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New physics in the muon magnetic moment?

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Abstract

The muon, a short-lived cousin of the electron, has provided a longstanding discrepancy between the standard model of particle physics and experimental measurements. This suggests that a not-yet-known particle or force perturbs the muon. The discovery of such "new physics" would have profound consequences on our understanding of Nature.

In 2021, the attention of the world was drawn to this discrepancy when the announcement of the independent confirmation of the experiment by Fermilab [1] coincided with the publication of our ab-initio calculation (Nature [2]). Our result dramatically updated the theoretical prediction, bringing it significantly closer to the experimental value: it may be possible to explain the Fermilab measurement without any new physics, even with the latest Fermilab update [3]. Our result established a new standard of precision for such calculations that has yet to be challenged; with uncertainties comparable to the experimental measurement and the reference, data-driven computations.

We are carrying out new simulations to reduce both the systematic and statistical uncertainties. Both improvements are required in order to match the precision of the final Fermilab measurement, to be obtained in the coming years. As such, these simulations will be critical to determine whether the muon's magnetic moment harbours new fundamental physics.

Keywords: particle physics; beyond Standard Model; lattice field theory; gpu acceleration

1. Introduction

The muon is an elementary particle, a short-lived cousin of the electron. For many years the calculation of its magnetic moment has disagreed with its measurement, suggesting that a not-yet-known particle or force perturbs the muon. Such a discovery would have profound consequences on our understanding of Nature. This is an extremely interesting time as there is a race between theoretical predictions and experimental findings, which might eventually lead to the discovery of a new interaction. If not, even that case would be an unprecedented triumph for modern

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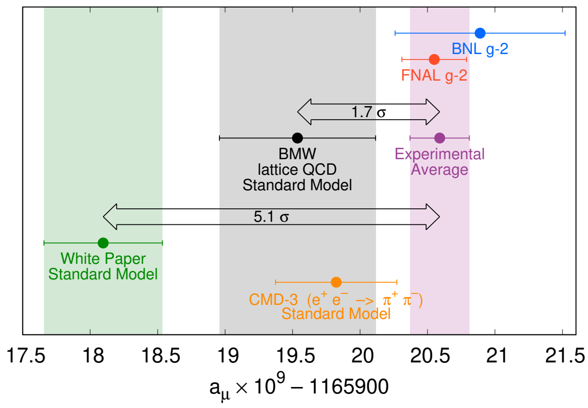

Fig. 1. Current status of theory and experiment for the anomalous contribution to the muon magnetic moment, denoted $a_\mu = \frac{1}{2}(g-2)$. Is there new physics or not? The blue dot shows the previous experimental result from Brookhaven National Laboratory (BNL) [4], while the red dot is the most recent result from the Fermilab experiment [3]. The purple band represents the current experimental average value. The green band represents the theoretical prediction given in the White Paper of the g-2 Theory Initiative [5], where the hadronic vacuum polarization (HVP) contribution is obtained using the data-driven approach. It has a 5.1σ discrepancy with the experimental value, indicating the existence of new physics, when taken at face value. The grey band shows the theoretical prediction, if the HVP contribution is replaced by the result of our lattice computation [2]. It reduces the tension between theory and experiment to 1.7σ. The orange point depicts the data-driven theoretical result, if the two-pion contribution to the HVP is taken from the recent CMD-3 measurements [6]. As of now it is not clear, how this new data should be included into an update of the theory prediction for the muon $g-2$.

physics, since the prediction of the Standard Model with three fundamental interactions would be proved up to an extremely high precision.

The experimental measurement at Fermilab National Laboratory is performing according to plan and in terms of precision has already outperformed the previous experiment from Brookhaven National Laboratory (BNL) [4]. As discussed above a new result from the Fermilab experiment was announced (called Fermilab/Run-2-3) [3]. These together with the earlier Fermilab (called Fermilab/Run-1) and BNL results are shown in Figure 1. In the next few years it is expected that Fermilab will reduce the uncertainty by a further factor of two. In addition, there is a facility under construction in Japan (called J-PARC) [7], which operates at a smaller beam energy but aims at the same precision.

The gold-standard of the theory computation is given by the White Paper of the g-2 Theory Initiative [5]. Its result is shown as the green dot and its error as the green band in Figure 1. This value differs from the combined Fermilab/Run1-2-3 and BNL experiments by 5.1σ, which is usually considered to be a very strong signal for physics beyond the Standard Model. As we discuss later the experiment did not announce this measurement as a sign of new physics, mainly because of our lattice result. The contribution to the Standard Model prediction with the largest uncertainty by far is the so-called hadronic vacuum polarization (HVP), shown in Figure 3. In the White Paper this is obtained from a phenomenological approach, sometimes called the R-ratio method. It uses a vast amount of data collected from electron-positron annihilation experiments.

There is an important new development in the direct measurements of the electron-positron cross-section. The CMD collaboration from Novosibirsk, Russia released their latest data (CMD-3) in the energy window 0.60–0.88 GeV for the two pion channel [6]. This channel provides approximately 70% of the hadronic vacuum polarization contribution into a_μ. The new result is shown in Figure 2, and it is in a strong, 4.4σ tension with the previous world average, which enters into the White Paper prediction. As of now it is not clear, how this new data should be included into an update of the theory prediction for the muon $g-2$. As can be seen in Figure 1, there is no tension with the direct magnetic moment measurement, if one takes the CMD-3 data at face value.

Finally, there is a soaring of lattice QCD determinations of the hadronic vacuum polarization contribution. These are ab-initio computations, and obviously do not rely on experimental determinations of the R-ratio. The first full

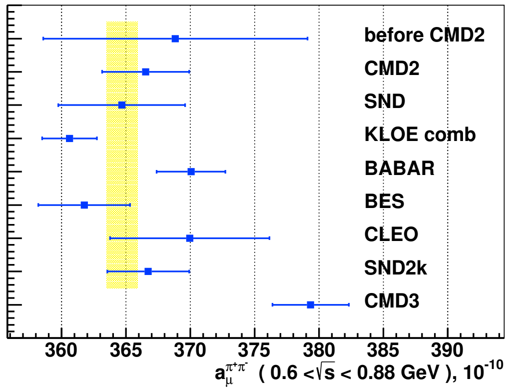

Fig. 2. Two-pion contribution to the hadronic vacuum polarization (HVP) obtained in different electron-positron annihilation experiments. Figure taken from [6].

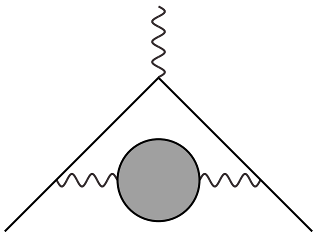

Fig. 3. The hadronic vacuum polarization (HVP) contribution to the muon magnetic moment: Before the muon (depicted as the straight line) interacts with the external magnetic field (shown as the vertical wavy line in the top of the figure), it emits a virtual photon (horizontal wavy line). This virtual photon polarizes the vacuum, temporarily creating the hadronic blob represented by the grey circle, then finally is recaptured by the leaving muon.

computation was done by our group and was published in Nature in 2021 [2]. This result is shown in Figure 1 by a black dot and its error by the grey band. It reduces the 5.1σ discrepancy to a mere 1.7σ.

To fully exploit the potential for new physics searches of the future Fermilab and J-PARC experiments, lattice computations must reach the same precision. To that end we have to have an even finer lattice than we have had until now: this is the aim of the current project.

2. Computation of the HVP contribution

The leading order, hadronic vacuum polarization (LO-HVP) describes how the propagation of a virtual photon is modified by the presence of quark and gluon fluctuations in the vacuum. The corresponding contribution to the muon magnetic moment, shown in Figure 3, dominates the uncertainty on the theoretical determination. Here we compute this LO-HVP contribution $a_\mu^{\mathrm{LO-HVP}}$, using ab initio simulations in quantum chromodynamics (QCD) and quantum electrodynamics (QED). In the present work, we include both QED and QCD in a lattice formulation, including four non-degenerate quark flavors (up, down, strange and charm). We also consider the tiny contributions of the bottom and top quarks.

In lattice quantum field theory a space-time grid is introduced, and both the fields as well as the equations of the theory are discretized. The resolution of the lattice is determined by the smallest distance a between neighbouring grid points, which is called the lattice spacing. The so obtained finite number of degrees of freedom are distributed on a supercomputer, and then the discretized equations of QCD and QED are solved on this lattice. In order to obtain results

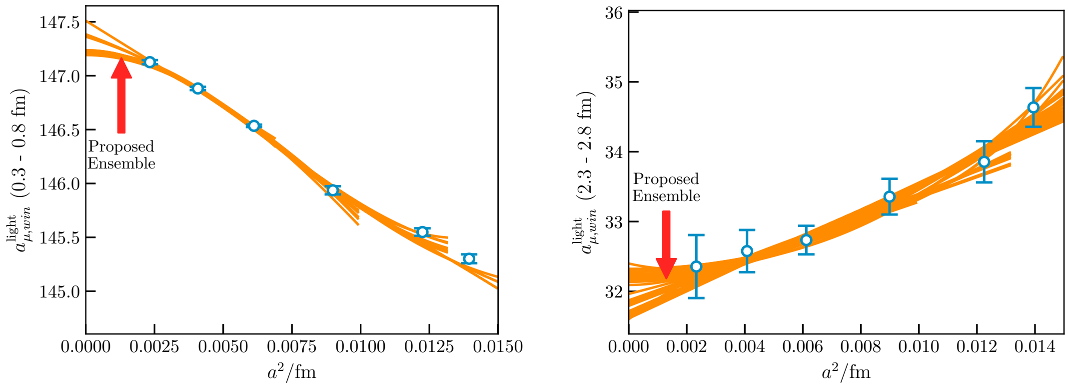

Fig. 4. Continuum limit in two different Euclidean time windows. On the left we have a short-distance contribution with no taste corrections applied. On the right we have a long-distance contribution with the SRHO taste correction from our previous work [2]. In both plots the orange curves show representative fits that contribute most strongly to the analysis. These curves are cut off where cuts have been applied to exclude the largest lattice spacings from the fit. The red arrows represent the position of the planned, additional lattice spacing, that helps reducing the uncertainty arising from the continuum limit procedure.

corresponding to the physical world, one has to take the continuum limit. This means that the computations have to be repeated at smaller and smaller lattice spacings, and the results have to be extrapolated to the $a = 0$ continuum case.

3. Continuum limit

The largest source of systematic error in our previous work [2] comes from the continuum extrapolation, 4.2 out of the total systematic error of 5.0, both in units of 10^{-10}. The continuum extrapolation error is obtained as a variation among various approaches to perform the continuum extrapolation. This includes changing the number of lattice spacings in the fit, changing the fit function from linear to quadratic in a^2, multiplying a^2 by powers of the strong coupling constant $\alpha_s(a)$, and applying different improvement procedures.

We can reduce the continuum extrapolation error by going to finer lattices. In a first step, we added a new lattice spacing $a = 0.048$ fm to our ensemble set, with lattice size $128^3 \times 192$. With previous computer time allocations we generated 5000 gauge configurations, a large part of which is already analysed for the hadron spectrum, a gradient flow scale and the current-current correlator, which gives the HVP contribution to a_μ. We are in the process of analysing these new data and preliminary results were reported at the lattice conferences in 2022 and 2023.

In the present phase of the project we are adding an even finer ensemble, $a = 0.036$ fm, with a $176^3 \times 264$ lattice size. This 30% decrease from our currently available finest lattice spacing will reduce the lattice artefacts by about 70% and will give an additional lever arm to our a^2 continuum extrapolations. With this new ensemble we will be able to use five different lattice spacings finer than $a = 0.1$ fm, with up to cubic continuum extrapolations in the a^2 scaling variable.

In Figure 4 we show the continuum extrapolation of the contribution to the full calculation within two different length scales. On the left, we show a short-distance contribution, where systematic errors dominate, and on the right a long-distance contribution which includes a model correction, and has larger statistical errors. In both cases, all existing lattice spacings including the new $a = 0.048$ fm are shown, and the proposed new lattice spacing is indicated by the red arrow. The location of the proposed point within a region where the representative fits (orange curves) are spreading out indicates that the new data at this point is expected to be very useful in constraining these fits.

4. Intermediate window

The RBC/UKQCD collaboration defined a particularly useful observable $a_{\mu,\rm win}$, in which the current propagator is restricted to an intermediate Euclidean time window using a smooth weight function [8]. The advantage of $a_{\mu,\rm win}$,

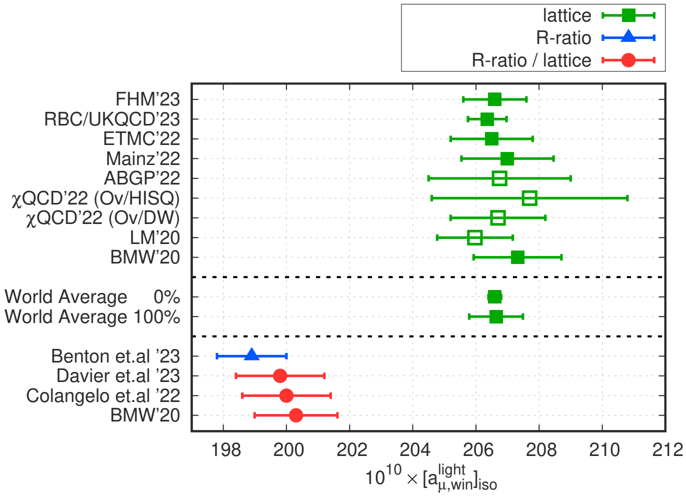

Fig. 5. Isospin-symmetric, light, connected component of the window observable, $[a^{\text{light}}_{\mu,\text{win}}]_{\text{iso}}$. The green squares denote lattice results. From each group only the most recent values are shown: FHM'23 [12], RBC/UKQCD'23 [13], ETMC'22 [14], Mainz'22 [15], ABGP'22 [16], χQCD'22 with overlap valence on HISQ and overlap valence on domain wall configurations [17], LM'20 [18], and our result BMW'20 [2]. The data points with filled squares give a set of independent results: each of these are obtained using a different set of configurations. The points World Average 0% and 100% represent their weigthed average, assuming 0% and 100% correlation between their systematics, respectively. The red dots show the R-ratio/lattice based determinations of Davier et.al. [10], Colangelo et.al. [9] and BMW'20 [2]. They are obtained by combining results from the data-driven approach for $a_{\mu,\text{win}}$ and lattice results for the non-light-connected contributions, as described in [2]. The blue triangle shows the R-ratio based determination of Benton et.al. [11]. This result is 6.9 sigmas away from the averaged lattice result assuming 0% correlation, and 5.6 sigmas away from the lattice average assuming 100% correlation.

which gives roughly 1/3 of the total HVP contribution, over a_μ, is that its calculation on the lattice is much less challenging than that of the full a_μ. The restriction to the intermediate window eliminates both the short-distance region, where large cutoff effects are present, and the long-distance region, where the statistical uncertainties and finite-size effects are large. Moreover, in the case of staggered fermions, the taste-breaking artefacts are reduced as well. This makes $a_{\mu,\text{win}}$ a strong benchmark quantity, allowing the comparison of the results of different lattice groups already at an early stage. The most recent value from each lattice collaboration is collected in Figure 5. As can be seen, all the lattice results agree nicely, and confirm our results of [2].

On the other hand, $a_{\mu,\text{win}}$ can be computed in the R-ratio approach as well [2, 9, 10, 11], which can then be compared to the values obtained on the lattice, providing a sharp comparison of the two methods. As Figure 5 shows, the R-ratio based determinations are in a strong tension with the lattice computations.

5. Code performance

In this project we use `dynqcd`, which has been developed continously for more than a decade. It includes state-of-the-art algorithms and has been deployed to several different architectures. We have optimized the code in a variety of ways:

- **Vectorization.** We designed our code from the beginning to enable auto-vectorization of time critical functions by the compiler. For this we fuse N sites in a given direction (`sitefusedir`) into a vector structure and all lattice fields are built up from such vectors. The GCC compiler does a good job when working with such structures: it automatically uses the SSE or AVX registers for operating on them.
- **Threading.** Currently the loops can be either parallelized using threads from the `pthreads` library or by OpenMP directives.

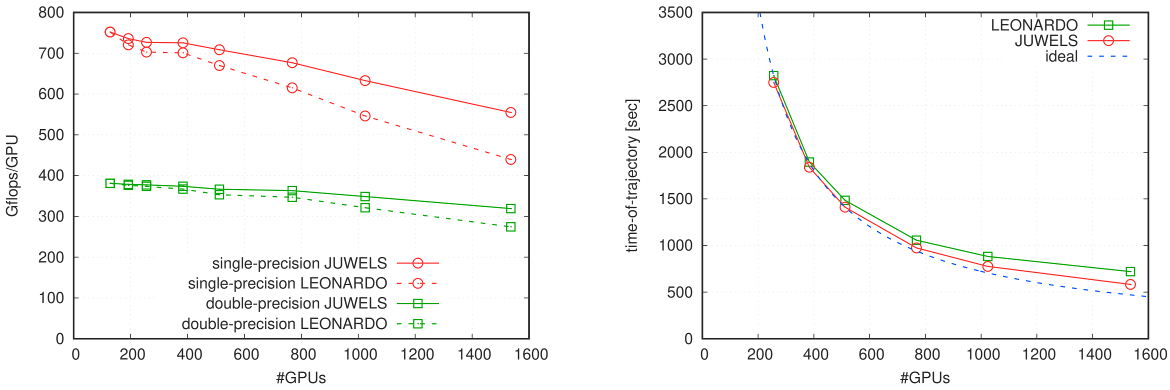

Fig. 6. Code performance as the function of the number of GPUs. The plots show strong scaling on a $176^3 \times 264$ lattice. Left: Gflops/GPU in multishift conjugate gradient iterations. Measurements performed on JUWELS Booster are connected with a solid line, estimates for the Leonardo Booster are connected with a dashed line. Right: time to solution in seconds, measurements on JUWELS and estimates on Leonardo are shown with different colors.

- **Communication.** The standard way of doing inter-node communication uses the MPI library. The code currently can split three out of the four directions for communication. We have also implemented the one-sided communication routines of the Global Address Space Programming Interface (GASPI).

We work at fixed lattice size, so only the strong scaling is relevant for our purposes. On the left panel of Figure 6 we show the performance of the multishift conjugate gradient inverter, which runs 90% of the time in the simulation. The numbers are obtained on JUWELS Booster, where we divided the lattice among the MPI processes to minimize the communication surface.

The local lattice size is large enough, so that the communication should not be an issue at least on the smaller partitions. This is also demonstrated by the fact that the performance numbers decrease very slowly with increasing the partition size, see the left panel of Figure 6.

Leonardo Booster nodes have half the number of network cards of that of the JUWELS Booster nodes. Therefore, we assume that the communication loss on Leonardo Booster is twice as large as on JUWELS Booster. This is how we obtained the numbers corresponding to Leonardo on Figure 6, denoted by dashed lines.

We also performed measurements on JUWELS, where we divided the lattice in a way to double the communication surface compared to the minimal case. In the case of the 384 and 512 GPU runs we saw a drop in performance below 1%. These are better than our estimates for the communication loss above.

6. Conclusions

This long term project was started nine years ago with the target to compute the leading order hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon, $a_\mu^{\mathrm{LO-HVP}}$. The project had two important milestones.

At the first milestone we were able to reach a relative precision of 2.7% and obtained the result

$$a_\mu^{\mathrm{LO-HVP}} = 711.0(7.5)(17.3)[18.9] \tag{1}$$

with statistical, systematic and total errors. This was published in Physical Review Letters [19] and the article was highlighted as an Editors' Suggestion.

At the second milestone we were able to improve the precision by several means and obtained the result

$$a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]\ . \tag{2}$$

The relative uncertainty is 0.8%, which is still far less than that of other lattice determinations, and is comparable to the errors of R-ratio based computations. The result was published in Nature [2] and was selected with the experimental measurement as one of the 10 ”Breakthroughs of the Year 2021” in all science by the journal Science.

Lattice field theory has now important challenges ahead. We have to explain the difference between the R-ratio method and our lattice determination. This is obviously necessary before drawing any conclusion about physics beyond the Standard Model in the muon magnetic moment. This will be our third milestone. We also have to increase our precision by a factor of four to reach the precision of the future Fermilab experiment, which will be the fourth and final milestone of this project.

7. Acknowledgements

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