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Flash temperature in sliding contacts: Comparing theory with experiments ^{EP}

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ABSTRACT

The temperature increase in the contact regions between solids in sliding contact has a huge influence on friction and wear. Here, we test an analytical theory for the flash temperature, valid for randomly rough surfaces with multiscale roughness, by comparing the theory predictions with the experimental results of Sutter and Ranc for steel sliding on steel. The theory, which is based on the study of stress and temperature correlation functions, is valid for randomly rough surfaces with roughness on an arbitrary number of decades in length scale. Within the uncertainty of the experimental data (mainly the surface roughness power spectrum and the steel penetration hardness), there is good agreement between the theory and the experimental results.

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I. INTRODUCTION

Friction between surfaces generates heat, leading to temperature increases at the contact points. This phenomenon is known as flash temperature, which is the high, localized, and brief temperature spike that occurs at the true points of contact between two rubbing solids. The rapid generation of heat at these contact points causes thermal spikes, resulting in intense flash temperatures that convert kinetic energy into thermal energy. These spikes can be extremely high, sometimes reaching over 1000 °C, but they are also incredibly brief, lasting only for the instant that the asperities are in contact. The process is so rapid that the generated heat has little time to conduct away into the bulk of the materials, trapping thermal energy and further elevating the temperature at the contact point.

In almost all cases, most of the dissipated energy in sliding friction ends up as thermal energy in the sliding solids. The temperature field in the solids can be written as $T(\mathbf{x}, t) = T_0(\mathbf{x}, t) + \Delta T(\mathbf{x}, t)$. The background temperature $T_0(\mathbf{x}, t)$ varies slowly in space and time, while the flash temperature $\Delta T(\mathbf{x}, t)$ varies very quickly in space and time. $\Delta T(\mathbf{x}, t)$ is non-zero only close to the asperity contact regions, so it is very localized in space.

Frictional heating is important in many applications, e.g., ice friction, rubber friction, and the friction between minerals. The flash

temperature can have a crucial influence on the friction force, usually reducing the friction force. This is the case for ice friction and for granite sliding on granite, where the granite (mainly quartz) melting temperature may be reached at the sliding speeds (on the order of ~1 m/s) involved in earthquakes. Rubber friction depends (exponentially) on the temperature, and an increase in the temperature shifts the friction coefficient master curve to higher sliding speeds, which usually reduces the friction but sometimes increases it. For rubber friction, relatively advanced theories for the flash temperature were developed in Refs. 1–3.

In Ref. 4, Rice studied the importance of flash heating on earthquake dynamics. He considered a model where the frictional shear stress is constant until the flash temperature reaches a temperature of the order of the pseudotachylite (rather than quartz or silica) melting temperature, after which the shear stress was assumed to be negligible. However, the high stresses and temperatures in the mineral contact regions are likely to strongly weaken the interface during slip already well before true melting occurs.⁵ Minerals like quartz, which are crystalline, may, during slip, become amorphous in the contact regions (quartz may be locally converted to silica) and will soften continuously with increasing temperature or sliding speed.⁵ This will result in a frictional shear stress that could decrease a lot even before true melting would occur, as also

observed for other crystalline materials, e.g., ice,⁶ which undergoes displacement-driven amorphization.⁷

One way the flash temperature can manifest itself in experiments is as sliding-induced phase transformations in materials. This was observed already in the classical studies by Bowden and Tabor and more recently in many other studies.^{8,9} However, in general, these transformations may be mechanochemical in nature, involving *both* the high contact stresses and the flash temperature. That is, the chemical or structural modifications result from stress-aided thermal excitations.

In this article, I test a recently developed multiscale theory for the flash temperature.¹⁰ The theory focuses on temperature-stress correlation functions, which contain information about the flash temperature. The correlation functions describe ensemble-averaged properties, i.e., averages over many realizations of the surface, where each realization is prepared in the same way experimentally or, if generated numerically, using the same procedure but with a different set of random numbers.

In the limiting case of roughness on one length scale, the theory is consistent with the classical theories of Jaeger, Archard, and Greenwood^{11–13} (see also Refs. 14–17). However, for multiscale roughness covering several decades in length scale, the classical theory fails severely.¹⁰

A multiscale flash temperature model was developed by Choudhry, Almqvist, and Larsson¹⁶ and tested in Ref. 18, but this approach is purely numerical and can only be applied to surfaces with roughness over a short wavelength region. The approach developed in Ref. 10 results in analytical equations that can be applied to systems with roughness over an arbitrary number of decades in wavelength. Here, the theory predictions are compared to experimental results for steel sliding on steel.¹⁹

In this study, all temperatures refer to the *increase* in temperature above the background temperature T_0 . Therefore, T_{flash} is a weighted average flash temperature, and the actual temperature in the contact regions is $T_0 + T_{\text{flash}}$. Similarly, temperature correlation functions like $\langle T(\mathbf{x})T(\mathbf{x}') \rangle$ are calculated with the background temperature T_0 subtracted from $T(\mathbf{x})$. Stated differently, all temperatures refer to actual temperatures if the background temperature vanishes.

II. MULTISCALE THEORY FOR THE CONTACT SPOT SIZE AND THE FLASH TEMPERATURE

In Refs. 10 and 20–22, we have derived analytical expressions for correlation functions involving the normal stress $\sigma(\mathbf{x})$ and the temperature $T(\mathbf{x})$. These results can be used to estimate the size of the contact regions^{21,23} and the temperature distribution in the contact region. Here, we summarize the most important results.

The stress–stress correlation function is defined by

$$g_\sigma = \langle \sigma(\mathbf{x}, t) \sigma(\mathbf{x}', t) \rangle, \quad (1)$$

where $\sigma(\mathbf{x}, t)$ is the stress at time t at the point $\mathbf{x} = (x, y)$ on the surface. Here, $\langle \dots \rangle$ stands for ensemble averaging. We focus on steady state, where g_σ is time independent. For surfaces with roughness with isotropic statistical properties, $g_\sigma(r)$ depends only on the separation $r = |\mathbf{x} - \mathbf{x}'|$. In Refs. 21 and 22, we have shown that

$$g_\sigma(r) = (E^*)^2 \frac{\pi}{2} \int_{q_0}^{q_1} dq q^3 C(q) W(q) J_0(qr), \quad (2)$$

where $E^* = E/(1 - \nu^2)$ is the effective modulus, J_0 is the zeroth-order Bessel function, $C(q)$ is the surface roughness power spectrum, and

$$W(q) = P(q)[\gamma + (1 - \gamma)P^2(q)],$$

where $\gamma \approx 0.4$ and

$$P(q) = \text{erf}\left(\frac{\sigma_0}{2\sqrt{G}}\right),$$

$$G = \frac{\pi}{4} (E^*)^2 \int_{q_0}^q dq q^3 C(q).$$

The surface has roughness components with wavenumbers $q_0 < q < q_1$, and the integral over q in Eq. (2) cover this wavenumber interval.

For a Hertzian-like contact with the diameter $2R$, the correlation function $g_\sigma(r)$ vanishes for $r > 2R$.²² For surfaces with multiscale roughness, the macroasperity contact regions consist, in general, of a compact central part surrounded by smaller nonconnected islands (see Fig. 1 and Refs. 22 and 24), which are elastically coupled. For this case, the $g_\sigma(r)$ function will have a tail extending beyond the diameter of the central compact region. For this case, one can define an effective diameter of the macroasperities by the condition $g_\sigma(r)/g_\sigma(0) = \alpha$, where $\alpha < 1$ is a small positive number, e.g., $\alpha = 0.1$.

A multiscale theory for the flash temperature was developed in Ref. 10, and here we review the most important results.

In applications to sliding friction and wear, it is the temperature in the asperity contact regions that matters, not the temperature distribution outside of the contact regions. One can define an effective flash temperature in the asperity contact regions using

$$T_{\text{flash}} = \frac{\langle T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle}, \quad (3)$$

where $\langle \sigma(\mathbf{x}) \rangle = \sigma_0$ is the nominal (or average) stress. Assuming that the frictional stress is proportional to the normal contact stress $\sigma(\mathbf{x})$, this definition of the flash temperature is optimal as it projects the temperature exactly on the regions that give the biggest contribution to the friction force. For viscoelastic materials like rubber, energy dissipation also occurs inside the material in the asperity

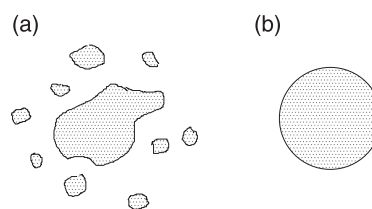


FIG. 1. (a) A macroasperity contact area and (b) the contact area in the Hertz approximation.

contact regions. However, the treatment presented in Ref. 10 can be generalized to this case too, as is discussed elsewhere.²⁵

Using the Persson contact mechanics theory, one can show that¹⁰

$$T_{\text{flash}} = \frac{\mu v}{\kappa \sigma_0} (E^*)^2 \int_{q_0}^{q_1} dq q^2 C(q) W(q) \times \text{Re} \int_0^{\pi/2} d\phi \frac{1}{[1 - i(v/Dq) \cos \phi]^{1/2}}, \quad (4)$$

where κ is the thermal conductivity and $D = \kappa/\rho C_p$ is the thermal diffusivity (ρ is the mass density and C_p is the heat capacity).

Equation (4), and more generally the theory developed in Ref. 10, assumes that the roughness occurs on one surface and that the other surface is smooth. If roughness of roughly equal amplitude occurs on both surfaces, then asperities will rapidly move in and out of contact during sliding. This will effectively result in time-dependent contact formation. In Ref. 26, I have generalized the theory of Ref. 10 to non-stationary sliding contacts.

For elastically stiff solids at not too high pressure, the contact involves the highest asperities, and then it is plausible that they stay in contact with the opposite surface for a sliding distance several times the diameters of the (macroasperity) contact regions, and then the theory of Ref. 10 should be approximately valid. However, more work on this topic is needed.

The temperature distribution in the macroasperity contacts during sliding is anisotropic and larger at the exit than at the front of the contact. This follows from the fact that the solid at the exit is already heated up by the contact with the asperity at the front of the contact. We can characterize this “tilt” of the temperature distribution using the correlation function,

$$\nabla T_{\text{flash}} = \frac{\langle \nabla T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle}. \quad (5)$$

Choosing the x axis along the sliding direction, only the x -component will be non-vanishing, which we write

$$T'_{\text{flash}} = \frac{\langle \partial_x T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle}, \quad (6)$$

where $\partial_x f = \partial f / \partial x$, and where the $'$ indicates the derivative of $T(\mathbf{x})$ with respect to x . We obtain

$$T'_{\text{flash}} = \frac{\mu v}{\kappa \sigma_0} (E^*)^2 \int_{q_0}^{q_1} dq q^3 C(q) W(q) \times \text{Re} \int_0^{\pi/2} d\phi \frac{i \cos \phi}{[1 - i(v/Dq) \cos \phi]^{1/2}}. \quad (7)$$

Since the flash temperature is higher at the exit than at the leading edge of the moving contact region, the temperature profile tilts upward toward the exit of the contact region. If the positive direction of the x axis is along the sliding direction, then $T'_{\text{flash}} < 0$ for a moving contact region, while for a stationary contact, by symmetry, $T'_{\text{flash}} = 0$.

It is easy to calculate similar averages of higher order derivatives of $T(\mathbf{x})$. The most important quantity is $\nabla^2 T_{\text{flash}}$, which can be used to estimate the size of the flash temperature hot spots. We define

$$\nabla^2 T_{\text{flash}} = \frac{\langle \nabla^2 T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle}, \quad (8)$$

we obtain

$$\nabla^2 T_{\text{flash}} = -\frac{\mu v}{\kappa \sigma_0} (E^*)^2 \int_{q_0}^{q_1} dq q^4 C(q) W(q) \times \text{Re} \int_0^{\pi/2} d\phi \frac{1}{[1 - i(v/Dq) \cos \phi]^{1/2}}. \quad (9)$$

III. SLIDING HERTZ CONTACT

When observed at low magnification, a macroasperity may appear smooth, and the contact region Hertzian-like; it is, therefore, interesting to correlate the results of the full theory in Sec. II with the known results for the sliding Hertz contact. For this case, we will also calculate $\langle \nabla^2 T \rangle$ and $\langle \partial_x T \rangle$, which have not been obtained before. We will only consider the case of stationary contact and sliding contact for high sliding speeds.

If we approximate a macroasperity contact region as a Hertz contact with the pressure distribution²⁷

$$\sigma(r) = \sigma_1 \left[1 - \left(\frac{r}{R} \right)^2 \right]^{1/2}, \quad (10)$$

then the heat source

$$\dot{q}(r) = \dot{q}_1 \left[1 - \left(\frac{r}{R} \right)^2 \right]^{1/2}, \quad (11)$$

where

$$\dot{q}_1 = \frac{3\dot{Q}}{2\pi R^2}, \quad (12)$$

where $\dot{Q}$ is the total power produced by the heat source. For a stationary contact, the heat diffusion equation can be easily solved. The temperature field for $r < R$ is given by

$$T(r) = T_1 \left[1 - \frac{1}{2} \left(\frac{r}{R} \right)^2 \right], \quad (13)$$

where $T_1 = \dot{q}_1 R \pi / 4\kappa$. Figure 2 shows $T(r)/T_1$ for this temperature distribution.

In what follows, $\langle \dots \rangle$ denotes integrating over the surface area. Using the results above, we obtain the weighted average temperature,

$$T_{\text{flash}} = \frac{\langle T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle} = T_1 \frac{\int_0^1 dr r (1-r^2)^{1/2} (1-r^2/2)}{\int_0^1 dr r (1-r^2)^{1/2}} = 0.8 T_1 = 0.3 \frac{\dot{Q}}{\kappa R}. \quad (14)$$

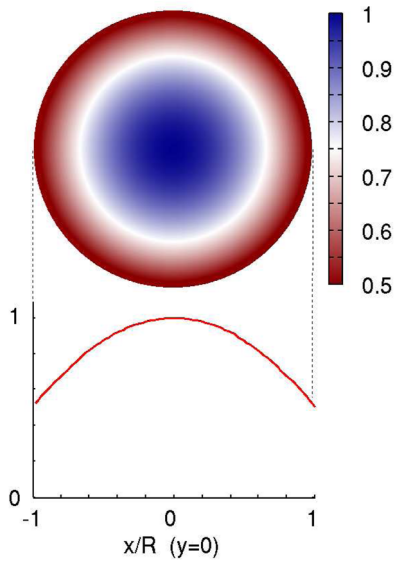


FIG. 2. Temperature distribution in a stationary Hertz contact. The top is an x, y 2D view, and the bottom curve shows the temperature profile through the centerline, $y = 0$.

The maximum temperature is

$$T_{\max} = T_1 = \frac{3}{8} \frac{\dot{Q}}{\kappa R} \approx 0.375 \frac{\dot{Q}}{\kappa R}, \quad (15)$$

and the average is

$$T_{\text{av}} = \frac{1}{\pi R^2} \int d^2x T(\mathbf{x}) = \frac{3}{4} T_1 \approx 0.281 \frac{\dot{Q}}{\kappa R}. \quad (16)$$

The flash temperature is, as expected, between T_{av} and $T_{\max}$.

Next, since

$$\nabla^2 T = -\frac{2}{R^2} T_1,$$

we obtain

$$\nabla^2 T_{\text{flash}} = \frac{\langle \nabla^2 T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle} = -\frac{2}{R^2} T_1, \quad (17)$$

and

$$\frac{\nabla^2 T_{\text{flash}}}{T_{\text{flash}}} = \frac{\langle \nabla^2 T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle T(\mathbf{x}) \sigma(\mathbf{x}) \rangle} = -\frac{2.5}{R^2}. \quad (18)$$

Finally, since the temperature distribution is independent of the angular coordinate ϕ , the inverse of the slope-length vanishes,

$$\frac{1}{d_{\text{flash}}} = \frac{T'_{\text{flash}}}{T_{\text{flash}}} = 0.$$

Next, let us consider high sliding speeds. In this case, one can neglect the lateral diffusion of the heat. Measuring the coordinates x and

y in units of the radius R , the temperature is given by the integral representation (see Appendix A and Ref. 28)

$$T(x, y) = \frac{\dot{q}_1}{\kappa} \left(\frac{DR}{\pi v} \right)^{1/2} f(x, y), \quad (19)$$

where

$$f(x, y) = \int_{-s}^x d\xi \frac{(1 - \xi^2 - y^2)^{1/2}}{(x - \xi)^{1/2}}, \quad (20)$$

where $s = (1 - y^2)^{1/2}$. An intensity map of $f(x, y)$ is shown in Fig. 3.

Using (19) and (20), one can calculate all quantities of interest using numerical integration. Here, we summarize the most important results (see also the Appendix).

The flash temperature is given by

$$T_{\text{flash}} \approx 0.352 \frac{\dot{Q}}{R\kappa} \left(\frac{D}{Rv} \right)^{1/2}. \quad (21)$$

The maximum temperature is

$$T_{\max} \approx 0.590 \frac{\dot{Q}}{R\kappa} \left(\frac{D}{Rv} \right)^{1/2}, \quad (22)$$

and the average flash temperature is

$$T_{\text{av}} \approx 0.323 \frac{\dot{Q}}{R\kappa} \left(\frac{D}{Rv} \right)^{1/2}, \quad (23)$$

which is a factor of ≈ 0.549 smaller than the maximal temperature. The flash temperature is, as expected, between T_{av} and $T_{\max}$.

In the Appendix, we show that

$$\frac{\nabla^2 T_{\text{flash}}}{T_{\text{flash}}} = \frac{\langle \nabla^2 T(\mathbf{x}) \sigma(\mathbf{x}) \rangle}{\langle T(\mathbf{x}) \sigma(\mathbf{x}) \rangle} \approx -\frac{4.93}{R^2}. \quad (24)$$

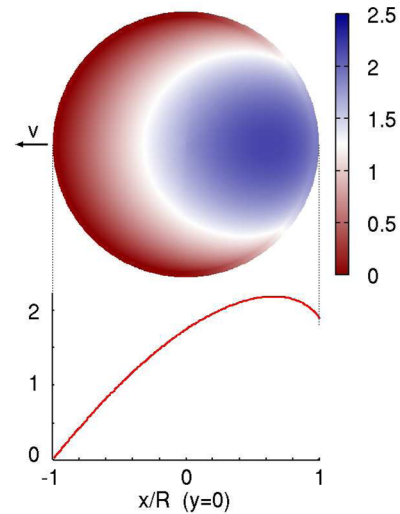


FIG. 3. Temperature distribution in a fast-moving Hertz contact. The top is an x, y 2D view, and the bottom curve is the temperature profile through the centerline, $y = 0$.

Note that the prefactor 4.93 is larger than in the static case, where it equals 2.5. This is intuitively plausible from the intensity maps in Figs. 2 and 3, where the hottest region is more concentrated in space for high sliding speeds.

We define the curvature-length D_{flash} and the slope-length d_{flash} by

$$D_{\text{flash}} = \left[\frac{-T_{\text{flash}}}{\nabla^2 T_{\text{flash}}} \right]^{1/2}, \quad (25)$$

$$d_{\text{flash}} = \frac{-T_{\text{flash}}}{T'_{\text{flash}}}. \quad (26)$$

For sliding Hertz contact,

$$D_{\text{flash}} = \alpha R,$$

where, using (18), $\alpha \approx 0.63$ for very low velocities (stationary contacts) and, using (24), $\alpha \approx 0.45$ for very high velocities. Here, high and low velocities refer to $v \gg v^*$ and $v \ll v^*$, with $v^* = D/R$, respectively.

In the Appendix, we derive the slope parameter for the sliding Hertzian contact,

$$d_{\text{flash}} \approx 1.36R,$$

while $d_{\text{flash}} = \infty$ for $v = 0$. Note that for high sliding speed, $d_{\text{flash}}/D_{\text{flash}} \approx 1.36/0.45 \approx 3.0$.

IV. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTS

The flash temperature was studied experimentally for steel sliding on steel, and here we will compare the theory predictions (Sec. II) with the experimental data presented in Ref. 19. We first briefly review the results obtained in Ref. 19. The experiments consisted of a steel block (substrate) sliding against a stationary steel block with a small hole (radius 1 mm) drilled in it. During sliding, the asperities on the stationary block will result in hot tracks on the substrate surface, which can be observed through the hole using a CCD camera as illustrated schematically in Fig. 4. After calibration of the camera, the detailed surface temperature profile can be measured with a spatial resolution of $4 \mu\text{m}$. Because of thermal diffusion, only the hot tracks from asperities on the stationary block, which contact the substrate close to the hole, can be observed, and the hottest tracks will result from asperity contacts very close to the hole. Therefore, the width of the hottest tracks at the edge of the hole will give an estimation of the effective width of the contact patches, which we will compare to the theory presented earlier.

Figure 5 shows the thermography of the hot tracks in two cases where the sliding speed was 33.9 m/s (test 3) and 34.5 m/s (test 5). Figure 6 shows the spatial temperature distribution along the y axis (orthogonal to the sliding direction) near point 5 for test 3. For this test, the asperity contact is assumed to be just outside the hole, so this figure gives an estimation of the width of the hot spots. A similar width was obtained for other asperity contacts when the asperities were located very close to the boundary of the drilled hole.

We will analyze the experimental data using the theory described in Sec. II. The power spectrum of the surface roughness of the steel surfaces was not given in Ref. 19, but only the R_a roughness

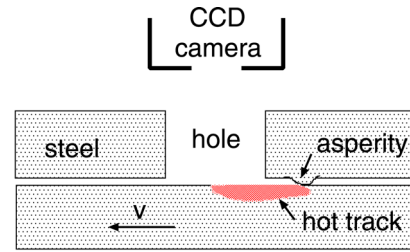


FIG. 4. Hot track on the sliding steel surface from the contact with an asperity on the stationary surface located close to the hole (schematic).

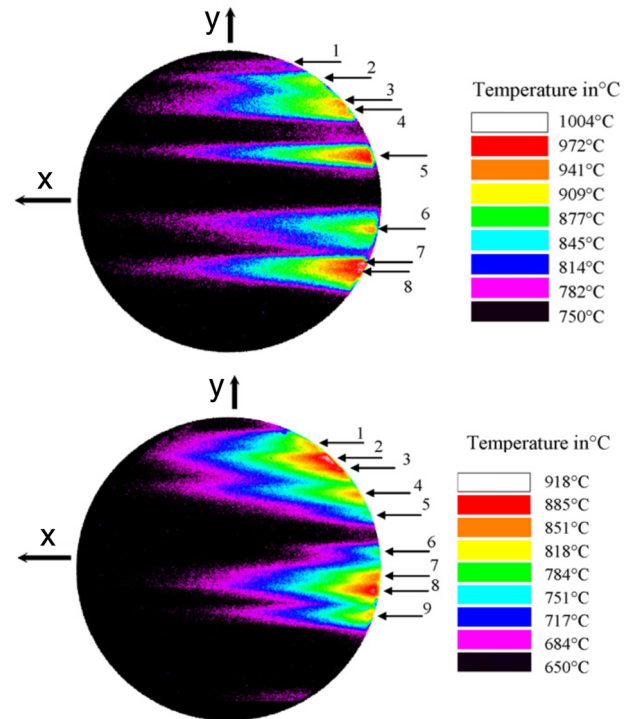


FIG. 5. Thermography of the sliding surface at the sliding speeds (a) 33.9 m/s (test 3) and (b) 34.5 m/s (test 5). Adapted from G. Sutter and N. Ranc, *Wear* **268**, 1237 (2010).

parameter was given, $R_a \approx 0.8 \mu\text{m}$. In the study below, we use the surface roughness power spectrum $C(q)$ of a machined steel surface with similar roughness as in Ref. 29. The black line in Fig. 7 gives the power spectrum of this steel surface.

For the medium-low carbon steel C22 used in Ref. 19, we use the following elastoplastic parameters: Young's modulus $E = 200 \text{ GPa}$, Poisson ratio $\nu = 0.3$, and for the penetration hardness, three values, namely $\sigma_p = 0.5, 1.2$, and 2.0 GPa . The Brinell hardness quoted in Ref. 19 was 120 (corresponding to $\sigma_p \approx 1.2 \text{ GPa}$), but the stress needed for plastic deformation of asperities may differ from the macroscopic penetration hardness for reasons discussed elsewhere.³⁰ Here, we therefore use three values for σ_p to show the dependency (sensitivity) of the results on the penetration hardness.

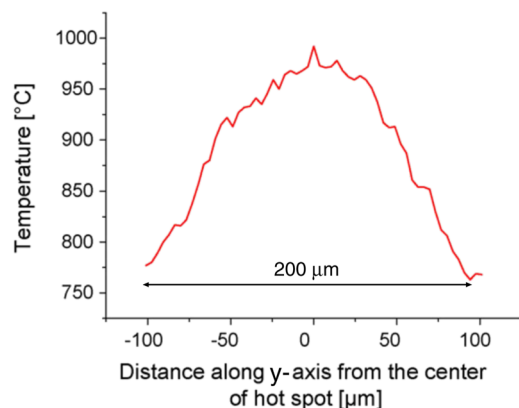


FIG. 6. Spatial temperature distribution along the x axis near point 5 for test 3. Adapted from G. Sutter and N. Ranc, *Wear* **268**, 1237 (2010).

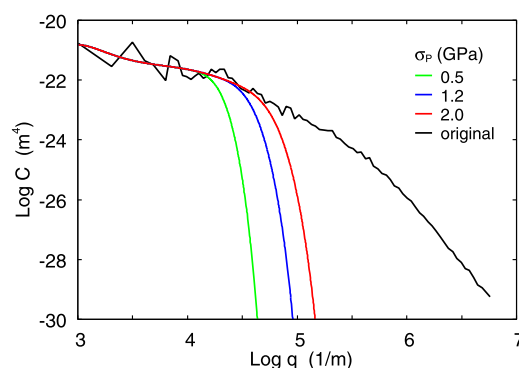


FIG. 7. Black line is the surface roughness power spectrum of a machined steel surface scaled by a factor of 2. The original (not scaled) surface has the root-mean-square roughness $h_{rms} \approx 0.90 \mu\text{m}$ and the rms slope $\xi \approx 0.30$. The green, blue, and red lines are the power spectra of the plastically smoothed profiles using the penetration hardness $\sigma_p = 0.5, 1.2$, and 2 GPa , respectively.

To take into account the contact between two steel surfaces, we use the effective modulus $E/2 = 100 \text{ GPa}$.

We use the Persson contact mechanics theory.³¹ We assume that the material deforms plastically without strain hardening in such a way that the maximum stress in the contact areas is σ_p . For this elastoplastic model, it has been shown that the theory accurately predicts both the elastic and plastic contact area.³² Since the theory discussed in Sec. II assumes elastic deformation, we replace the original surface with a surface where the short-wavelength roughness is smoothed by the plastic deformations. This smoothing is performed everywhere, i.e., also in the non-contact area, which is necessary in order to have a randomly rough surface also after the smoothing.

Plastic smoothing can be performed in different ways, and here we use a simple procedure that gives a smoothed surface where the (elastic) contact area as a function of magnification is virtually the same as the elastoplastic contact area using the original surface.^{33–35} The basic idea is that if the applied stress is removed and then applied again, the surface will deform elastically as long as the applied stress is smaller than the stress used to plastically deform the surface.

The power spectrum $C_{sm}(q)$ of the smoothed surface is obtained from the power spectrum $C(q)$ of the original surface using

$$C_{sm}(q) = \left[1 - \left(\frac{A_{pl}(\zeta)}{A_{pl0}} \right)^6 \right] C(q). \quad (27)$$

Here, $A_{pl}(\zeta)$ is the plastic surface area predicted by the elastoplastic calculation when the interface is observed at the magnification $\zeta = q/q_0$ (where q_0 is the smallest wavenumber) and $A_{pl0} = F_N/\sigma_p$ is the plastic contact area observed at the highest magnification, where all contact regions have yielded plastically. The smoothing (27) consists of removing or reducing the amplitude of the short-wavelength roughness components that deform plastically.

The black line in Fig. 7 shows the surface roughness power spectrum of the original steel surface scaled by a factor of 2 to take into account the contact between two rough surfaces with identical power spectra but uncorrelated roughness. The green, blue, and red lines are the power spectra of the plastically smoothed profiles using the penetration hardness $\sigma_p = 0.5, 1.2$, and 2 GPa , respectively.

For the penetration hardness of 1.2 GPa , Fig. 8 shows the elastic contact area (red curve) and the contact area where the solid deforms plastically (green curve). The result was obtained using the elastoplastic mechanics contact theory (Ref. 36), which was tested in detail in Ref. 32. The blue curve is the result of the theory for elastic solids with the smoothed power spectrum (blue line in Fig. 7) determined by (27). The prediction of the elastic contact mechanics with the plastically smoothed power spectrum (blue curve in Fig. 8) agrees well with the prediction of the elastoplastic theory (sum of the red and green curves in Fig. 8), except for a small difference in the region where the asperities are fully plastically deformed.

Figure 9 shows the stress–stress correlation function $g_\sigma(r) = \langle \sigma(\mathbf{x})\sigma(\mathbf{x}') \rangle$ as a function of the separation $r = |\mathbf{x} - \mathbf{x}'|$ between the points $\mathbf{x}$ and $\mathbf{x}'$ for the surface plastically smoothed, assuming the penetration hardness $\sigma_p = 0.5$ (green line), 1.2 (blue line), and 2 GPa (red line). For a Hertzian contact, $g_\sigma(r) = 0$ for separations r larger than the diameter $2R$ of the contact area.²² In the present

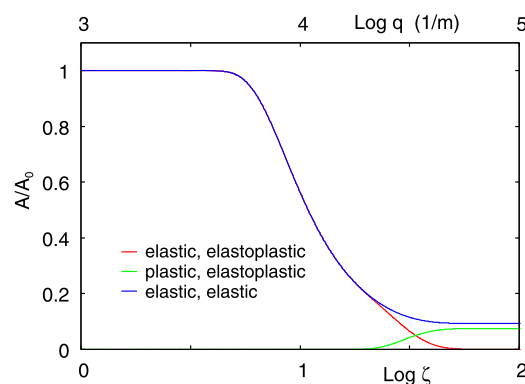


FIG. 8. Contact area as a function of the magnification (lower scale) or the wavenumber cutoff (upper scale) used in calculating the contact area. The red and green curves are the elastic and plastic contact areas obtained using the elastoplastic contact mechanics theory. The blue curve is the contact area obtained using the elastic contact mechanics theory with the plastically smoothed power spectrum (blue line in Fig. 7).

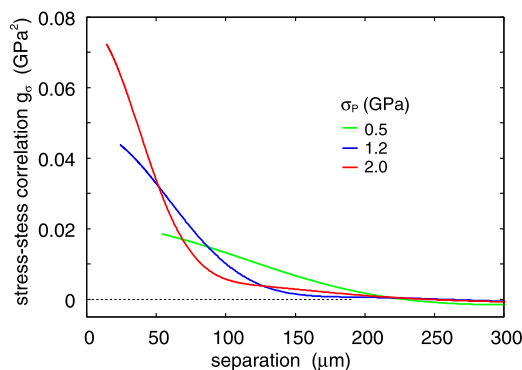


FIG. 9. Stress–stress correlation function $g_\sigma(r) = \langle \sigma(\mathbf{x})\sigma(\mathbf{x}') \rangle$ as a function of the separation $r = |\mathbf{x} - \mathbf{x}'|$ between the points $\mathbf{x}$ and $\mathbf{x}'$ for the surface plastically smoothed, assuming the penetration hardness $\sigma_P = 0.5$ (green line), 1.2 (blue line), and 2 GPa (red line).

case, the contacts are not circular, and island-like contact regions will surround the main compact inner region (see Fig. 1); in this case, there is no sharp cutoff in the $g_\sigma(r)$ function. Nevertheless, we can define an effective radius of the macroasperity contact area using $g_\sigma(r)/g_\sigma(0) \approx 0$. Using such a definition, the figure shows that the diameter of the macroasperity contact regions is of order $\approx 200 \mu\text{m}$ if σ_P is chosen as the Brinell hardness.

Next, we focus on the result of the calculation of the flash temperature. In what follows we use the measured nominal contact pressure $\sigma_0 \approx 90 \text{ MPa}$ and the following thermal parameters for steel: mass density $\rho = 7900 \text{ kg/m}^3$, thermal conductivity $\kappa = 30.0 \text{ W/Km}$, and thermal heat capacity $C_P = 500.0 \text{ J/kg K}$. We note that the thermal conductivity at room temperature for the used steel is about 50.0 W/Km , but for the higher temperatures relevant here it is about 30 W/Km . (The thermal conductivity of materials decreases at high temperatures due to increased scattering of the thermal carriers. Therefore, as temperature rises, the phonon and electron mean free path becomes shorter, which reduces the efficiency of heat transfer.)

Here, we note that Reddyhoff *et al.*³⁷ have shown that the thermal conductivity of solids (mainly metals) close to the surface may differ from that in the bulk. Therefore, work hardening, which produces disordered lattice structures and defects, e.g., dislocations or point defects, decreases the surface thermal conductivity. This is due to scattering of the thermal carriers (phonons and electrons) from the lattice imperfections, which result in a reduced carrier mean free path and smaller thermal conductivity. These defects, however, have only a small influence on the mass density and the thermal heat capacity, and these quantities are expected to be nearly the same in the surface region as in the bulk.

Reddyhoff *et al.* found that the measured thermal conductivity of AISI 52100 steel (21 W/mK) is less than half the value cited in the literature (46 W/mK). This discrepancy arises from a reduction in thermal conductivity of AISI 52100 due to work-hardening. They pointed out that the thermal conductivity value generally cited and used in the literature represents that of soft, annealed alloys, but work-hardened AISI 52100, which is generally employed in rolling bearings and for lubricant testing, appears to have a much lower surface thermal conductivity.

The measured friction coefficient was not constant during the sliding tests, and here we use an average $\mu \approx 0.11$. This friction coefficient is much smaller than typically found for steel sliding on steel at low sliding speeds (typically $\mu \approx 0.7$ for $v \approx 1 \text{ mm s}^{-1}$ ³⁸) and is likely a result of the high temperatures ($\sim 1000^\circ\text{C}$) prevailing in the asperity contact regions, which result from the high sliding speed (about 35 m/s).

If one assumes that T_{flash} is continuous at the interface, then if s is the fraction of energy going into the sliding block (substrate) and $1 - s$ into the stationary block, then $(1 - s)T_{\text{flash}}(\text{stationary}) = sT_{\text{flash}}(\text{moving})$, where $T_{\text{flash}}(\text{stationary})$ and $T_{\text{flash}}(\text{moving})$ are calculated assuming that all have thermal energy goes into each block. Using this equation for the sliding speed used in the experiments, $v = 35 \text{ m/s}$, assuming the penetration hardness $\sigma_P = 1.2 \text{ GPa}$, the theory predicts that the fraction of the frictional energy going into the sliding block is $s \approx 0.938$ and the rest goes into the stationary block. In what follows, we reduce the frictional energy by a factor of $s \approx 0.938$ in order to take this effect into account when calculating the flash temperature on the sliding substrate surface.

Figure 10(a) shows the calculated flash temperature T_{flash} and (b) the curvature-length D_{flash} as a function of the logarithm of the sliding speed for the three plastically smoothed surfaces. For the sliding speeds in the experiments (about 35 m/s), indicated by the vertical dotted lines, the temperature increase is 259°C for the yield stress of 1.2 GPa . The measured temperature increase is about $\sim 200 \text{ K}$, but the actual maximum temperature increase may be slightly larger since the temperature measurements are for some short distance away from the hot spots. For the yield stress of 1.2 GPa

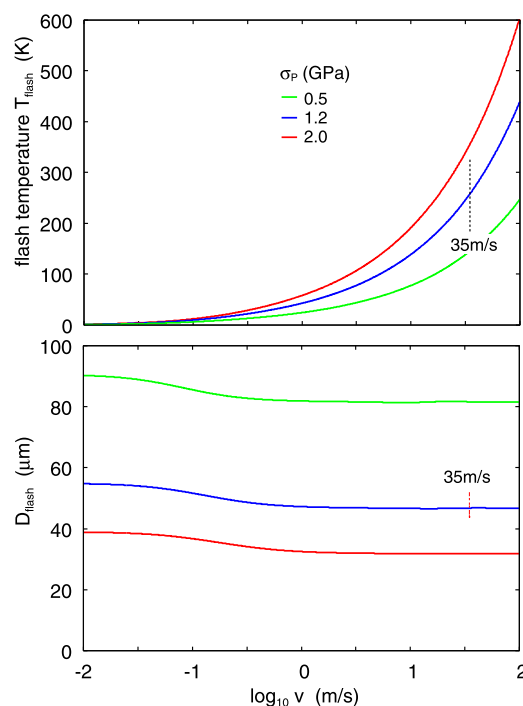


FIG. 10. (a) The flash temperature T_{flash} and (b) the curvature-length D_{flash} of the hot spots as a function of the logarithm of the sliding speed for the three plastically smoothed surfaces.

and sliding speed $v = 35$ m/s, the curvature-length $D_{\text{flash}} = 46.58 \mu\text{m}$. Assuming the result (25) derived for a sliding Hertz contact with $\alpha = 0.45$ (since $v \gg v^*$ in the present case) gives $2R \approx 207 \mu\text{m}$. This is similar to the diameter as predicted from the stress correlation function and also observed in the experiments.

In Fig. 11, we show the slope-length d_{flash} as a function of the logarithm of the sliding speed for the three plastically smoothed surfaces. For the sliding speed of 35 m/s, this gives $118 \mu\text{m}$ if $\sigma_P = 1.2$ GPa. For Hertzian contact, the theory predicts the slope length $d_{\text{flash}} \approx 0.68 \times [2R]$ so that if the diameter is $\approx 200 \mu\text{m}$, then the Hertz contact theory would predict $136 \mu\text{m}$, which is consistent with the full theory. Hence, it appears that the sliding Hertz contact results give useful results for the present system. The reason for this is that plastic deformation occurs already at relatively long length scales, and only roughness over a relatively narrow length-scale region matters, which is the limit where the classical flash temperature descriptions are approximately valid (see Ref. 10). However, when roughness on many length scales is important, the classical theory fails even qualitatively, as was the case for granite sliding on granite in Ref. 10.

We now study the sensitivity of the results to the surface roughness power spectrum. I note that power spectra prepared using similar methods tend to have similar fractal-like roughness but may differ in the roll-off region.²⁹ For most surfaces, it is the region around the onset of the roll-off that gives the largest contribution to the rms roughness.²⁹ Figure 12 shows (a) the flash temperature T_{flash} and (b) the curvature-length D_{flash} as a function of the logarithm of the sliding speed for the three different plastically smoothed surfaces. The red, blue, and green lines are after plastic smoothing (with $\sigma_P = 1.2$ GPa) of the power spectrum of the original surface scaled with the factors 0.5, 1, and 2, respectively. As expected, the flash temperature increases when the surface roughness amplitude increases. At the same time, the curvature-length D_{eff} increases, which is expected as the plastic deformation starts at longer wavelengths, resulting in bigger asperity contact regions. Since the total contact area is unchanged (it is determined by the penetration hardness, which is not changed), the contact consists of a lower concentration of bigger asperity contact regions with the same contact pressure σ_P as for the original surface. Hence, the flash temperature will increase as observed.

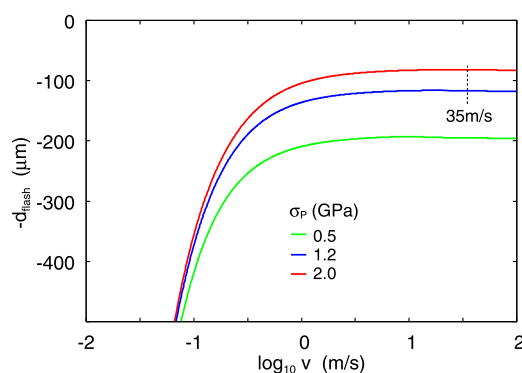


FIG. 11. Temperature slope-length d_{flash} as a function of the logarithm of the sliding speed for the three plastically smoothed surfaces.

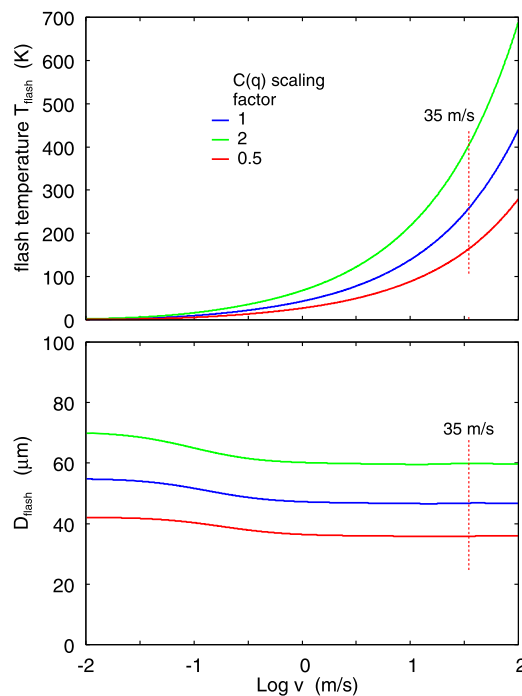


FIG. 12. (a) The flash temperature T_{flash} and (b) the curvature-length D_{flash} as a function of the logarithm of the sliding speed for the three different plastically smoothed surfaces. The red, blue, and green lines are after plastic smoothing (with $\sigma_P = 1.2$ GPa) of the power spectrum of the original surface scaled with the factors 0.5, 1, and 2, respectively.

For the case where we change the penetration hardness, the opposite effect is observed: the lower penetration hardness, $\sigma_P = 0.5$ GPa, gives bigger asperity contact regions than when $\sigma_P = 1.2$ GPa, but since the contact pressure is lower, the frictional energy produced per unit surface area is lower, and the net result is that the flash temperature is lowest for the lowest penetration hardness.

Finally, we note that Fig. 5 shows that the surface temperature away from the hot tracks behind the asperity contact regions is about 600 – 700 °C, i.e., much higher than room temperature. This background temperature is the cumulative result of the flash temperature. It formally corresponds to the $q = 0$ contribution in the Fourier decomposition of the temperature profile. It can be calculated assuming a uniform heat source, $\dot{q}_0 = \mu\sigma_0 v$, with an onset at the time $t = 0$ of the start of sliding. This results in a background surface temperature (here defined as the increase in the temperature relative to the starting temperature, which may be the room temperature) that is given by

$$T_0(t) = \frac{1}{\kappa} \left(\frac{D}{\pi} \right)^{1/2} \int_0^t dt' \frac{\dot{q}(t')}{(t-t')^{1/2}} = \frac{2\dot{q}_0}{\kappa} \left(\frac{D}{\pi} \right)^{1/2} \sqrt{t}.$$

If the sliding solid (the bottom steel block) has been in contact with the upper stationary block for a distance $L = vt$ before entering the hole, then the background temperature,

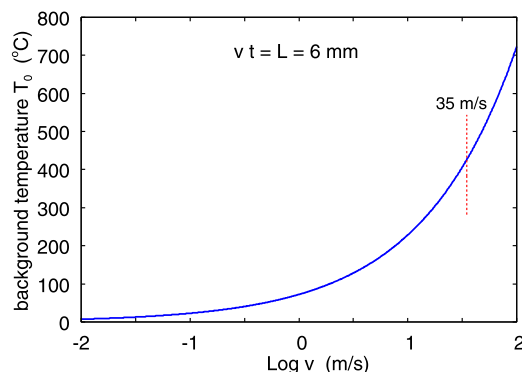


FIG. 13. Calculated background temperature as a function of the sliding speed for the sliding distance $L = 6$ mm.

$$T_0 = \frac{2\dot{q}_0}{\kappa} \left(\frac{D}{\pi} \right)^{1/2} \left(\frac{L}{v} \right)^{1/2}. \quad (28)$$

The upper block with the hole is rather short in the sliding direction (12 mm), with the hole in the middle so that $L = 5$ –6 mm. Using $L = 6$ mm in (28) gives the background temperature shown in Fig. 13. Here, we have assumed that after some short sliding distance, all frictional power is transferred to the substrate. This may be a good approximation because the surface of the bottom block is much longer than the upper block, and hence, after some small sliding distance, the bottom surface of the upper block will be very hot, and then all the frictional power may be transferred to the bottom block as it enters the contact region cold.

The background temperature shown in Fig. 13 assumes that the temperature before the start of sliding is 0°C , which certainly is not the case, but the initial temperature was not specified in Ref. 19 and could be higher than room temperature, as the experiments were repeated many times. However, even adding 50°C to the temperature in Fig. 13 results in a background temperature that is at least 100°C smaller than observed. I have no explanation for this discrepancy, but it may reflect uncertainty in the thermal parameters. Since the background temperature scales with the thermal conductivity as $1/\sqrt{\kappa}$, a reduction in κ from 30 to 20 W/mK, which is similar to what was found by Reddyhoff *et al.* (Ref. 37), would increase T_0 from ≈ 420 to 515°C or $\approx 540^\circ\text{C}$, assuming the starting temperature is 25°C .

V. SUMMARY AND CONCLUSION

In this study, I have tested a recently developed theory for the size of the contact regions and for the flash temperature for solid blocks in sliding contact.¹⁰ The theoretical predictions are consistent with the experimental observations by Sutter and Ranc¹⁹ for steel sliding on steel. However, an “exact” comparison between theory and experiments is not possible because the surface roughness power spectrum of the steel surfaces used in Ref. 19 is not known, and also the (macroscopic) Brinell hardness may differ somewhat from the penetration hardness involved in plastic deformations of asperities at the micrometer length scale.³⁰ The theory is based on multiscale contact mechanics and is valid for surfaces with roughness on an arbitrary number of decades in length scale.

We have found that the flash temperature parameters D_{flash} and d_{flash} for the sliding Hertz heat source are consistent with the result of the full theory and with the experiments for steel sliding on steel.¹⁹ The reason for this is that the steel deforms plastically already at relatively long length scales so that effectively roughness occurs only on a relatively narrow wavelength region, and in particular, the short wavelength roughness on the original surfaces is irrelevant for the flash temperature. However, this is not the case in general, and in Ref. 10, it was shown that the classical theory for the flash temperature fails severely for surfaces with roughness extending over two decades or more of length scale. This was the case for granite sliding on granite,¹⁰ a system of interest in earthquake dynamics.

AUTHOR DECLARATIONS

Conflict of Interest

The author has no conflicts to disclose.

Author Contributions

B. N. J. Persson: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Resources (equal); Software (equal); Validation (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

APPENDIX: SLIDING HERTZIAN CONTACT

We consider a sliding Hertz contact in the limit of very high sliding speeds where one can neglect the lateral diffusion of the heat. In this case, if a material point experiences the time dependent heat source $\dot{q}(t)$ for $t > 0$, then the temperature becomes

$$T(t) = \frac{1}{\kappa} \left(\frac{D}{\pi} \right)^{1/2} \int_0^t dt' \frac{\dot{q}(t')}{(t-t')^{1/2}}.$$

Figure 14 shows the Hertz circular contact region moving with the velocity v along the negative x axis. A point (x, y) has been in contact with the heat source for the time $t = (s+x)/v$ or the distance $s+x$ (dotted line). If we write $x' = -s + vt'$, where $s = (R^2 - y^2)^{1/2}$, we obtain

$$T(t) = \frac{1}{\kappa} \left(\frac{D}{\pi v} \right)^{1/2} \int_{-s}^x dx' \frac{\dot{q}_1}{(x-x')^{1/2}} \left(1 - \frac{x'^2 + y^2}{R^2} \right)^{1/2}.$$

Writing $x' = R\xi$ and measuring x and y in units of R gives²⁸

$$T(x, y) = \frac{\dot{q}_1}{\kappa} \left(\frac{DR}{\pi v} \right)^{1/2} f(x, y), \quad (A1)$$

where

$$f(x, y) = \int_{-s}^x d\xi \frac{(1 - \xi^2 - y^2)^{1/2}}{(x - \xi)^{1/2}}, \quad (A2)$$

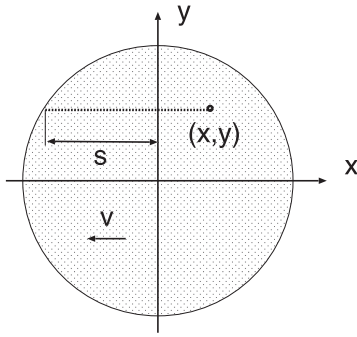


FIG. 14. Hertz circular contact region moves with the velocity v to the left along the negative x axis. A point (x, y) has been in contact with the heat source for the time $t = (s + x)/v$ or the sliding distance $s + x$ (dotted line).

where $s = (1 - y^2)^{1/2}$. An intensity map of $f(x, y)$ is shown in Fig. 3. Using (A1) and (A2), one can calculate

$$T_{\text{flash}} = \frac{\langle T(\mathbf{x})\sigma(\mathbf{x}) \rangle}{\langle \sigma(\mathbf{x}) \rangle} = 1.305 \frac{\dot{Q}}{\kappa} \left(\frac{DR}{\pi v} \right)^{1/2}.$$

Using $\dot{q}_1 = 3\dot{Q}/2\pi R^2$, we can write

$$T(x, t) = \frac{\dot{Q}}{R\kappa} \left(\frac{D}{Rv} \right)^{1/2} \frac{3}{2\pi\sqrt{\pi}} f(x, y),$$

and

$$T_{\text{flash}} \approx 0.352 \frac{\dot{Q}}{R\kappa} \left(\frac{D}{Rv} \right)^{1/2}. \quad (\text{A3})$$

The maximum temperature is

$$T_{\text{max}} \approx 0.590 \frac{\dot{Q}}{R\kappa} \left(\frac{D}{Rv} \right)^{1/2}, \quad (\text{A4})$$

and the average flash temperature is

$$T_{\text{av}} \approx 0.323 \frac{\dot{Q}}{R\kappa} \left(\frac{D}{Rv} \right)^{1/2}, \quad (\text{A5})$$

which is a factor of ≈ 0.549 smaller than the maximal temperature. The flash temperature is, as expected, between T_{av} and T_{max} .

Next, we calculate $\nabla^2 T_{\text{flash}}$. We write

$$\langle \nabla^2 T(\mathbf{x})\sigma(\mathbf{x}) \rangle = \int d^2x \nabla \cdot (\nabla T \sigma) - \int d^2x \nabla T \cdot \nabla \sigma.$$

Using the Gauss divergence theorem and the fact that $\sigma(\mathbf{x})$ vanishes for $r = R$, it follows that the first term in this equation vanishes. For the second term, we use that $\sigma(r)$ only depends on $r = |\mathbf{x}|$ so that $\nabla \sigma = \sigma'(r)\mathbf{x}/r$. Therefore, we obtain

$$\langle \nabla^2 T(\mathbf{x})\sigma(\mathbf{x}) \rangle = - \int_0^R dr r \sigma'(r) \frac{\partial}{\partial r} \int_0^{2\pi} d\phi T(r, \phi).$$

Using this, we can calculate

$$\frac{\nabla^2 T_{\text{flash}}}{T_{\text{flash}}} = \frac{\langle \nabla^2 T(\mathbf{x})\sigma(\mathbf{x}) \rangle}{\langle T(\mathbf{x})\sigma(\mathbf{x}) \rangle} \approx - \frac{4.93}{R^2}, \quad (\text{A6})$$

which is larger than in the static case.

Finally, consider

$$\langle \partial_x T(\mathbf{x})\sigma(\mathbf{x}) \rangle = \int d^2x \partial_x T \sigma = \int d^2x [\partial_x (T\sigma) - T \partial_x \sigma].$$

The first term can be written as

$$\int_{-R}^R dy \int_{-h(y)}^{h(y)} dx \partial_x (T\sigma) = \sigma(R) \int_{-R}^R dy [T(h(y), y) - T(-h(y), y)]$$

where $h(y) = (R^2 - y^2)^{1/2}$ and where we used that $\sigma(r)$ only depends on $r^2 = h^2(y) + y^2 = R^2$. Since $\sigma(R) = 0$, the first term vanishes. The second term can be written as

$$- \int d^2x T \partial_x \sigma = - \int_0^R dr r \sigma'(r) \int_0^{2\pi} d\phi \cos \phi T(r \cos \phi, r \sin \phi).$$

Using this equation, one obtains

$$\frac{1}{d_{\text{flash}}} = - \frac{T'_{\text{flash}}}{T_{\text{flash}}} \approx \frac{0.734}{R}. \quad (\text{A7})$$

REFERENCES

- B. N. J. Persson, "Rubber friction: Role of the flash temperature," *J. Phys.: Condens. Matter* **18**, 7789 (2006).
- G. Fortunato, V. Ciaravola, A. Furno, B. Lorenz, and B. N. J. Persson, "General theory of frictional heating with application to rubber friction," *J. Phys.: Condens. Matter* **27**, 175008 (2015).
- B. N. J. Persson, "Role of frictional heating in rubber friction," *Tribol. Lett.* **56**, 77 (2014).
- J. R. Rice, "Heating and weakening of faults during earthquake slip," *J. Geophys. Res.: Solid Earth* **111**, 2005JB004006, <https://doi.org/10.1029/2005jb004006> (2006).
- S. V. Sukhomlinov, M. H. Müser, and B. N. J. Persson, "Granite sliding on granite: Friction, wear rates, surface topography, and the scale-dependence of rate-state effects," *Rep. Prog. Phys.* **89**, 038301 (2025).
- S. L. Bore, B. N. J. Persson, and H. A. Sveinsson, "Why ice is so slippery," *arXiv:2603.11539* (2026).
- A. Atila, S. V. Sukhomlinov, and M. H. Müser, "Cold self-lubrication of sliding ice," *Phys. Rev. Lett.* **135**, 066204 (2025).
- T. E. Fischer, "Tribocchemistry," *Annu. Rev. Mater. Sci.* **18**, 303 (1988).
- M. Kalin and J. Vižintin, "High temperature phase transformations under fretting conditions," *Wear* **249**, 172 (2001).
- M. H. Müser and B. N. J. Persson, "On the flash temperature in sliding contacts," *arXiv:2603.29547* (2026).
- J. C. Jaeger, "Moving sources of heat and the temperature at sliding contacts," *J. Proc. R. Soc. New South Wales* **76**, 203 (1943).
- J. F. Archard, "The temperature of rubbing surfaces," *Wear* **2**, 438 (1959).
- J. A. Greenwood, "An interpolation formula for flash temperatures," *Wear* **150**(1–2), 153 (1991).
- J. R. Barber, "Distribution of heat between sliding surfaces," *J. Mech. Eng. Sci.* **9**, 351 (1967).
- F. E. Kennedy, "Thermal and thermomechanical effects in dry sliding," *Wear* **100**, 453 (1984).
- J. Choudhry, A. Almqvist, and R. Larsson, "A multi-scale contact temperature model for dry sliding rough surfaces," *Tribol. Lett.* **69**, 128 (2021).
- X. Tian and F. E. Kennedy, "Maximum and average flash temperatures in sliding contacts," *J. Tribol.* **116**, 167 (1994).
- J. Choudhry, A. Almqvist, and R. Larsson, "Validation of a multi-scale contact temperature model for dry sliding rough surfaces," *Lubricants* **10**, 41 (2022).

- ¹⁹G. Sutter and N. Ranc, "Flash temperature measurement during dry friction process at high sliding speed," *Wear* **268**, 1237 (2010).
- ²⁰B. N. J. Persson, "On the elastic energy and stress correlation in the contact between elastic solids with randomly rough surfaces," *J. Phys.: Condens. Matter* **20**, 312001 (2008).
- ²¹B. N. J. Persson, R. Xu, and N. Miyashita, "Rubber wear: Experiment and theory," *J. Chem. Phys.* **162**, 074704 (2025).
- ²²N. Miyashita and B. N. J. Persson, "Tire tread block dynamics," *Tribol. Lett.* **74**, 62 (2026); [arXiv: 2602.22078](#).
- ²³M. H. Müser and A. Wang, "Contact-patch-size distribution and limits of self-affinity in contacts between randomly rough surfaces," *Lubricants* **6**, 85 (2018).
- ²⁴S. Hyun, L. Pei, J. F. Molinari, and M. O. Robbins, "Finite-element analysis of contact between elastic self-affine surfaces," *Phys. Rev. E* **70**, 026117 (2004).
- ²⁵B. N. J. Persson, "On the flash temperature in sliding rubber contacts," [arXiv:2607.23562](#) (2026).
- ²⁶B. N. J. Persson, "On the flash temperature in accelerated sliding contacts," [arXiv:2606.1183](#) (2026).
- ²⁷K. L. Johnson, *Contact Mechanics* (Cambridge University Press, Cambridge, 1985).
- ²⁸H. A. Francis, "Interfacial temperature distribution within a sliding Hertzian contact," *ASLE Trans.* **14**, 41 (1971).
- ²⁹B. N. J. Persson, "On the fractal dimension of rough surfaces," *Tribol. Lett.* **54**, 99 (2014).
- ³⁰A. Tiwari, A. Almqvist, and B. N. J. Persson, "Plastic deformation of rough metallic surfaces," *Tribol. Lett.* **68**, 129 (2020).
- ³¹B. N. J. Persson, "Contact mechanics for randomly rough surfaces," *Surf. Sci. Rep.* **61**, 201 (2006).
- ³²A. Almqvist and B. N. J. Persson, "Multiscale contact mechanics for elastoplastic contacts," *Phys. Rev. E* **113**, 015503 (2026).
- ³³The smoothing procedure (27) was first used in Ref. 34. It results in an area of real contact that is virtually identical to the result of the theory for elastoplastic contact for all magnifications. However, it is not accurate for describing the separation between the surfaces in the non-contact surface area. For the interfacial separation, another more cumbersome approach has been developed.³⁵ However, for the flash temperature, the contact area is most important, and it is accurately described using (27).
- ³⁴B. N. J. Persson, B. Lorenz, and A. I. Volokitin, "Heat transfer between elastic solids with randomly rough surfaces," *Eur. Phys. J. E* **31**, 3 (2010).
- ³⁵A. Almqvist and B. N. J. Persson, "Surface separation in elastoplastic contacts," *Phys. Rev. E* **113**, 025509 (2026).
- ³⁶B. N. J. Persson, "Theory of rubber friction and contact mechanics," *J. Chem. Phys.* **115**, 3840 (2001).
- ³⁷T. Reddyhoff, A. Schmidt, and H. Spikes, "Thermal conductivity and flash temperature," *Tribol. Lett.* **67**, 22 (2019).
- ³⁸R. Xu, H. Ren, A. Clerc, G. Mollon, W. Sheng, F. Zhou, and B. N. J. Persson, "Sliding contact creates universal self-affine fractal surfaces," [arXiv: 2606.00626](#) (2026).