FZJ-IKP-TH-2010-03, HISKP-TH-10/05

Novel analysis of the decays $\psi' \to h_c\pi^0$ and $\eta_c' \to \chi_{c0}\pi^0$

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Abstract

We show that in the transitions $\psi' \to h_c\pi^0$ and $\eta_c' \to \chi_{c0}\pi^0$ the contributions from charmed meson loops are highly suppressed, in contrast to various other charmonium decays. We calculate the width of the $\psi' \to h_c\pi^0$, which agrees with the recent BES-III data, and predict the width of the $\eta_c' \to \chi_{c0}\pi^0$, $\Gamma(\eta_c' \to \chi_{c0}\pi^0) = 1.5 \pm 0.4$ keV. A confirmation of this prediction would also provide additional support for a recent analysis of $\psi' \to J/\psi\pi^0(\eta)$, where loops are claimed to play a prominent role.

PACS numbers: 13.25.Gv, 14.40.Pq, 12.39.Fe

I. INTRODUCTION

Recently, in a variety of calculations it was shown that charmed meson loops contribute importantly to the decays of charmonia with and without isospin breaking (for a overview, see [1]). For instance, using an effective Lagrangian approach (ELA), the intermediate meson loop contributions are found to be essential for understanding the puzzling $\psi(3770)$ non-$D\bar{D}$ decays [2, 3]. They are also important in the J/ψ decays into a vector and a pseudoscalar mesons [4] and in the $M1$ radiative transitions between two charmonia [5]. For isospin breaking decays, besides $\pi^0 - \eta$ mixing and electromagnetic (e.m.) effects, one also expects the mass difference between the neutral and charged charmed mesons in the intermediate states (i.e. in the meson loops) to play a role. This effect, known to be of particular importance near the continuum thresholds, was studied in the decays $\phi \to \omega\pi^0$ [6, 7], $J/\psi \to \phi\eta\pi^0$ [8, 9], and $D_{s0}^*(2317) \to D_s\pi^0$ [10–12].

In Ref. [13] a non–relativistic effective field theory (NREFT) was described, that allows one to study the role of charmed meson loops in charmonium decays in a systematic way. Applied to the reaction $\psi' \to J/\psi\pi^0(\eta)$, the formalism revealed that the contribution from charmed meson loops is enhanced by a factor of $1/v$, with $v \simeq 0.5$ being the charmed meson velocity[41], compared to the tree-level one. In this letter we apply NREFT to the decays $\psi' \to h_c\pi^0$ and $\eta_c' \to \chi_{c0}\pi^0$. We will show that in these two decays, the loop contributions are highly suppressed, and hence the tree-level terms dominate the decay amplitudes. Testing experimentally the predictions that emerge here, especially for the partial decay width for $\eta_c' \to \chi_{c0}\pi^0$, would provide a non–trivial test of the NREFT and is thus of high importance towards an understanding of the properties of charmonia.

In this context it is instructive to compare the $\psi' \to J/\psi\pi^0$ decays to $\psi' \to h_c\pi^0$ on the basis of power counting. Here we follow the reasoning of Ref. [13]. For the former decay, which happens in a p–wave, the tree level amplitude scales as $m_q q$, where q denotes the momentum of the final particles in the ψ' rest-frame and a quark mass factor appears since the reaction is isospin violating. For the same reaction the loops on the other hand scale as $v^3/v^4(m_q/v^2)[qv^2] = qm_q/v$, where the the factor v^3 comes from the the non–relativistic integral measure and the $1/v^4$ from the two non–relativistic two–meson propagators. Further, the term in the round brackets emerges, since pulling out a factor of m_q, which is an energy scale, has to be balanced by a factor which characterizes the intrinsic energy, v^2. In addition, the $[qv^2]$ term contains the vertex factors from the external pion coupling, q, and from the two p–wave vertices in the loop. Thus, heavy meson loops are enhanced by a factor $1/v \sim 2$. For the s–wave decay $\psi' \to h_c\pi^0$ on the other hand, the tree level scales as m_q, while the loop here scales as $v^3/v^4(m_q/v^2)[q/M_D]^2$. All factors that appear are analogous to those discussed above up to the factor q^2/M_D^2. The origin of this is, on the one hand, that the pion is produced in a p-wave and, on the other hand, that there is now only one p–wave vertex in the loop (the ψ' decay) giving rise to a momentum factor at that vertex. To obtain a non–vanishing result, however, that momentum has to be proportional to $\vec{q}$ (c.f. Eq. (15) below). The factor $1/M_D^2$ is then introduced to match dimensions. Thus, for the reaction $\psi' \to h_c\pi^0$ loops appear to be suppressed kinematically, on the amplitude level, by a factor $q^2/(M_D^2 v^3) \sim 1/30$. In case of the reaction $\eta_c' \to \chi_{c0}\pi^0$, we find from the same analysis a suppression of the loops by factor 1/10 — here the suppression is weaker due to a larger phase space. Thus we find on the basis of the same power counting that heavy meson loops are enhanced compared to the tree level amplitudes in the isospin violating p–wave decays like $\psi' \to J/\psi\pi^0$, while they are strongly suppressed in s–wave decays like $\psi' \to h_c\pi^0$. The latter observation allows us to predict the partial decay width of $\eta_c' \to \chi_{c0}\pi^0$. A confirmation of this prediction would at the same time provide a strong support for the analysis of Ref. [13].

The $h_c(^1P_1)$ has been the last charmonium state below the $D\bar{D}$ threshold that was confirmed experimentally; for a comprehensive review of the charmonium physics, see Ref. [14]. It was first established in $p\bar{p}$ annihilation by the E760 Collaboration at Fermi Lab in 1992 [15]. With $J^{PC} = 1^{+-}$, this state cannot be produced in e^+e^- annihilation directly. Furthermore, due to the phase space restriction, it cannot be accessed by ψ' decays into $h_c\eta$. Instead, the only open strong decay to h_c is via the isospin-violating $\psi' \to h_c\pi^0$ process. As a consequence, this branching ratio is strongly suppressed. Recently, the CLEO-c Collaboration with 24.5×10^6 ψ' events succeeded in measuring precisely the product of two branching ratios $\mathcal{B}(\psi' \to \pi^0 h_c) \times \mathcal{B}(h_c \to \gamma\eta_c) = (4.19 \pm 0.32 \pm 0.45) \times 10^{-4}$ in [16]. This combined branching ratio was confirmed by the BES-III Collaboration [17] with 110 million ψ' events in the same channel, $\mathcal{B}(\psi' \to \pi^0 h_c)\times\mathcal{B}(h_c \to \gamma\eta_c) = (4.58\pm0.40\pm0.50)\times10^{-4}$. Furthermore, they reported the first measurement of the absolute value of the branching ratio for the $\psi' \to h_c\pi^0$ as $\mathcal{B}(\psi' \to \pi^0 h_c) = (8.4 \pm 1.3 \pm 1.0) \times 10^{-4}$ [18]. Using the PDG value for the total width of the ψ', $\Gamma(\psi') = 309 \pm 9$ keV [19], the partial width of the $\psi' \to h_c\pi^0$ is

$$\Gamma(\psi' \to h_c\pi^0) = 0.26 \pm 0.05 \text{ keV}. \tag{1}$$

This experimental progress makes it possible to study physics of the h_c to some extent of precision. In particular, its production in the decay $\psi' \to h_c\pi^0$ appears to be an ideal channel for investigating the isospin-violating mechanisms and the pertinent non-perturbative QCD dynamics. The reaction $\psi' \to h_c\pi^0$ was first studied theoretically more than thirty years ago using $\pi^0 - \eta$ mixing [20]. In order to get an estimate of the width, the authors estimated the coupling of the ψ' to the h_c and η by assuming it to be equal to the $\psi' J/\psi\eta$ coupling, which was extracted from the measured $\psi' \to J/\psi\eta$ width. As a result, they obtained $5\ldots30$ keV for the partial decay width of the $\psi' \to h_c\pi^0$, which overshoots the measurement significantly.

The QCD multipole expansion (QCDME) was applied to this problem [21–24], and the following typical expression was obtained

$$\Gamma[\psi' \to h_c\pi^0] = 0.12\frac{\alpha_M}{\alpha_E} \text{ keV} \tag{2}$$

where α_E and α_M are the coupling constants for the color electric dipole and magnetic dipole gluon radiation, respectively. The phenomenological determination of the ratio α_M/α_E has large uncertainties. By taking the ratio in a range of $1\ldots3$, the partial width is about $0.12\ldots0.36$ keV, consistent with the experimental result. In contrast, a later calculation [25] gives a larger value of 0.84 keV, and the estimate by Voloshin gives a much smaller value of 15 eV [26]. Suffering from a poor knowledge of the coupling constants [26, 27], the QCDME results should rather be regarded as an order-of-magnitude estimate.

In this work, we shall further investigate the isospin violation mechanisms of $\psi' \to h_c\pi^0$ and its analogue, the reaction $\eta_c' \to \chi_{c0}\pi^0$. In Section II, we will give the tree level decay amplitudes by constructing the effective chiral Lagrangian. In Section III we present the NREFT [13] analysis for the heavy meson loops. As we will demonstrate the explicit calculation supports the scale arguments presented above, that heavy meson loops are highly suppressed for the reactions under consideration. We also checked that the results are consistent with a calculation using the ELA [2, 6, 7] — details will be presented elsewhere [28]. In Section IV, the results for the decay widths are given. In particular, the width of the $\eta_c' \to \chi_{c0}\pi^0$ is predicted. Some discussions and a summary are given in the last section.

II. TREE LEVEL CONTRIBUTION

Since the mass difference between the initial and final charmonia is small, the emitted pion is soft. Hence one can construct an effective chiral Lagrangian considering the charmonia as matter fields. Since the charmonia are isoscalar and the pion has isospin one, the transitions violate isospin symmetry. Isospin breaking has two sources. One is the mass difference between the up and down quarks, and the other one is of e.m. origin. For the transitions considered here, the e.m. effect can be neglected (for details, see [28]). Defining $\chi = 2B_0 \cdot \text{diag}\,(m_u, m_d)$, the quark mass difference is contained in the operator χ_- which contains an odd number of pion fields,

$$\chi_- = u^\dagger\chi u^\dagger - u\chi^\dagger u, \tag{3}$$

where $B_0 = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2$, F_π is the pion decay constant in the chiral limit, and u parameterizes the pion fields as the Goldstone bosons of the spontaneously broken $\text{SU}(2)_L\times\text{SU}(2)_R$,

$$u = \exp\left(\frac{i\phi}{\sqrt{2}F_\pi}\right), \quad \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}. \tag{4}$$

Using the two-component notation of Ref. [29], the field for the S-wave charmonia ψ' and η_c' reads

$$J' = \vec{\psi}' \cdot \vec{\sigma} + \eta_c', \tag{5}$$

with $\vec{\psi}'$ and η_c' annihilating the ψ' and η_c' states, and $\vec{\sigma}$ the Pauli matrices. The field for the P-wave charmonia is

$$\chi^i = \sigma^j\left(-\chi_{c2}^{ij} - \frac{1}{\sqrt{2}}\epsilon^{ijk}\chi_{c1}^k + \frac{1}{\sqrt{3}}\delta^{ij}\chi_{c0}\right) + h_c^i, \tag{6}$$

where χ_{c2}^{ij}, χ_{c1}^i, χ_{c0} and h_c annihilate the χ_{c2}, χ_{c1}, χ_{c0} and h_c states, respectively.

The leading order (LO) chiral Lagrangian for the transitions $\psi' \to h_c\pi^0$ and $\eta_c' \to \chi_{c0}\pi^0$ reads

$$\mathcal{L} = \frac{i}{4}C\left[\left\langle\vec{\chi}^\dagger \cdot \vec{\sigma}J'\right\rangle + \left\langle J'\vec{\sigma}\cdot\vec{\chi}^\dagger\right\rangle\right](\chi_-)_{aa} + \text{h.c.}, \tag{7}$$

where C is an unknown coupling constant, $\langle\cdots\rangle$ is the trace in spinor space, the subscript $a = u, d$ is a flavor index, and repeating indices aa means evaluating the trace in flavor space. The spin symmetry is violated due to the presence of the Pauli matrices between the two heavy quarkonium fields. Note that the Lagrangian was first proposed in Ref. [30] in four-component notation, and the coupling constant C here is $-4/B_0$ times the one defined in that paper. Working out the traces in both the spinor and flavor space, one finds that only the two transitions considered in this paper are allowed as dictated by conservation of angular momentum, parity and charge conjugation invariance,

$$\mathcal{L} = iC\left(\vec{\psi}'\cdot\vec{h}_c^{\,\dagger} + \sqrt{3}\eta_c'\chi_{c0}^\dagger\right)(\chi_-)_{aa} + \text{h.c.}, \quad (8)$$

and, after taking into account the $\pi^0-\eta$ mixing which contributes to the isospin breaking transitions,

$$(\chi_-)_{aa} = 6i\frac{B_0}{F_\pi}(m_d - m_u)\tilde{\pi}^0 + \dots\ , \quad (9)$$

where $\tilde{\pi}^0 = \pi^0 + \epsilon_{\pi^0\eta}\eta$ is the physical pion field with $\epsilon_{\pi^0\eta}$ being the $\pi^0-\eta$ mixing angle. In the above equation, we have neglected the multi-pion terms. Now it is easy to write out the tree level amplitudes,

$$\begin{aligned}\mathcal{M}(\psi'\to h_c\pi^0) &= \sqrt{M_{\psi'}M_{h_c}}\frac{6}{F_\pi}C\vec{\varepsilon}(\psi')\cdot\vec{\varepsilon}(h_c)\\ &\quad\times B_0(m_d-m_u),\\ \mathcal{M}(\eta_c'\to\chi_{c0}\pi^0) &= \sqrt{M_{\eta_c'}M_{\chi_{c0}}}\frac{6\sqrt{3}}{F_\pi}CB_0(m_d-m_u). \quad (10)\end{aligned}$$

Note that the above amplitudes were multiplied by a factor $\sqrt{M_f M_i}$, with $M_{i(f)}$ being the mass of the initial (final) charmonium, to account for the non-relativistic normalization of the heavy fields in the Lagrangian.

III. CHARMED MESON LOOPS

In this section we investigate the contribution of charmed meson loops to the two decays. The relevant diagrams for the reaction $\psi'\to h_c\pi^0$ with neutral intermediate states are shown in Fig. 1. The coupling of pion to the charmed mesons is described by heavy meson chiral perturbation theory [31–33] (for a review, see Ref. [34]). The fields for the pseudoscalar and vector charmed mesons in the same spin multiplet can be written as $H_a = \vec{V}_a\cdot\vec{\sigma} + P_a$ with V_a and P_a denoting the vector and pseudoscalar charmed mesons [29], respectively, where a is the flavor index with $\{P_u, P_d\} = \{D^0, D^+\}$ and similar for the vector mesons. The LO chiral effective Lagrangian for the axial coupling is [29]

$$\mathcal{L}_\phi = -\frac{g}{2}\left\langle H_a^\dagger H_b\vec{\sigma}\cdot\vec{u}_{ba}\right\rangle, \quad (11)$$

where the axial current is $\vec{u} = -\sqrt{2}\vec{\partial}\phi/F_\pi + \mathcal{O}(\phi^3)$, and g the pertinent coupling constant.

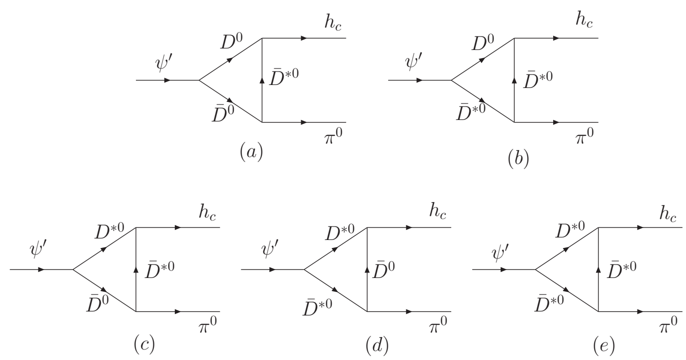

FIG. 1: Diagrams of neutral intermediate meson loops contributing to $\psi'\to h_c\pi^0$. Their charge conjugate diagrams are implied.

The LO Lagrangian for the coupling of the S- or P-wave charmonium fields to the charmed and anti-charmed mesons can be constructed respecting parity, charge conjugation and spin symmetry. The one for the ψ' and η_c' reads [13]

$$\mathcal{L}_\psi = i\frac{g_2'}{2}\left\langle J'^\dagger H_a\vec{\sigma}\cdot\overleftrightarrow{\partial}\bar{H}_a\right\rangle + \text{h.c.}, \quad (12)$$

where $A\overleftrightarrow{\partial}B \equiv A(\vec{\partial}B) - (\vec{\partial}A)B$, and $\bar{H}_a = -\vec{\bar{V}}_a\cdot\vec{\sigma} + \bar{P}_a$ is the field for anti-charmed mesons [35]. The LO Lagrangian for the P-wave charmonia spin multiplet is [35]

$$\mathcal{L}_\chi = i\frac{g_1}{2}\left\langle\chi^{\dagger i}H_a\sigma^i\bar{H}_a\right\rangle + \text{h.c.}. \quad (13)$$

These Lagrangians were introduced in Ref. [36] in four-component notation. The values of the coupling constants g_1 and g_2 in that paper are half of those introduced here.

Using these Lagrangians, one can work out the decay amplitudes. For simplicity, we focus on diagram (a) in Fig. 1 — the analysis of the other diagrams is analogous. The amplitude for $\psi'\to h_c\pi^0$ from the diagram (a) reads (for the full amplitudes and calculation details, we refer to Ref. [28])

$$\mathcal{M}(\psi'\to h_c\pi^0)_{(\text{a})} = 2\frac{g}{F_\pi}g_1g_2'\vec{q}\cdot\vec{\varepsilon}(h_c)\vec{q}\cdot\vec{\varepsilon}(\psi')R\left[I^{(1)}(q,D^0,D^0,D^{*0}) - I^{(1)}(q,D^\pm,D^\pm,D^{*\pm})\right], \quad (14)$$

where q is the pion three-momentum in the rest-frame of the initial charmonium, $R = \sqrt{M_{\psi'} M_{h_c}} M_D^2 M_{D^*}$ is included to account for the non-relativistic normalization of the heavy fields, and we have labeled the loop function $I^{(1)}(q)$ by the intermediate charmed mesons. It is obvious that the isospin breaking comes from the neutral and charged meson mass differences. The loop function, evaluated in the rest-frame of the decaying particle, is defined by

$$
\begin{aligned}
q^i I^{(1)}(q) &= \frac{i}{8 m_1 m_2 m_3} \int \frac{d^4 l}{(2\pi)^4} \frac{l^i}{\left(l^0 - T_1(|\vec{l}|)\right)\left(P^0 - l^0 - T_2(|\vec{l}|)\right)\left(l^0 - q^0 - T_3(|\vec{l} - \vec{q}|)\right)} \\
&= \frac{1}{8 m_1 m_2 m_3} \int \frac{d^3 l}{(2\pi)^3} \frac{l^i}{\left(E_i - T_2(|\vec{l}|) - T_1(|\vec{l}|)\right)\left(E_f - T_2(|\vec{l}|) - T_3(|\vec{l} - \vec{q}|)\right)}
\end{aligned} \quad (15)
$$

where $T_i(p) = p^2/2m_i$ denotes the kinetic energy for the charmed mesons with masses m_1, m_2 and m_3, $E_i = M_i - m_1 - m_2$ and $E_f = M_f - m_2 - m_3 - E_\pi$ denote the energies available for the first (before pion emission) and second (after pion emission) two heavy meson intermediate state. For diagram (a), $m_1 = m_2 = M_D$, $m_3 = M_{D^*}$, $M_i = M_{\psi'}$, and $M_f = M_{h_c}$. The loop function is convergent [13, 28]. Defining $c = 2\mu_{12} b_{12}$ and $c' = 2\mu_{23} b_{23}$, with $\mu_{ij} = m_i m_j/(m_i + m_j)$ being the reduced mass, $b_{12} = m_1 + m_2 - M_{\psi'}$ and $b_{23} = m_2 + m_3 + E_\pi - M_{\psi'}$, the loop function can be approximated by

$$
I^{(1)}(q) = N \frac{\mu_{23}}{m_3} \frac{2\left(\sqrt{c'} + 2\sqrt{c}\right)}{3\left(\sqrt{c'} + \sqrt{c}\right)^2} \quad (16)
$$

with $N = \mu_{12}\mu_{23}/(16\pi m_1 m_2 m_3)$, where terms of order $\vec{q}^{\,2}/c'$ and higher have been neglected. The approximation is reasonable because for either of the two decays considered here, the pion momentum is small and fulfills $\vec{q}^{\,2} \ll c'$.

We can now have another, more refined look, at the order of magnitude estimate for the loop function. Since $\sqrt{c}$ and $\sqrt{c'}$ are approximately the momenta of the charmed mesons in the loop, we count them as $M_D v$ with v being the velocity of the charmed mesons. It follows that $b_{12} \sim b_{23} \sim M_D v^2$. For an order-of-magnitude estimate, one may neglect the difference between c and c', and denoting them by $2\mu b$. Then, $I^{(1)}(q) \sim N/(4\sqrt{2\mu b})$. Denoting the mass difference between the charged and neutral charmed mesons by δ, we have $\mu_c = \mu_n + \delta/2$ and $b_c = b_n + 2\delta$, where the lower-index $n(c)$ means neutral (charged). The amplitude for diagram (a) scales as

$$
\begin{aligned}
\mathcal{M}(\psi' \to h_c \pi^0)_{(a)} &\sim N_{(a)} \vec{q}^{\,2} \frac{N}{4} \left(\frac{1}{\sqrt{2\mu_n b_n}} - \frac{1}{\sqrt{2\mu_c b_c}} \right) \\
&\sim N_{(a)} \frac{N}{4} \delta v \frac{\vec{q}^{\,2}}{b_n^2} \\
&\sim N_{(a)} \frac{N}{4} \delta \frac{1}{v^3} \frac{\vec{q}^{\,2}}{M_D^2},
\end{aligned} \quad (17)
$$

where $N_{(a)} = 2(g/F_\pi) g_1 g_2' R \vec{q} \cdot \vec{\varepsilon}(h_c) \vec{q} \cdot \vec{\varepsilon}(\psi')/\vec{q}^{\,2}$. Thus, we confirm the parametric behavior derived in the introduction on the basis of the NREFT. For the transition from the ψ' to the h_c, the pion momentum in the ψ' rest-frame is $q = q_1 = 86$ MeV, and hence $\vec{q}^{\,2}/M_D^2 \simeq 2 \times 10^{-3}$. Taking into account that the velocity v may be roughly estimated as $\sqrt{[2M_{\hat{D}} - (M_{\psi'} + M_{h_c})/2]/M_{\hat{D}}} \simeq 0.4$ with $M_{\hat{D}}$ being the averaged charmed meson mass, the dimensionless factor

$$
\frac{1}{v^3} \frac{\vec{q}^{\,2}}{M_D^2} \simeq 0.03 \quad (18)
$$

produces a significant suppression compared to the tree level amplitude. A similar though more moderate suppression happens in case of the $\eta_c' \to \chi_{c0}\pi^0$ also. For this decay, the momentum of the pion is $q = q_2 = 171$ MeV, and the suppression factor is

$$
\frac{1}{v^3} \frac{\vec{q}^{\,2}}{M_D^2} \simeq 0.1. \quad (19)
$$

The numerical results support the above power counting argument. If we only consider the contribution from the charmed-meson loops, the widths of the $\psi' \to h_c \pi^0$ and $\eta_c' \to \chi_{c0}\pi^0$ are

$$
\begin{aligned}
\Gamma(\psi' \to h_c \pi^0)_{\rm loop} &= 2.1 \times 10^{-7} g_1^2 g_2'^{\,2} \text{ keV}, \\
\Gamma(\eta_c' \to \chi_{c0} \pi^0)_{\rm loop} &= 1.0 \times 10^{-5} g_1^2 g_2'^{\,2} \text{ keV},
\end{aligned} \quad (20)
$$

where the $\pi^0 - \eta$ mixing has been taken into account, and the values of g_1 and g_2' are given in units of $\text{GeV}^{-1/2}$ and $\text{GeV}^{-3/2}$, respectively. We checked that the ELA gives similar results, which confirms our analysis.

One may ask if the values of the coupling constants are so large that the suppression gets invalidated. In fact, because all the charmonia considered here are below the $D\bar{D}$ threshold, the couplings cannot be extracted directly from the decay widths. However, one may get a feeling about their values from other sources or from model calculations. Assuming the coupling of the ψ' to the charmed mesons has similar strength as the one of the

J/ψ, we obtain $g_2' \simeq 2\ \mathrm{GeV}^{-3/2}$ from the $\psi' \to J/\psi\pi^0(\eta)$ where the charmed meson loops dominate [13]. Values of the same order of magnitude were obtained from various model calculations, see e.g. Refs. [36–38]. In Ref. [36], the authors estimated g_1 using vector meson dominance, which gives $g_1 = -4.2\ \mathrm{GeV}^{-1/2}$. Using these values, the resulting width $\Gamma(\psi' \to h_c\pi^0)_{\mathrm{loop}} \simeq 1 \times 10^{-5}$ keV. It is smaller by 4 orders of magnitude compared to the BES-III measurement, and it confirms our power counting estimate presented before.

In addition, even without any assumption on the coupling constants, from our analysis we can predict

$$\frac{\Gamma(\eta_c' \to \chi_{c0}\pi^0)_{\mathrm{loop}}}{\Gamma(\psi' \to h_c\pi^0)_{\mathrm{loop}}} = 48\ . \tag{21}$$

As we will see, this ratio as derived solely from the loop contributions, is much larger than the corresponding one derived from including the tree level amplitudes only. Note, however, that because $v \simeq 0.5$, there might be sizeable corrections to this result. Thus, for the mentioned decays it can be tested experimentally, if there is a dominance from the loops or from the tree level contribution.

IV. DECAY WIDTHS

As shown in the last section, the charmed meson loops can be neglected. Hence, the LO decay amplitudes are given by Eq. (10). Then the decay widths are

$$\Gamma\left(\psi' \to h_c\pi^0\right) = \frac{q_1 M_{h_c}}{8\pi M_{\psi'}} C^2 \left[\frac{6}{F_\pi} B_0(m_d - m_u)\right]^2,$$
$$\Gamma\left(\eta_c' \to \chi_{c0}\pi^0\right) = \frac{3q_2 M_{\chi_{c0}}}{8\pi M_{\eta_c'}} C^2 \left[\frac{6}{F_\pi} B_0(m_d - m_u)\right]^2 \tag{22}$$

The pion momenta q_i where introduced in the previous section. The ratio of these two widths is free of any parameter

$$\frac{\Gamma\left(\eta_c' \to \chi_{c0}\pi^0\right)}{\Gamma\left(\psi' \to h_c\pi^0\right)} = 3\frac{q_2}{q_1}\frac{M_{\chi_{c0}} M_{\psi'}}{M_{\eta_c'} M_{h_c}} = 5.86 \pm 0.94, \tag{23}$$

where the 15% uncertainty comes from neglecting higher order terms in either the heavy quark expansion or the chiral expansion

$$\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right) \sim \mathcal{O}\left(\frac{m_\pi}{\Lambda_\chi}\right) \sim 15\%,$$

where $\Lambda_\chi \simeq 1$ GeV, as well as heavy meson loops. Using the experimental value of $\Gamma\left(\psi' \to h_c\pi^0\right)$, we predict the width of the $\eta_c' \to \chi_{c0}\pi^0$ as

$$\Gamma\left(\eta_c' \to \chi_{c0}\pi^0\right) = 1.5 \pm 0.3 \pm 0.2\ \mathrm{keV}, \tag{24}$$

where the first uncertainty is experimental and the second theoretical due to neglecting higher orders. With the total width of the η_c', $\Gamma(\eta_c') = 14 \pm 7$ MeV [19], the branching fraction of the isospin-breaking transition is

$$\mathcal{B}\left(\eta_c' \to \chi_{c0}\pi^0\right) = (1.1 \pm 0.6) \times 10^{-4}. \tag{25}$$

Note, as a contrast, if the reactions were dominated by heavy meson loops, the predicted branching fraction would be larger by a factor of about 5. The prediction, and therefore the dynamics underlying the decays, is testable with $\overline{\mathrm{P}}$ANDA at FAIR [39].

Furthermore, one may check if the tree-level contribution gives the right order of magnitude of the decay width $\Gamma\left(\psi' \to h_c\pi^0\right)$ through dimensional analysis. The tree–level amplitudes of Eq. (10) are proportional to the dimensionless factor $\sqrt{M_f M_i}C$. Because the spin symmetry is violated as can be seen from the presence of the Pauli matrices in Eq. (7), one may write

$$\sqrt{M_f M_i}C = \tilde{C}\frac{\Lambda_{\mathrm{QCD}}}{m_c}\ , \tag{26}$$

with the dimensionless parameter $\tilde{C}$ being a number of natural size, i.e. of order one. Using the current knowledge of the quark mass ratio [40]

$$r \equiv \frac{m_u}{m_d} = 0.47 \pm 0.08, \tag{27}$$

and the LO relation between the pion mass and the quark masses $m_{\pi^0}^2 = B_0(m_u + m_d)$ (neglecting strong isospin violation), we get

$$B_0(m_d - m_u) = \left(\frac{1-r}{1+r}\right) m_{\pi^0}^2 = (6.6 \pm 2.0) \times 10^{-3}\ \mathrm{GeV}^2, \tag{28}$$

where the uncertainty is dominated by that of the quark mass ratio. Using the value given in Eq. (28), the width for the $\psi' \to h_c\pi^0$ is

$$\Gamma\left(\psi' \to h_c\pi^0\right) = (0.9 \pm 0.6)\tilde{C}^2\ \mathrm{keV}. \tag{29}$$

Since $\tilde{C}$ is of order 1, the above result agrees with the BES-III measurement 0.26 ± 0.05 keV well. The agreement in turn supports the tree-level dominance argued for in this paper. It is also consistent with the QCDME results reported in Refs. [24, 25].

V. DISCUSSION AND SUMMARY

We demonstrated that, based on a non–relativistic effective field theory (NREFT), charmed meson loops are highly suppressed by a factor $v\vec{q}^{\,2}/(M_D v^2)^2 \ll 1$ in $\psi' \to h_c\pi^0$ and $\eta_c' \to \chi_{c0}\pi^0$, which are transitions between one P-wave and one S-wave charmonia. The reason for the suppression is that, due to the small phase spaces of the two transitions, the pion momentum is much smaller than the approximate kinetic energy of the intermediate charmed mesons. The situation is completely different for the transitions between two S-wave charmonia. For the $\psi' \to J/\psi\pi^0(\eta)$, the charmed meson loops are enhanced by a factor of $1/v$ compared with the tree level contribution [13]. There is no factor proportional to $\vec{q}^{\,2}$, and hence the relative size of the pion momentum to the kinetic energy of the virtual charmed mesons does not have an impact. The difference in these two cases is a consequence of the difference in the quantum numbers of the J/ψ and h_c which determine their coupling to the charmed mesons.

We note in passing that it follows from our findings that approximating the $\psi' h_c \eta$ coupling by that for $\psi' J/\psi\eta$, as done in Ref. [20], is not justified. In the light of this one then understands why the estimate made there is much larger than the measurement.

In summary, we have shown that intermediate charmed meson loops are highly suppressed in the decays $\psi' \to h_c\pi^0$ and $\eta_c' \to \chi_{c0}\pi^0$, which is completely different from the situation of the $\psi' \to J/\psi\pi^0(\eta)$. We confirmed the general power counting arguments given by explicit calculations both within the NREFT as well as an effective Lagrangian approach. By constructing the LO chiral Lagrangian for the decays, and employing the experimental result for $\Gamma(\psi' \to h_c\pi^0)$, we give a model-independent prediction for the width of the $\eta_c' \to \chi_{c0}\pi^0$, which can be measured at $\overline{\mathrm{P}}$ANDA, $\Gamma(\eta_c' \to \chi_{c0}\pi^0) = 1.5 \pm 0.4$ keV, where the experimental and theoretical uncertainties have been summed in quadrature. Note, were the transitions dominated by the heavy meson loop contributions, the predicted partial decay width would be larger by a factor of about 8. An experimental confirmation of our prediction would provide a strong support for the NREFT employed. Having available an effective field theory that allows one to study both direct transitions as well as those mediated via heavy loops is an important step towards a detailed theoretical understanding of charmonium states.

Acknowledgments

Q.Z. and G.L. acknowledge the supports, in part, from the National Natural Science Foundation of China (Grants No. 10675131 and 10491306), Chinese Academy of Sciences (KJCX3-SYW-N2), and Ministry of Science and Technology of China (2009CB825200). The work of F.K.G., C.H. and U.G.M. is supported by the Helmholtz Association through funds provided to the virtual institute "Spin and strong QCD" (VH-VI-231) and by the DFG (SFB/TR 16, "Subnuclear Structure of Matter"), and the European Community-Research Infrastructure Integrating Activity "Study of Strongly Interacting Matter" (acronym HadronPhysics2, Grant Agreement n. 227431) under the Seventh Framework Programme of EU.

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[41] Here the velocity is defined via the analytic continuation of the standard definition, namely $v = \sqrt{-E/M_D}$, with E measured relative to the open charm threshold.