



Introduction to Quantum Computing

**Invited Lecture at the Escuela Técnica Superior de Ingeniería de Caminos, Canales y Puertos,
University of Castilla–La Mancha, Spain**

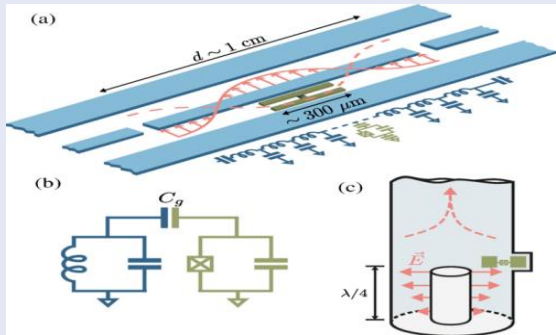
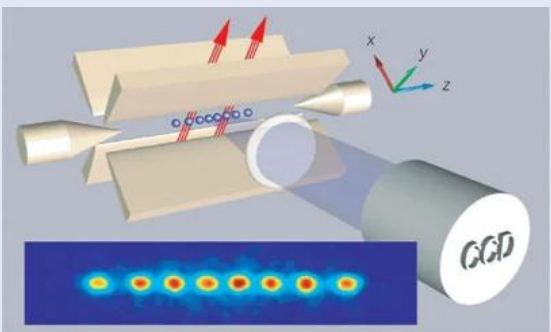
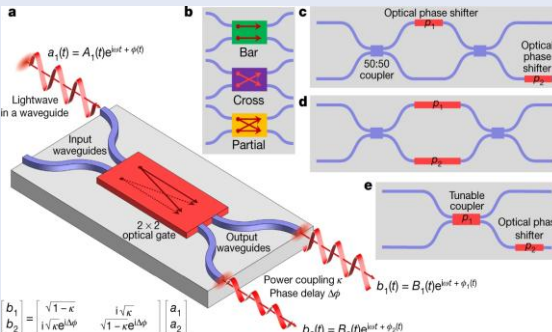
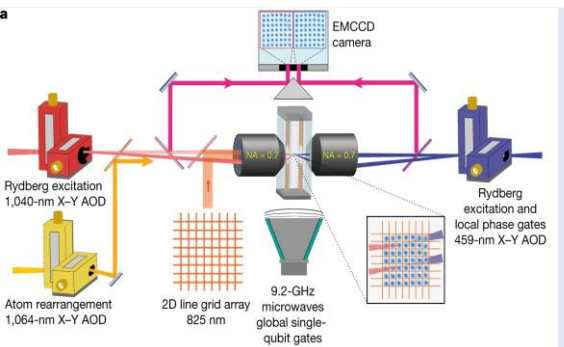
July 15, 2026 | Dr. Yanjun Ji | Jülich Supercomputing Centre (JSC), Forschungszentrum Jülich, Germany

Quantum applications and physical realizations

Quantum computing is an emerging field that leverages principles of quantum mechanics, offering the potential to solve certain complex problems significantly faster than classical computers.

Quantum Optimization	Quantum Chemistry	Quantum Machine Learning
- Portfolio Optimization - Traffic Flow Optimization ...	- Material Design - Drug Discovery ...	- Neural Networks - Support Vector Machines ...

Physical realizations of quantum computers

Superconducting Circuits	Trapped Ions	Photonic	Neutral Atom
 Rev. Mod. Phys. 93, 025005 IBM, Google, ...	 Nature 453, 1008–1015 IonQ, Quantinuum, ...	 Nature 586, 207–216 Xanadu, ...	 Nature 604, 457–462 Pasqal, QuEra, ...

Outline

1. Computation as a Physical Process
2. The Quantum Bit
3. Quantum Circuits
4. Near-Term Quantum Computing
5. From Noisy Qubits to Fault-Tolerant Computation
6. Summary and Outlook

Computation is a controlled physical process

Classical computation

bits + reversible / irreversible logic

Quantum computation

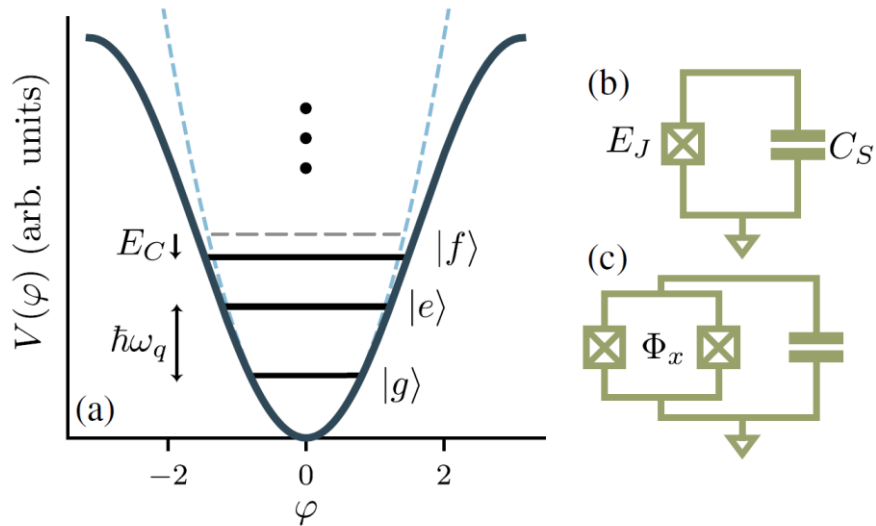
state preparation + controlled
evolution + measurement

- Information is always embodied in a physical system.
- Classical and quantum carriers obey different transformation and measurement rules.
- A quantum computer is a different physical model of computation.

Physics determines the state space, controllable transformations, measurement rules, and noise.

A qubit is a controllable two-level system

Superconducting transmon



Blais et al. Rev. Mod. Phys. 93, 025005

Quantum bit (qubit)

A qubit is the fundamental unit of information in quantum computing.

Computational basis:

$$|0\rangle = |g\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

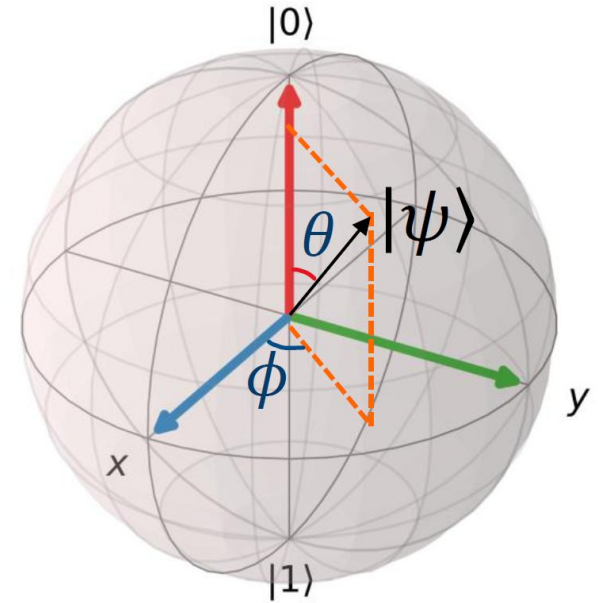
$$|1\rangle = |e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The Bloch sphere visualizes one pure qubit

Pure single-qubit state

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

- $\theta \in [0, \pi]$ controls measurement bias in the computational basis
- $\phi \in [0, 2\pi)$ is a relative phase



Useful for one pure qubit; it does not visualize a generic many-qubit state.

Measurement turns amplitudes into classical outcomes

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1 \quad \xrightarrow{\text{measure } Z} \quad \begin{aligned} p(0) &= |\langle 0|\psi\rangle|^2 = |\alpha|^2, \\ p(1) &= |\langle 1|\psi\rangle|^2 = |\beta|^2. \end{aligned}$$

One ideal projective shot

returns one classical outcome

post-measurement state: $|0\rangle$ or $|1\rangle$

Repeated independent shots

estimate probabilities or expectation values

standard sampling uncertainty decreases only statistically
($\propto 1/\sqrt{\text{shots}}$)

Measurement is the standard quantum-to-classical interface.

Single-qubit gates

Quantum gates implement unitary transformations

Pauli-X flips the state of the qubit from $|0\rangle$ to $|1\rangle$ and vice versa:

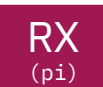
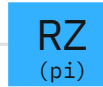
$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

Hadamard gate creates a superposition state from a basis state:

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$

Gate type	Matrix	Symbol
Pauli-X (X)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	 
Pauli-Y (Y)	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	 
Pauli-Z (Z)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	 
Hadamard (H)	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	

- Quantum gates are **unitary matrices**: $U^\dagger U = I$, $U^\dagger := (U^T)^*$
- Multi-qubit gates act on **tensor-product state spaces**

Interference turns relative phase into a deterministic bit

Two states with identical Z-basis statistics — are they the same state?

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

$$Z \text{ measurement: } p(0) = p(1) = \frac{1}{2}$$

$$\text{Apply } H: |+\rangle \rightarrow |0\rangle$$

$$|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$$

$$Z \text{ measurement: } p(0) = p(1) = \frac{1}{2}$$

$$\text{Apply } H: |-\rangle \rightarrow |1\rangle$$

Phase is invisible in one basis, but controls how amplitudes recombine in another.

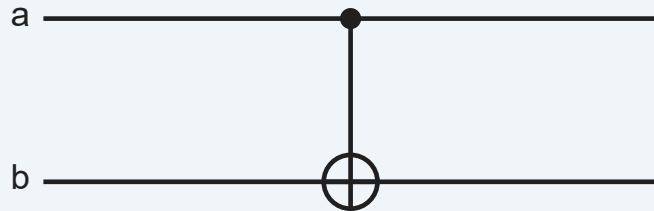
CNOT: controlled quantum logic

Two qubits live in $\mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$, basis $\{ |00\rangle, |01\rangle, |10\rangle, |11\rangle \}$; for n qubits, tensor-product composition gives a 2^n -dimensional state space $|\psi_n\rangle = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle$.

$$\text{CNOT } |a, b\rangle = |a, b \oplus a\rangle$$

a: control (unchanged)

b: target (flipped iff $a = 1$)



in	out
$ 00\rangle$	$ 00\rangle$
$ 01\rangle$	$ 01\rangle$
$ 10\rangle$	$ 11\rangle$
$ 11\rangle$	$ 10\rangle$

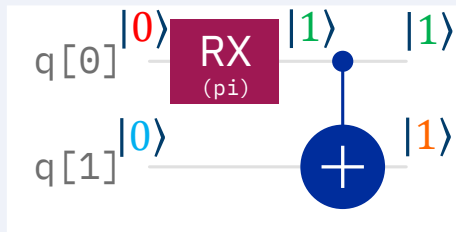
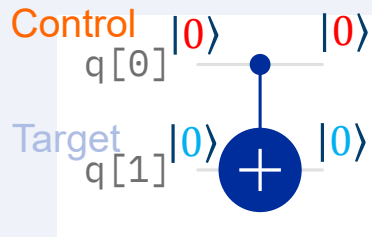
CNOT or CX can create entanglement from suitable product inputs.

Two-qubit gates

CX gate is crucial for creating entanglement:

$$\text{CX}|00\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle$$

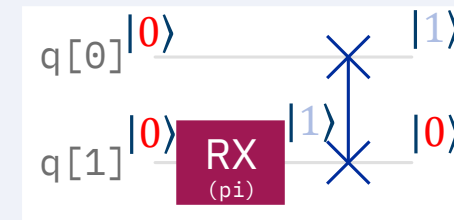
$$\text{CX}|10\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |11\rangle$$



SWAP gate swaps the states of two qubits:

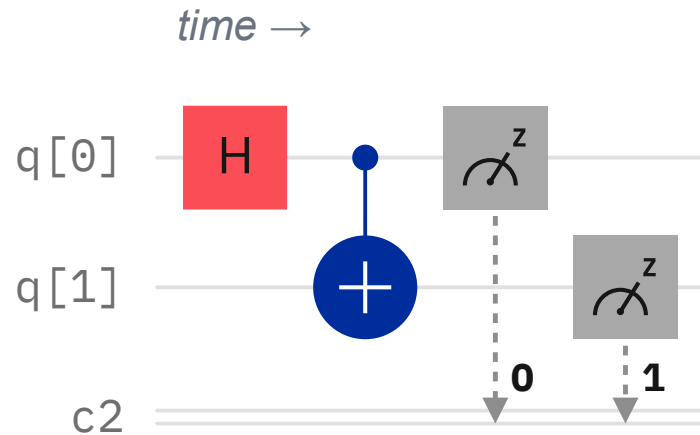
$$\text{SWAP}|01\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = |10\rangle$$

$$\text{SWAP}|10\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = |01\rangle$$



Circuit notation: wires, gates, and measurements

Bell state

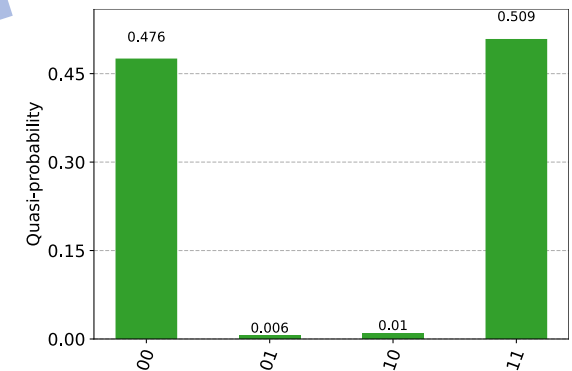
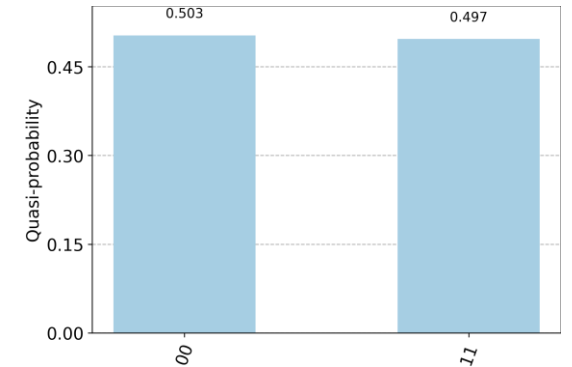
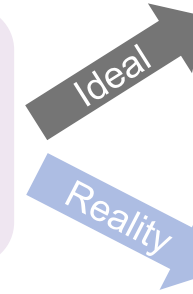


$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$ for any single-qubit states

Entanglement = non-product structure

$$H \otimes I |00\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle)$$

$$CX \left(\frac{1}{\sqrt{2}} (|00\rangle + |10\rangle) \right) = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$



- Each wire is one qubit evolving through time (left to right)
- Boxes are gates
- Measurements produce classical outcomes

Generic quantum states outgrow classical state-vector descriptions

Generic state-vector description

An n -qubit pure state $|\psi_n\rangle = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle$ needs 2^n complex amplitudes in the generic representation.

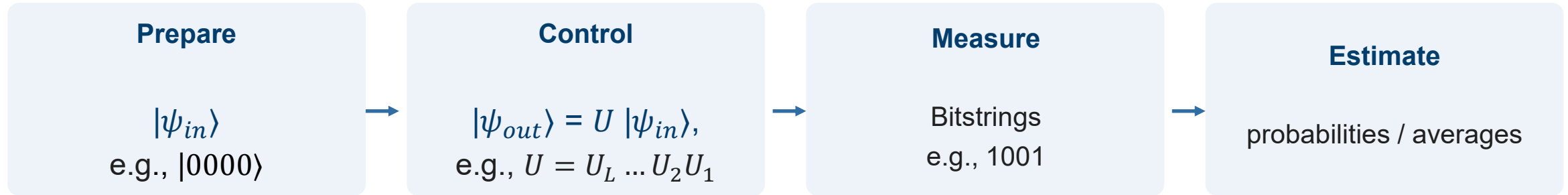
This is a description-size statement—not a universal hardness theorem.

30 qubits $\approx 10^9$ amplitudes

50 qubits $\approx 10^{15}$ amplitudes

300 qubits $\approx 10^{90}$ amplitudes

A quantum circuit is unitary linear algebra



Time-dependent Schrödinger equation

$$i\hbar \frac{d|\psi(t)\rangle}{dt} = H(t)|\psi(t)\rangle$$

Hardware controls the Hamiltonian $H(t)$; the circuit model packages the resulting evolution as local gates.

The difficulty is controlling high-dimensional coherent evolution physically.

Universality

Single-qubit gates + two-qubit entangling gates $\rightarrow$ universal gate sets (e.g., {H, T, CX})

A universal gate set is **sufficient for any quantum computation**, enabling a quantum computer to approximate arbitrary unitaries to desired precision.

Quantum algorithms exploit different kinds of structure

Simulation

evolve quantum systems

estimate energies and observables

Search

amplitude amplification

quadratic query improvement

Spectral structure

phase estimation
period and eigenphase information

Variational / hybrid

parameterized circuits

classical optimization loop

Sampling / estimation

return selected statistics

not the full quantum state

Every speedup is task- and model-specific; no family gives access to all 2^n amplitudes as classical answers.

An advantage claim needs an evidence label

Example	Formal or empirical status	Claim that is safe to make
Grover	Theorem in the black-box query model	Quadratic and optimal query improvement for unstructured search
Shor	Polynomial-time quantum algorithm	Classical factoring hardness is believed, not proved; fault tolerance is required
Quantum simulation	Problem-dependent complexity results	Preparation, observable, precision, and verification determine total cost
Sampling experiments	Complexity assumptions + experiment	A sampling separation is not automatically an application benefit
VQA / QML	Heuristic and empirical evidence	No generic scalable end-to-end advantage has been established

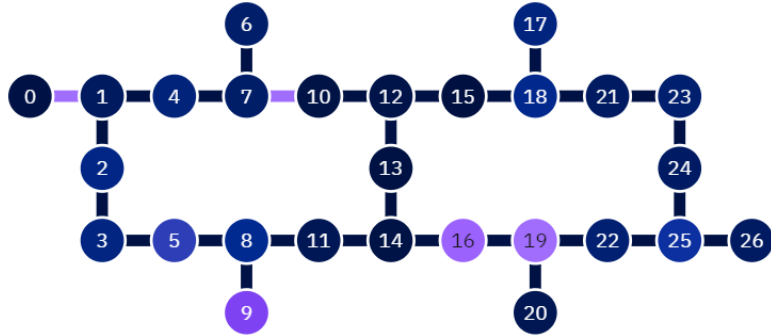
Fault tolerant algorithms

... designed to operate correctly even in the presence of errors.

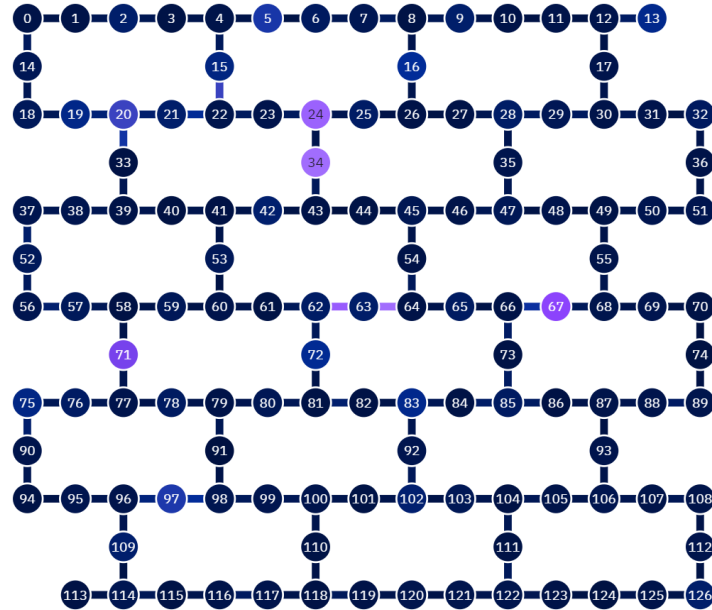
Dominantly near-term hybrid approaches

... designed for noisy quantum hardware using quantum–classical workflows.

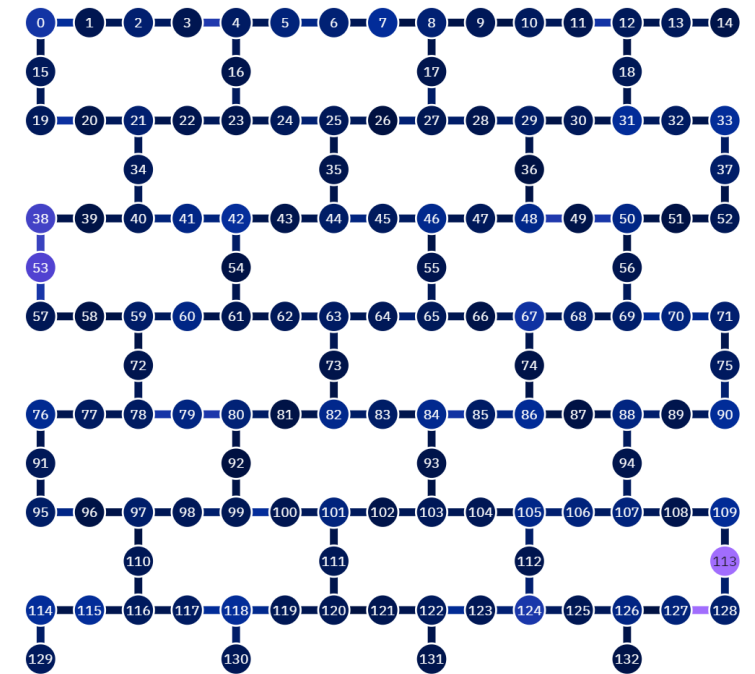
Near-term devices impose machine-specific constraints



QPU: ibm_cairo
Qubits: 27
Basis gates: {CX, ID, RZ, SX, X}

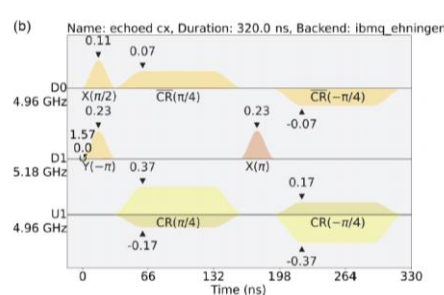
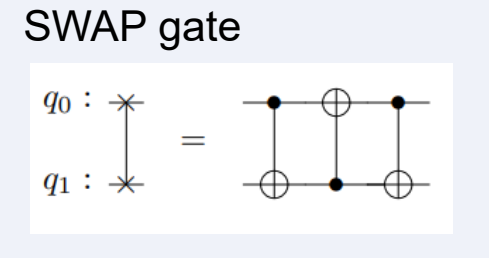
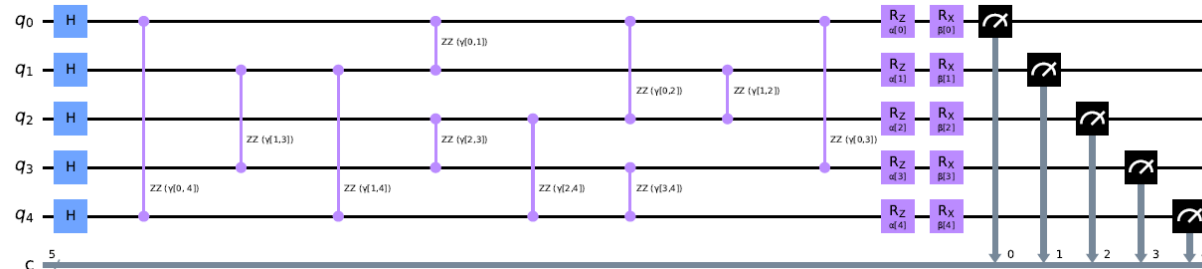
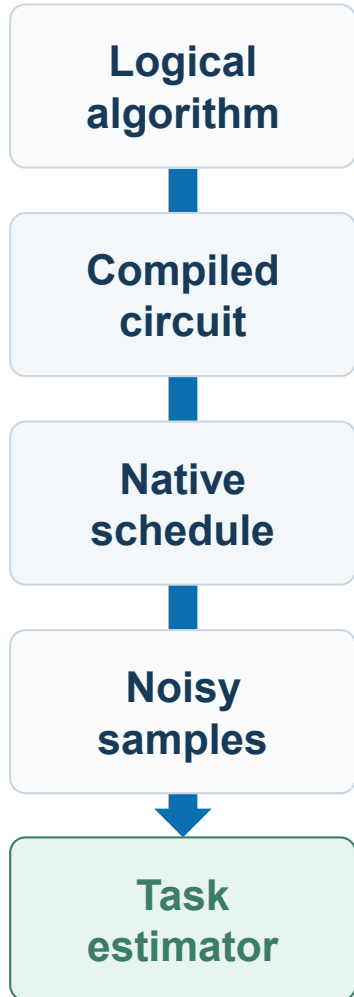


ibm_sherbrooke
127
{ECR, ID, RZ, SX, X}

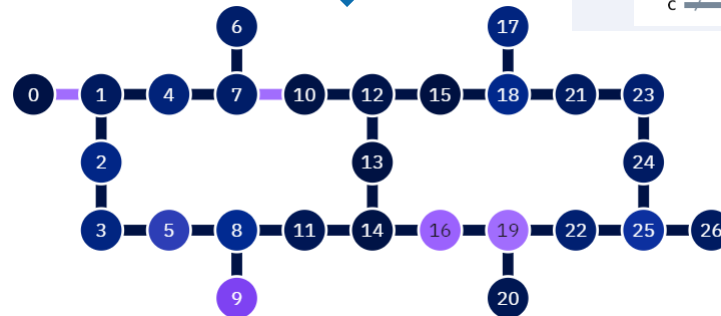
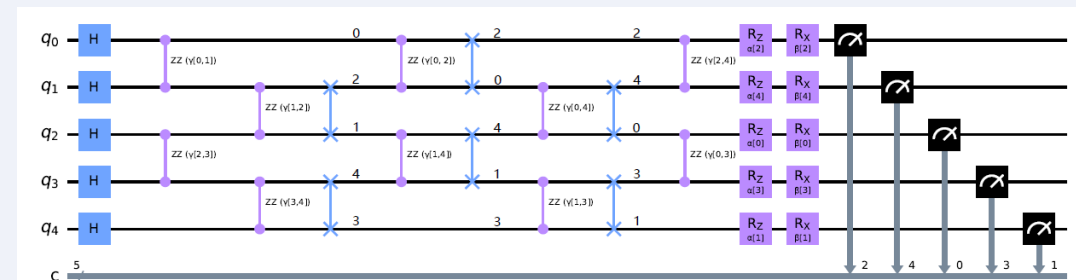


ibm_torino
133
{CZ, ID, RZ, SX, X}

Compilation turns abstract circuits into hardware workloads

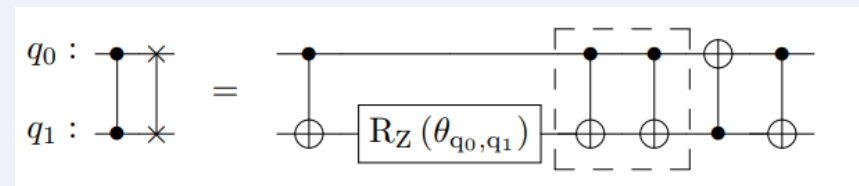


Routing



Sparse connectivity + {CX, ID, RZ, SX, X} + Calibration drift

Native-gate decomposition and optimization



Variational quantum algorithms

Quantum Approximate Optimization Algorithm (QAOA) [1] for quadratic unconstrained binary optimization (QUBO)

$$H_m = \sum_{i=1}^n X_i, \quad H_c = \sum_{i<j} J_{ij} Z_i Z_j + \sum_i h_i Z_i$$

where H_m is the mixer Hamiltonian and H_c is the problem/cost Hamiltonian.

Adiabatic Quantum Computation (AQC) [2]

$$H(t) = (1 - \lambda(t))H_m + \lambda(t)H_c$$

where $\lambda(0) = 0$, $\lambda(T) = 1$, and T is the total annealing time.

Trotterization

$$e^{-it\sum_{i=1}^L H_i} = \lim_{p \rightarrow \infty} \left(\prod_{i=1}^L e^{-\frac{iH_i t}{p}} \right)^p = \left(\prod_{i=1}^L e^{-\frac{iH_i t}{p}} \right)^p + \mathcal{O}(L^2 t^2 / p)$$

Trotter error

Here p denotes the product-formula step count. QAOA depth plays an analogous role in digitized-adiabatic viewpoints, but standard QAOA layers are variational rather than a fixed Trotterization.

[1] Farhi et al., arXiv:1411.4028 (2014)

[2] Albash & Lidar, Rev. Mod. Phys. 90, 015002 (2018)

Goal: Minimize the expectation value of the cost Hamiltonian

$$E = \langle \psi_{\vec{\gamma}, \vec{\beta}} | H_c | \psi_{\vec{\gamma}, \vec{\beta}} \rangle$$

with $|\psi_{\vec{\gamma}, \vec{\beta}}\rangle = \prod_{k=1}^p e^{-i\beta_k H_m} e^{-i\gamma_k H_c} |\psi_0\rangle$. Task score: objective value, approximation ratio (E/E_0), or success probability.

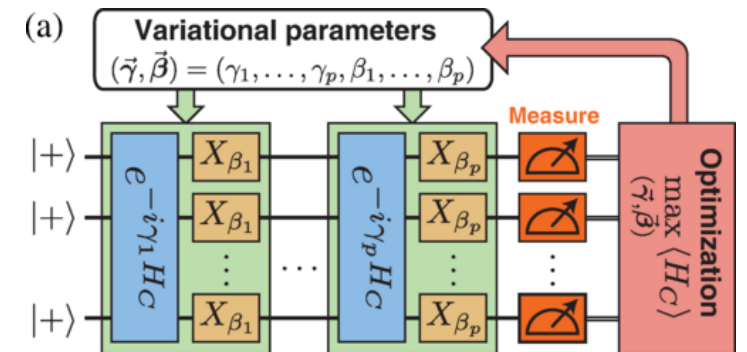


Image from Zhou et al., Phys. Rev. X 10, 021067

Noise changes unitary evolution into a quantum channel

Ideal circuit model

$$\rho \rightarrow U\rho U^\dagger$$

ideal closed-system model

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|, p_i \geq 0, \sum_i p_i = 1$$

Physical processor

$$\rho \mapsto \mathcal{E}(\rho)$$

Coherent errors

Systematic control
imperfections
Crosstalk

Incoherent errors

Random fluctuations
Depolarizing noise

Finite-shot uncertainty

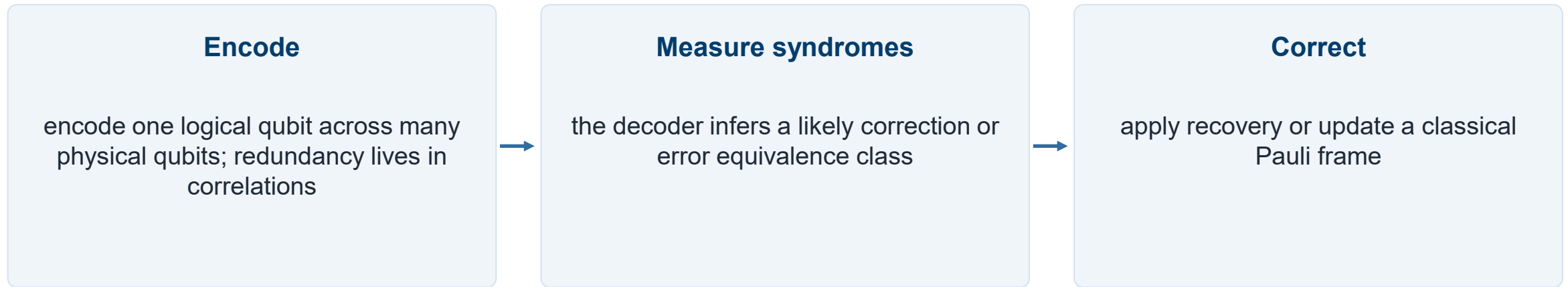
Probabilistic nature of
quantum mechanics

Reliability depends on the full circuit: fault locations, depth, coherent accumulation, and spatial or temporal correlations.

Scalable quantum computing is systems engineering: control + characterization + codes + decoding.

Quantum error correction (QEC) extracts error syndrome

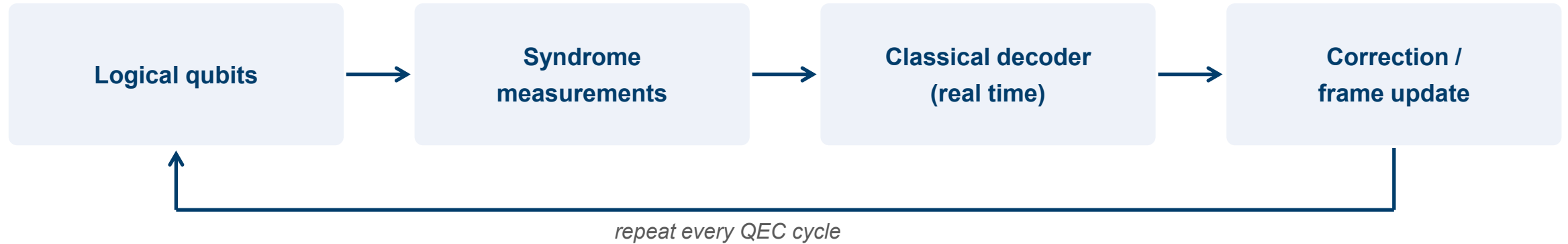
Encode correlations; measure errors indirectly; correct without learning α or β .



Syndrome extraction is indirect and repeated; it preserves the logical superposition.

QEC turns fragile quantum information into a monitored engineering object.

Fault tolerance protects the correction process itself



- Syndrome extraction, decoding, and frame updates repeat throughout the computation.
- Fault-tolerant operations prevent a single physical fault from spreading into an uncorrectable logical error.
- Threshold theorem: under specified local-noise and fault-tolerant-operation assumptions, sufficiently low physical noise permits arbitrarily long computation with scalable overhead.
- The threshold and overhead depend on code, operations, architecture, decoder, and noise model.

Summary and Outlook

- Information is physical; quantum computing is a distinct physical model of computation.
- Qubits encode amplitudes and relative phases; measurement reveals only selected classical statistics.
- Quantum algorithms engineer interference to expose problem structure—superposition alone is not enough.
- Quantum speedups are task-specific and depend on the full resource model.
- Near-term devices are constrained by noise, connectivity, native gates, compilation, and sampling costs.
- Scalable quantum computation requires fault-tolerant logical qubits and operations.

Closing thesis

Quantum computing = preparation + controlled quantum evolution + measurement.

The goal is to make one selected property influence a measurable distribution through interference and joint-state structure.

The frontier question is: for which tasks can this be done reliably, verifiably, and at lower total cost?

Resources

Nielsen & Chuang Quantum Computation and Quantum Information

Preskill's lecture notes preskill.caltech.edu/ph229/

IBM Quantum Learning learning.quantum.ibm.com — hands-on courses (successor of the Qiskit Textbook)

Further reading

1. Algorithms and advantage

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3. NISQ, hybrid methods, and software

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4. Error correction and hardware

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Google Quantum AI, Nature 638, 920 (2025)

Thank you!

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