J. Phys. G: Nucl. Part. Phys. 18 (1992) 1455–1466. Printed in the UK

Pushing the Nambu–Jona-Lasinio soliton and the zero-point energy

P V Pobylitsa†‡, E Ruiz Arriola†§, Th Meissner†, F Grümmer†, K Goeke† and W Broniowski||

† Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-4630 Bochum 1, Federal Republic of Germany

‡ Nuclear Physics Institute, Gatchina, St. Petersburg, 188350, Russia

§ Departamento de Fisica Moderna, Universidad de Granada, E-18071 Granada, Spain

|| Institute of Nuclear Physics, Radzikowskiego 152, 31-342 Cracow, Poland

Received 25 March 1992, in final form 26 May 1992

Abstract. We consider the collective quantization of translational degrees of freedom of the hedgehog soliton in the Nambu–Jona-Lasinio Model. We show explicitly that for an O(4)-invariant regularization scheme in Euclidean space–time the translational mass obtained in the perturbative pushing model coincides with the static mass. Zero-point energies for translation and rotation are evaluated numerically. The nucleon mass appears to be around 900 MeV.

1. Introduction

Spontaneously broken chiral symmetry seems to be the dominant mechanism for describing hadronic phenomena at low or intermediate energies. One hopes that QCD can be simulated by an effective quark, quark–meson or even purely mesonic theory which incorporates this feature. Among all these effective chiral models, the theory of Nambu and Jona-Lasinio (NJL) [1] has received recently. It realizes the spontaneous breakdown of chiral symmetry in the formally simplest and most natural way and it has been applied quite successfully to various phenomena resulting from chiral symmetry breaking in the vacuum and the mesonic sector [2].

In addition it turned out that baryons can also be described within the model [3–6]. They appear in a natural way as a solitonic bound state of $N_C = 3$ valence quarks in a chiral (time-independent) $SU(n)$-flavour meson field $U(\boldsymbol{x})$. In the mean-field approximation $U(\boldsymbol{x})$ arises as the stationary point of the effective fermionic action $S_{\text{eff}}(U(\boldsymbol{x}))$, which is the fermion determinant obtained after formally integrating out the quark degrees of freedom. In contrast to non-relativistic many-body systems, it is essential that $S_{\text{eff}}(U(\boldsymbol{x}))$ is ultraviolet divergent and has therefore to be provided with a finite UV cutoff within a proper regularization scheme.

It is clear that these solitonic mean-field solutions break the rotational, isotopic and translational symmetries of the original NJL action explicitly. In order to restore them, a semi-classical collective quantization has to be performed [7]. If one assumes a hedgehog shape for the chiral field $U(\boldsymbol{x})$, which can be shown to be a self-consistent symmetry of the system [8], isotopic and rotational motion are coupled trivially, so it is enough to quantize one of them. This is done in the well known cranking approach [7–11], which for the NJL model has been described in detail in [12, 13]. In fact this quantization of the rotational degrees of freedom is necessary to identify the quantum numbers of the nucleon and to evaluate observables and form factors.

Of course the quantization of the translational degrees of freedom also has to be performed. Pushing methods corresponding to the cranking approach of the (iso-) rotational degrees of freedom, as well as the Peierls–Yoccoz momentum projection, have been developed in non-relativistic nuclear many-body physics [7] and have also been used in the context of two-dimensional kink soliton solutions [14]. In the last few years it has turned out that with some modification these methods can also be applied in the framework of relativistic effective soliton models, especially in various versions of the bag model, the soliton bag model of Friedberg and Lee, the Skyrme model, the chiral sigma model of Gell-Mann and Levi and the colour dielectric model [15–24]. A series of investigations concerning the translational degrees of freedom in these models have been carried out using projection, boosting and also pushing techniques and it has turned out that the centre-of-mass corrections to nucleon observables (e.g. masses and radii) are significant [18–20, 23, 24].

None of these relativistic effective models, in which the translational degrees of freedom have been studied, have up to now taken the polarized Dirac sea into account. Actually, this is done in the NJL model, which incorporates the polarization of the Dirac sea from the beginning in the form of the UV-divergent regularized fermion determinant. It is the objective of the present paper to review the status of the semi-classical quantization of the translational degrees of freedom and to evaluate the corresponding inertial parameter for the NJL model using the regularized form of the theory explicitly from the beginning. Furthermore, we will consider zero-point energies associated with rotation and translation in order to evaluate an improved nucleon mass.

The paper is constructed as follows: section 2 reviews the conceptual background of rotational and translational cranking; section 3 evaluates the translational mass; section 4 studies zero-point energies of translational and rotational collective motion; and section 5 summarizes the paper and provides some conclusions.

2. Concepts of rotational and translational cranking

For the quantization of rotational and iso-rotational degrees of freedom, one performs an adiabatic $SU(2)$ rotation of the chiral field $U(\boldsymbol{x})$ with the angular frequency ω:

$$U(\boldsymbol{x}) \to U_{\mathrm{rot}}(\boldsymbol{x}, t) = \mathrm{e}^{\mathrm{i}\frac{1}{2}\omega^a\tau^a t} U(\boldsymbol{x}) \mathrm{e}^{-\mathrm{i}\frac{1}{2}\omega^a\tau^a t} \tag{2.1}$$

and expands the effective action $S_{\mathrm{eff}}(U_{\mathrm{rot}}(\boldsymbol{x}, t))$ in powers of ω^a:

$$S_{\mathrm{eff}}(U_{\mathrm{rot}}(\boldsymbol{x}, t)) = S_{\mathrm{eff}}(U(\boldsymbol{x})) + T\omega^a\langle T^a\rangle + \tfrac{1}{2}T\omega^a\Theta^{ab}\omega^b + \mathcal{O}(\omega^3) \tag{2.2}$$

where T denotes an (Euclidean) time interval. The expectation value of the isospin vanishes for hedgehog configurations $\langle T^a\rangle = 0$. The corresponding inertial parameter of Thouless and Valatin is called the moment of inertia

$$\Theta^{ab} = \delta^{ab}\Theta = \frac{1}{T}\frac{\delta^2 S(U_{\mathrm{rot}})}{\delta\omega^a\delta\omega^b}. \tag{2.3}$$

It can be calculated from the single-particle spectrum of the self-consistent U field by the well known Inglis formula [7], which of course appears modified due to the regularization of S_{eff} [12, 13]. Moreover, care has to be taken in performing the Wick rotation of the angular velocity ω from Minkowski to Euclidean space in order to guarantee a vanishing moment of inertia for the vacuum configuration $U = 1$. After collective quantization of the (iso-)rotational degrees of freedom, one obtains finally for the energy of a system with isospin T:

$$E_T = M + \frac{T(T+1)}{2\Theta} \tag{2.4}$$

where M denotes the self-consistent mean-field energy:

$$M = \frac{1}{\mathcal{T}} \min_{U(\boldsymbol{x})} S_{\text{eff}}(U(\boldsymbol{x})). \tag{2.5}$$

In the present paper we want to consider the translational degrees of freedom by *pushing* the space coordinates of the soliton with the velocity $\boldsymbol{v}$:

$$U(\boldsymbol{x}) \to U_{\text{tr}}(\boldsymbol{x}, t) = U(\boldsymbol{x} - \boldsymbol{v}t) \tag{2.6}$$

and expanding $S_{\text{eff}}(U_{\text{tr}})$ in powers of $\boldsymbol{v}$:

$$S_{\text{eff}}(U_{\text{tr}}(\boldsymbol{x}, t))] = S_{\text{eff}}(U(\boldsymbol{x})) + \mathcal{T}\boldsymbol{v}\langle \boldsymbol{P}\rangle + \tfrac{1}{2}\mathcal{T} v_i {M^*}_{ij} v_j + \mathcal{O}(v^3). \tag{2.7}$$

As in non-relativistic quantum mechanics the expectation value of the linear momentum $\langle \boldsymbol{P}\rangle$ vanishes for static mean-field configurations, which is just a consequence of Ehrenfest's theorem:

$$\langle\lambda|p|\rangle\lambda = \frac{\mathrm{i}}{2}\langle|\lambda[p^2, \boldsymbol{x}]|\rangle\lambda = \frac{\mathrm{i}}{2}\langle|\lambda[h^2, \boldsymbol{x}]|\rangle\lambda = 0 \tag{2.8}$$

where h denotes the stationary single-particle Hamiltonian for a quark with constituent mass m:

$$h = \alpha p + \beta m U(\boldsymbol{x}) \tag{2.9}$$

with the spectrum

$$h|\lambda\rangle = \epsilon_\lambda|\lambda\rangle. \tag{2.10}$$

In second-order $\boldsymbol{v}$, one obtains the Thouless–Valatin parameter of the translational motion, which is called the *translational mass*:

$${M^*}_{ij} = \delta_{ij} M^* = \frac{1}{\mathcal{T}} \frac{\delta^2 S(U_{\text{rot}})}{\delta v^i \delta v^j}. \tag{2.11}$$

It is instructive to have a glance at the non-relativistic analogue first. For an N-particle system interacting by a purely local time- and velocity-independent one-body force, we have the static and the *pushed* Hamiltonians:

$$H = \sum_{k=1}^{N} \frac{{p_{(k)}}^2}{2m_{(k)}} + V(\boldsymbol{x}^{(k)}) \qquad H_v(t) = \sum_{k=1}^{N} \frac{{p_{(k)}}^2}{2m_{(k)}} + V(\boldsymbol{x}^{(k)} - \boldsymbol{v}t)$$

respectively. The corresponding solutions of the time-dependent Schrödinger equations

$$\mathrm{i}\partial_t|\Psi(t)\rangle = H|\Psi(t)\rangle \qquad \mathrm{i}\partial_t|\Psi_v(t)\rangle = H|\Psi_v(t)\rangle$$

are simply connected by the unitary generators of the Galilean transformation [25]:

$$|\Psi_v(t)\rangle = \mathrm{e}^{\mathrm{i}\frac{1}{2}Mv^2t}\mathrm{e}^{-\mathrm{i}\boldsymbol{P}vt}\mathrm{e}^{\mathrm{i}M\boldsymbol{R}v}|\Psi(t)\rangle \tag{2.12}$$

with

$$\boldsymbol{P} = \sum_{k=1}^{N}\boldsymbol{p}_{(k)} \qquad M = \sum_{k=1}^{N} m_{(k)} \qquad \boldsymbol{R} = \frac{1}{M}\sum_{k=1}^{N} m_{(k)}\boldsymbol{x}_{(k)}$$

and the total energies

$$E = \langle\Psi(t)H\rangle\Psi(t)E_v = \langle\Psi_v(t)H_v(t)\rangle\Psi_v(t)$$

in the static and the pushed system are exactly related by

$$E_v = E + \tfrac{1}{2}Mv^2. \tag{2.13}$$

For relativistic theories, Lorentz invariance guarantees generally that under a Lorentz boost transformation in the $\hat{v}$ direction:

$$U(\boldsymbol{x}) \to U_{\mathrm{boost}}(\boldsymbol{x}, t) = U(\cosh\omega\ \boldsymbol{x}_{\|} + \boldsymbol{x}_{\perp} - \sinh\omega\ t) \tag{2.14}$$

with

$$\boldsymbol{x}_{\|} = (\boldsymbol{x}\circ\hat{\boldsymbol{v}})\hat{\boldsymbol{v}} \qquad \boldsymbol{x}_{\perp} = \boldsymbol{x} - \boldsymbol{x}_{\|} \qquad \omega = \operatorname{arctanh} v. \tag{2.15}$$

The energy of the system transforms like the zeroth component of a four-vector:

$$E(U_{\mathrm{boost}}(\boldsymbol{x}, t)) = E_{\mathrm{stat}}(U(\boldsymbol{x}))\cosh\omega = M\cosh\omega. \tag{2.16}$$

In the limit of small velocities v the boost (2.14) and the pushing (2.6) transformations are equivalent. Therefore, one expects that the expansion of (2.16) up to second order in v gives:

$$E(U_{\mathrm{tr}}(\boldsymbol{x}, t)) = M + \tfrac{1}{2}Mv^2 + \mathcal{O}(v^3) \tag{2.17}$$

or, in other words, that translational mass M^* and the static mean-field energy M coincide. It should be emphasized that the transformation (2.16), and therefore also (2.17), only hold if $M = E_{\mathrm{stat}}(U(\boldsymbol{x}))$ is the energy of the *self-consistent* meson profile

$$M = \frac{1}{T}\min_{U(\boldsymbol{x})} S_{\mathrm{eff}}[U(\boldsymbol{x})]$$

in contrast to the non-relativistic analogue, where the one-body potential can be chosen completely arbitrarily as long as it is not velocity dependent. This reflects the well known fact that for a relativistic field theory the energy $E = \int d^3x\, T^{00}$ and the momentum $P^i = \int d^3x\, T^{0i}$ form a Lorentz four-vector due to the vanishing four-divergence of the energy momentum tensor $T^{\mu\nu}$, which only holds if the classical Euler–Lagrange equations are satisfied.

An explicit proof of the transformation property (2.16) has been given in the Friedberg–Lee soliton model [15]. The second-order result (2.17) was also established for sine-Gordon kink solitons [14] as well as the Skyrme model [17] In doing so, the Lorentz group generators, instead of those of the Galilean group, (2.12) have to be used. One ends up with the desired expression for the Lorentz boosted or pushed energy, $\cosh\omega\; M$ or $\frac{1}{2}Mv^2$, respectively, plus additional terms which either vanish exactly for static configurations like the expectation value of the linear momentum (2.8) or can be annihilated by using a relativistic virial theorem [26], if the meson fields are self-consistent. In our model, we are faced now with the additional difficulty that one has to start from a properly regularized effective fermionic determinant. Moreover, most of these regularization schemes [27] are working in Euclidean space–time, which makes the algebra more involved.

In the next section we will give proof of the equality $M^* = M$ using the pushing ansatz for the NJL soliton. We will find that $M^* = M$ also holds in this case, as long as the regularization scheme respects Lorentz invariance in Minkowski or O(4)-invariance in Euclidean space–time, respectively. In the next step we will consider the spurious zero-point energies of the translational and (iso-)rotational motions. To get an estimate of their magnitude we will use the corresponding formulae from non-relativistic many-particle physics as well as from projection theories. It turns out that those energies are quite high (≈ 100 MeV for the (iso-)rotational and ≈ 300 MeV for the translational degrees of freedom), which, with a mean-field energy of ≈ 1300 MeV [4, 6], give a nucleon mass quite close to its experimental value.

3. Pushing the Nambu–Jona-Lasinio soliton; translational mass

The Nambu–Jona-Lasinio Lagrangian [1] in the $SU(2)$-flavour sector, with scalar and pseudo-scalar couplings, reads

$$\mathcal{L} = \mathrm{i}\bar{q}\gamma^\mu\partial_\mu q + \frac{G}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau})^2] \tag{3.1}$$

where q represents a Dirac quark field with N_c colours and two flavours. This Lagrangian can be transformed, after Wick rotation, bosonization and integration of the quarks, into the following effective action [28]

$$S_{\rm eff} = -N_c\,\mathrm{Sp}\log D + \frac{g^2}{2G}[\sigma^2 + \boldsymbol{\pi}^2] \tag{3.2}$$

where only meson fields σ and $\boldsymbol{\pi}$ appear. Here the Dirac operator

$$D = \mathrm{i}\gamma^\mu\partial_\mu + mU(x) \tag{3.3}$$

with

$$U(x) = (\sigma + \mathrm{i}\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5) \qquad m = gf_\pi$$

has been introduced.

$S_{\rm eff}$ is UV divergent and therefore has to be provided with an UV cutoff, Λ, within a given regularization scheme. We concentrate on the scalar and pseudo-scalar sector of $SU(2)$, where only the real part of the quark effective action is essential, so, instead of $\mathrm{Tr}\log D$ we proceed with $(\det D^\dagger D)^{1/2}$. We demand that the regularization scheme preserves the Lorentz invariance or the O(4)-invariance, respectively, of the effective action (3.2). This means, in fact, that for any unitary operator $\mathcal{U}(\omega)$ of the Lorentz or O(4)-group in an appropriate representation, the equality

$$\mathrm{Sp}\log|_{\rm reg}(D^\dagger D) = \mathrm{Sp}\log|_{\rm reg}[\mathcal{U}^{-1}(\omega)(D^\dagger D)\mathcal{U}(\omega)] \tag{3.4}$$

which formally holds for unregularized traces ($\mathrm{Sp}\log = \log\det$), is also maintained in the regularized case. For convenience, we will carry out our explicit calculations in the Pauli–Villars regularization

$$S_{\rm eff} = -N_c/2\sum_i c_i\,\mathrm{Sp}\log(D^\dagger D + \Lambda_i^2) \tag{3.5}$$

where $c_0 = 1$ and $\Lambda_0 = 0$ [27, 29, 30]. For the sake of simplicity, we will, in the following, implicitly assume the summation over the Pauli–Villars regulators. To construct a system with baryon number $B = 1$, one has to add explicitly $N_c = 3$ valence particles [31]

$$E^{B=1} = \frac{1}{T}S_{\rm eff} + \eta_{\rm val}\epsilon_{\rm val} \tag{3.6}$$

with

$$\eta_{\rm val} = \begin{cases} 1 & \text{if}\,\epsilon_{\rm val} \geqslant 0 \\ 0 & \text{if}\,\epsilon_{\rm val} < 0. \end{cases} \tag{3.7}$$

The stationary points of $E^{B=1}$ with respect to σ and π determine the self-consistent mean-field configurations:

$$\delta_\sigma E^{B=1} = \delta_\pi E^{B=1} = 0. \tag{3.8}$$

It turns out to be essential to put the non-linear constraint of the chiral circle, $\sigma^2 + \pi^2 = f_\pi^2$, to the meson fields, i.e. parametrizing the hedgehog σ and π by

$$\sigma(\boldsymbol{x}) = \cos\theta(r) \qquad \boldsymbol{\pi}(\boldsymbol{x}) = \sin\theta(r)\hat{\boldsymbol{x}} \tag{3.9}$$

and varying $E^{B=1}$ with respect to the chiral angle $\theta(r)$, otherwise no solitonic solution exists [32].

Because of (3.4), the Dirac operator for the pushed-soliton ansatz (2.6) is unitary, equivalent to the static Dirac operator with an additional term

$$D \to D - \mathrm{i}\gamma_4 v_k \frac{\mathrm{d}}{\mathrm{d}x_k}. \tag{3.10}$$

We first concentrate on the sea part, i.e. the regularized fermion determinant. Putting this into (3.5), we get

$$S_{\text{eff}} = -N_c \frac{T}{2} \int \frac{d\omega}{2\pi} \operatorname{Sp} \log \left\{ \left(h + i\omega + v_k \frac{d}{dx_k} \right) \left(h - i\omega - v_k \frac{d}{dx_k} \right) + \Lambda^2) \right\} \tag{3.11}$$

with the one-particle Hamiltonian h. The v_k is assumed to be real in Euclidean space, in accordance with the rotational analogue of [12]. Expanding the effective action (3.11) in powers of the soliton velocity up to second order, we get

$$S_{\text{eff}} = M_{\text{sea}} T + \tfrac{1}{2} \int dt \, M^*_{\text{sea}} v^2 + \ldots \tag{3.12}$$

with the sea parts of the static mass

$$M_{\text{sea}} = N_c \int \frac{d\omega}{2\pi} \omega^2 \operatorname{Sp} K_\Lambda^{-1}(\omega) \tag{3.13}$$

and the translational mass

$$M^*_{\text{sea}} \delta_{kl} = N_c \int \frac{d\omega}{2\pi} \operatorname{Sp} \left\{ K_\Lambda^{-1}(\omega) \frac{d}{dx_k} \frac{d}{dx_l} + \tfrac{1}{2} K_\Lambda^{-1}(\omega) B_k(\omega) K_\Lambda^{-1}(\omega) B_l(\omega) \right\} \tag{3.14}$$

respectively, where we have used the notation

$$K_\Lambda(\omega) = h^2 + \omega^2 + \Lambda^2 \tag{3.15}$$

$$B_k(\omega) = -2i\omega \frac{d}{dx_k} + \left[\frac{d}{dx_k}, h \right]. \tag{3.16}$$

The next step is to represent the operator $B_k(\omega)$ in the form

$$B_k(\omega) = [\Delta_k(\omega), K_\Lambda(\omega)] \tag{3.17}$$

where

$$\Delta_k(\omega) = -i\omega x_k - \tfrac{1}{2}\gamma_4 \gamma_k. \tag{3.18}$$

The commutator representation (3.17) allows us to write down

$$K_\Lambda^{-1}(\omega) B_k(\omega) K_\Lambda^{-1}(\omega) = -[\Delta_k(\omega), K_\Lambda^{-1}(\omega)]. \tag{3.19}$$

With this relation, our expression for the translational mass M^* can be re-written as

$$M^*_{\text{sea}} \delta_{kl} = N_c \int \frac{d\omega}{2\pi} \operatorname{Sp} \left\{ K_\Lambda^{-1}(\omega) \ \left(\frac{d}{dx_k} \frac{d}{dx_l} + \tfrac{1}{2} [\Delta_k(\omega), B_l(\omega)] \right) \right\} \tag{3.20}$$

The commutator that has appeared here is

$$[\Delta_k(\omega), B_l(\omega)] = 2\omega^2\delta_{kl} + g\gamma_k\partial_l(\sigma + \mathrm{i}\pi\cdot\tau\gamma_5). \tag{3.21}$$

Due to the rotational symmetry of the hedgehog soliton, only the $k = l$ part of this commutator gives a contribution to (3.20). Comparing (3.13) and (3.20), we get

$$(M^* - M)_{\mathrm{sea}} = N_c/3 \int \frac{\mathrm{d}\omega}{2\pi}\,\mathrm{Sp}\left\{K_\Lambda^{-1}(\omega)\left[\frac{\mathrm{d}^2}{\mathrm{d}x_k^2} + \frac{1}{2}g\gamma_k\partial_k(\sigma + \mathrm{i}\pi\cdot\tau\gamma_5)\right]\right\}. \tag{3.22}$$

The right-hand side of (3.22) is nothing but the sea part of the expectation value

$$\frac{1}{3}\left\langle \int \mathrm{d}^3x\,\left[\frac{\mathrm{d}^2}{\mathrm{d}x_k^2} + \frac{1}{2}g\gamma_k\partial_k(\sigma + \mathrm{i}\pi\cdot\tau\gamma_5)\right]\right\rangle$$

in the static self-consistent mean-field configuration. It is easy to convince oneself that (3.22) holds as well, if one considers the total values (sea and valence contributions) on both sides, i.e.

$$(M^* - M) = \mathcal{T}\frac{1}{3}\left\langle \int \mathrm{d}^3x\left[\frac{\mathrm{d}^2}{\mathrm{d}x_k^2} + \frac{1}{2}g\gamma_k\partial_k(\sigma + \mathrm{i}\pi\cdot\tau\gamma_5)\right]\right\rangle_{\mathrm{tot}}. \tag{3.23}$$

In order to prove that the right-hand side of (3.23) vanishes, we make use of a relativistic virial theorem [26]. It is based on the fact that the meson fields are the stationary points of the action (3.8). In the Pauli–Villars regularization, this is expressed by the equation

$$\int \frac{\mathrm{d}\omega}{2\pi}\,\mathrm{Tr}\{K_\Lambda^{-1}(\omega)\delta h^2\} = 0. \tag{3.24}$$

In particular, we can take here the dilatation variation of the soliton fields

$$\delta\sigma = x_k\partial\sigma/\partial x_k \qquad \delta\pi = x_k\partial\pi/\partial x_k. \tag{3.25}$$

One should notice that this variation respects the non-linear constraint of the chiral circle because

$$\delta(\sigma^2 + \pi^2) = 2(\sigma\delta\sigma + \pi\delta\pi) = x_k\partial_k(\sigma^2 + \pi^2) = 0. \tag{3.26}$$

The corresponding variation of the squared Dirac Hamiltonian is

$$\delta h^2 = x_k[\partial_k, h^2] - g\gamma_k\partial_k(\sigma + \mathrm{i}\pi\cdot\tau\gamma_5). \tag{3.27}$$

Using the formula

$$x_k[\partial_k, h^2] = [x_k\partial_k, h^2] - 2\partial_k^2 \tag{3.28}$$

together with the fact that

$$\langle\lambda[x_k\partial_k, h^2]\rangle\lambda = 0 \tag{3.29}$$

it follows that the stationary condition (3.24) with respect to the dilatation variation (3.25) annihilates the right-hand side of (3.23) and we get

$$M = M^*. \tag{3.30}$$

4. Zero-point energies of the translational and rotational motion

For both the translational as well as the (iso-)rotational motion, one expects spurious zero-point energies, which have to be subtracted from the total energy when a semi-classical quantization is performed. This is due to the fact that, even at mean-field level, there are already finite expectation values for the two-particle operators $\boldsymbol{P}^2$ and $\boldsymbol{T}^2$. Up to now it has not been clear how to treat these spurious zero-point energies in such a model consistently, but we expect that their order of magnitude can be estimated well by using the corresponding expressions obtained in non-relativistic many-particle physics [7] and which also appear in chiral soliton models in the context of projection techniques for linear and angular momenta [24–33]. Actually, this means that, after semi-classical quantization of the rotational degrees of freedom [9] and with $M^* = M$, the mass of a system with isospin T at rest ($P^i = 0$), $M_{T,\boldsymbol{P}=0}$, reads

$$M_{J,\boldsymbol{P}=0} = M + \frac{T(T+1)}{2\Theta} - \frac{\langle \boldsymbol{T}^2\rangle}{2\Theta} - \frac{\langle \boldsymbol{P}^2\rangle}{2M}. \tag{4.1}$$

Using the proper time-regularization scheme [31], the hedgehog mass M and the rotational moment of inertia Θ can be taken from [6, 13] as a function of the constituent mass m. Actually, the operator $T^2 = N_c\frac{1}{2}(\frac{1}{2}+1)B$ is proportional to the baryon number operator and we therefore handle it in the same way, i.e. we do not regularize it because it is finite [6]. For an NJL soliton in the hedgehog approximation, the sea part vanishes identically and we have [10, 11]

$$\langle \boldsymbol{T}^2\rangle = \langle \boldsymbol{T}^2\rangle_{\text{val}} = N_c\tfrac{3}{4} = \tfrac{9}{4}. \tag{4.2}$$

Hence we obtain for the rest masses of the nucleon ($J = T = \frac{1}{2}$) and the Δ ($J = T = \frac{3}{2}$), respectively:

$$M_{\text{N}} = M + \frac{3}{8\Theta} - \frac{9}{8\Theta} - \frac{\langle \boldsymbol{P}^2\rangle}{2M} \tag{4.3a}$$

$$M_{\Delta} = M + \frac{15}{8\Theta} - -\frac{9}{8\Theta} - \frac{\langle \boldsymbol{P}^2\rangle}{2M}. \tag{4.3b}$$

The $\langle \boldsymbol{P}^2\rangle$ is evaluated in the present paper for the first time:

$$\langle \boldsymbol{P}^2\rangle = \langle \text{val}|\boldsymbol{P}^2|\text{val}\rangle + \sum_{\lambda}\left(R(\epsilon_\lambda,\Lambda)\langle\lambda|\boldsymbol{P}^2|\lambda\rangle - R(\epsilon_{\lambda_{\text{VAK}}},\Lambda)\langle\lambda_{\text{VAK}}|\boldsymbol{P}^2|\lambda_{\text{VAK}}\rangle\right) \tag{4.4}$$

where $|\lambda\rangle$ and ϵ_λ denote the eigenvector and the eigenvalue of the single-particle Hamiltonian h, respectively (for details see [6, 8]), and the regularization function $R(\epsilon_\lambda, \Lambda)$ is given by

$$R(\epsilon_\lambda,\Lambda) = (-)\frac{1}{\sqrt{4\pi}}\int_1^\infty \mathrm{d}\tau\ \tau^{-\frac{1}{2}}\mathrm{e}^{-\tau(\epsilon_\lambda/\Lambda)^2}\left(\frac{\epsilon_\lambda}{\Lambda}\right). \tag{4.5}$$

It should be noted that the valence part has to be suppressed if $\epsilon_{\text{val}} < 0$ [31]. The expectation values $\langle\lambda|\boldsymbol{P}^2|\lambda\rangle$ can easily be evaluated by expanding $|\lambda\rangle$ into the free spherical wave basis of [36]:

$$\boldsymbol{P}^2|k_n, G, M\rangle = {k_n}^2|k_n, G, M\rangle. \tag{4.6}$$

Furthermore, it turns out that for the unpolarized plane-wave vacuum, equation (4.4) vanishes. If $\epsilon_{\text{val}} > 0$, $\langle\boldsymbol{P}^2\rangle$ is clearly dominated by the valence quark. For example, at a constituent mass of $m = 400$ MeV, we find $\langle\boldsymbol{P}^2\rangle = 865^2\ (\text{MeV})^2 = 867^2\ (\text{MeV})^2 + (-)66^2\ (\text{MeV})^2$ for valence and seaquarks, respectively. Figure 1 shows the zero-point energies $\langle\boldsymbol{P}^2\rangle/2M$ and $\langle T^2\rangle/2\Theta = 9/8\Theta$. The masses of the hedgehog M, the nucleon mass M_{N} and the Δ mass M_Δ are presented in figure 2. One notices that the zero-point energy of the translational motion is of the order of magnitude one encounters in, for example, the projected mean-field approaches to the Gell-Mann–Levi chiral σ-model and similar approaches mentioned above [18–24], and which also appear in non-relativistic quark models ([37] and references therein). Furthermore, it can be seen that for small constituent masses near the critical value, $m \approx 350$ MeV, the nucleon rest mass M_{N} is around 900 MeV — close to its experimental value. Apparently, the nucleon is clearly bound, i.e. $M_{\text{N}} < N_{\text{c}}m$. It is very gratifying that the above corrections due to spurious rotational and translational motion of the soliton bring the nucleon energy down to its experimental value and at the same time guarantee its stability against decay into three free quarks. However, it is a bit disturbing that the zero-point energies amount altogether to something like 30% of the total rest mass, which is by no means a small correction. One has to consider how to go beyond the present perturbation expansions in the collective velocities.

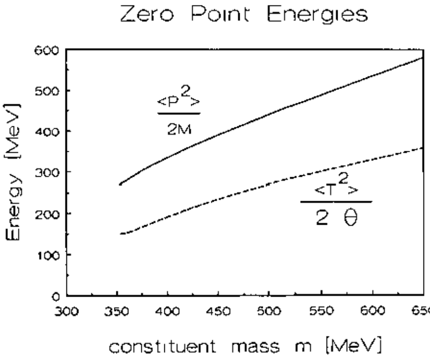

Figure 1. The spurious zero-point energies (correlation energies) for the translational and the (iso-) rotational motion $\langle\boldsymbol{P}^2\rangle/2M$ and $\langle T^2\rangle/2\Theta = 9/(8\Theta)$, respectively, as a function of the constituent mass m.

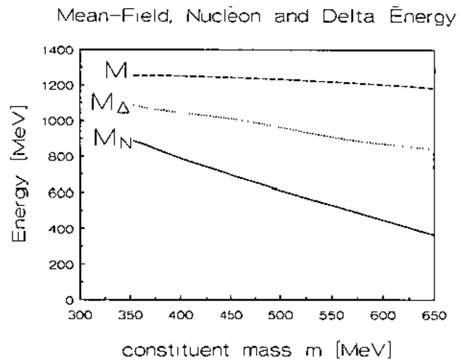

Figure 2. The mass of the hedgehog (mean-field energy) M as well as the total mass of the nucleon M_{N} and the $\Delta\, M_\Delta$ as a function of the constituent mass m.

5. Conclusion

We can summarize our points as follows:

(i) We have shown explicitly that the static and the translational masses of the regularized NJL soliton are identical for Lorentz or O(4)-invariant regularization schemes, respectively.

(ii) Zero-point energies for the rotational and translational motion are around 100 MeV and 300 MeV, respectively, yielding a nucleon rest mass of about 900 MeV.

Acknowledgments

Useful discussions with D I Diakonov, V Yu Petrov, V Dimitrov, H Reinhardt and A Gorski are acknowledged. The work has been supported partially by the Deutsche Forschungsgemeinschaft, the Soviet Academy of Science and the Bundesministerium für Forschung und Technologie, Bonn.

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