area.

plowing at the edges of the bent rubber segments. In addition, compacted snow can enter the sipes, resulting in a snow-snow interface (of compacted snow) that is sheared during slip. These effects significantly enhance snow grip.^{127,129,130} Tread depth and void ratio also affect performance. Comparable observations have been reported for footwear on snow surfaces.¹⁴¹ Analogous to artificial tread structures, natural surface adaptations can also play a role in snow friction.

Recent experiments on bear paw pads have highlighted the importance of papillae morphology for frictional performance on snow.¹⁴⁴ Polar bears exhibit papillae that are significantly taller than those of brown and black bears, leading to a higher height-width (H/D) ratio. Friction tests with three-dimensional printed model surfaces demonstrated that shear stress on snow increases with increasing H/D , an effect comparable with the function of spikes in human footwear. As a result, polar bear paw pads achieve a 1.33–1.50 times higher shear stress compared with other bear species. Notably, polar bear paw pads also exhibit smoother roughness at short wavelengths. This is attributed to the presence of a thin water film at the ice-snow interface, which prevents roughness at these scales from penetrating the film and contributing to friction. Therefore, during evolution, polar bears did not develop pronounced short-wavelength roughness.

These observations emphasize the general principle that surface microgeometry, whether engineered in rubber treads or evolved in biological systems, can critically influence snow friction through its effect on interfacial shear stress.

Finally, we note that measurements of rubber-ice friction on ice surfaces with a low concentration of snow crystals or with a thin snow film show drastically reduced friction coefficients.¹⁴² Pilots have also reported that ice-covered runway surfaces are much more slippery when loose snow is present.^{121,124}

It is not clear how these results can be understood since ice sliding on ice generally gives relatively large friction coefficients, unless the temperature is close to the ice melting point or the sliding speed is so high that frictional heating becomes important.^{117,143}

X. SUMMARY AND CONCLUSIONS

This review has presented a comprehensive analysis of rubber friction on rigid, randomly rough surfaces, combining theoretical formulations and experimental comparisons.

Rubber friction has two primary contributions: viscoelastic energy dissipation due to time-dependent deformations induced by surface asperities, and adhesive contribution from the interfacial shear stress acting within the real area of contact. The analytical theory of rubber friction is established based on the Persson contact mechanics framework and provides quantitative predictions of frictional behavior over a wide range of conditions, incorporating the surface roughness power spectrum and the frequency-dependent viscoelastic modulus. When additional mechanisms such as strain softening and frictional heating are included, the theoretical predictions show good agreement with experimental results under stationary sliding conditions.

For non-stationary sliding (dynamic friction), other mechanisms become important and manifest as transient friction phenomena, such as breakloose friction. This can be explained by the interplay of two dynamic effects: the flash temperature, which

influences the mechanical properties in the local contact area, and elastic effects, which enable pre-slip and alter the frictional response compared to uniform slip across the entire contact area.

The rubber friction theory is also applicable to different geometries (rolling or sliding) and conditions (with or without third-body layers). Rolling friction for cylinders and spheres can be interpreted as a limiting case where the contribution from shearing the contact area is absent and friction arises solely from viscoelastic dissipation. A theory for rolling friction,⁷⁴ which is in good agreement with experimental data, was reviewed. This approach can also describe the sliding of solid objects with arbitrary shapes on rubber if the contribution from the frictional shear stress in the area of real contact can be neglected, as may be the case for lubricated surfaces.

Open problems include the role of adhesion-induced enhancement of the real contact area and energy dissipation due to crack opening, both of which may be important for surfaces with small roughness. For rough surfaces, there is still no definitive understanding of the origin of the short-distance cutoff to be used when calculating the viscoelastic contribution to the friction force. Future research should aim to integrate these factors, as well as other possible effects (e.g., wear), into a unified model supported by systematic experimental validation across different materials, geometries, and environmental conditions.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

B. N. J. Persson: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **R. Xu:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX: ADHESIVE CONTRIBUTION TO FRICTION FROM BINDING-BREAKING-BINDING CYCLES

Schallamach developed a fundamental theory for the adhesive contribution to sliding friction. This theory was originally presented as a model for the adhesive contribution to sliding friction, but is of more general validity. The basic idea is illustrated in Fig. 70.

Consider a solid block (green in Fig. 70) sliding on a substrate (velocity v). A molecular segment of a small (nano-sized) region of

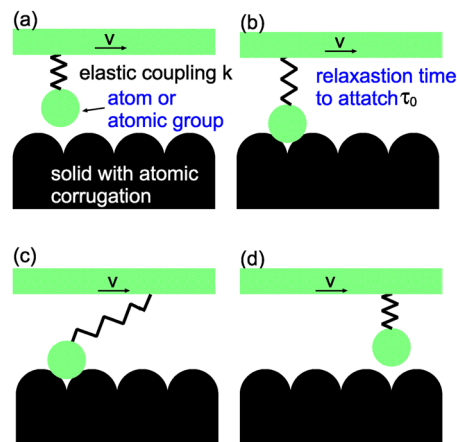


FIG. 70. Molecular segment of a small (nano-sized) region of the green solid is in a detached state (a), moves into contact due to thermal fluctuations (b), remains attached while the solid locally deforms elastically (c), and finally detaches when the force in the stretched “bond” becomes large enough (d). Detachment is a thermally activated and stress-aided process. The elastic energy stored in the stretched bond in panel (c) is dissipated as heat when the bond breaks.

the block, which we denote as an “atom” in what follows, is first in a detached state (a). To bring it into adhesive contact with the substrate, a free energy barrier must be overcome, which requires a sufficiently large thermal fluctuation. The average time that the atom spends in the detached state is denoted by τ_0 . After attachment, the atom remains attached for an average period τ_1 , while the solid locally deforms elastically (c). When the force or stress in the stretched “bond” becomes large enough, the bond breaks. This is a thermally activated and stress-aided process. When the bond breaks, the elastic energy stored in the stretched bond is dissipated as heat.

Consider sliding at speed v . The fraction of time that an atom spends in the attached state is $\tau_1/(\tau_0 + \tau_1)$. If $\langle f \rangle$ is the average (tangential) force of the stretched bond, then the average tangential force acting on the sliding block from the atom is $F = \tau_1 \langle f \rangle / (\tau_0 + \tau_1)$. If there are c atoms per unit surface area that can participate in the attach–detach process, then the frictional shear stress is

$$\sigma_f = \frac{c \langle f \rangle \tau_1}{\tau_0 + \tau_1}. \quad (\text{A1})$$

Let P be the probability for the “atom” to be in the attached state at time t , assuming $P(0) = 1$. We have

$$\frac{dP}{dt} = -wP, \quad (\text{A2})$$

where w is the probability per unit time for going from the attached state to the detached state.

Assume that the atom attaches to the substrate at time $t = 0$. The probability that it is still in the same attached state at time t is $P(t)$. Hence, the average time in the attached state is

$$\tau_1 = \int_0^\infty dt P(t). \quad (\text{A3})$$

If $f(t)$ is the force in the spring at time t , the average force during the attachment is

$$\langle f \rangle = \frac{1}{\tau_1} \int_0^\infty dt P(t) f(t).$$

If the binding potential keeping the atom attached to the substrate is short ranged, so that the displacement of the atom from the equilibrium position is small compared to the extension of the spring at the point of detachment, then the spring force at time t is $f(t) = kv t$, giving

$$\langle f \rangle = \frac{kv}{\tau_1} \int_0^\infty dt P(t) t, \quad (\text{A4})$$

which Schallamach approximated by $\langle f \rangle \approx kv \tau_1$. Using this in (A1) gives

$$\sigma_f \approx \frac{ckv\tau_1^2}{\tau_0 + \tau_1} = ckv\tau_0 \frac{(\tau_1/\tau_0)^2}{1 + (\tau_1/\tau_0)}. \quad (\text{A5})$$

The fraction N/N_0 of the total number of atoms N_0 that are in the attached state is

$$\frac{N}{N_0} = \frac{\tau_1}{\tau_0 + \tau_1} = \frac{(\tau_1/\tau_0)}{1 + (\tau_1/\tau_0)}. \quad (\text{A6})$$

The attachment [step (a) to (b)] may involve rearrangements of the atomic groups, which require overcoming a free energy barrier ε_0 . The attachment rate is

$$w_0 = v_0 e^{-\beta \varepsilon_0}, \quad (\text{A7})$$

which we denote by $1/\tau_0$. In (A7), $\beta = 1/k_B T$ and v_0 is the “attempt frequency,” which we treat as a constant. If ε_1 is the energy barrier to break the atom–substrate bond and $U(t)$ the elastic energy in the stretched bond, then

$$w = v_1 e^{-\beta[\varepsilon_1 - U(t)]}. \quad (\text{A8})$$

If $u(t)$ is the elongation of the spring k , then $U(t) = ku^2(t)/2$. If an atom attaches at time $t = 0$, then assuming a short-ranged interaction potential, $u(t) = vt$, giving $U(t) = k(vt)^2/2$. Schallamach instead assumed a linear time dependency, $U(t) = \lambda kv t$, where λ is a length of molecular dimension. This assumption simplifies the mathematical analysis but will not be used here.

Introducing

$$q(t) = \int_0^t dt' v_1 e^{-\beta[\varepsilon_1 - U(t')]},$$

we can write (A2) as

$$\frac{d}{dt} [P e^{q(t)}] = 0$$

or

$$P(t) = C e^{-q(t)}.$$

The integration constant C is determined by the initial condition $P(0) = 1$, which gives $C = 1$, and

$$P(t) = e^{-q(t)}. \quad (\text{A9})$$

Thus,

$$\tau_1 = \int_0^\infty dt e^{-q(t)}. \quad (\text{A10})$$

If we write $\zeta = w_0 t = t/\tau_0$, then (A10) takes the form

$$\frac{\tau_1}{\tau_0} = \int_0^\infty d\zeta e^{-q(\zeta)}, \quad (\text{A11})$$

where

$$q(\zeta) = Q \int_0^\zeta d\zeta' e^{R(\zeta')^2}, \quad (\text{A12})$$

with

$$Q = \tau_0 v_1 e^{-\beta \varepsilon_1} = \frac{v_1}{v_0} e^{-\beta(\varepsilon_1 - \varepsilon_0)} = \frac{v_1}{v_0} e^{-\beta \Delta E} \quad (\text{A13})$$

and

$$R = \beta \frac{1}{2} kv^2 \tau_0^2. \quad (\text{A14})$$

Writing $x = \zeta \sqrt{R}$ and $y = \zeta' \sqrt{R}$, from (A11), we get

$$\frac{\tau_1}{\tau_0} = \frac{1}{\sqrt{R}} \int_0^\infty dx e^{-q(x)}, \quad (\text{A15})$$

$$q(x) = \frac{Q}{\sqrt{R}} \int_0^x dy e^{y^2}. \quad (\text{A16})$$

We can write

$$ckv\tau_0 = \sqrt{R} \left(\frac{2kc^2}{\beta} \right)^{1/2}. \quad (\text{A17})$$

We define a reference velocity v^* as

$$\sqrt{R} = v/v^*. \quad (\text{A18})$$

Using this, we get

$$v^* = \left(\frac{2k_B T}{k\tau_0^2} \right)^{1/2}. \quad (\text{A19})$$

If we define the reference stress,

$$\sigma^* = \left(\frac{2kc^2}{\beta} \right)^{1/2}, \quad (\text{A20})$$

we can write (A5) as

$$\frac{\sigma_f}{\sigma^*} \approx \frac{v}{v^*} \frac{(\tau_1/\tau_0)^2}{1 + (\tau_1/\tau_0)}. \quad (\text{A21})$$

Let us apply the results above to rubber sliding on a hard counter-surface. If a small region (patch) of radius R of the rubber adheres to the substrate, the effective spring constant $k \approx ER$, where E is the elastic modulus. Using $c = 1/R^2$, this gives

$$\sigma^* \approx \left(\frac{2Ek_B T}{R^3} \right)^{1/2} \quad (\text{A22})$$

and

$$v^* = \left(\frac{2k_B T}{E R \tau_0^2} \right)^{1/2}. \quad (\text{A23})$$

Figure 71 shows the normalized frictional shear stress σ_f/σ^* , and the ratio N/N_0 of atoms in the attached state, as a function of the logarithm of the sliding speed for several different binding energies $\Delta E = \varepsilon_1 - \varepsilon_0$.

Let us estimate σ^* and v^* for rubber at $T = 300$ K. We assume that the adhesion involves the rubber bead unit adhering to the substrate so that $R \approx 1$ nm. Using (A22) with $E \approx 10$ MPa, we get $\sigma^* \approx 9$ MPa. Assuming that τ_0 is of the order of the shortest Rouse relaxation time, which is the relaxation time characterizing the movement of the bead units, typically 10^{-6} s, we get $v^* \approx 1$ mm/s.

Figure 72 shows that for $\Delta E/k_B T = 3$ (a typical bead–substrate binding energy resulting from weak van der Waals interactions), σ_f/σ^* is well-approximated by a Gaussian,

$$\frac{\sigma_f}{\sigma^*} = a \exp \left(-b \left[\log_{10} \left(\frac{v}{v^*} \right) \right]^2 \right),$$

with $a = 1.172$ and $b = 0.558$. The maximum of the frictional shear stress $\sigma_f/\sigma^* \approx 1$, and the position of the maximum is roughly at $v/v^* = 1$. Thus, at room temperature, we expect the adhesive friction coefficient to be a broad Gaussian as a function of $\log_{10} v$ with height

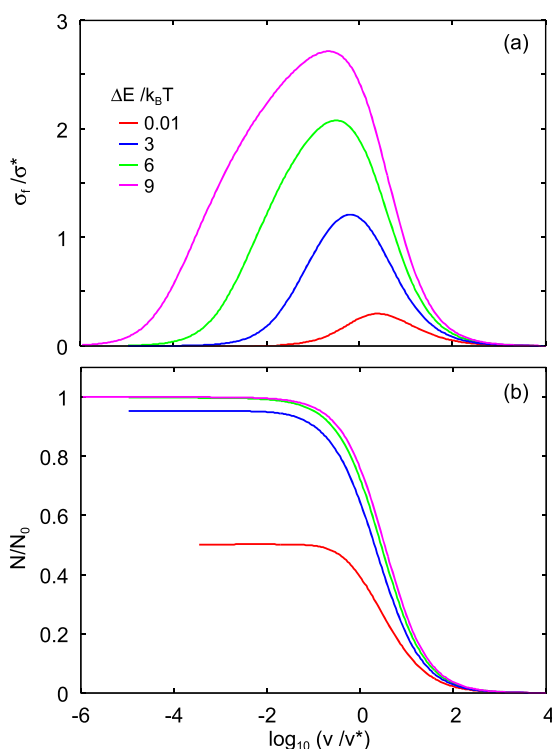


FIG. 71. (a) Normalized frictional shear stress σ_f/σ^* and (b) the ratio N/N_0 of the atoms in the attached state, as a function of the logarithm of the sliding speed. The results are shown for several different binding energies $\Delta E = \varepsilon_1 - \varepsilon_0$ and assuming $v_0 = v_1$.

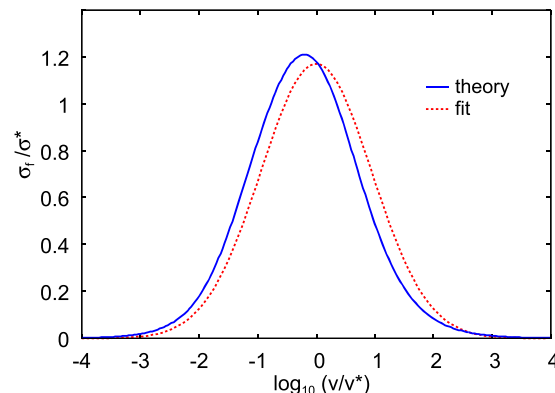


FIG. 72. Normalized frictional shear stress σ_f/σ^* as a function of the logarithm of the sliding speed for $\Delta E/k_B T = 3$. The blue curve represents the theory prediction (from Fig. 71), and the dashed red curve indicates a Gaussian fit function.

$\approx \sigma^* \approx 10$ MPa and centered at $v \approx v^* \approx 1$ mm/s. These results are consistent with experimental data, but the width (full width at half maximum) of the Gaussian is typically larger than predicted by the Schallamach theory (typically, ~ 4 velocity decades rather than ~ 2 as in Fig. 72). This can be explained by a more complete theory that accounts for the viscoelastic nature of rubber.²¹

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