

SIMULATION OF A Z2 LATTICE GAUGE THEORY ON A D-WAVE QUANTUM PROCESSING UNIT

01.11.26

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Member of the Helmholtz Association

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Shaping Change

GAUGE THEORIES

In the standard model of particle physics gauge theories describe interactions between particles

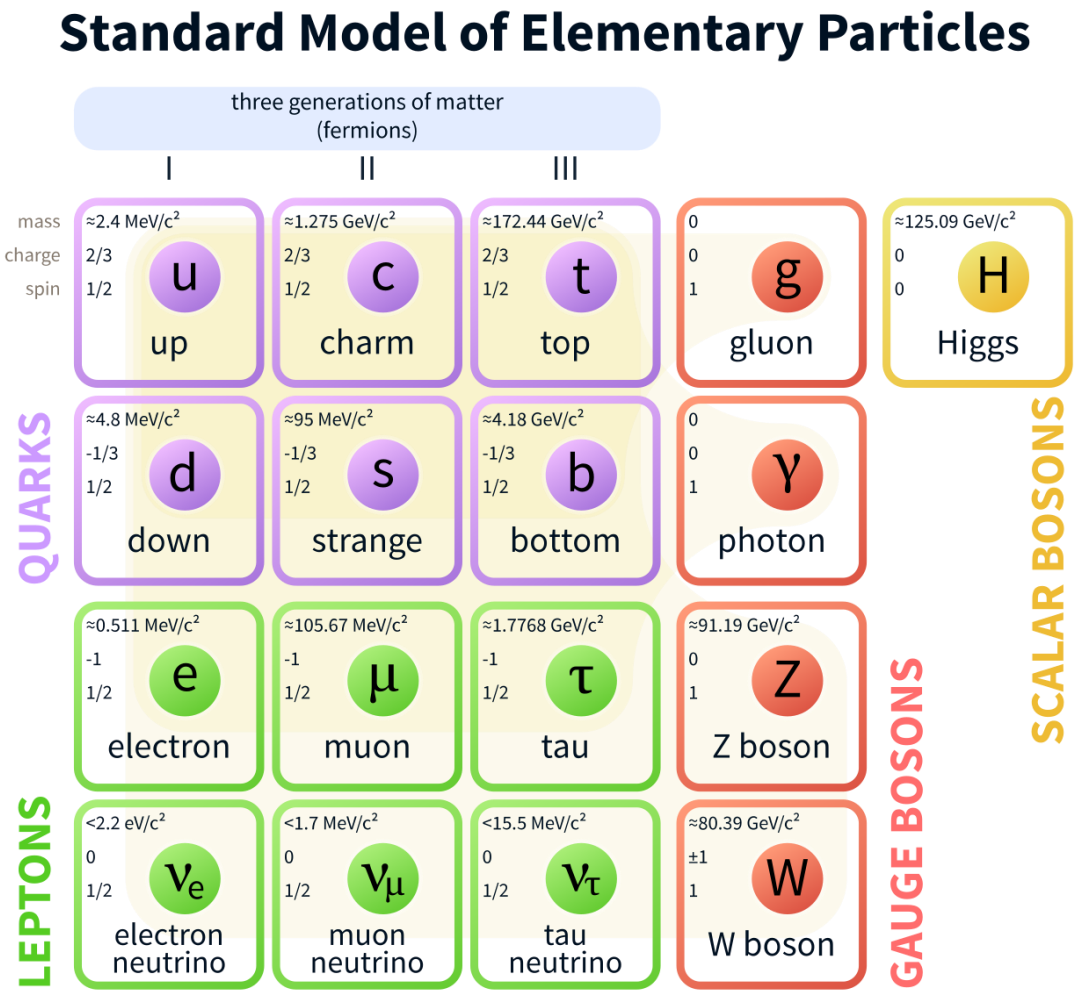


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- Electromagnetism

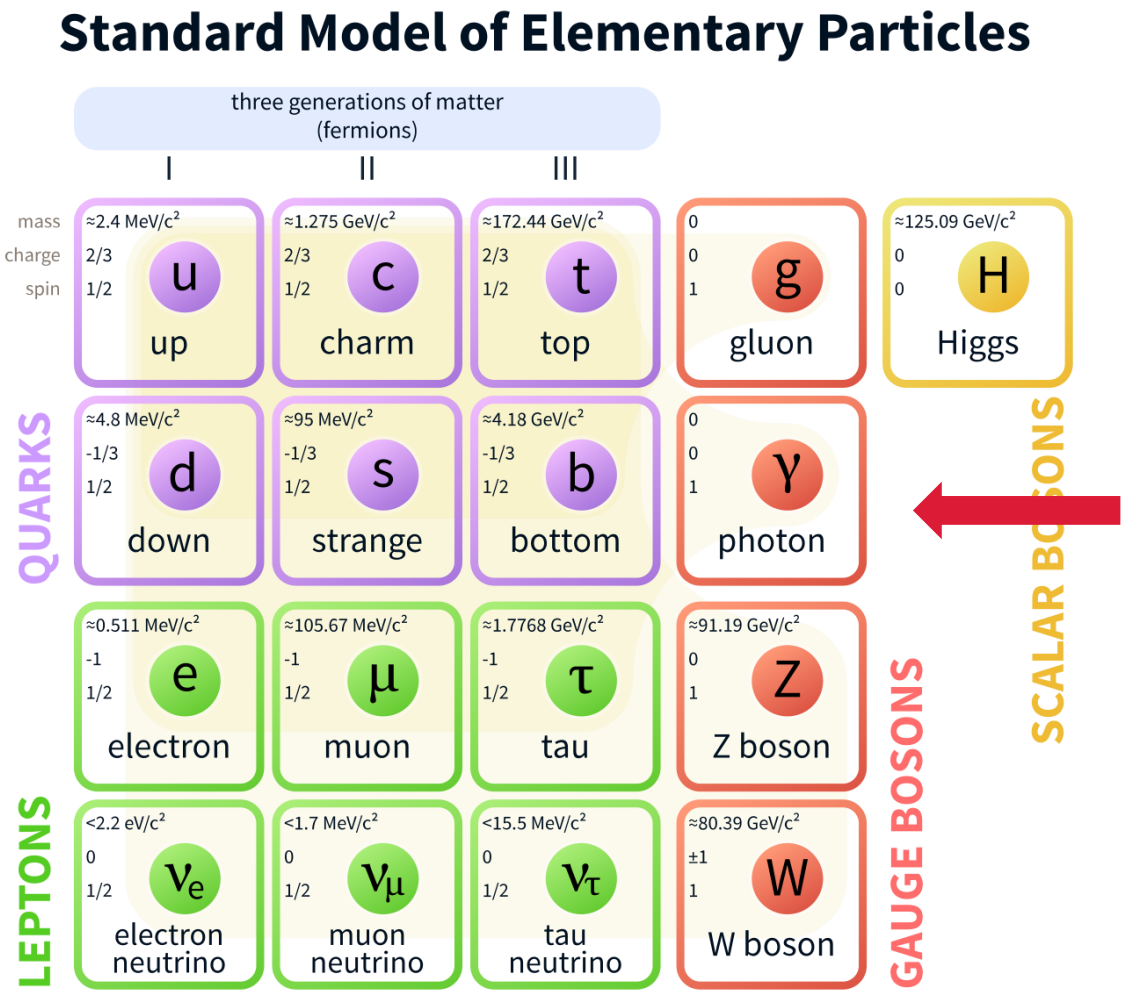


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GAUGE THEORIES

In the standard model of particle physics gauge theories describe interactions between particles

- Electromagnetism
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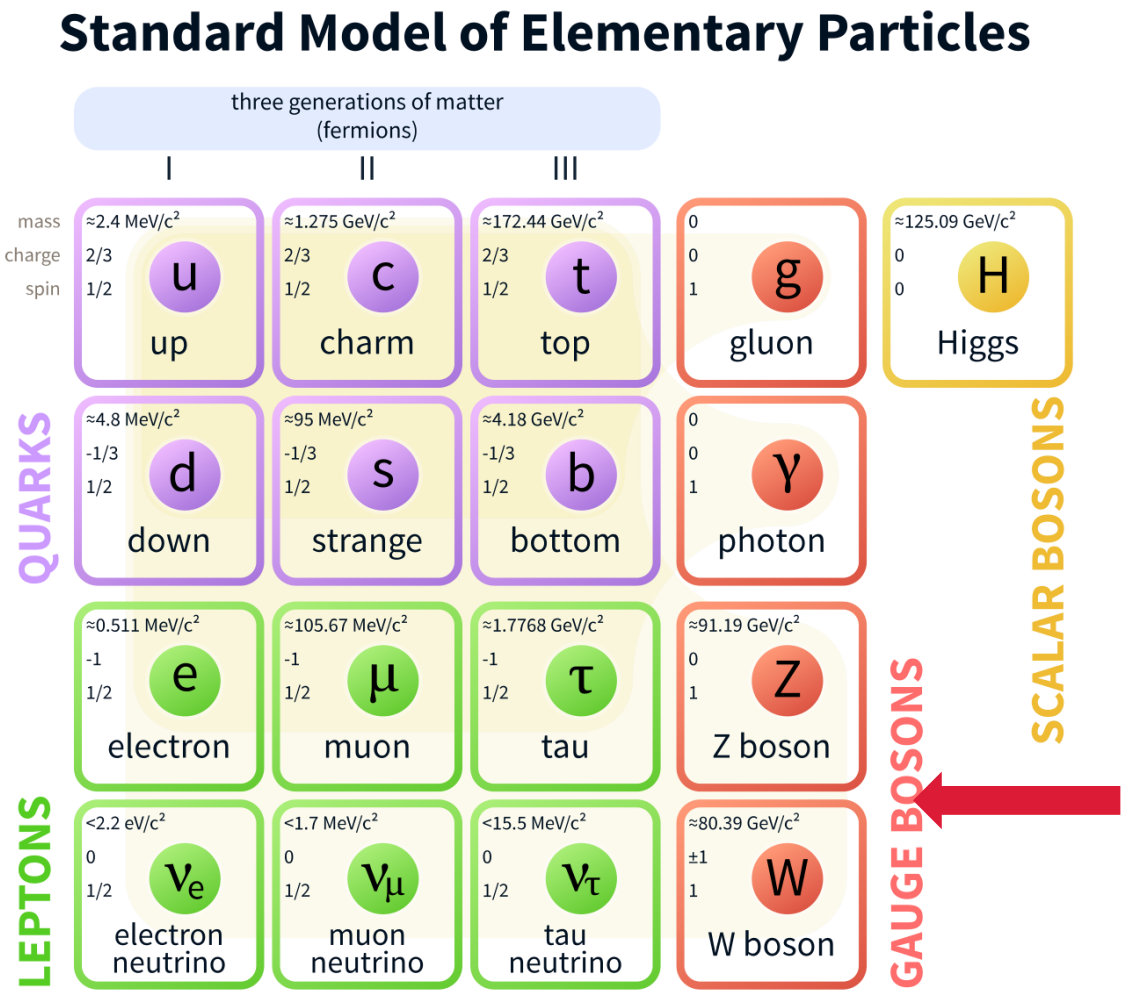


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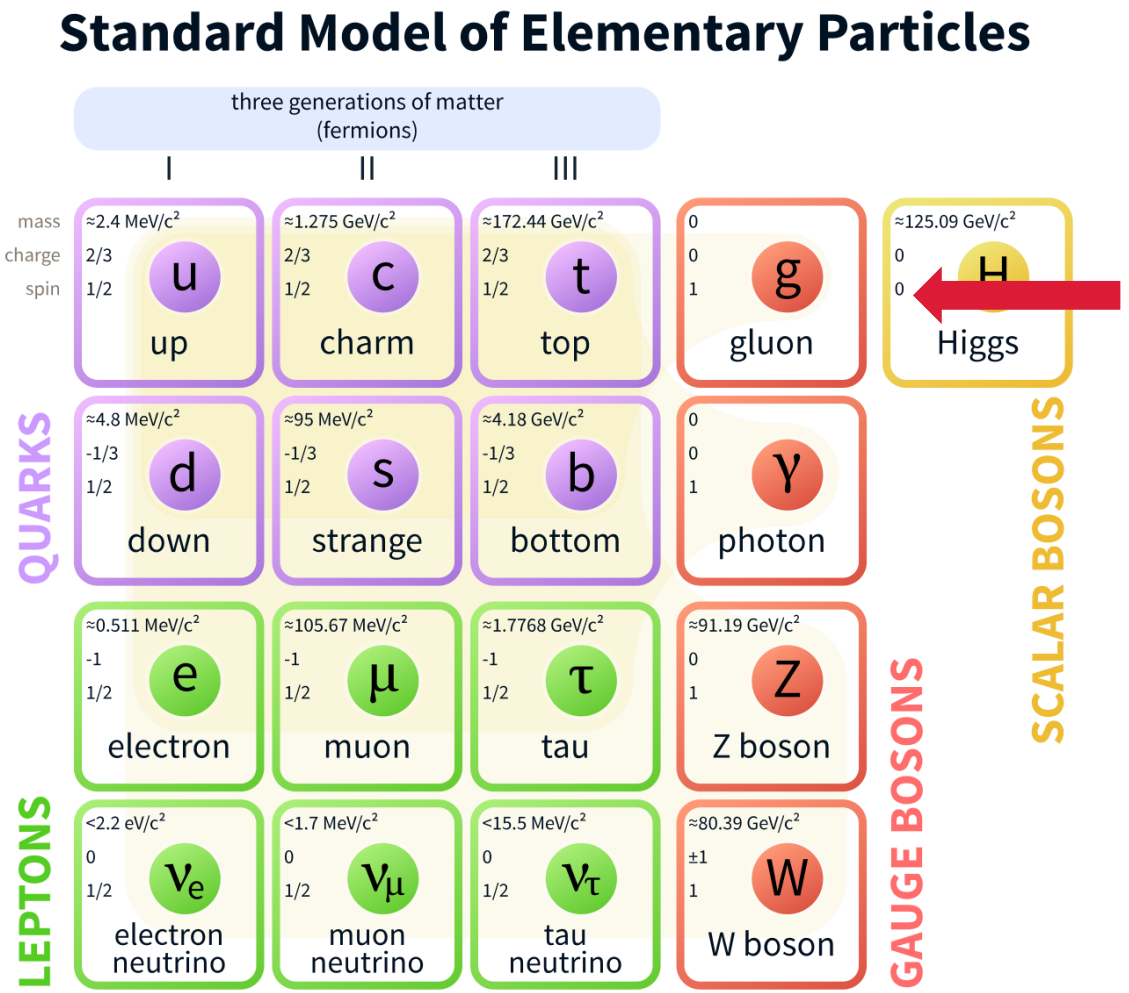
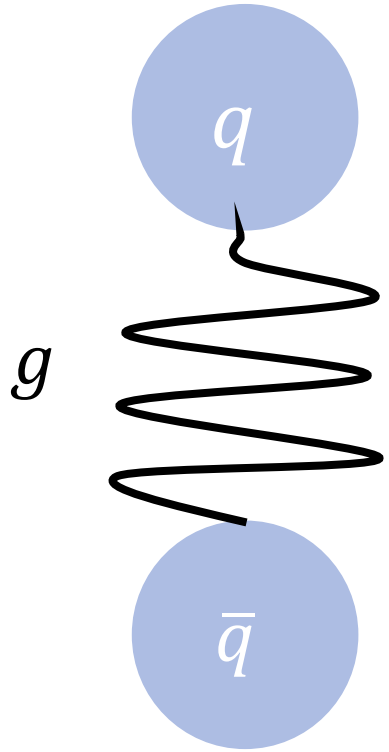


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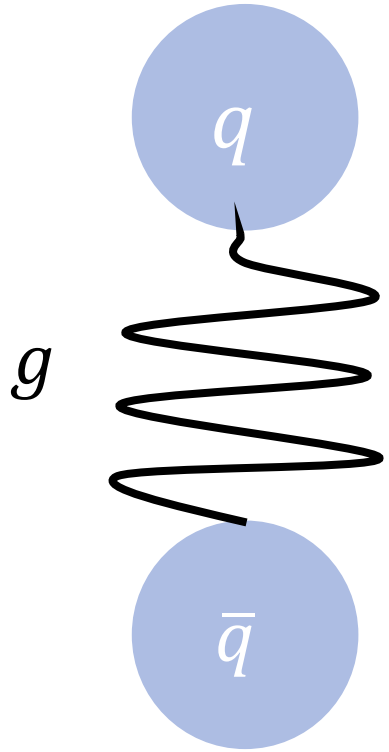


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Historically, confinement couldn‘t be described perturbatively



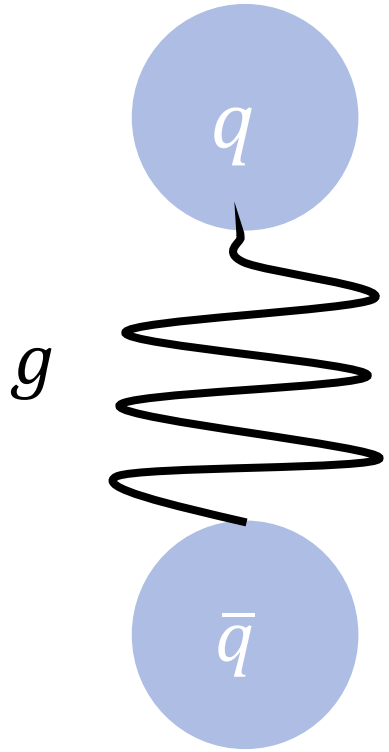
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➢ Lattice gauge theory



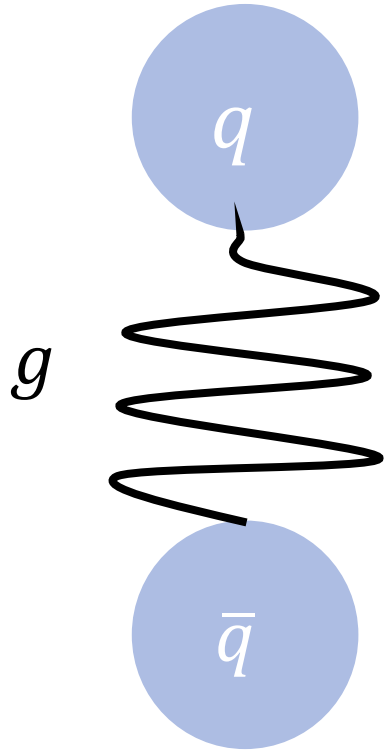
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- ➢ Lattice gauge theory
- ➢ Z2 lattice gauge theories



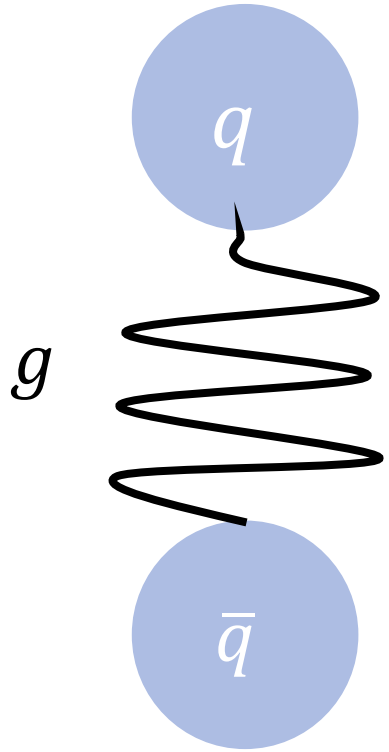
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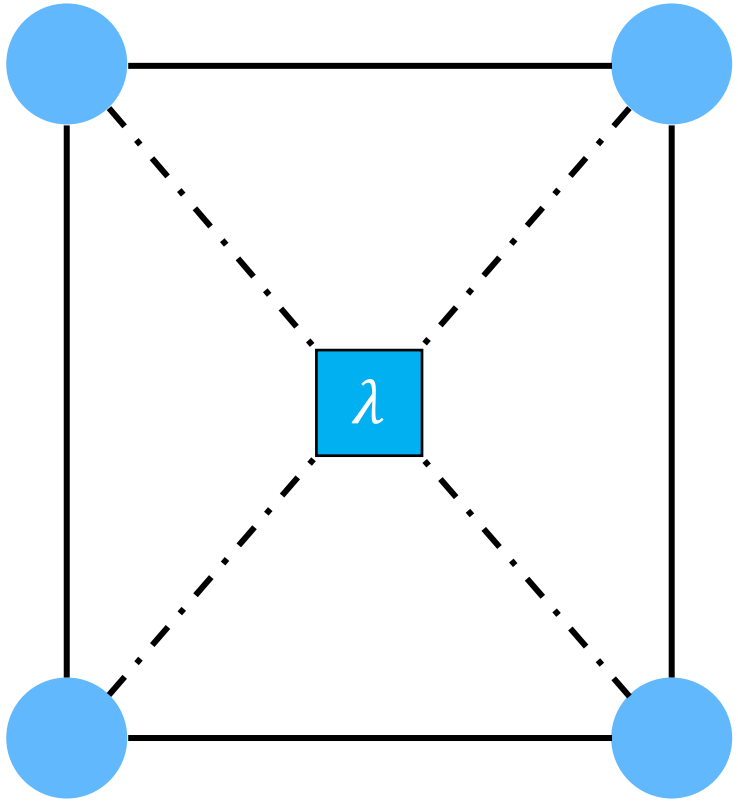
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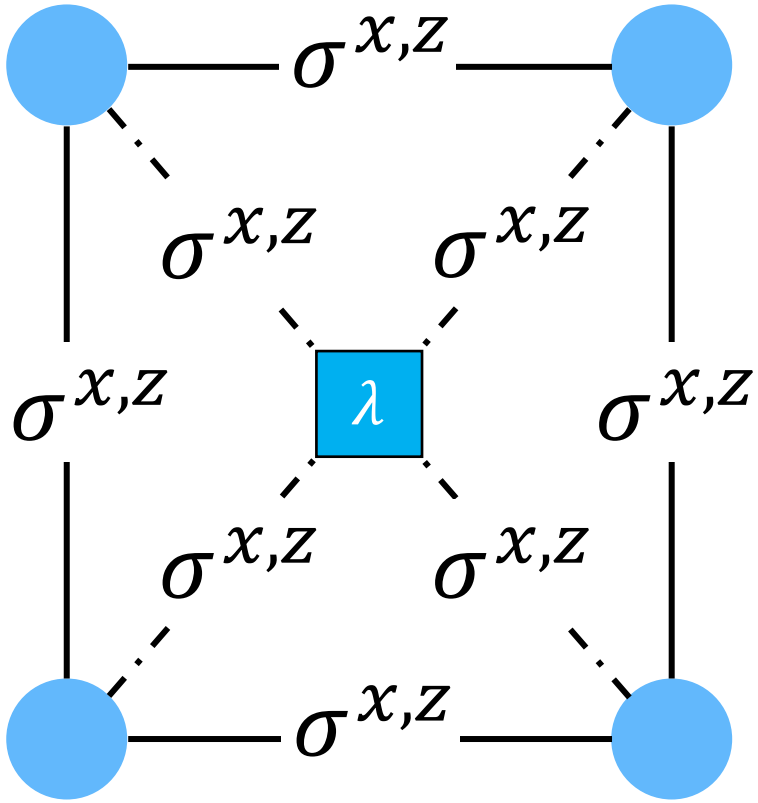
- ➢ Lattice gauge theory
- ➢ Z2 lattice gauge theories
  - ➢ show similar phenomenology such as confinement and string breaking



Z2 LATTICE GAUGE THEORY

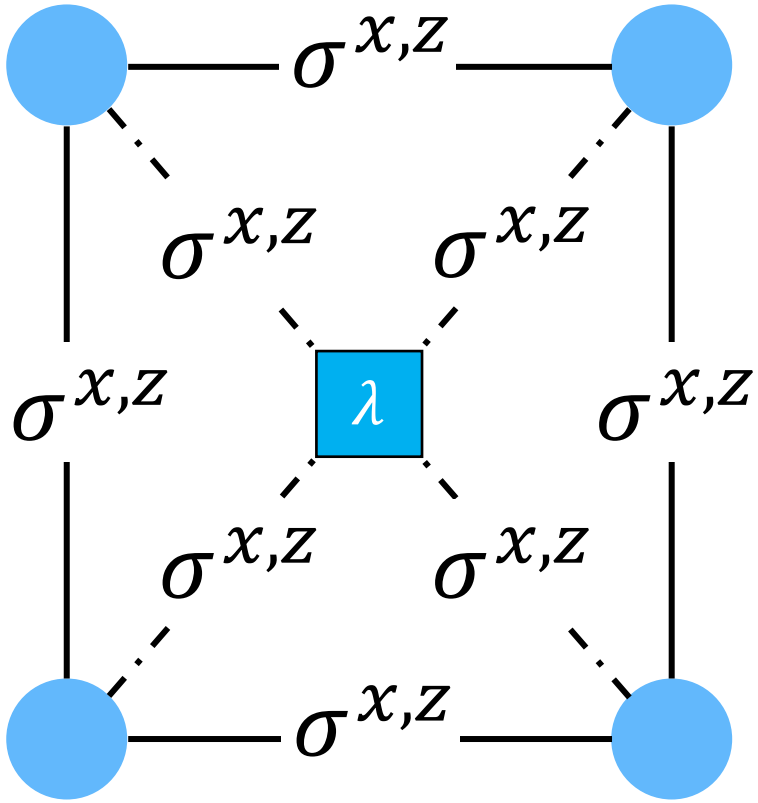


Z2 LATTICE GAUGE THEORY



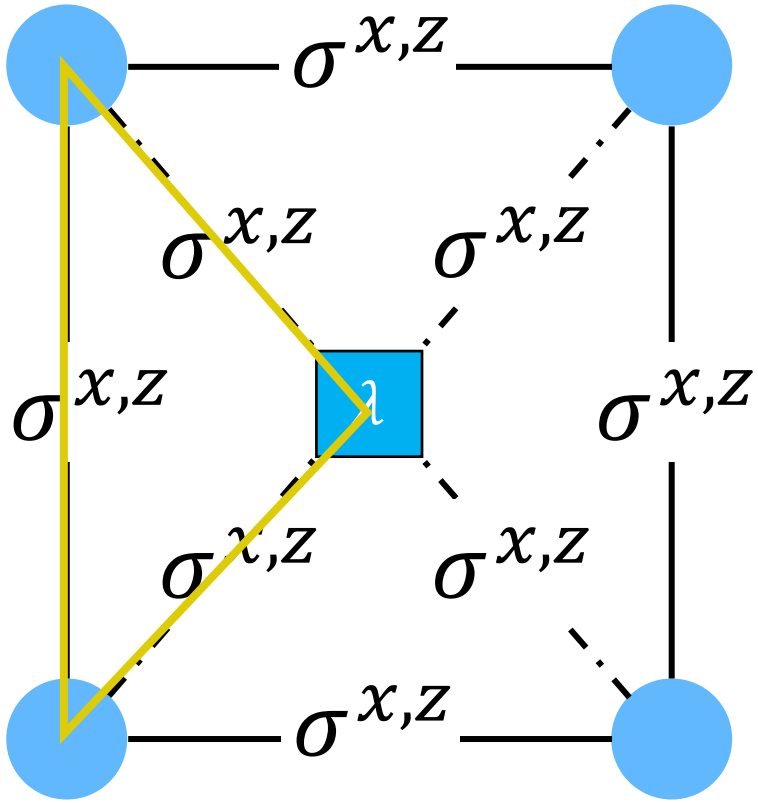
Z2 LATTICE GAUGE THEORY

$$H = -\mathrm{b}\sum \sigma^{x} - \mathrm{a}\sum_{\Delta} \sigma^{z}\,\sigma^{z}\sigma^{z}$$



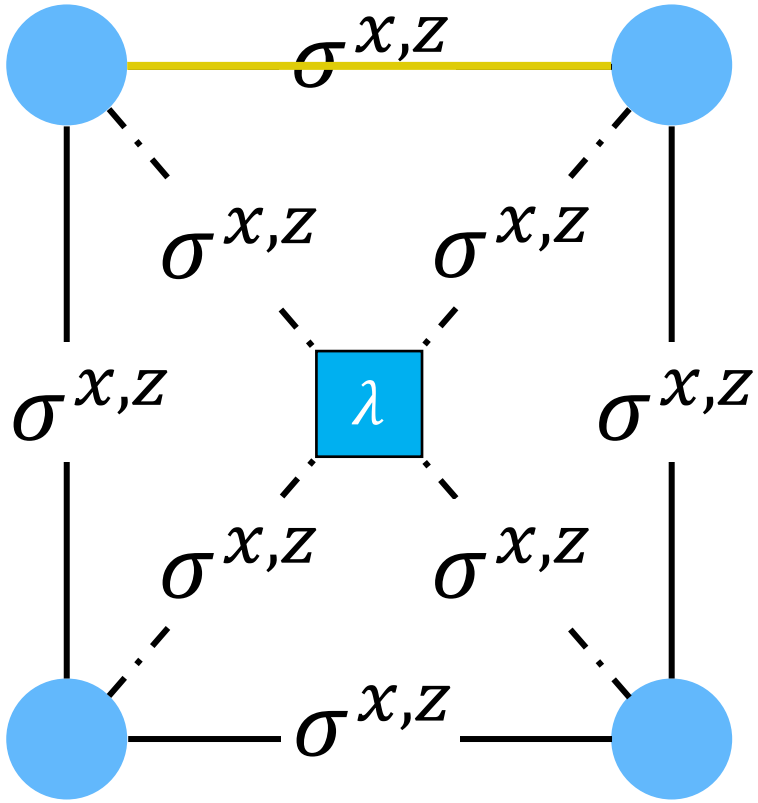
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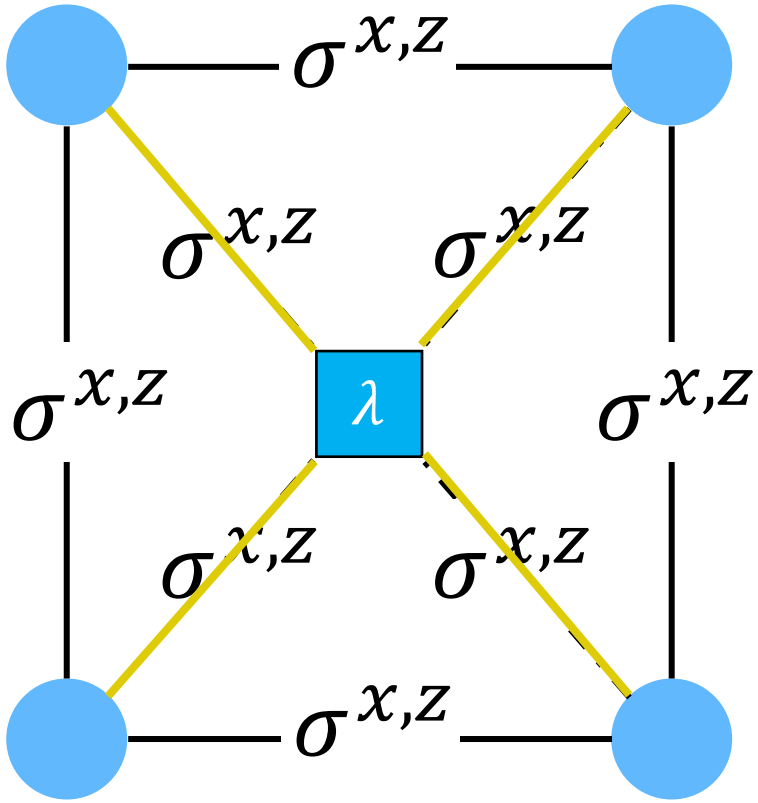
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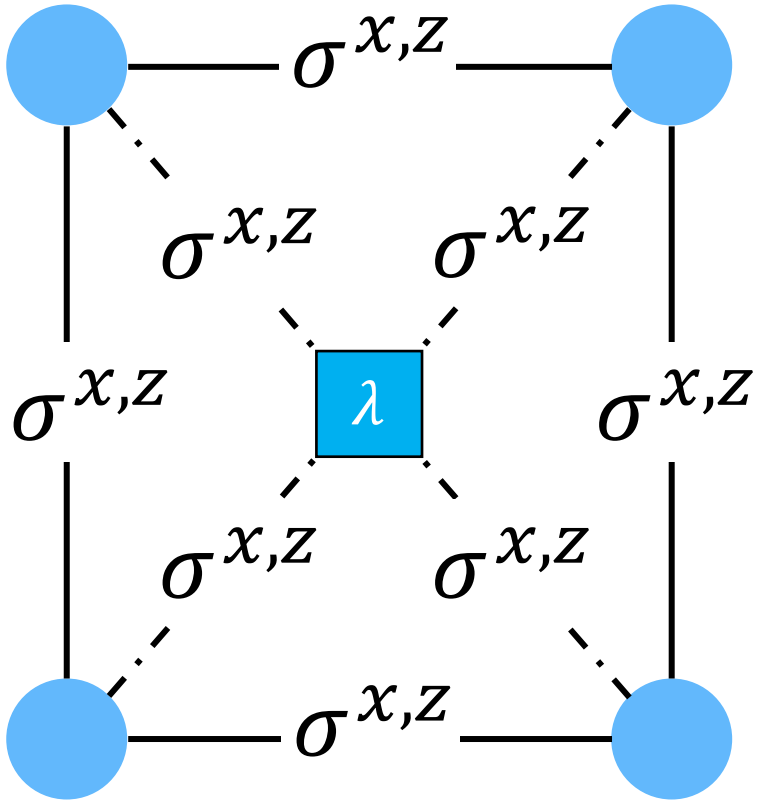
$$G_{\lambda} = \prod_{\partial\lambda} \sigma^{x}$$



Z2 LATTICE GAUGE THEORY

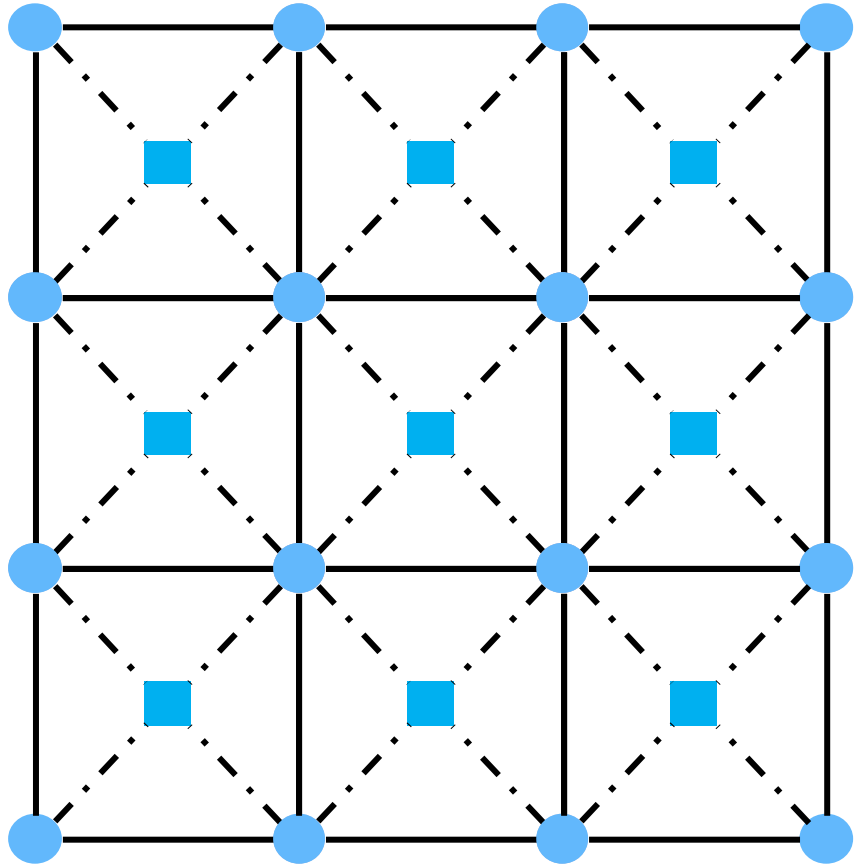
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$$G_{\lambda} = \prod_{\partial\lambda} \sigma^{x} \qquad [G_{\lambda}, H] = 0$$



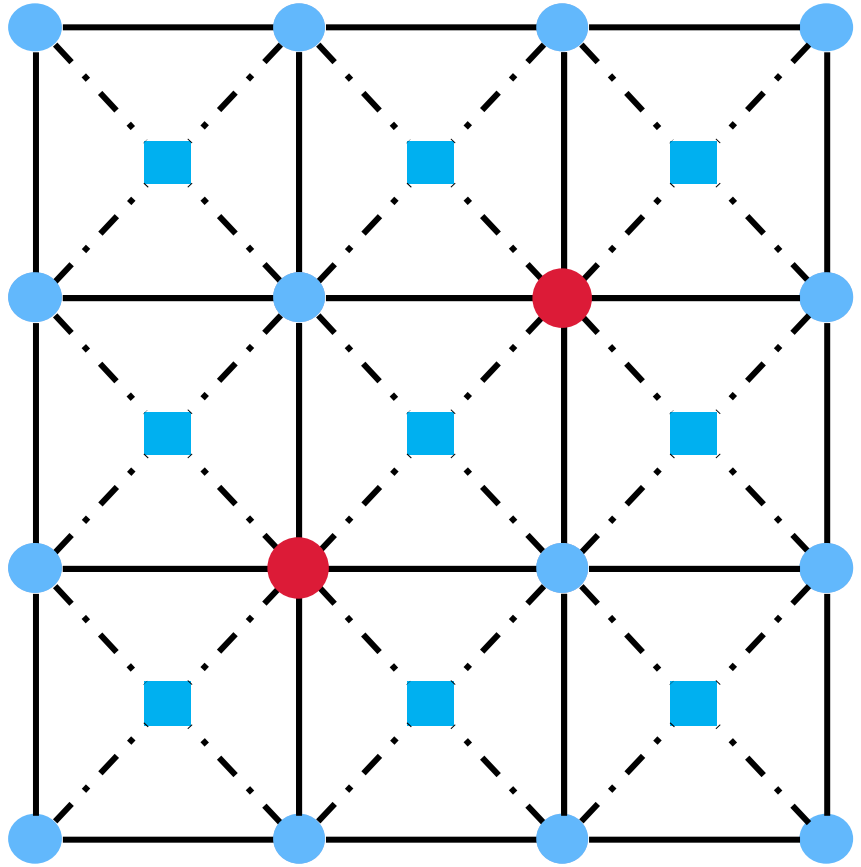
DUAL TRANSFORMATION

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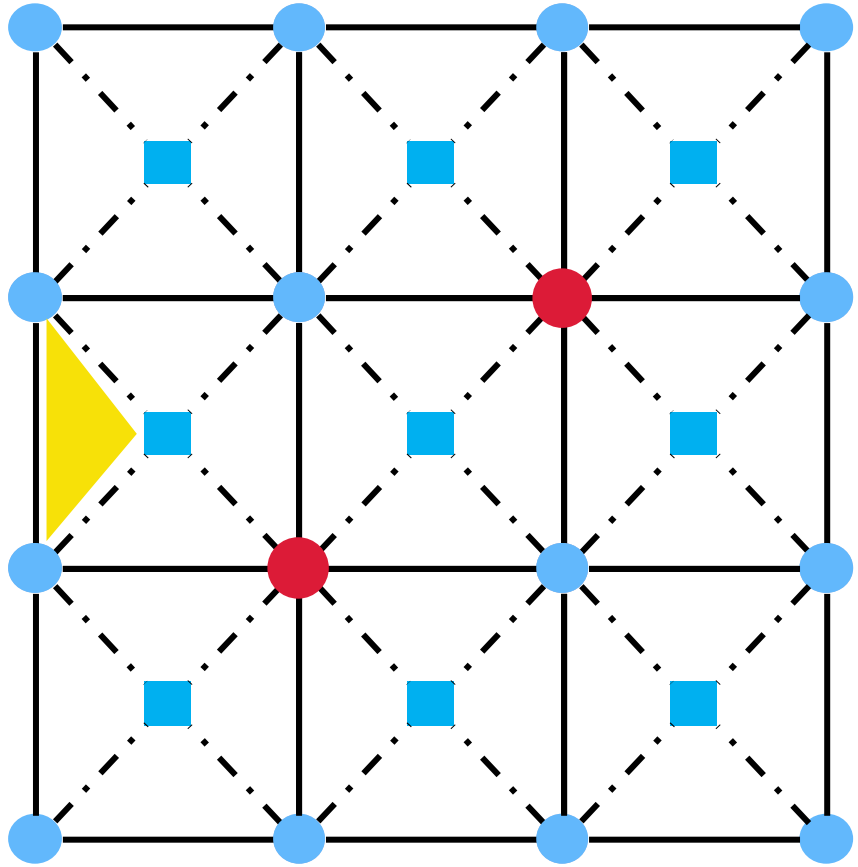
$g = -1$ $g = 1$

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DUAL TRANSFORMATION

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$$\sigma^z \sigma^z \sigma^z = \mu^x$$



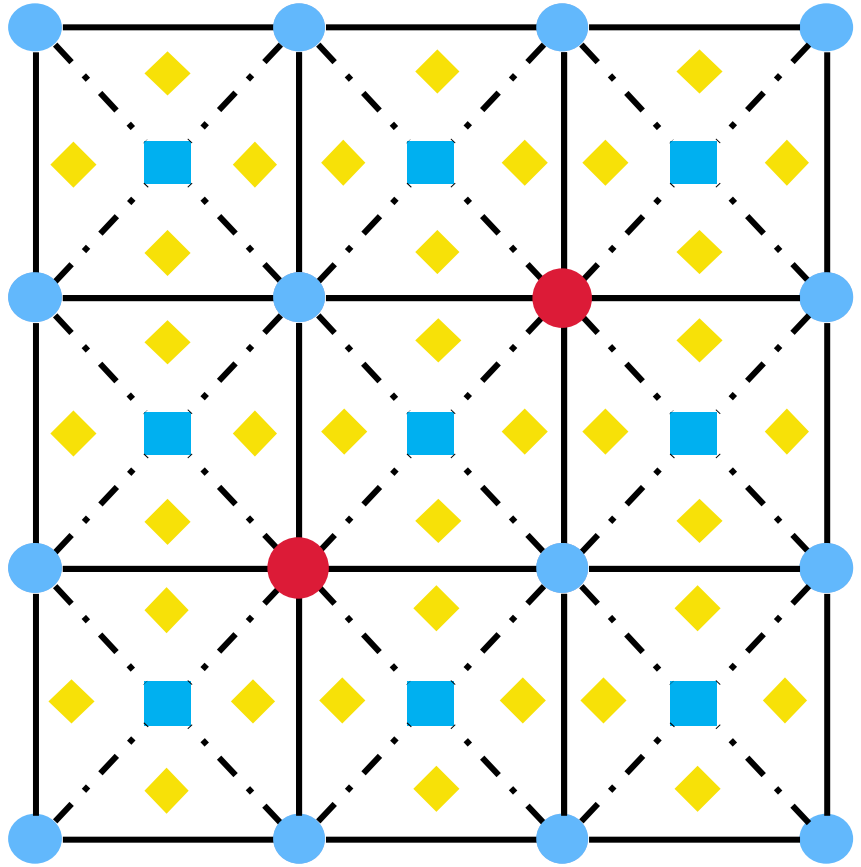
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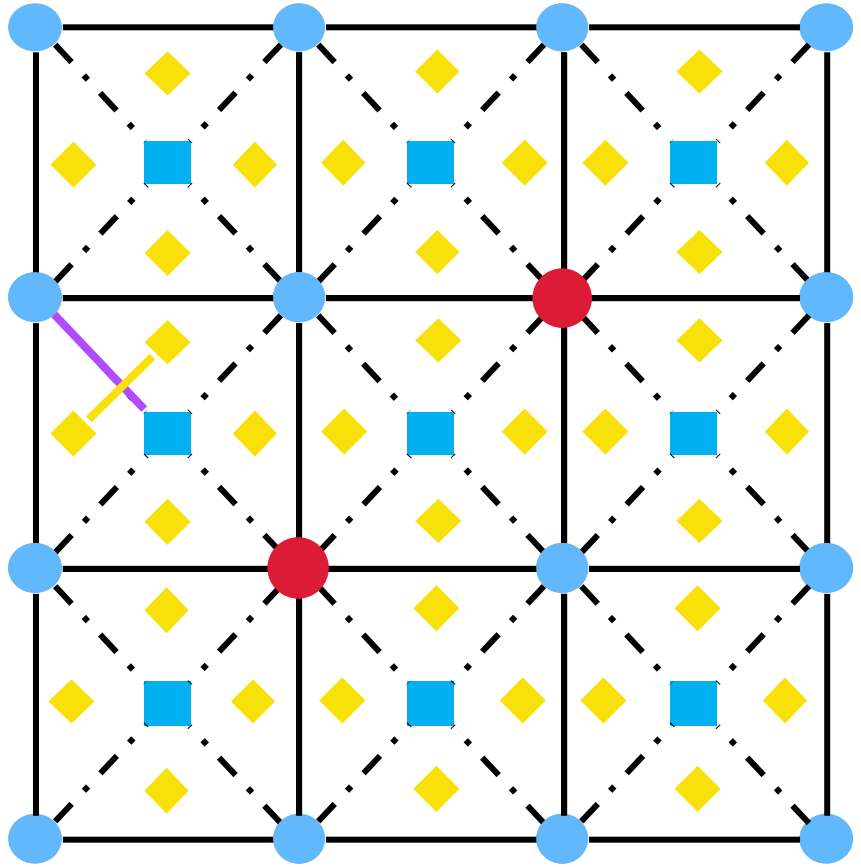
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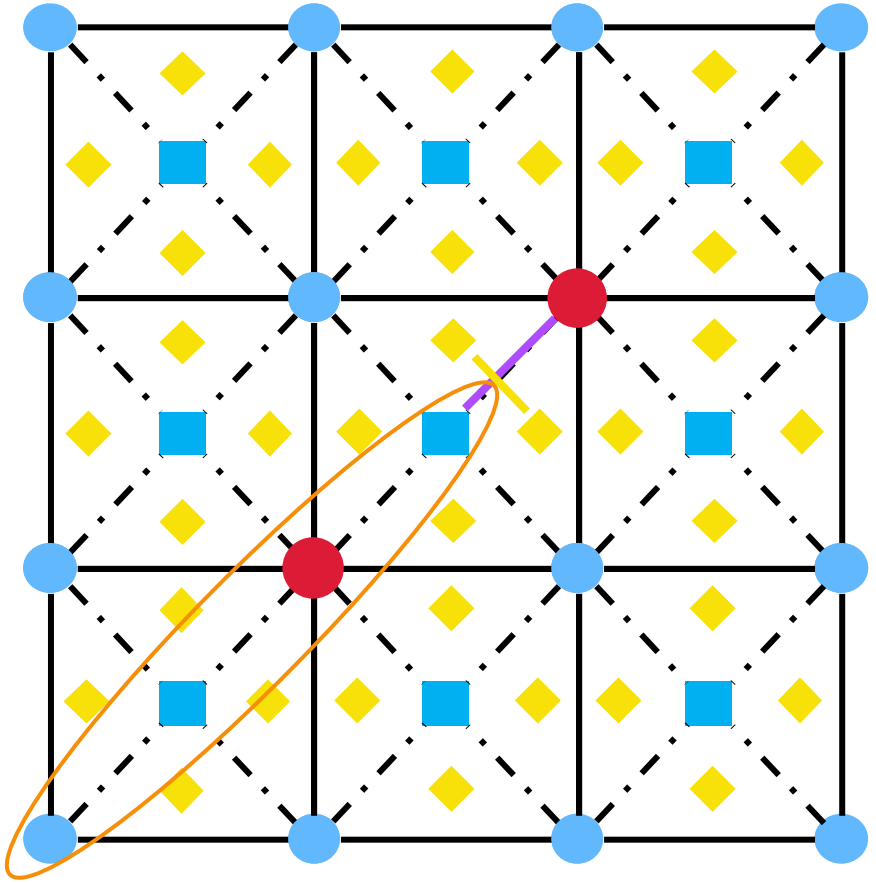
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$$\sigma^z\sigma^z\sigma^z = \mu^x$$

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$$g = -1 \qquad g = 1$$

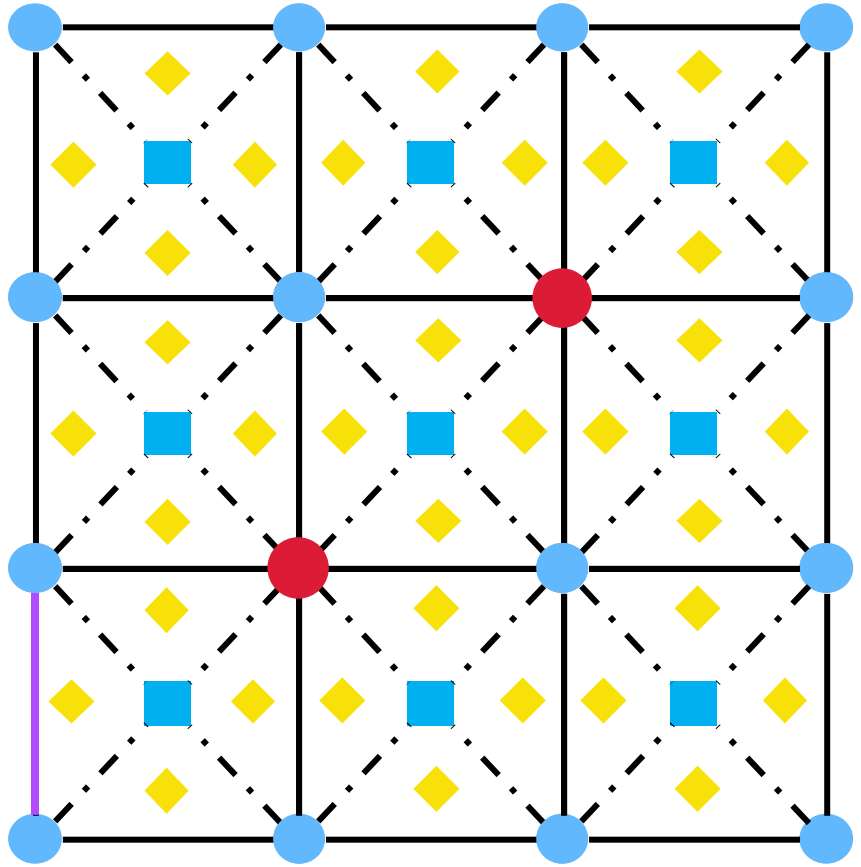
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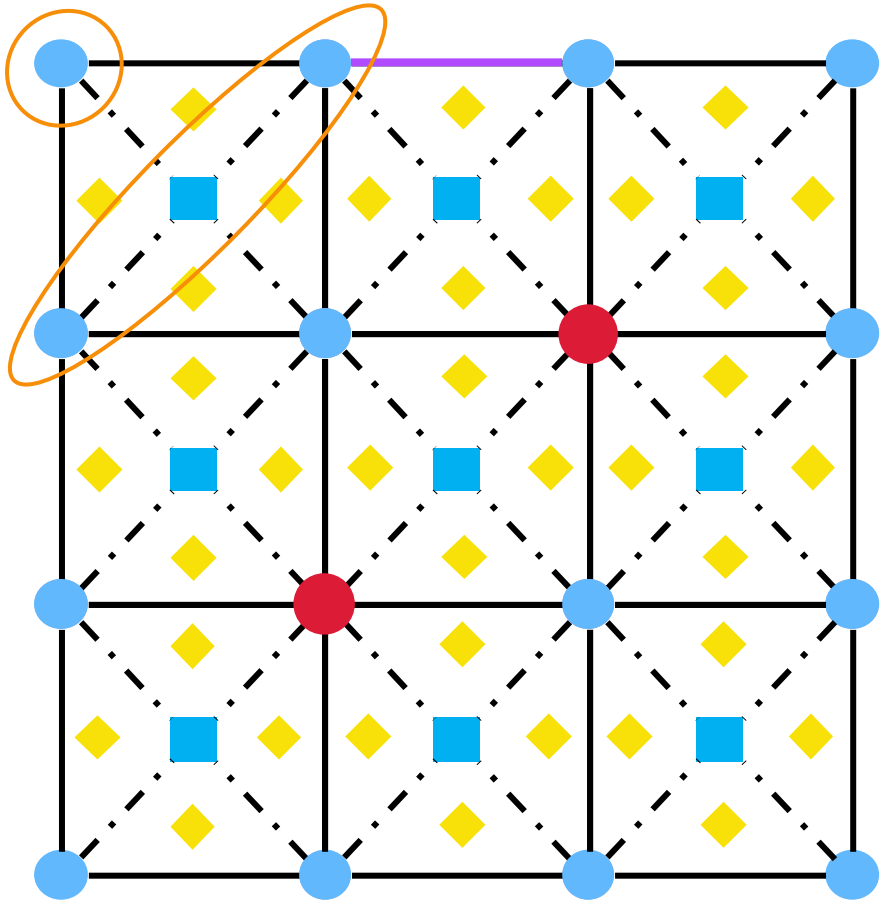
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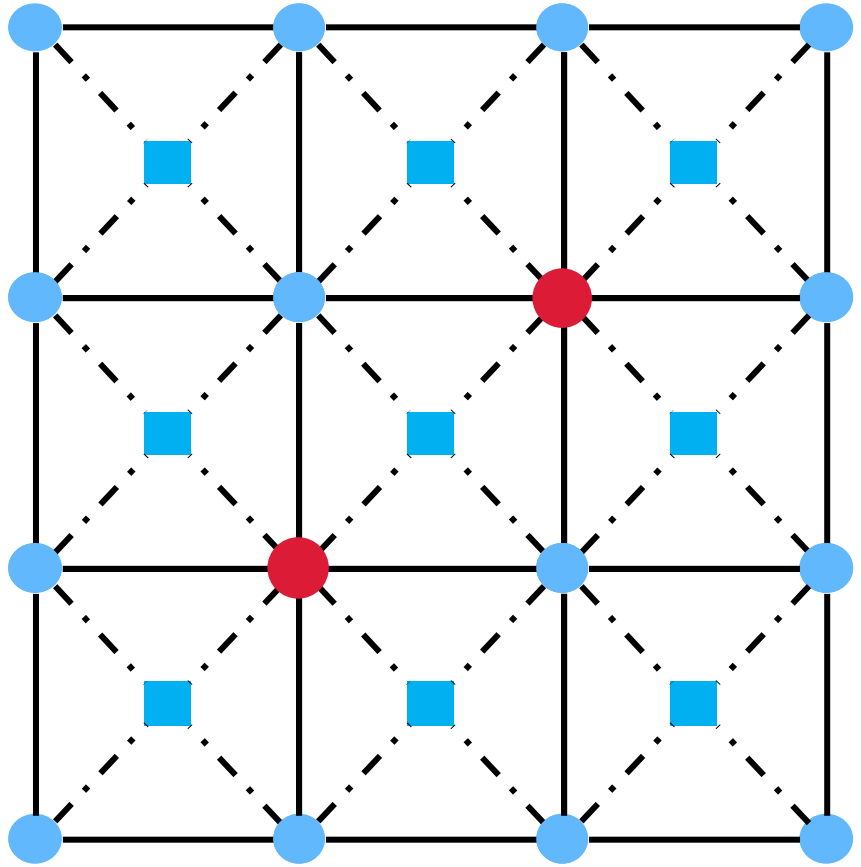


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$$H = -\mathrm{b}\sum \sigma^x - \mathrm{a}\sum_{\Delta} \sigma^z \sigma^z \sigma^z$$

$$H = -a\sum_i \mu_i^x - b\left(\sum_{<i,j>} \eta\,(i,j)\mu_i^z \mu_j^z + \sum_i \eta\,(i,i)\mu_i^z\right)$$



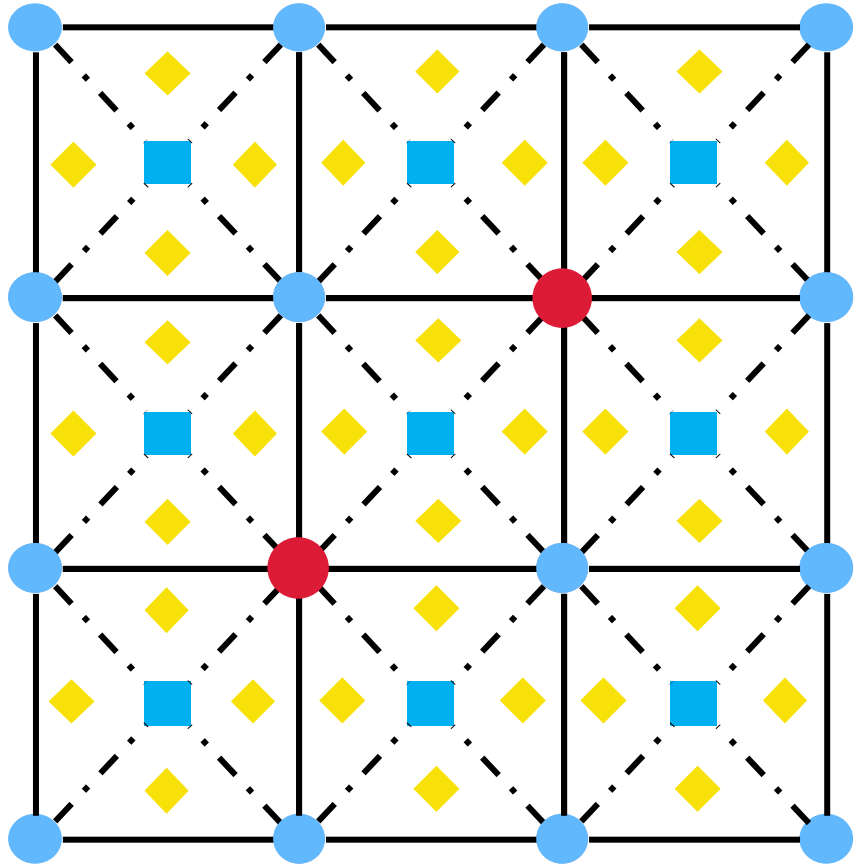
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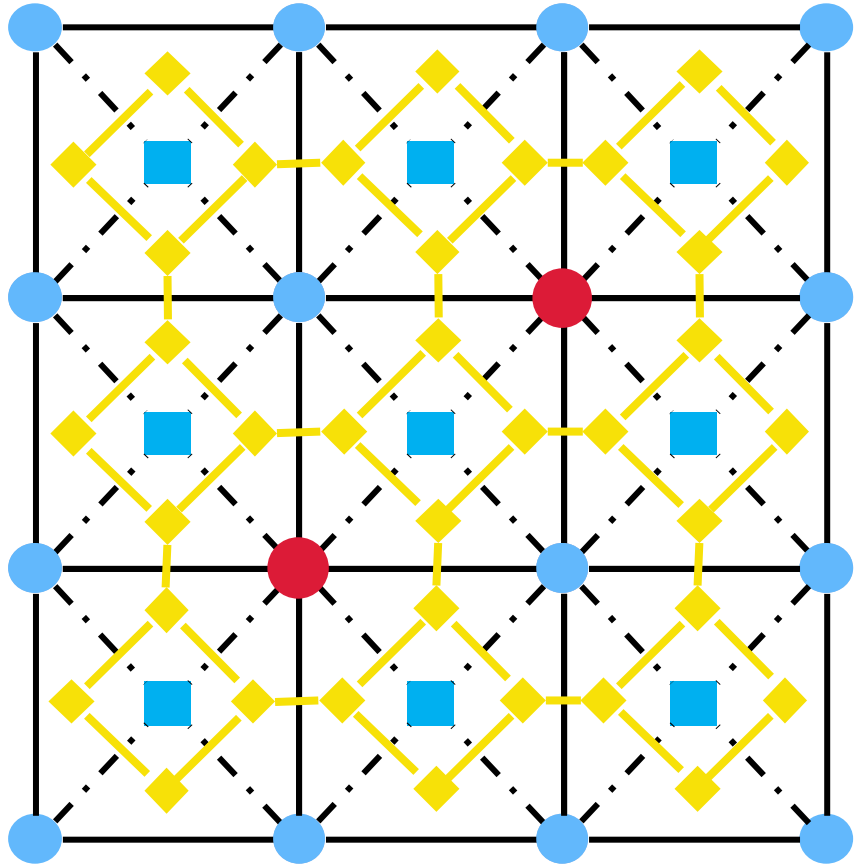
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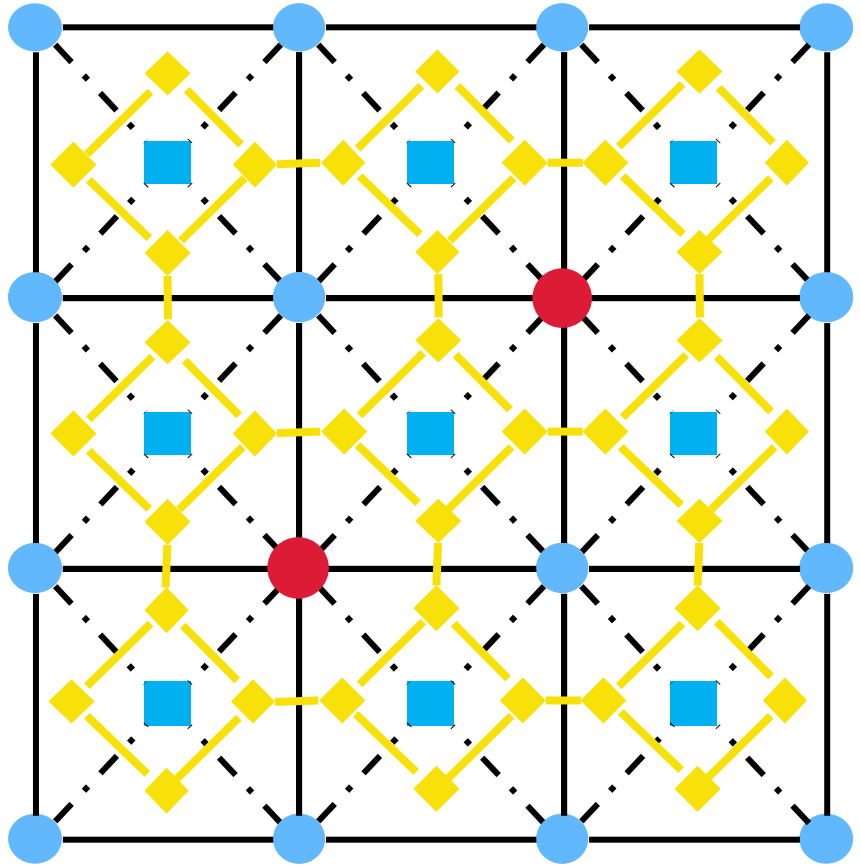
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DWave Hamiltonian:

$$\widehat{H} = -\frac{A(t)}{2}\sum_i \mu_i^x + \frac{B(t)}{2}\left(\sum_{<i,j>} J_{i,j}\,\mu^z\mu_j^z + \sum_i h_i\,\mu_i^z\right)$$



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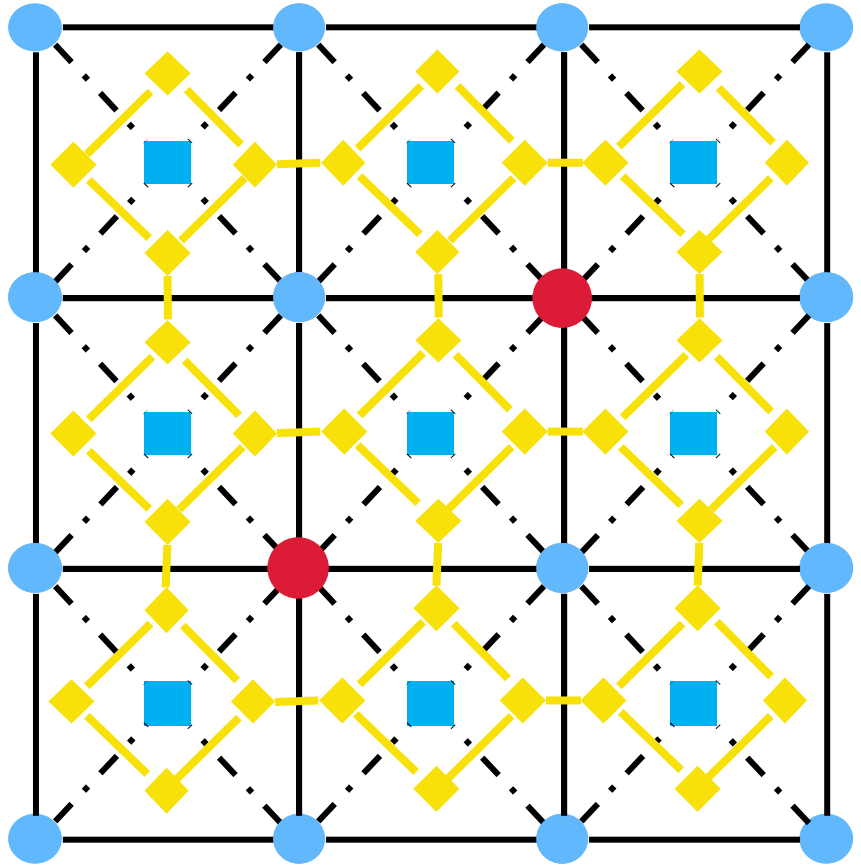
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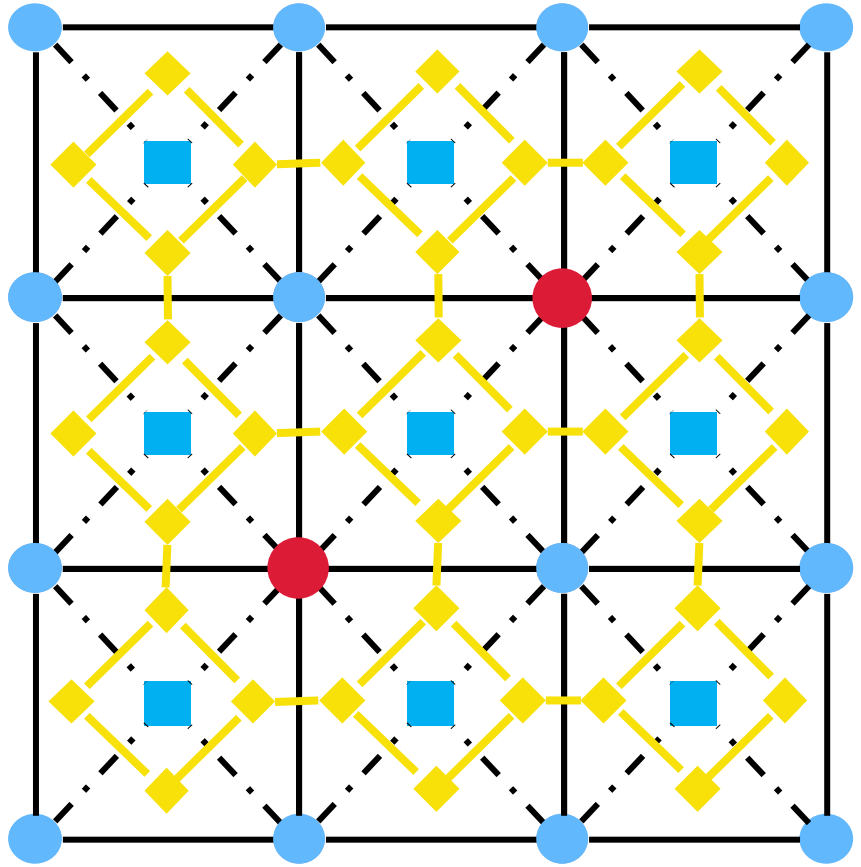
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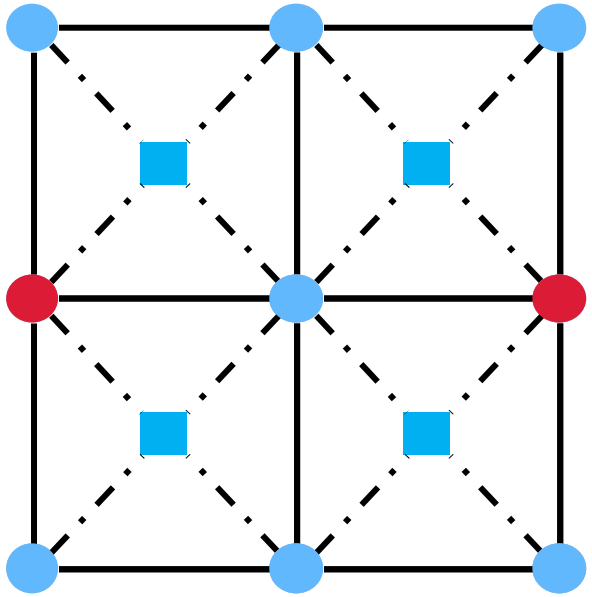
$$J_{i,j} = -\eta(i,j) \qquad h_i = -\eta(i,i) \qquad a = \frac{A(t)}{2} \qquad b = \frac{B(t)}{2}$$



$g = -1$ $g = 1$

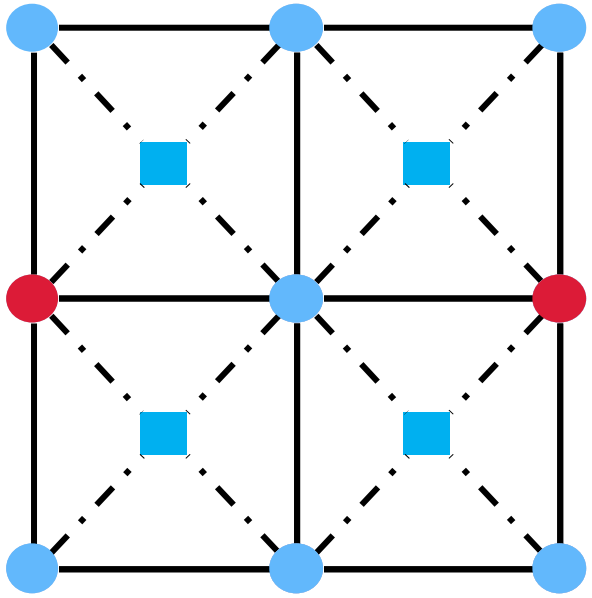
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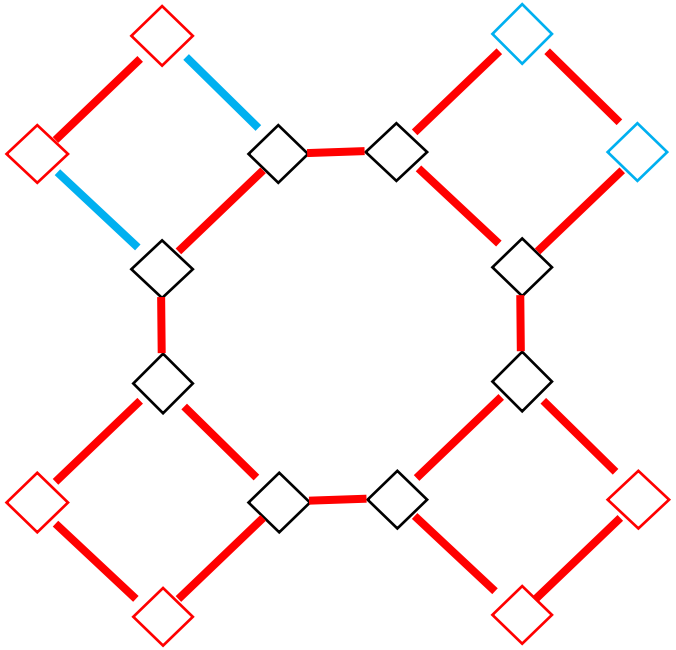
STATE TRANSFORMATION



$g = -1$ $g = 1$

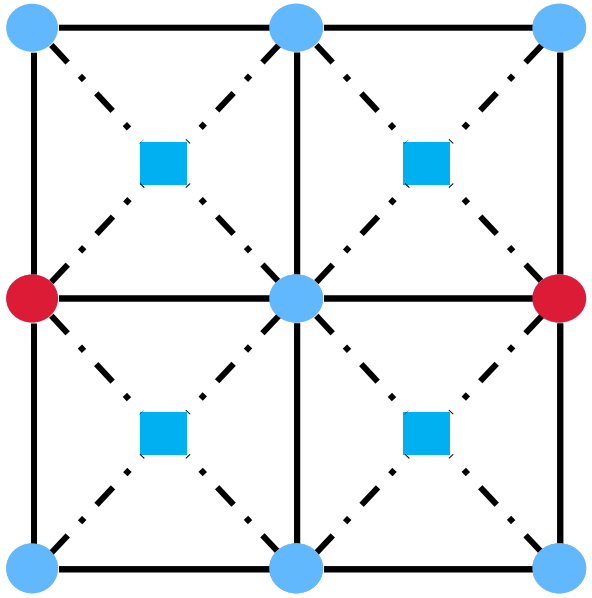
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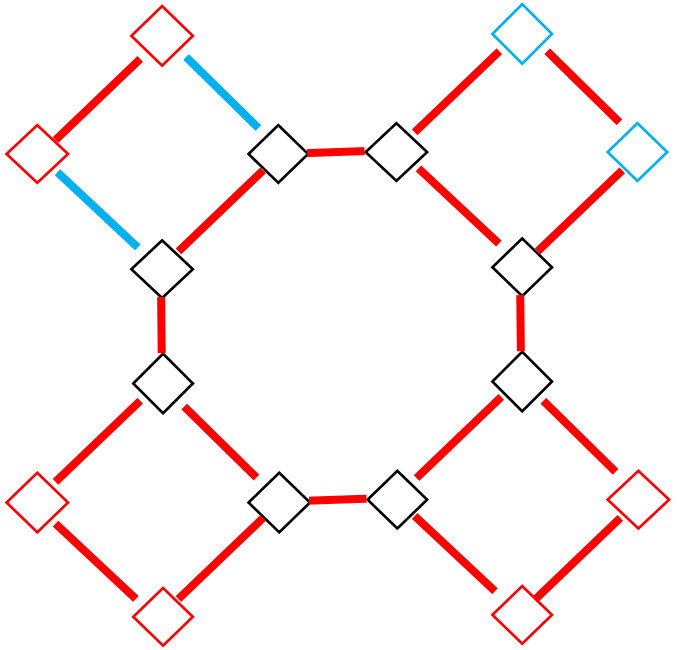


$g = -1$ $g = 1$

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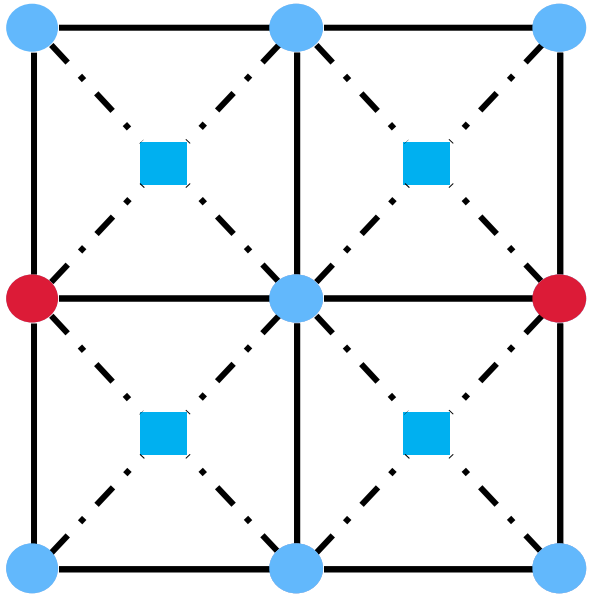


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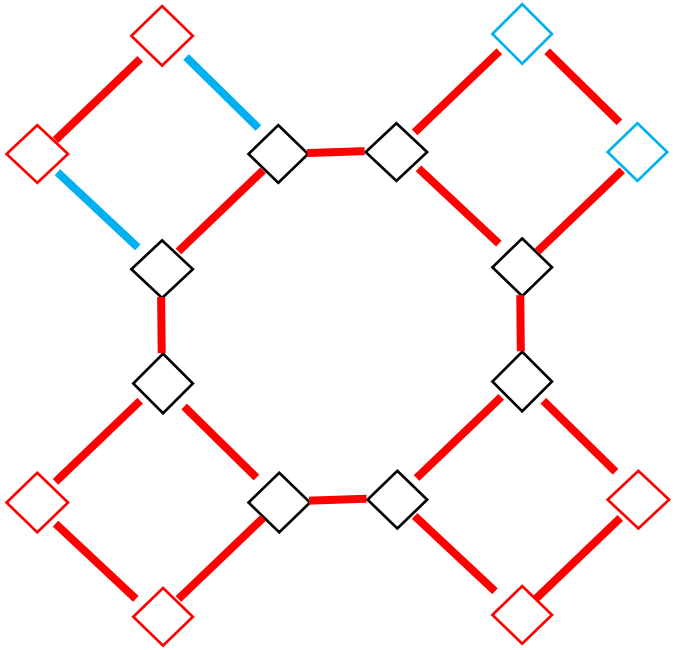


$h = 0$

STATE TRANSFORMATION



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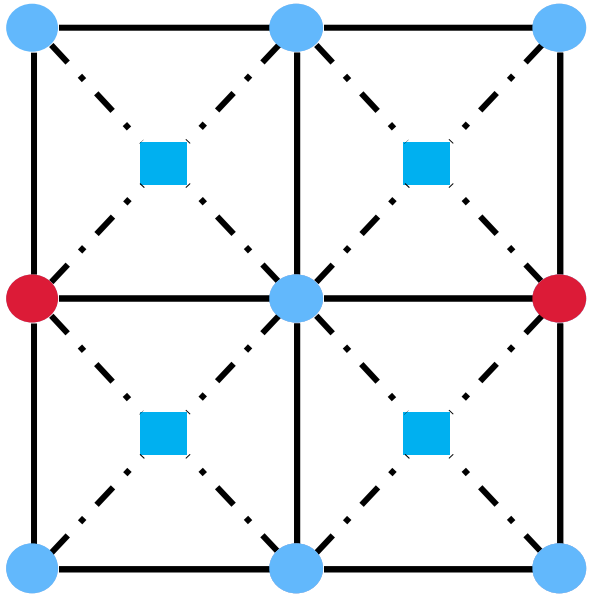


$h = 0$ $h = -1$

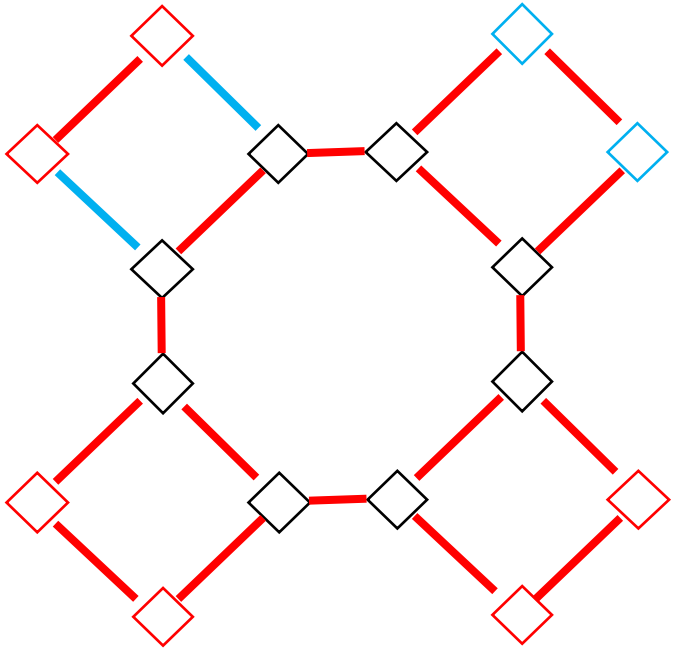
$J = -1$

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STATE TRANSFORMATION



$g = -1$ $g = 1$

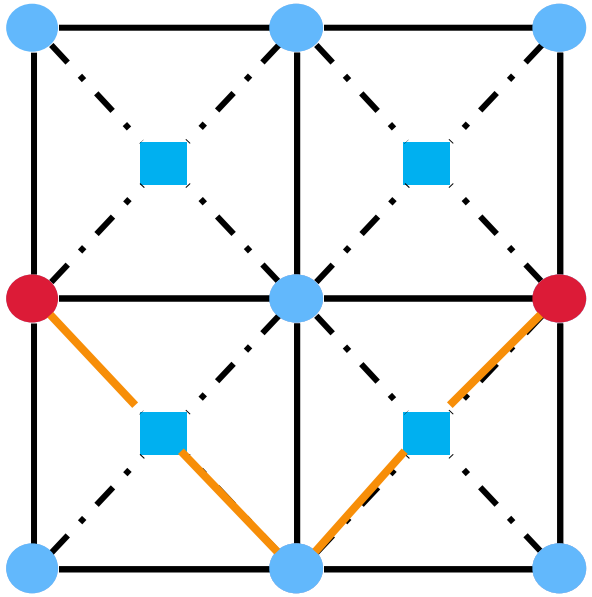


$h = 0$ $h = -1$ $h = +1$

$J = -1$ $J = +1$

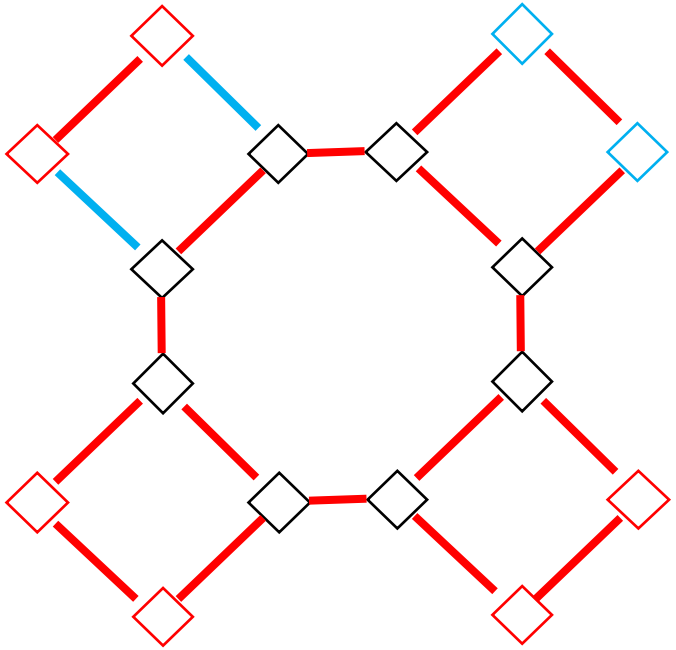
JÜLICH Forschungszentrum

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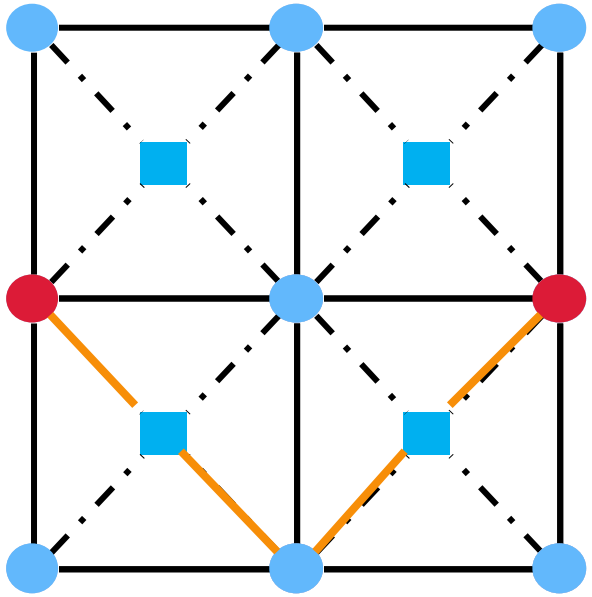


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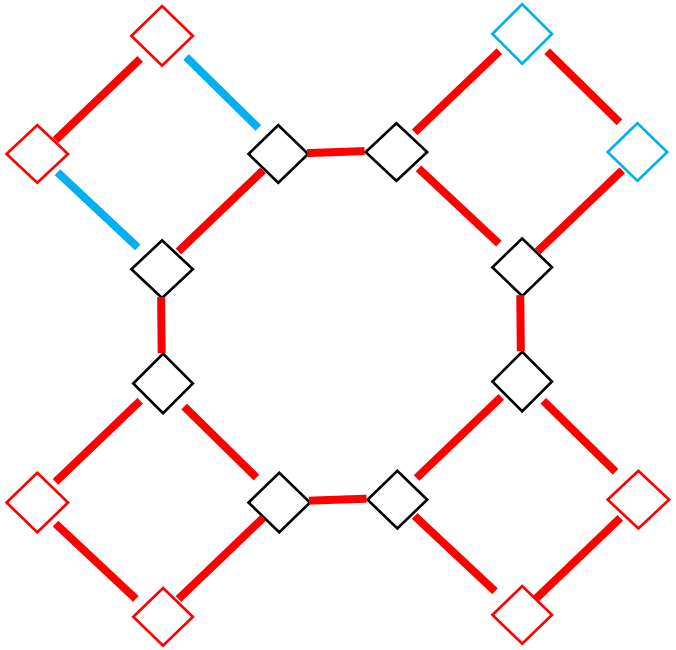
JÜLICH Forschungszentrum

STATE TRANSFORMATION



$$\sigma^x = -J\mu^z\mu^z$$

$$\sigma^x = -h\mu^z$$



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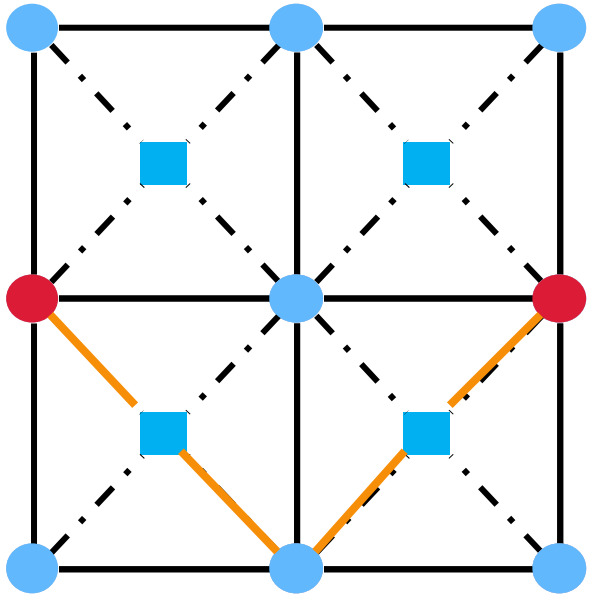
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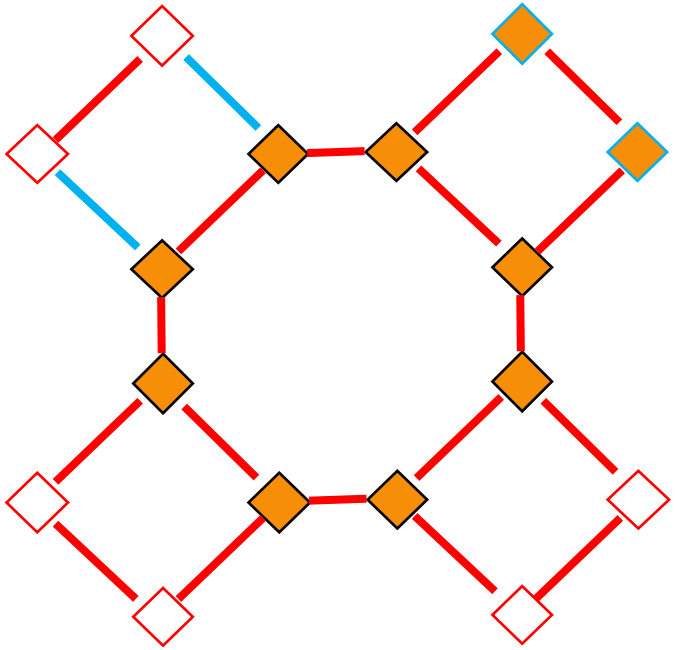
JÜLICH Forschungszentrum

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$\blacklozenge = |1\rangle$ $\lozenge = |0\rangle$

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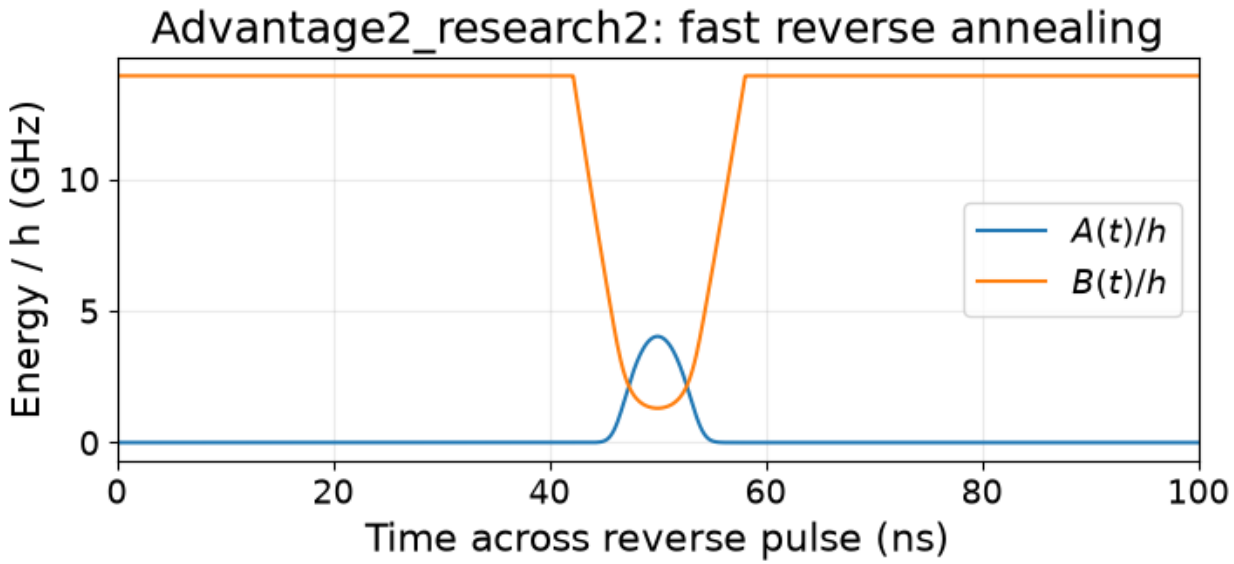
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FAST REVERSE ANNEALING

$$\hat{H} = -\frac{A(t)}{2}\sum_i \mu_i^x + \frac{B(t)}{2}\left(\sum_{\langle i,j\rangle} J_{i,j}\,\mu^z\mu_j^z + \sum_i h_i\,\mu_i^z\right)$$

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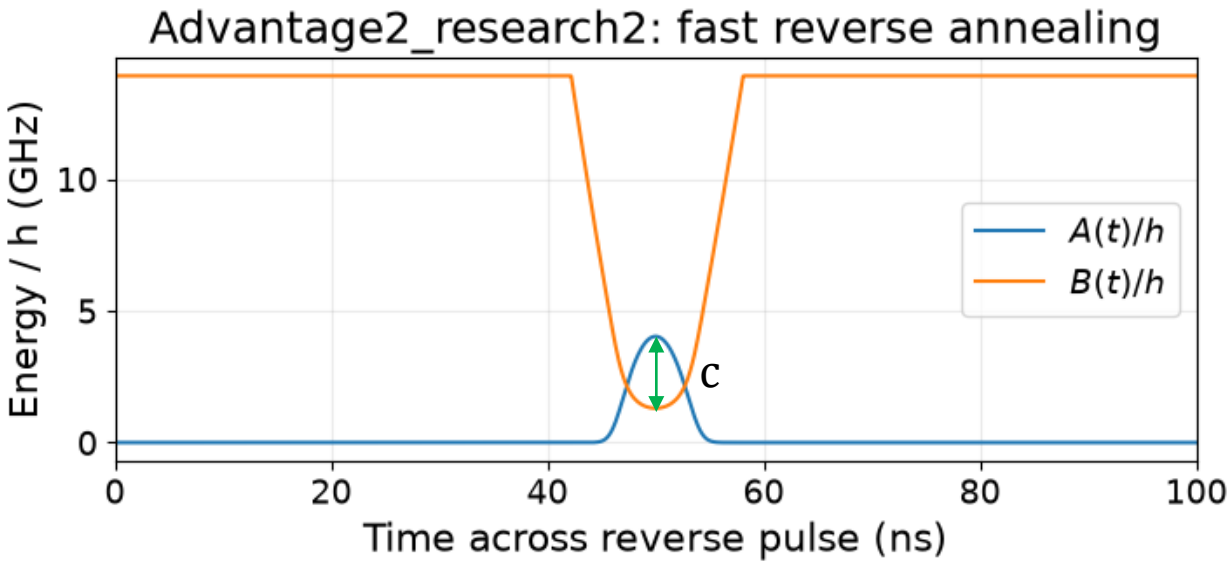
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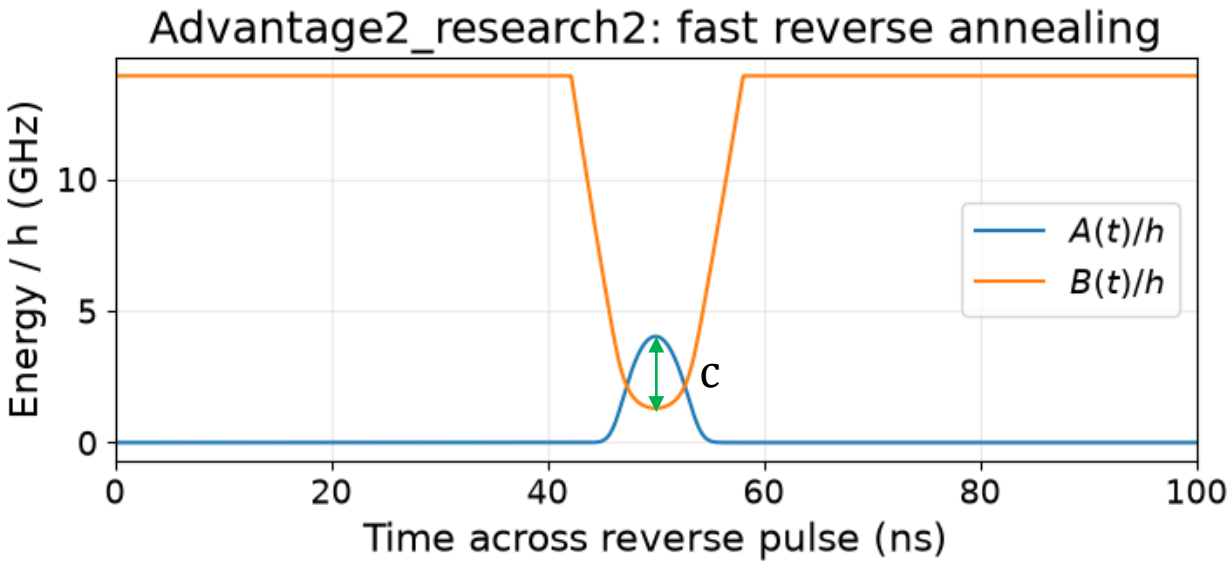
- Pulseform is fixed but we can specify a $c \in [0,1]$



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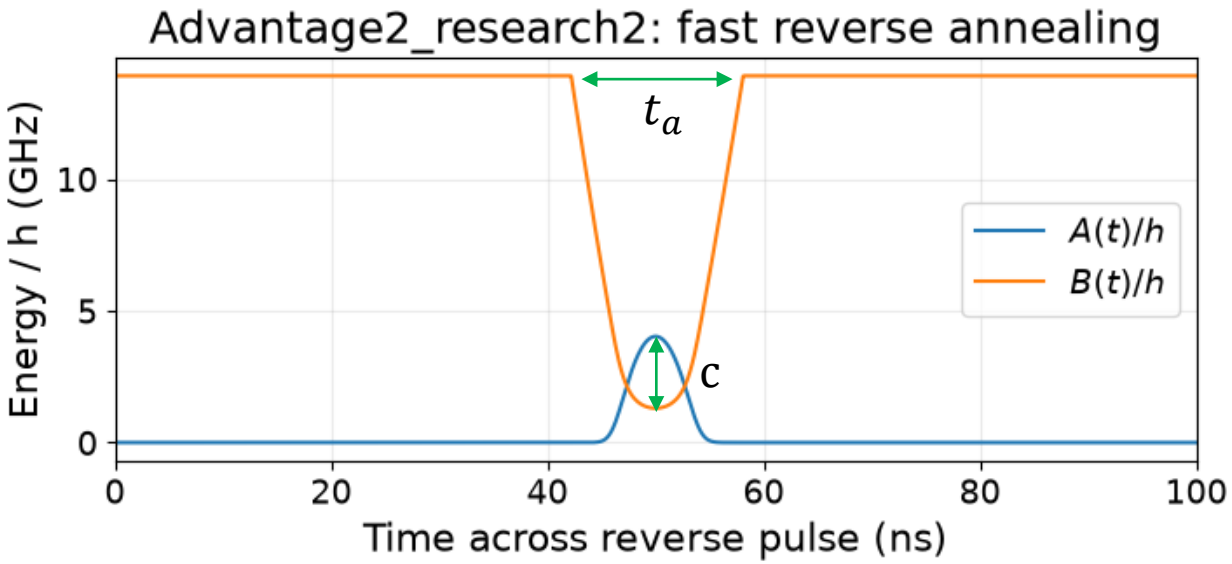
- Pulseform is fixed but we can specify a $c \in [0,1]$
  - c ≡ measure of balance between A(t) and B(t)



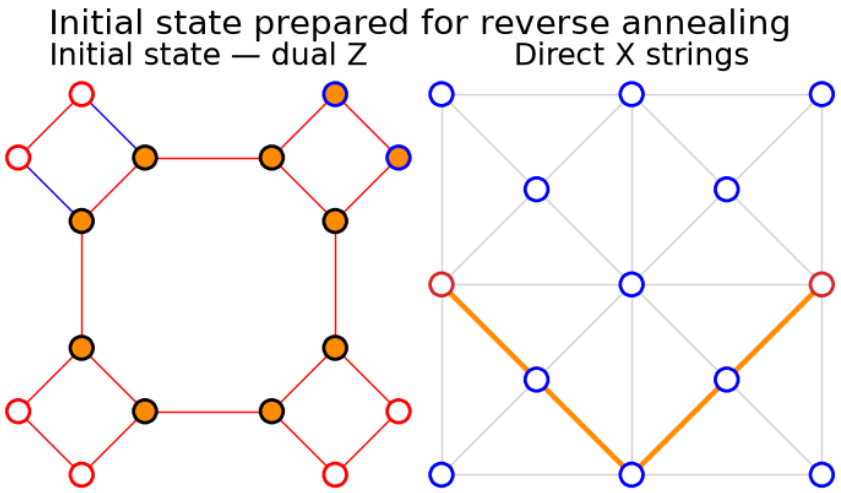
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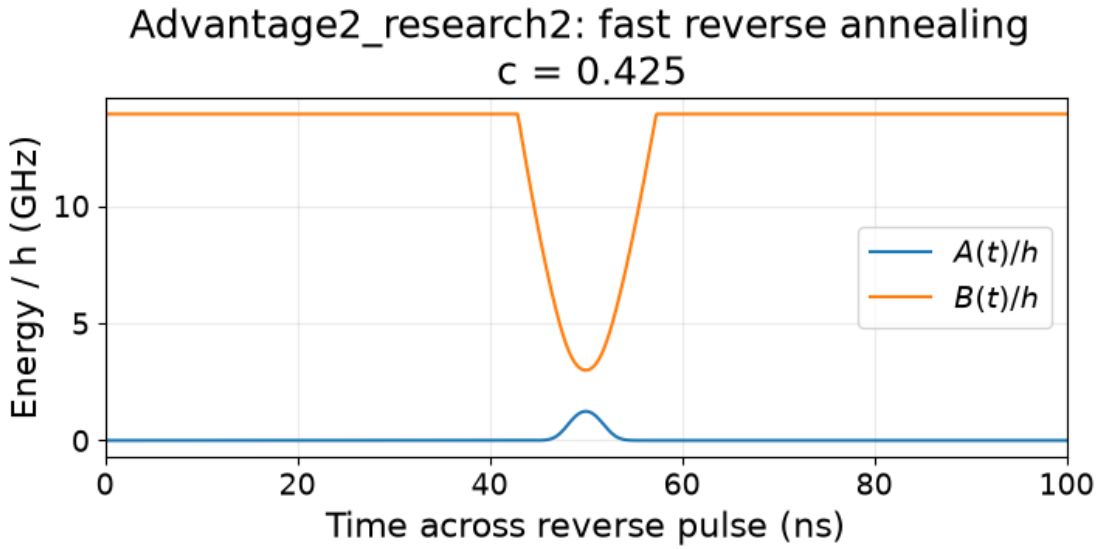
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- Pulse width determines the annealing time t_a

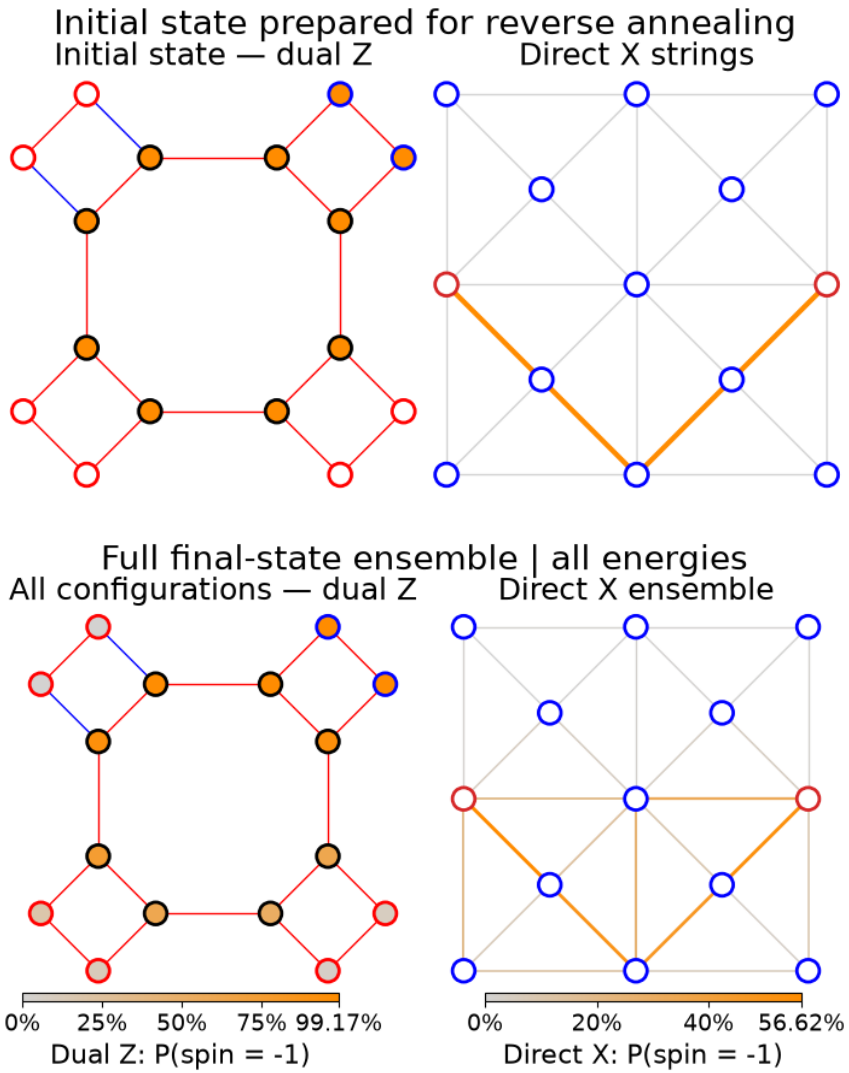


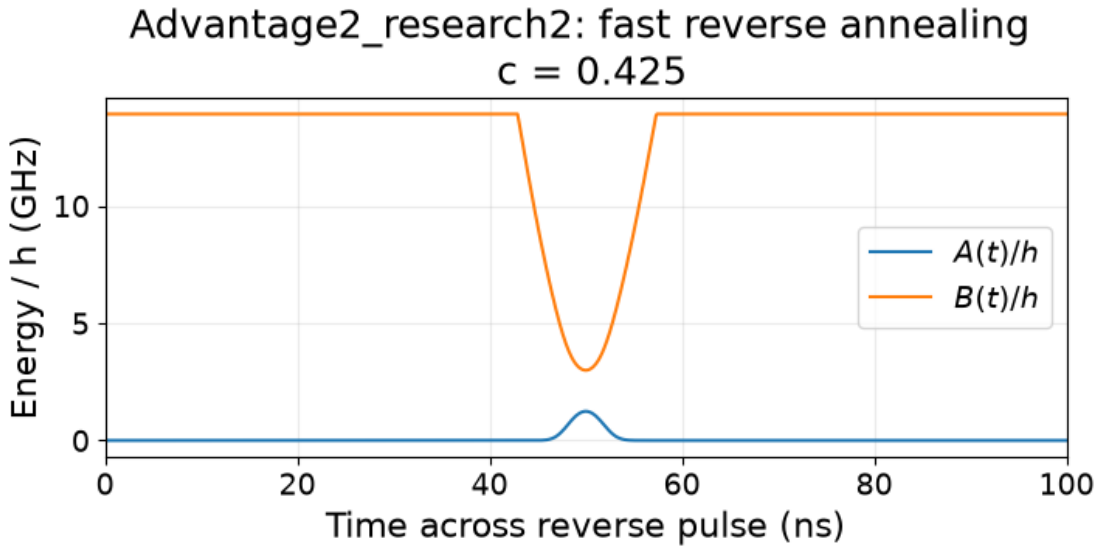
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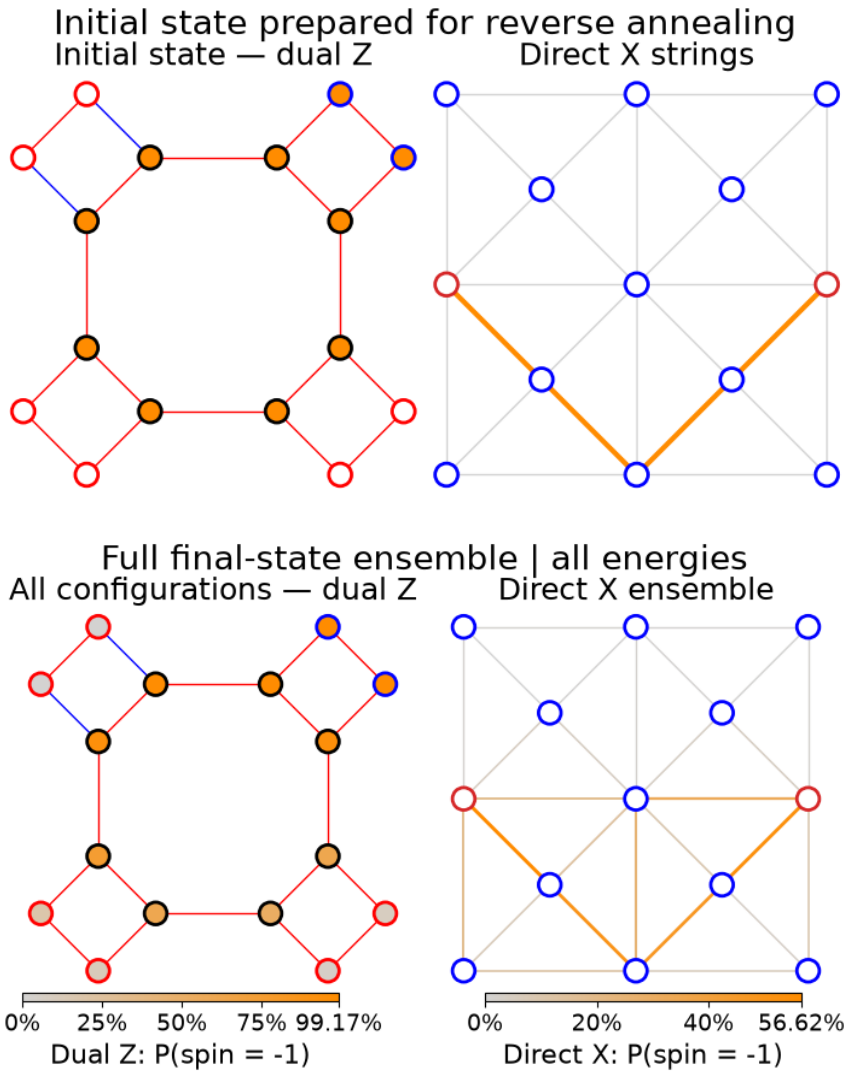


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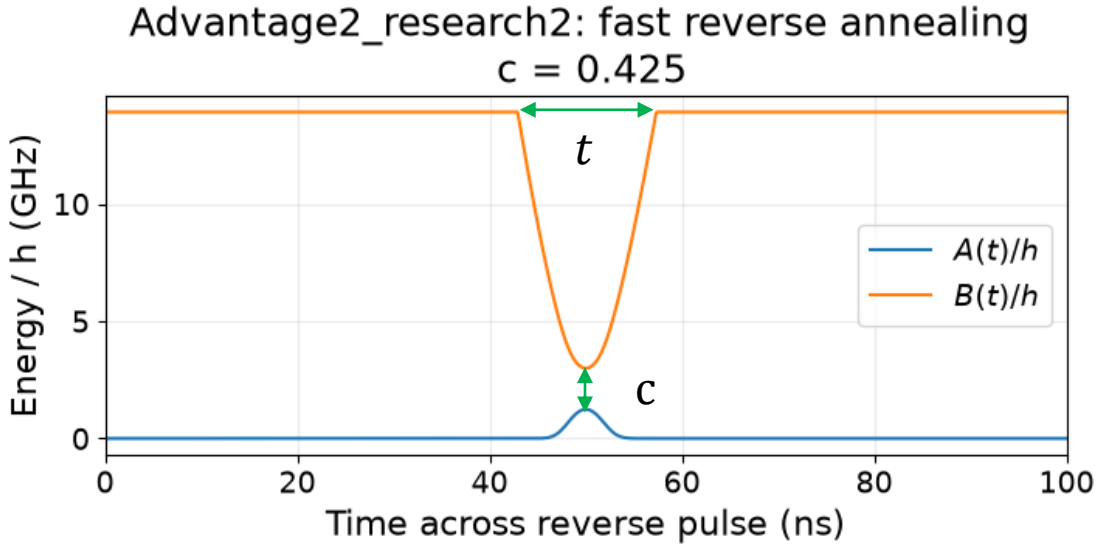




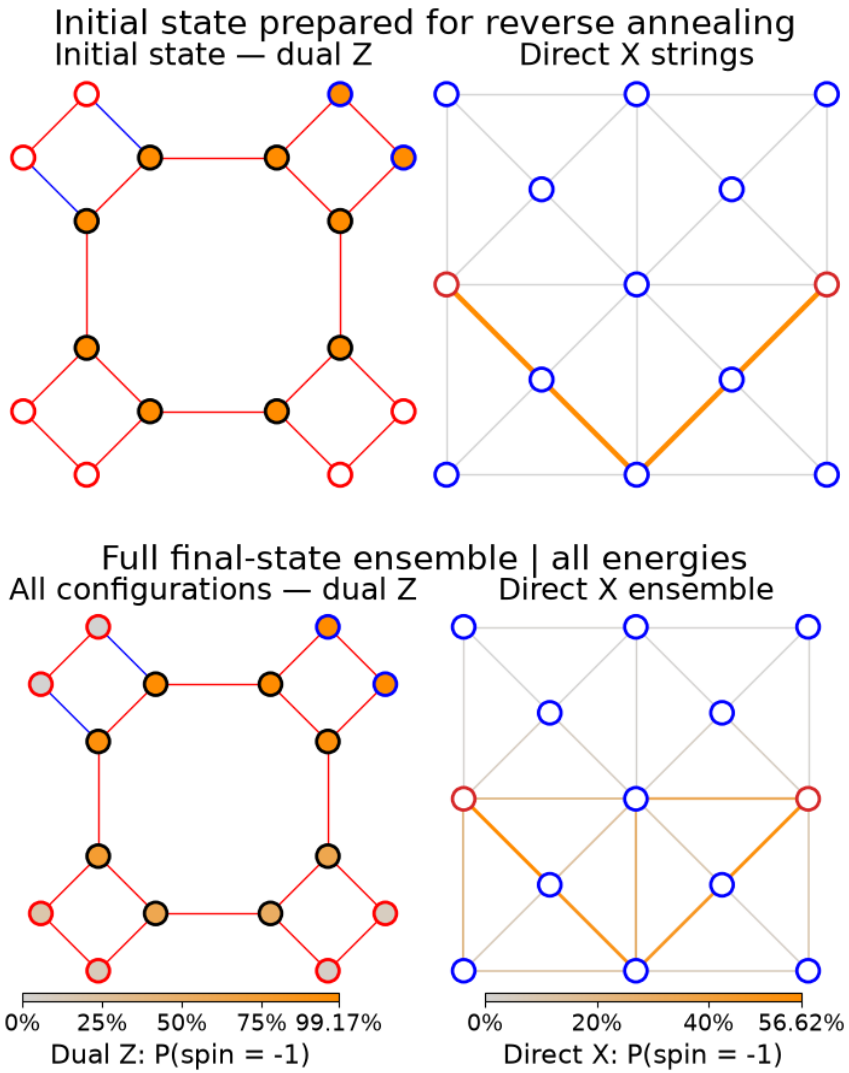
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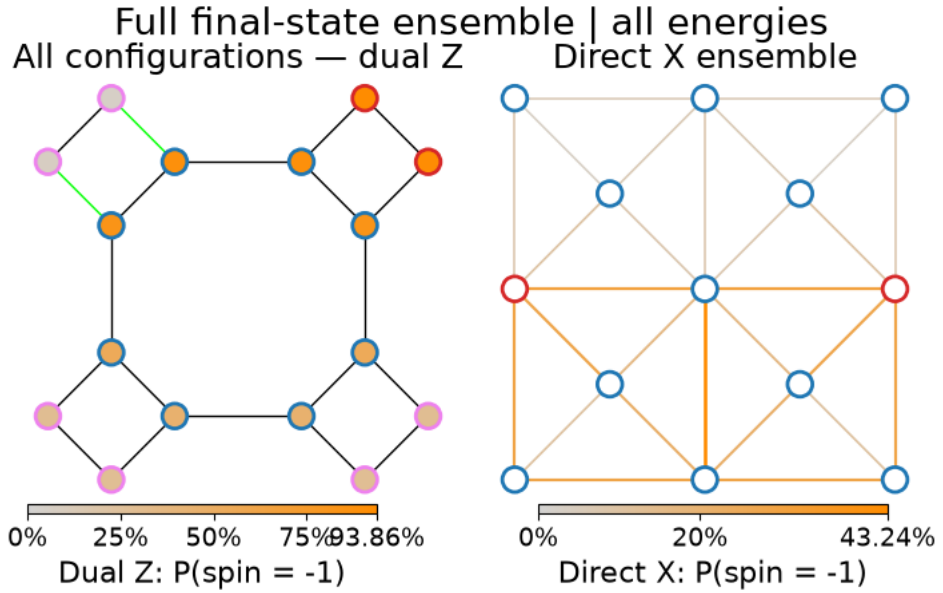
$$|\phi\rangle_t = e^{-i\hat{H}_c t}|\phi\rangle_0$$



FAST REVERSE ANNEALING



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Numerical simulation for $t = 14.4ns$

NUMERICAL SIMULATIONS

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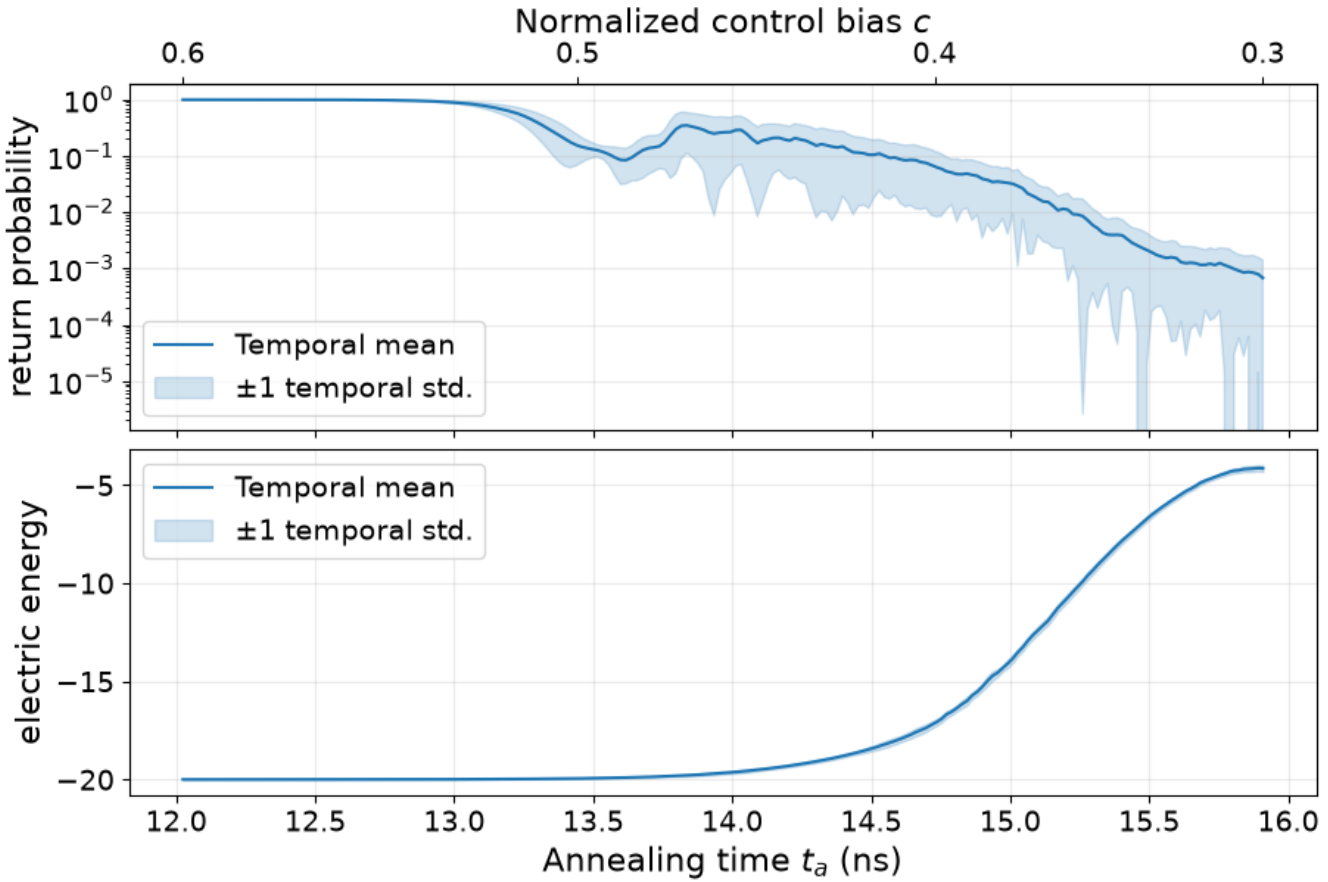
$$\langle H_{\mathrm{E}}\rangle = \left\langle \sum_{\langle i,j\rangle} J_{i,j}\,\mu^z\mu_j^z + \sum_i h_i\,\mu_i^z \right\rangle$$

NUMERICAL SIMULATIONS

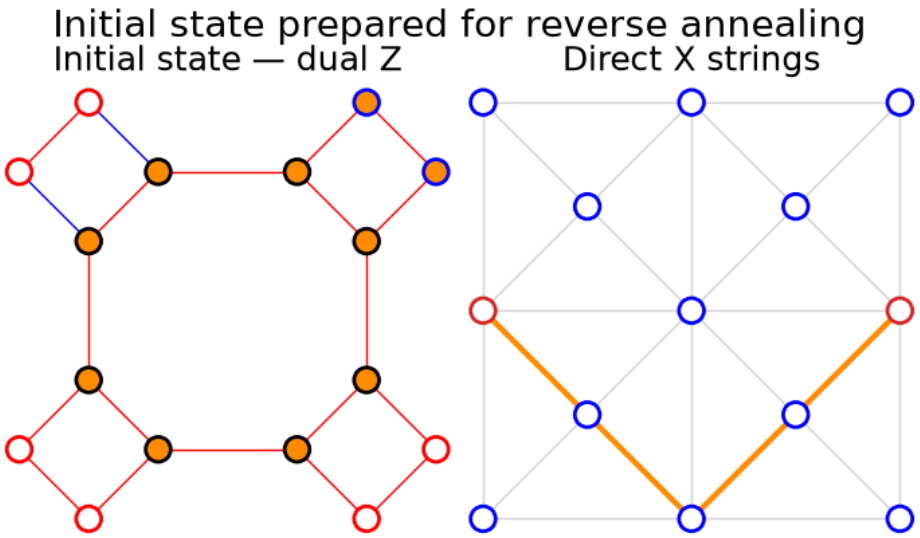
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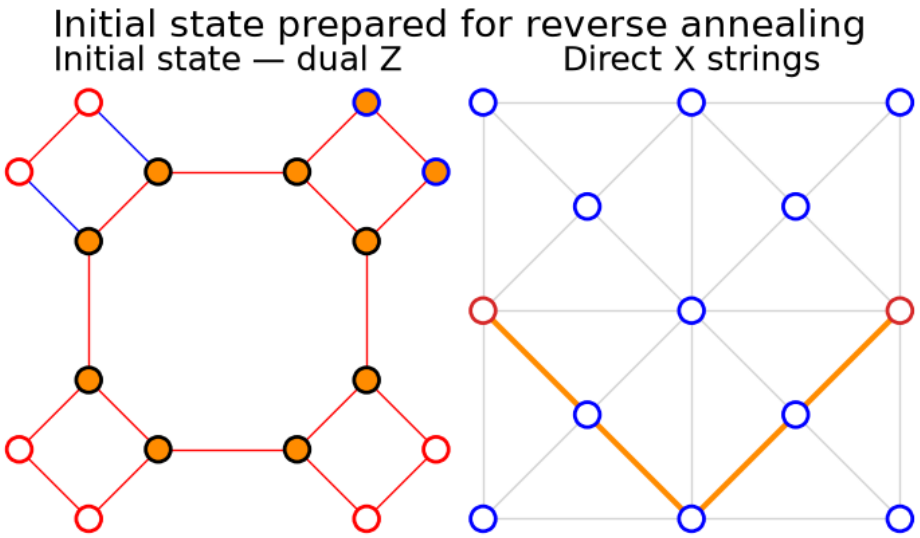
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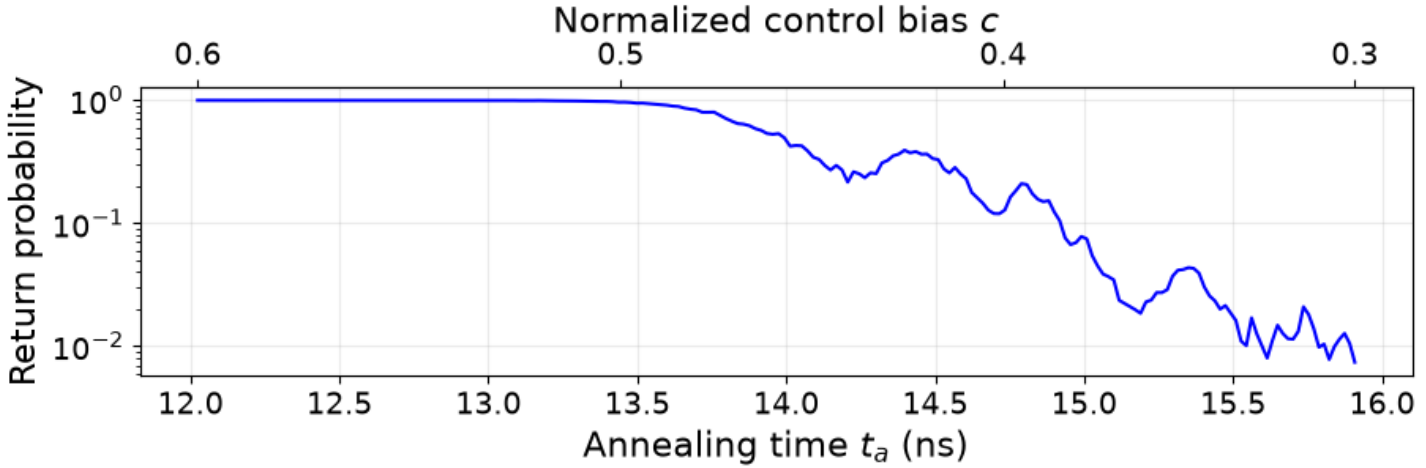


DWAVE RESULTS

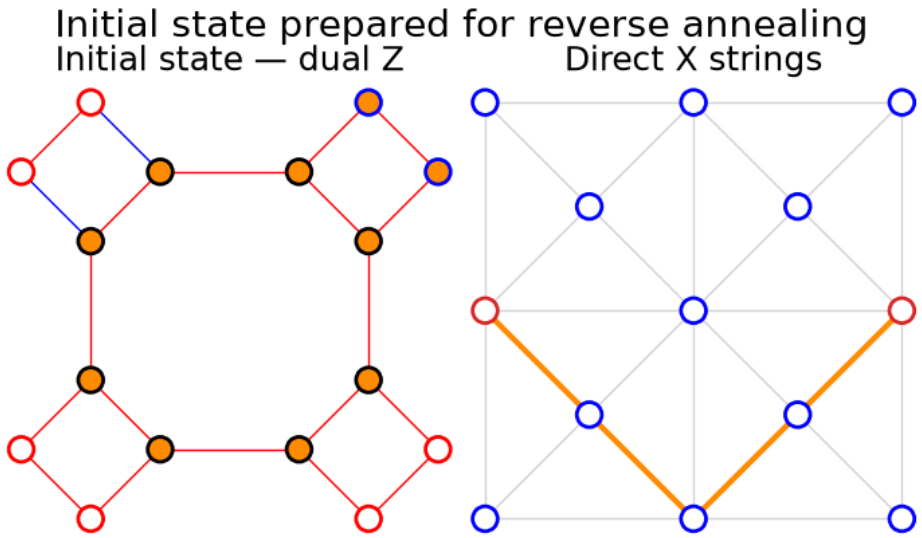


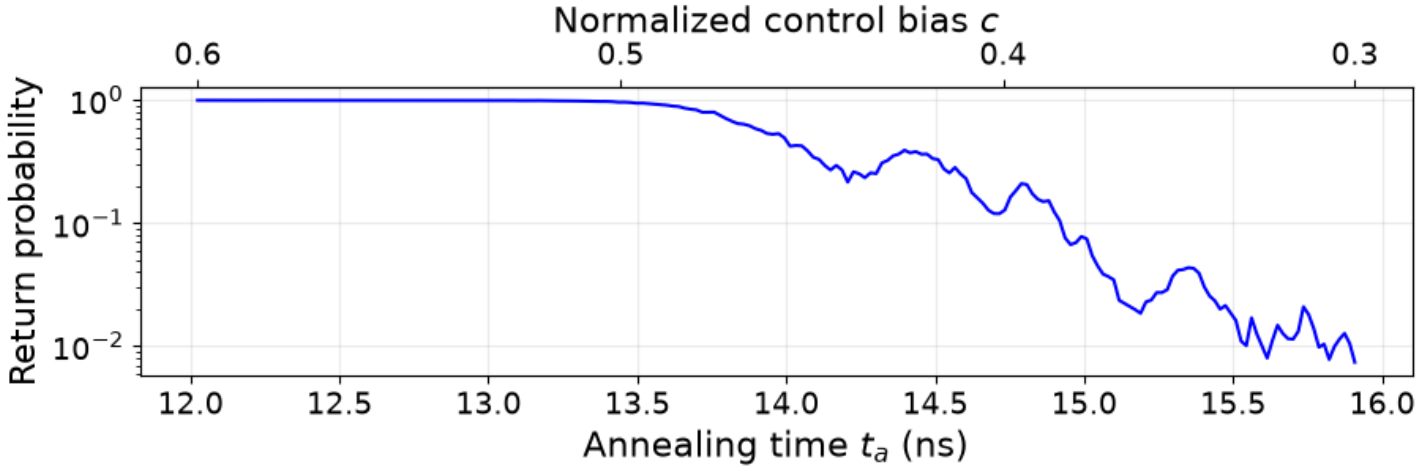
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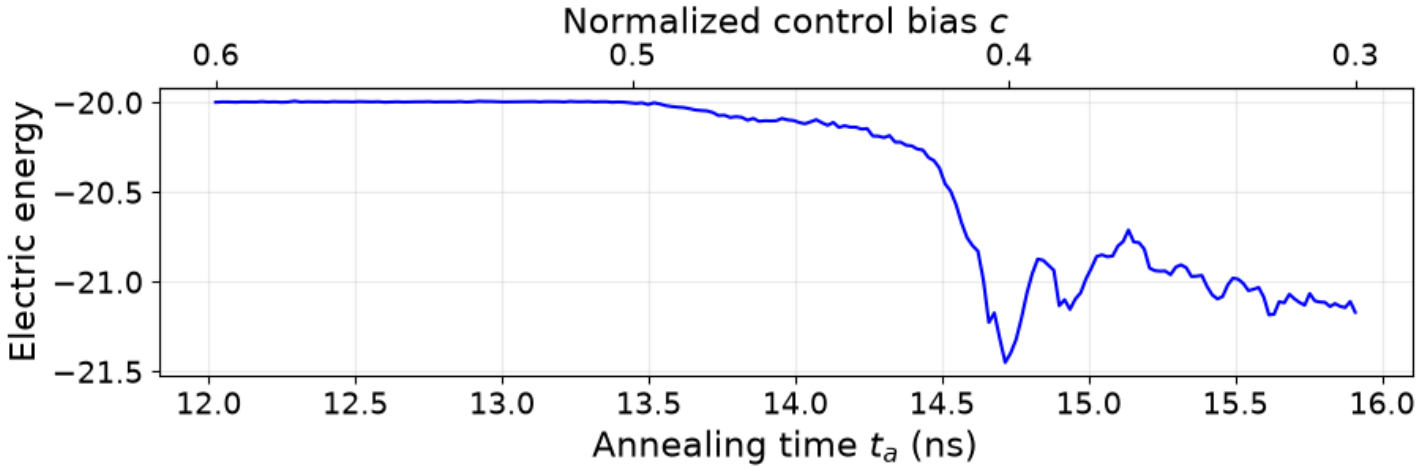




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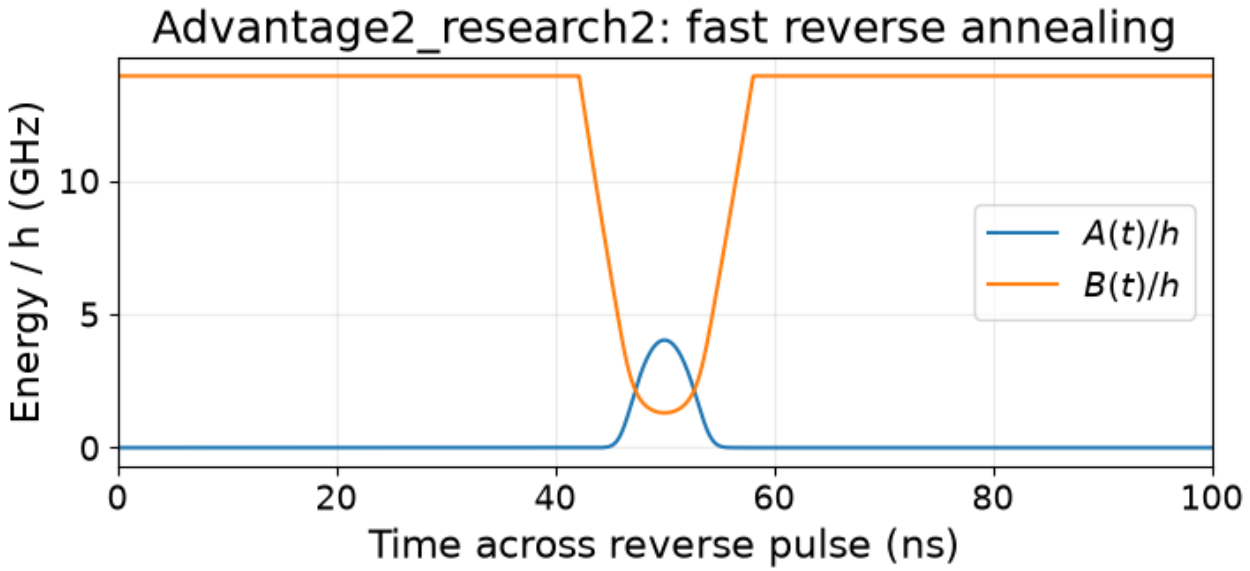






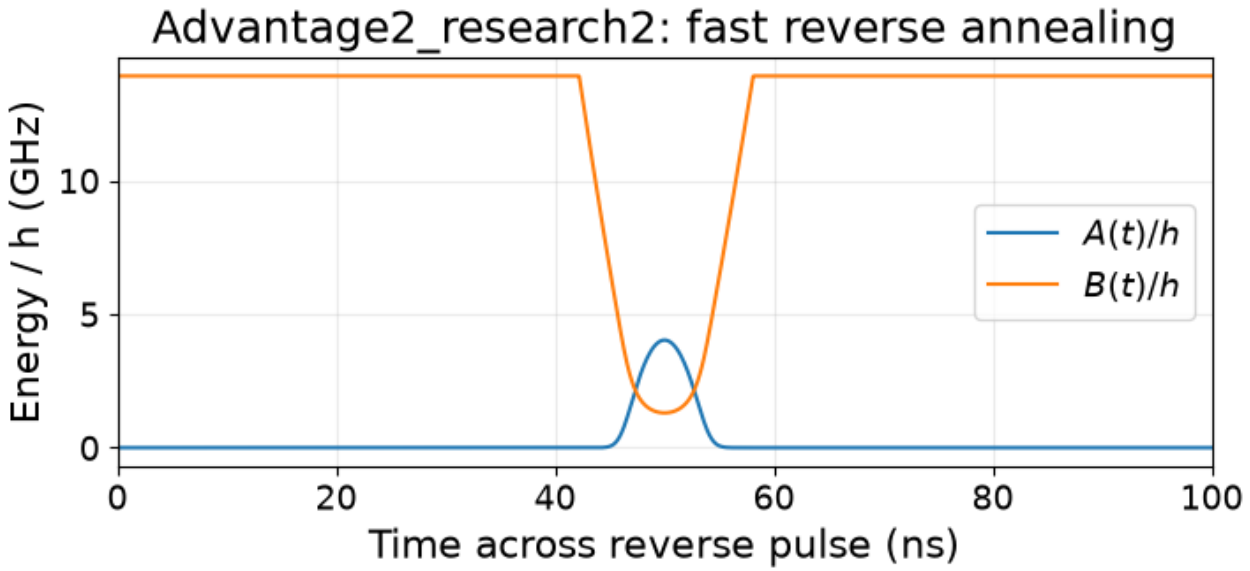
CAVEATS

- DWave Hamiltonian is time-dependent



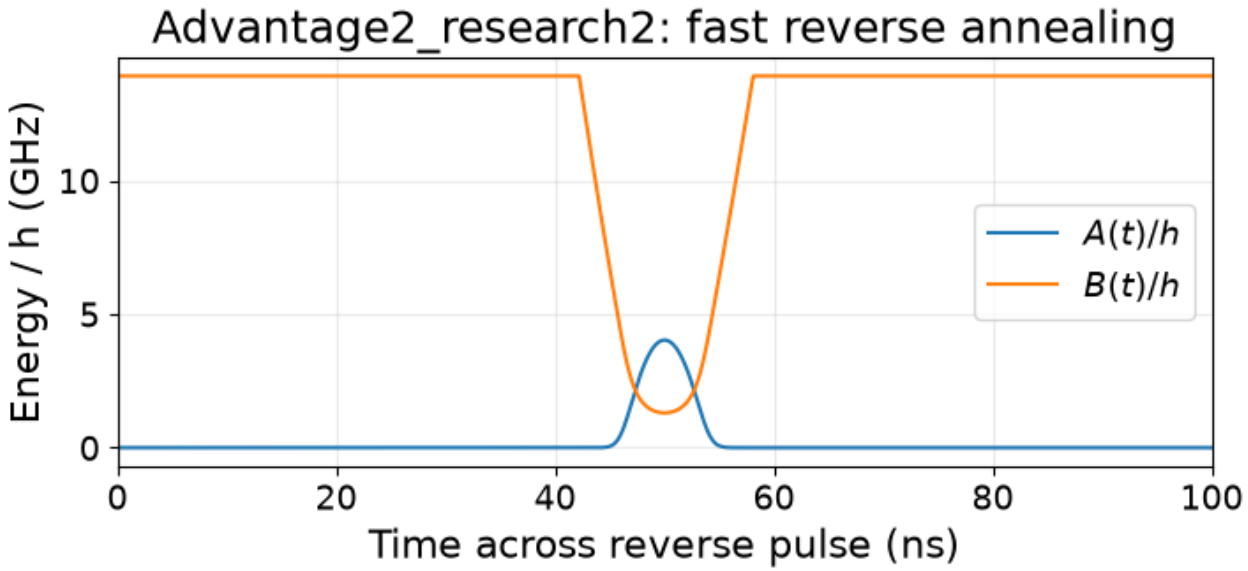
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  - ➢ One has to emulate them, e.g. by using constant flux biases



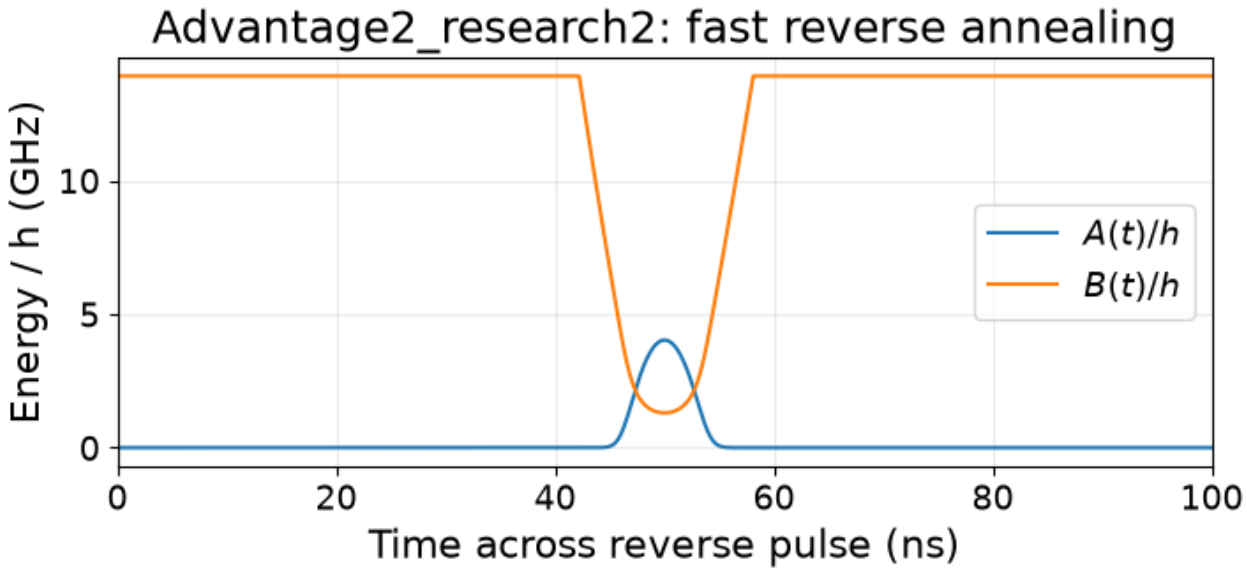
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  - Different annealing schedule offsets per qubit, flux biases…



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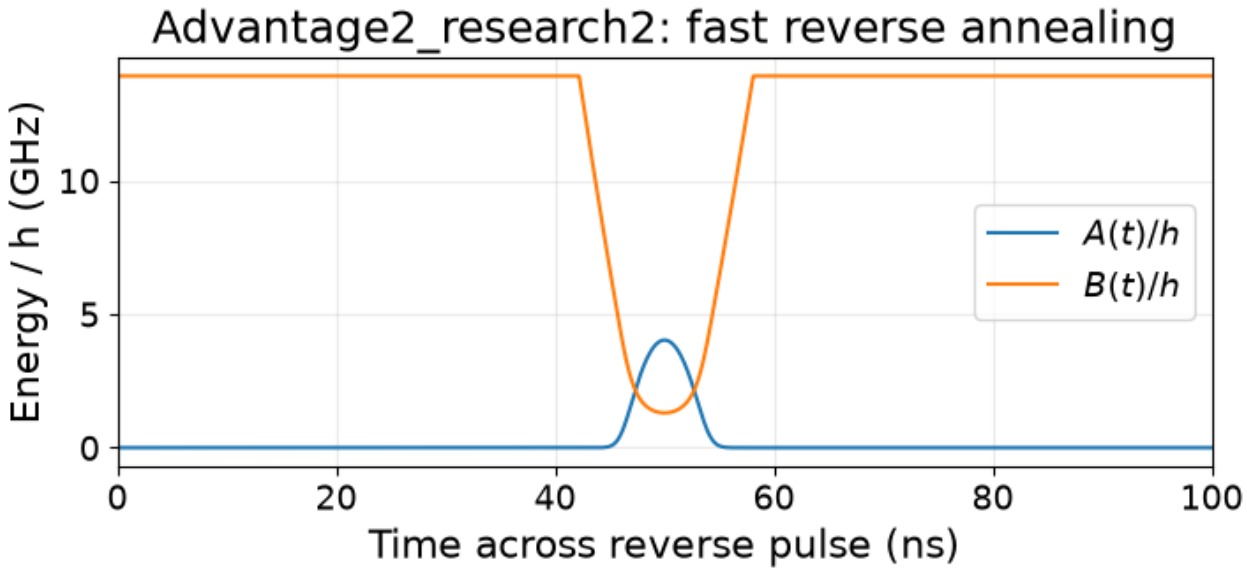
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➡ Conclusion: Fast Reverse Annealing faces difficulties for this type of quantum simulation

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➡ Conclusion: Fast Reverse Annealing faces difficulties for this type of quantum simulation

➡ Ongoing Work: Make use of Multicolor Annealing to deal with the issues

Thank you for your attention!