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Roper resonances and generator coordinate method in the chiral-soliton model

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The nucleon and Δ Roper resonances are described by means of the generator coordinate method in the framework of the nontopological chiral-soliton model. Solitons with various sizes are constructed with a constrained variational technique. The masses of all known Roper resonances come out to within 150 MeV of their experimental values. A nucleon compression modulus of about 4 GeV is extracted. The limits of the approach due to the polarization of the Dirac vacuum are displayed.

I. INTRODUCTION

The description of the ground-state nucleon and Δ isobar by means of relativistic chiral topological and nontopological soliton models has enjoyed increasing popularity among nuclear and particle theorists. In particular the Skyrme model[1-6] and the linear chiral-soliton model[7-13] have been shown to reproduce static properties of the nucleon within 30% accuracy. It is interesting now to investigate whether properties of baryonic excited states can also be obtained from these models. In the framework of the Skyrme model this question has already been addressed by several groups either in the framework of a scaling approach[14-16] or within a semiclassical approximation for the vibrational modes around the mean field.[17-19]

In the present paper we will show how the collective vibrational degrees of freedom can also be treated by means of the generator coordinate method[20,21] (GCM) in the nontopological chiral-soliton model. The GCM is a well-established approach in nuclear structure studies for small- and large-amplitude collective motion.[21] Its application here consists of superimposing solitons of different sizes obtained by a constraint technique. Because of this our description of the Roper resonance is fully quantum mechanical and does not introduce any additional collective parameters over and above those in the original chiral-soliton Lagrangian. The simplest way to apply the GCM technique would consist of calculating the solitons in the mean-field approximation and to associate coherent states with the boson fields. Since, however, rotation-vibration couplings seem to be important we employ the coherent-pair approximation[22] (CPA) to construct states of good angular momentum and isospin.

The outline of the paper is as follows. Section II reviews the coherent-pair approximation and the techniques needed to apply it to the nucleon and Δ. Section III describes the constraint technique, presents some results, and reviews the generator coordinate method. The kernels for the GCM are derived in Sec. IV. The final energies and observables of the nucleon and of the various monopole vibrations on top of the nucleon and of the Δ are presented in Sec. V and compared to experimental data and other approaches. A summary and some conclusions are given in Sec. VI.

II. THE LINEAR CHIRAL-SOLITON MODEL AND THE COHERENT-PAIR APPROXIMATION

This section reviews the linear chiral-soliton model, originating from Gell-Mann and Levy,[7] and the coherent-pair approximation[12,21] (CPA) to the static nucleon and Δ ground state as far as it is necessary for the application of the generator coordinate method to the breathing mode. Details can be found in Refs. 12, 22, and 23. Actually some recent investigations[22] show that the CPA has a weakness concerning the Goldberger-Treiman relation. Although this is not in favor of the method as such, it is believed that this problem is not relevant for the energies of the Roper resonance. In Ref. 24 various nontopological calculations with similar problems are compared and it turns out that, except for $g_{\pi NN}$, most of the other observables are rather unaffected by the degree of violation of the Goldberger-Treiman relation. We believe the CPA provides a useful and not unphysical approach to demonstrate the ingredients and results of the GCM applied to the chiral-soliton model.

The Lagrangian density may be written as

$$\mathcal{L}=\bar{q}[i\partial\!\!\!/-g(\sigma+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]q + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma+\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}-U(\sigma,\boldsymbol{\pi}) , \tag{2.1}$$

with

$$U(\sigma,\boldsymbol{\pi})=\frac{\lambda^2}{4}(\sigma^2+\boldsymbol{\pi}^2-\nu^2)^2-m_\pi^2 f_\pi \sigma+U_0 \ . \tag{2.2}$$

Here the q, σ and $\boldsymbol{\pi}$ are the quark, sigma (scalar, isoscalar), and pion (pseudoscalar, isovector) fields, respectively. Remote from a hadron, the quark and pion fields are assumed to vanish and the σ field reverts to the pion decay constant $f_\pi=93$ MeV which essentially sets the energy and length scales. The nonzero value of the pion mass $m_\pi=139.6$ MeV explicitly breaks the chiral symmetry. The parameter λ is related to the mass of the σ field by $m_\sigma^2=2\lambda^2 f_\pi^2+m_\pi^2$ and the ν is given by $\nu^2=f_\pi^2-m_\pi^2/\lambda^2$. Both the m_σ and the coupling constant g are free parameters. The U_0 is given by $U_0=f_\pi^2 m_\pi^2(2m_\sigma^2-3m_\pi^2)/2(m_\sigma^2-m_\pi^2)$.

In the CPA we consider the expansion of the pion field as

$$\pi^t(r)=\left[\frac{1}{\pi}\right]^{1/2}\int dk\, k^2\omega_\pi(k)^{-1/2}\sum_{lm} j_l(kr)Y^*_{lm}(\Omega_r)[b^\dagger_{lmt}(k)+(-1)^{t+m}b_{l-m-t}(k)] \ , \tag{2.3}$$

with $\omega_\pi(k)=(k^2+m_\pi^2)^{1/2}$. The basis operators $b^\dagger_{lmt}(k)$ create a free massive pion with spherical isospin component t and orbital angular momentum (l,m). For the Fock state of the pion field we will use p-wave pions whose creation operators are

$$D^\dagger_{mt}=\int dk\, k^2 c(k) b^\dagger_{1mt}(k) \ , \tag{2.4}$$

with a trial function $c(k)$. The $D^\dagger_{mt}$ are used to construct multipion coherent-pair states[25] $|P^{00}_{00}\rangle$ and $|P^{11}_{mt}\rangle$, having zero and one unpaired pion, respectively. The $|P^{11}\rangle$ state therefore has the p-wave isovector character of a single pion with creation operator $D^\dagger_{mt}$. The $|P^{JT}\rangle$ are constructed by requiring them to satisfy the recurrence relation

$$\begin{aligned}&(-1)^{m+t}D_{-m-t}|P^{00}_{00}\rangle=a|P^{11}_{mt}\rangle \ ,\\ &\sum_{mt} D_{mt}|P^{11}_{mt}\rangle=9b|P^{00}_{00}\rangle \ ,\end{aligned} \tag{2.5}$$

from which follows the coherence property

$$D{:}D|P\rangle=x|P\rangle \ . \tag{2.6}$$

Here $|P\rangle$ can be $|P^{00}\rangle$ or $|P^{11}\rangle$ and $x=9ab$ serves as a free coherence parameter. The symbol $D{:}D$ indicates the coupling to a scalar and isoscalar. Explicitly one obtains, for example,

$$|P^{00}\rangle=\sum_n \frac{f_n}{(2n)!}(D^\dagger{:}D^\dagger)^n|0\rangle \ , \tag{2.7}$$

with $f_{n+1}=x(2n+1)f_n/(9+2n)$ and f_0 given by the normalization of $|P^{00}\rangle$ (see Refs. 21 and 22 for details). The a and b can be expressed in terms of x from the normalization $\langle P^{11}_{mt}|P^{11}_{mt}\rangle=1$. We use for the physical nucleon a Fock state of the form

$$\begin{aligned}|NJ_zT_3\rangle=[&\alpha(|n\rangle\times|P^{00}\rangle)_{J_zT_3}+\beta(|n\rangle\times|P^{11}\rangle)_{J_zT_3}\\ &+\gamma(|\delta\rangle\times|P^{11}\rangle)_{J_zT_3}]|\Sigma\rangle \ .\end{aligned} \tag{2.8}$$

Here $|\Sigma\rangle$ is a spherical coherent state of the σ field, $|n\rangle$ and $|\delta\rangle$ refer to the conventional SU(2)×SU(2) three-quark structures with nucleon and the Δ quantum numbers, respectively, having the common $1s$ orbit $(u(r),iv(r)\boldsymbol{\sigma}\cdot\mathbf{r})$ with $\boldsymbol{\sigma}$ here referring to the conventional Pauli spin matrices. In principle a more accurate Fock state could be considered by including in the sum in Eq. (2.8) states having more unpaired pions.[25] We consider here the smaller (and easier) truncation in order to concentrate on the later generator coordinate technique to get an idea of the crucial effects.

In a first step, the total energy of the system is evaluated with the pion degrees of freedom represented equivalently by

$$\Phi(r)=\frac{1}{(2\pi)}\int dk\, k^2 c(k)\omega_\pi(k)^{-1/2}j_1(kr) \tag{2.9}$$

instead of $c(k)$. In a second step, for a given coherence parameter x and given mixing parameters α, β, and γ, the total energy is made stationary by variation with respect to the amplitudes of the quarks $u(r)$, $v(r)$, the pion-field amplitude $\Phi(r)$, and the σ-field amplitude $\sigma(r)=\langle\Sigma|\sigma(\mathbf{r})|\Sigma\rangle$ subject to normalization conditions for the quark and pion amplitudes. This leads to four coupled nonlinear differential equations with two Lagrange multipliers ϵ and ξ:

$$\frac{du}{dr}=-(g\sigma+\epsilon)v-\tfrac{2}{3}g a\delta(a+b)\Phi u \ , \tag{2.10a}$$

$$\frac{dv}{dr}=-\frac{2}{r}v-(g\sigma-\epsilon)u-\tfrac{2}{3}g\alpha\delta(a+b)\Phi v \ , \tag{2.10b}$$

$$\begin{aligned}\frac{d^2\sigma}{dr^2}=&-\frac{2}{r}\frac{d\sigma}{dr}+\lambda^2(\sigma^2-\nu^2)\sigma+2\lambda^2(x+N_\pi)\Phi^2\sigma\\ &+3g(u^2-v^2)-m_\pi^2 f_\pi \ ,\end{aligned} \tag{2.10c}$$

$$\begin{aligned}\frac{d^2\Phi}{dr^2}=&-\frac{2}{r}\frac{d\Phi}{\delta r}+\frac{2}{r^2}\Phi+\frac{1}{2}\left[1-\frac{x}{N_\pi}\right]m_\pi^2\Phi\\ &+\frac{1}{2}\left[1+\frac{x}{N_\pi}\right]\lambda^2(\sigma^2-\nu^2)\Phi+\frac{2x}{N_\pi}(\tfrac{2}{3}x+N_\pi)\lambda^2\Phi^3\\ &-\frac{\alpha\delta(a+b)}{N_\pi}guv-\xi\Phi_p \ ,\end{aligned} \tag{2.10d}$$

with $\delta=5(\beta+4\gamma\sqrt{2/5})/\sqrt{3}$ and N_π being the average pion number $N_\pi=9\alpha^2a^2+9(\beta^2+\gamma^2)c^2$. The Φ_p results from Φ by a double folding

$$\Phi_p(r)=\int \omega_1(r,r')\Phi(r')r'^2dr' , \tag{2.11}$$

with

$$\omega_1(r,r')=\frac{2}{\pi}\int dk\, k^2\omega_\pi(k)j_1(kr)j_1(kr') . \tag{2.12}$$

The kernel $\omega_1(r,r')$ is in principle ill defined; however, if it is applied to a well-behaved pion field $\Phi(r)$, the corresponding field $\Phi_p(r)$ does exist. The Lagrange parameter ξ in Eq. (2.10d) is fixed by the normalization condition

$$8\pi\int \Phi(r)\Phi_p(r)r^2dr=1 , \tag{2.13}$$

which guarantees that the $D^\dagger_{mt}$ are proper bosons. The Lagrange multiplier ϵ is equivalent to the quark energy eigenvalue, cf. Eqs. (2.10a) and (2.10b), and originates from the normalization condition $4\pi\int(u^2+v^2)r^2dr=1$. The above set of nonlinear differential equations is solved for given x, α, β, and γ by the program package COLSYS in a way described in Refs. 26 and 27. In principle we deal with integro-differential equations, since $\Phi_p(r)$ is an integral transformation of $\Phi(r)$; however, in practice the system may be solved by using an iterative procedure keeping $\Phi_p(r)$ fixed for each iteration. After adjusting ϵ and ξ by the normalization conditions the α, β, and γ are determined by diagonalization. Their new values are fed again into the system of differential equations yielding new fields, and so on, until convergence is achieved. The x is found by minimizing the nucleon mass yielding always $x\simeq 1.1$.

The electromagnetic observables considered are calculated as suitable matrix elements of the corresponding currents.[22,23] Reasonable values for the ground-state energies of the nucleon and Δ and of the nucleon observables are obtained for a g around 5.6 and an $m_\sigma \geq 0.3$ GeV. Typical results are presented in Refs. 12 and 22.

III. CONSTRAINT TECHNIQUE AND THE GENERATOR COORDINATE METHOD

In the present paper the Roper resonance is considered as a collective monopole vibration of the nucleon. In order to formulate this quantum mechanically without semiclassical appropriations, we construct first solitons of different radii around the ground-state radius. These Fock states, which all have proper spin and isospin quantum numbers through the use of CPA, are then superimposed by the generator coordinate method (GCM). This means one considers them as a (nonorthogonal) basis of a collective Hilbert space in which the total Hamiltonian is diagonalized.

In our approach the Fock states with various radii are constructed by introducing into the CPA variational principle for the total energy a Lagrange multiplier χ and a function $r^2F(r)$ such that the mean quadratic radius R of the quarks is constrained:

$$\delta\int d^3r\langle NT_3J_z|:\mathcal{H}(\mathbf{r})-\epsilon q^\dagger(\mathbf{r})q(\mathbf{r})-\xi\Phi(r)\Phi_p(r)$$
$$-\chi r^2F(r)q^\dagger(\mathbf{r})q(\mathbf{r}):|NT_3J_z\rangle=0 , \tag{3.1}$$

with

$$R^2=4\pi\int dr\, r^4(u^2+v^2) . \tag{3.2}$$

Here $:\mathcal{H}(\mathbf{r}):$ is the normal product of the Hamiltonian operator. Since r^2 is an unbounded operator, we artificially force it to be bounded by multiplying it with the Fermi function $F(r)$ with appropriately chosen parameters ($r_0=1$ fm, $a=0.25$ fm). The Euler-Lagrange equations of the new constrained variational principle are easily obtained by making the substitution in Eq. (2.10):

$$\epsilon\rightarrow\epsilon+\chi r^2F(r) . \tag{3.3}$$

For various values of χ we solved the now modified Eq. (2.10) in the way described in Sec. II yielding solutions $|NJ_zT_3\rangle_R$ with varying value of the quark radius R. The equilibrium solution was found at $R_0=0.722$ fm for $\chi=0$. The resulting nucleon energy as a function of the quark radius R is displayed in the upper part of Fig. 1 and shows some very interesting features which are related to the fact that we deal with a relativistic system rather than with a nonrelativistic one. First, for values of R smaller than $R\simeq 0.6$ fm there was no localized solution to be found. Second, the energy does exhibit a local minimum at the equilibrium solution; however, it also exhibits a maximum at $R=0.84$ fm beyond which it is con-

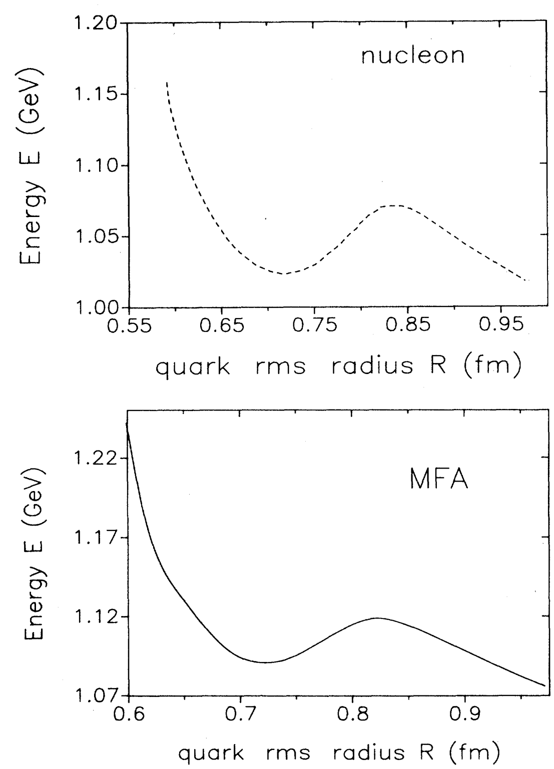

FIG. 1. The total energy of the soliton in the hedgehog–mean-field approach (MFA) and of the nucleon in the coherent-pair approximation (nucleon) vs the root-square radius of the quark distribution. Qualitatively both approaches show need for the effects of polarization of the sea quarks for $R\geq 0.85$ fm. The calculations are done with $m_\sigma=0.7$ GeV and $g=5.37$.

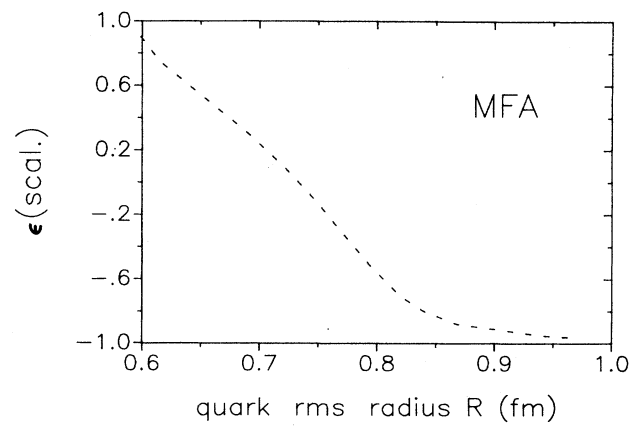

FIG. 2. The scaled quark eigenvalue $\epsilon'=\epsilon/gf_\pi$ vs the root-squared quark radius. The calculations are performed in the hedgehog–mean-field approach with $g=5.32$ and $m_\sigma=0.7$ GeV.

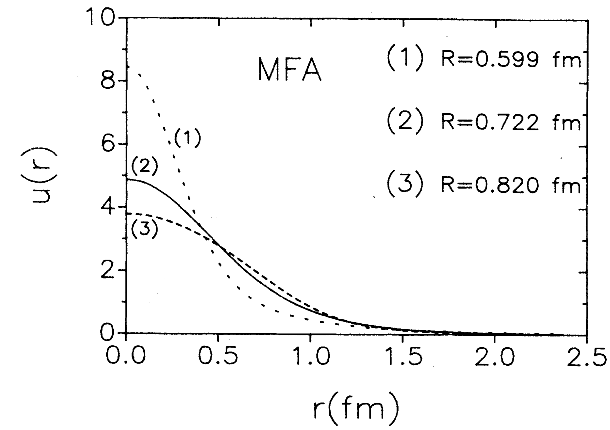

FIG. 4. The radial distribution of the upper quark component for three different root-squared quark radii R. The parameters are given in Fig. 2.

tinuously decreasing, showing even negative values for large R. In order to explain these features and to discuss the effects in detail we performed for this purpose simpler mean-field calculations using the hedgehog ansatz rather than solving the full coherent-pair model. The qualitative results and the trends of both approaches with varying R are identical, as the lower part of Fig. 1 shows.

As shown in Refs. 12 and 22, the boundary condition of $u(r)$ and $v(r)$ for having localized solutions is that at large distances r from the center of the soliton

$$r\sqrt{gf_\pi-\epsilon}+\frac{1}{\sqrt{gf_\pi-\epsilon}}u-\sqrt{gf_\pi+\epsilon}\,rv=0 \ . \qquad (3.4)$$

The Lagrange multiplier χ does not enter in Eq. (3.4) because of the form factor $F(r)$. The scaled quark energy $\epsilon'=\epsilon/gf_\pi$ is bounded, $-1\le\epsilon'\le+1$. This ϵ' is plotted in Fig. 2 and is shown to reach the limit $\epsilon'=+1$ around $R=0.6$ fm. Thus the bound solution is pushed into the continuum, where it is no longer localized and hence no longer supports a solitonic solution.

Another effect occurs at large R. Here the ϵ' is close to the negative-energy continuum where the occupied (negative-energy) orbitals are noticeably polarized. As one can see from Fig. 3, taken from Kahana, Ripka, and Soni,[8,28] with increasing soliton size an increasing number of sea quark orbitals emerge from the filled Dirac sea ($\epsilon'<-1$) and enter the domain $\epsilon'>-1$. The lifting of these levels costs energy which has not entirely been taken into account in the present calculation. This is the

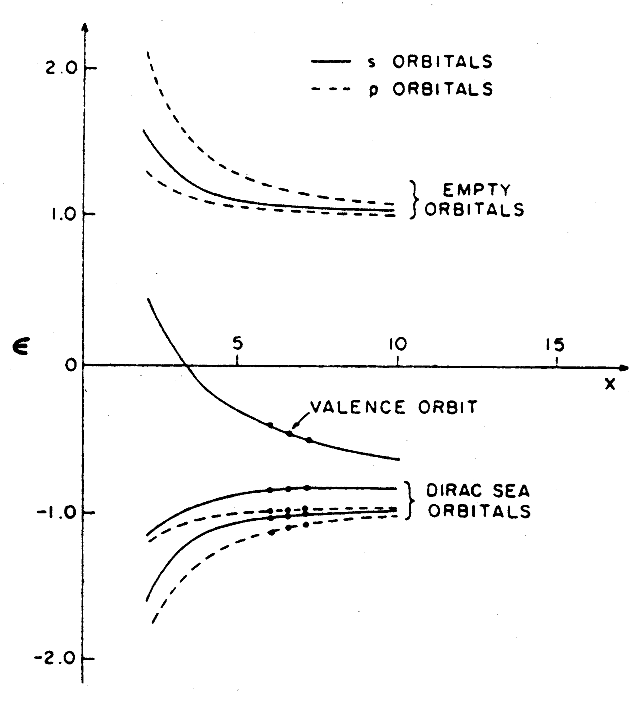

FIG. 3. The behavior of valence- and sea-quark eigenvalues with respect to the size of the soliton given in the scaled unit X. The figure is taken from Kahana, Ripka, and Soni (Ref. 8).

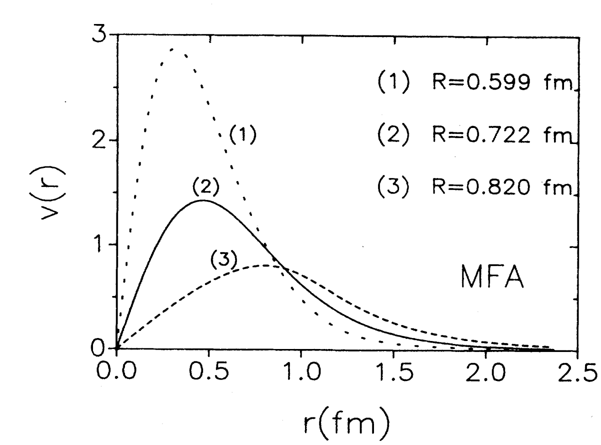

FIG. 5. As in Fig. 4 for the lower quark components.

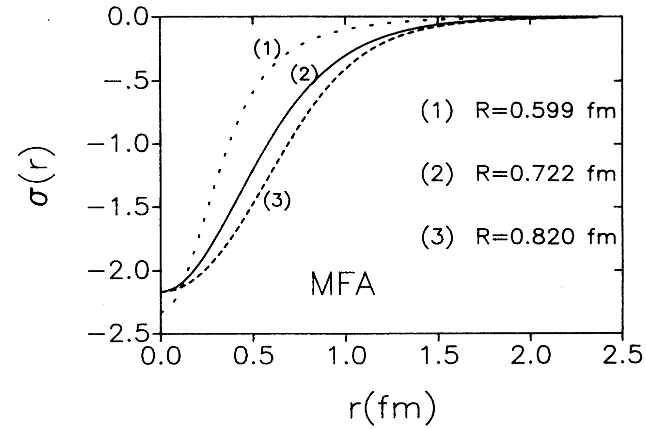

FIG. 6. As in Fig. 4 for the σ-field $\sigma(r)$ distributions.

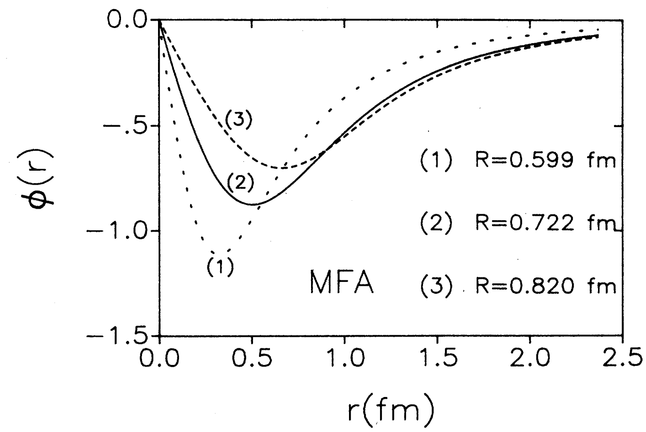

FIG. 7. As in Fig. 4 for the pion-field $\phi(r)$ distributions.

reason we believe our calculated energy behaves in the way that it does at $R>0.85$ fm. Since the approximation of having only valence quarks obviously breaks down at $R>0.85$ fm we limit the subsequent GCM to the region $0.58<R<0.85$ fm. Actually the positive-energy orbits are polarized as well for large R, however they are not occupied and hence do not contribute to the total energy. In the region $0.58\le R\le 0.85$ fm the system behaves as expected. Some curves of $u(r)$, $v(r)$, $\sigma(r)$, and $\Phi(r)$ for three different R values are displayed in Figs. 4–7.

In the generator coordinate method[20,21] one constructs new Fock states of the total system by the superposition

$$|\Psi^{(\alpha)}_{NJ_zT_3}\rangle=\int dR\, f_\alpha(R)|NJ_zT_3\rangle_R\ , \tag{3.5}$$

with the normalization condition

$$\int dR\, dR' f^*_\alpha(R) f_\alpha(R')\,{}_R\langle NJ_zT_3|NJ_zT_3\rangle_{R'}=1\ . \tag{3.6}$$

By the Ritz variational principle

$$\delta(\langle\Psi^\alpha_{NJ_zT_3}|{:}H{:}|\Psi^\alpha_{NJ_zT_3}\rangle-E\langle\Psi^\alpha_{NJ_zT_3}|\Psi^\alpha_{NJ_zT_3}\rangle)=0 \tag{3.7}$$

one determines the weight function $f(R)$ from the resulting Hill-Wheeler equation[20]

$$\int dR'\,{}_{R'}\langle NJ_zT_3|{:}H{:}-E|NJ_zT_3\rangle_R f_\alpha(R)=0\ . \tag{3.8}$$

If we discretize the continuous parameter R we obtain a Hermitian, nonorthogonal matrix eigenvalue problem:

$$\sum_j (H^{ij}-E^{(\alpha)}N^{ij})f^{(\alpha)}_j=0\ , \tag{3.9}$$

where

$$H^{ij}=\int dr\, r^2 H^{ij}(r)\ , \tag{3.10}$$

$$N^{ij}={}_i\langle NJ_zT_3|NJ_zT_3\rangle_j\ , \tag{3.11}$$

and

$$H^{ij}(r)=\int d\Omega\,{}_i\langle NJ_zT_3|{:}\mathcal{H}(\mathbf{r}){:}|NJ_zT_3\rangle_j\ . \tag{3.12}$$

Equation (3.9) is solved in a standard way. Stable results are obtained for five (or more) equally distributed R points around the equilibrium point. The lowest solution ($\alpha=0$) corresponds to the new ground state $|\Psi^{(\alpha=0)}_{NJ_zT_3}\rangle$ being correlated compared to $|NJ_zT_3\rangle_{R_0}$. The first excited state ($\alpha=1$) corresponds to the first Roper resonance (seen at 1470 MeV for the nucleon), that with $\alpha=2$ to the second, etc. The emerging $E^{(\alpha)}$ are obtained as eigenvalues of Eq. (3.9). If one solves the constrained CPA Eq. (2.10) for the Δ, yielding $|\Delta J_zT_3\rangle_i$, one obtains, after the GCM calculation, the new Δ ground-state and its vibrational monopole modes.

IV. THE OVERLAP KERNELS

In this section we provide all the tools to derive the overlap kernels N^{ij} and H^{ij} and the corresponding quantities for the observables. The final expression for N^{ij} is presented in Eq. (4.17). The others are rather lengthy and can be found in Ref. 23.

A. The pion overlap

We start from Eq. (2.3) for the pion field and the corresponding expression for its conjugate momentum:

$$P^t_\pi(\mathbf{r})=i\left[\frac{1}{\pi}\right]^{1/2}\int dk\, k^2\sqrt{\omega_\pi(k)}\sum_{lm} j_l(kr)Y^*_{lm}(kr)[b^\dagger_{lmt}(k)-(-1)^{m+t}b_{l-m-t}(k)]\ , \tag{4.1}$$

with

$$[b_{lmt}(k),b^\dagger_{l'm't'}(k')]=\frac{1}{k^2}\delta(k-k')\delta_{ll'}\delta_{mm'}\delta_{tt'}\ . \tag{4.2}$$

The coherent-pair states are constructed from the D_{mt} of Eq. (2.4). If we label them by i and j, to indicate that they belong to solitons with different radii R_i and R_j, we have

$$[D^i_{mt},D^{j\dagger}_{m't'}]=\pi^{ij}\delta_{mm'}\delta_{tt'}\ , \tag{4.3}$$

with

$$\pi^{ij}=\int dk\, k^2 c^i(k) c^j(k)\ . \tag{4.4}$$

Normalization requires the c's to satisfy $\pi^{ii}=1$. These relations can be transformed into coordinate space by Eqs. (2.9) and (2.11).

The off-diagonal overlaps can then be expressed as

$$\pi^{ij}=\pi^{ji}=8\pi\int dr\, r^2\phi^i(r)\phi_p^j(r)\ . \tag{4.5}$$

By these means the off-diagonal overlaps between the states $|P_{00}^i\rangle$ can (after some algebra) be evaluated to yield

$$P_0^{ij}=\langle P_{00}^i|P_{00}^j\rangle=f_0^2 2^{\kappa-1}(\tilde{L}-2)!!(\partial_{\tilde{y}})^{\kappa-1}\cosh\tilde{x}\ , \tag{4.6}$$

$$P_1^{ij}=\langle P_{mt}^i|P_{m't'}^j\rangle$$
$$=\delta_{mm'}\delta_{tt'}f_0^2\pi^{ij}\frac{x^2}{\tilde{L}a^2}2^{\kappa}(\tilde{L}-2)!!(\partial_{\tilde{y}})^{\kappa}\cosh\tilde{x}\ , \tag{4.7}$$

with $\tilde{x}=x\pi^{ij}$ and $\tilde{y}=\tilde{x}^2$. Here $\tilde{L}\equiv 9=3\times 3$ and $\kappa=(\tilde{L}+1)/2=5$ corresponds to a p-wave $(2L+1=3)$ isovector $(2T+1=3)$. Expressions for $f_0=f_0(x)$ and $a(x)$ both depending on the coherence parameter x can be found in Ref. 22.

B. The σ overlap

For the σ field we start from expressions analogous to Eqs. (2.3) and (4.1) (but for isoscalar field) in terms of σ creation and annihilation operators $a_{lm}^{\dagger}(k)$, respectively. We assume the Fock state

$$|\Sigma^i\rangle=N^i\exp\left[\int dk\, k^2\eta^i(k)a_{00}^{\dagger}(k)\right] \tag{4.8}$$

and

$$\sigma^i(r)=\langle\Sigma^i|\sigma(\mathbf{r})|\Sigma^i\rangle\ .$$

For off-diagonal overlaps we also need a $\sigma_p^i(r)$ with

$$\sigma_p^i(r)=\int dr' r'^2\omega_0(r,r')\sigma^i(r') \tag{4.9}$$

and

$$\omega_0(r,r')=\frac{2}{\pi}\int dk\, k^2\omega_0(k)j_0(kr)j_0(kr') \tag{4.10}$$

and

$$\omega_0(k)=(k^2+m_\sigma^2)^{1/2}\ ,$$

$$\langle\Sigma^i|:[\sigma(\mathbf{r})]^n:|\Sigma^j\rangle=\left[\frac{\sigma^i+\sigma^j}{2}\right]^n\sigma^{ij}\ , \tag{4.11a}$$

$$\langle\Sigma^i|:[\nabla\sigma(\mathbf{r})]^n:|\Sigma^j\rangle=\left[\frac{\nabla\sigma^i+\nabla\sigma^j}{2}\right]^n\sigma^{ij}\ , \tag{4.11b}$$

with $\sigma^{ij}=\langle\Sigma^i|\Sigma^j\rangle$ and

$$\sigma^{ij}=\sigma^{ji}=\frac{\exp\left[2\pi\int dr\, r^2\sigma^i(r)\sigma_p^j(r)\right]}{\exp\left[\pi\int dr\, r^2(\sigma^i\sigma_p^i+\sigma^j\sigma_p^j)\right]}\ . \tag{4.12}$$

Sometimes one prefers to express the asymptotic value of the σ field as $\sigma(r)=\bar{\sigma}(r)+f_\pi$ with $\bar{\sigma}(r\to\infty)=0$. The above formulas hold equally well for $\bar{\sigma}$ as for σ.

C. The quark and baryon overlap

The quark states are characterized by the 1s-orbital spinor

$$q^j(\mathbf{r})=\begin{bmatrix} u^j(r) \\ iv^j(r)\boldsymbol{\sigma}\cdot\mathbf{r}\end{bmatrix} \tag{4.13}$$

and a three-quark spin-flavor function, for example, in the case of the Δ isobar,

$$\langle r|\delta,j_z=\tfrac{3}{2},t_3=\tfrac{3}{2}\rangle_j=[q^j(r)]^3|\uparrow\uparrow\uparrow\rangle|uuu\rangle\ . \tag{4.14}$$

Thus the overlap can be written as

$$_i\langle fj_zt_3|f'j_z't_3'\rangle_j=q^{ij}\delta_{ff'}\delta_{t_3t_3'}\delta_{j_zj_z'}\ , \tag{4.15}$$

where f indicates whether we have a bare nucleon or Δ. Here the q^{ij} are

$$q^{ij}=4\pi\int dr\, r^2(u^iu^j+v^iv^j) \tag{4.16}$$

and $q^{ii}=1$. Using the orthogonality relations of the Clebsch-Gordan coefficients in Eq. (2.8) one obtains, with $N^{ij}={}_i\langle NJ_zT_3|NJ_zT_3\rangle_j$,

$$N^{ij}=N^{ji}=(\alpha^iP_0^{ij}\alpha^j+\beta^iP_1^{ij}\beta^j+\gamma^iP_1^{ij}\gamma^j)(q^{ij})^3\sigma^{ij}\ . \tag{4.17}$$

V. RESULTS AND DISCUSSION

The results of the coherent-pair approximation are known to be rather insensitive to the value of the σ mass if this is chosen to be larger than ≈ 0.6 GeV. We assume the same feature for the GCM calculation and hence use a value of $m_\sigma=0.7$ GeV. The quark-meson coupling constant g is chosen such that the nucleon energy is approximately reproduced in the GCM calculations. We use $g=5.37$ corresponding to an asymptotic quark mass of $m_q=500$ MeV. For all values of R the coherence parameter x is taken to be $x=1.1$, the value resulting from the minimization of the nucleon mass in the coherent-pair approach.

The GCM results for the nucleon ground state are presented in Table I and compared with the uncorrelated coherent-pair approximation. The nucleon mass decreased by 57 MeV as a result of the monopole correlations. The other observables are similarly only little affected by the vibrations such that basically the GCM results are similar to the ones from the coherent-pair approximation. One gets a qualitative overall agreement with experiment. Actually the Δ-nucleon splitting is too small, leaving some room for an explicit color-magnetic interaction. There remain the problems relating to the pion-nucleon coupling constant and the Goldberger-Treiman relation.[22] Nevertheless, we believe the numbers are good enough to justify the application of the formalism to the excited vibrational states.

The alternative to this approach would have been to project states of good spin and isospin from a mean-field (generalized hedgehog) state. Although the Goldberger-Treiman relation would have been satisfied, the additional complexity of both a projection and the GCM require-

TABLE I. Comparison of the coherent-pair approximation, the generator coordinate method, and the experimental values. Listed are the nucleon mass, the Δ-nucleon splitting, the quadratic charge radius of the proton, the proton magnetic moment, its ratio to the neutron magnetic moment, and the axial-vector-coupling constant.

	Coherent pair	GCM	Expt.
E_N (MeV)	1027	970	938
E_Δ-E_N (MeV)	201	190	294
$\langle r^2 \rangle_p$ (fm^2)	0.53	0.53	0.68
μ_p (μ_N)	1.97	1.93	2.79
μ_p/μ_n	-1.48	-1.48	-1.46
g_A/g_V	1.44	1.41	1.23

ment did not seem to be justified in these initial investigations.

The ground-state energies and the excitation energies of the first three vibrational modes for both the nucleon and Δ as given in Table II. Considering the model has but one significant parameter (g), which was determined from ground-state considerations alone, and has only pion and σ fields, the fair agreement with observed data should be considered nontrivial. The results show that collective vibrations cannot be ignored when considering the structure of Roper resonances and that such effects change but little the properties of the ground state.

The present value of the Roper excitation energy is 390 MeV compared to the experimental value of 500 MeV. Although there is only fair agreement we have nevertheless extracted the compression modulus for the nucleon

$$K_N = R_0^2 \left[\frac{d^2E}{dR^2} \right]_{R_0} ,$$

where $E(R)$ is the energy surface of the coherent-pair approximation and R_0 the equilibrium quark radius. Using the upper curve of Fig. 1 one obtains, qualitatively,

$$K_N = 4 \text{ GeV} .$$

This figure can be compared with those given by Bhaduri, Dey, and Preston[29] (about 1.3 GeV) from a nonrelativistic model with a linear and modified color-hyperfine interactions. These authors have also shown that in the MIT bag model the compression modulus is equivalent to the mass of the nucleon. The compression modulus from the chiral-soliton model which includes pions indicates a far greater incompressibility than these earlier estimates by a factor of 3–4. It is perhaps worthwhile noting that in nucleon matter, the nucleon compression modulus is in the range of 200–250 MeV. If one considers that the Roper resonance is the lowest mode of an effective harmonic oscillator, the required collective mass[21] M_{coll} would be given by

$$M_{\text{coll}}(R_0) = \left[\frac{d^2E}{dR^2} \right]_{R_0} \Big/ (E_{\text{Roper}} - E_{\text{nucl}})^2 .$$

If we denote by M_N the nucleon mass one gets, with the compression modulus of the CSM,

$$M_{\text{coll}}(R_0) \approx 2M_N .$$

This value should be contrasted with the value used by Brown, Durso, and Johnson.[30] They considered nonrelativistic collective vibrations of a bag model including the perturbative value of the pion contribution to the energy and "arbitrarily set $M_{\text{coll}} = 0.48\, M_N$ as would follow from homologous motion with the ground-state quark wave functions." The present calculations do not support this assumption. Indeed the quark wave functions of Figs. 4 and 5 do not exhibit homologous motion.

This conclusion was also reached in a paper by Durso and Meissner[31] who investigated the role of relativity in the approach of Brown, Durso, and Johnson.[30] In both approaches the potential-energy function is that from the MIT bag model with a term related to the finite size of the pion. Because of this term the functional form of this potential is vastly different from that of the diagonal elements of the GCM energy kernel exhibited in Fig. 1. It therefore appears to be impossible to compare the internal details of these approaches with our calculation. Their resulting estimates of the lowest monopole vibrations are shown in Table III.

Fiebig[32,33] has shown how to quantize the bag without introduction of *ad hoc* parameters by using periodic boundary condition in the collective Lagrangian. He deduces a mass parameter of about 500 MeV. In his approach the ground-state and the Roper (and higher) resonances for both the nucleon and Δ result from different modes of solutions to the equations of motion, with the results given in Table III. Actually the two modes have not been recognized in our GCM calculations.

Two types of calculations have been performed within the framework of the topological Skyrme model. In the first[14–16] quantum modes are determined from a scaling of the static Skyrme solution. In the second[5,17–19] solutions are sought from the Lagrangian expanded to leading order in the semiclassical approximation for the vibrational modes around the mean-field solution. The former approaches[14–16] lead to predicted excitation states with definite energies (as in the present approach) whereas in the latter[5,17–19] no bound-state solutions were found and the Roper resonances were interpreted in terms of the behavior of the calculated phase shifts: all the latter authors agree that phase shifts barely pass

TABLE II. The absolute energies (masses) of the nucleon, the Δ, and their various Roper resonances are given for the present generator coordinate method (GCM) and comparison with experimental data. The GCM calculations are performed using the constrained coherent-pair approximation with parameters $g = 5.37$ and $m_\sigma = 0.7$ GeV. The widths of the experimental P_{11} and P_{33} pion-nucleon resonances are indicated.

	Nucleon		Δ	
(MeV)	GCM	Expt. (Γ)	GCM	Expt. (Γ)
E_0	970	938	1160	1232±120
E_1	1360	1470±200	1560	1690±160
E_2	1800	1710±120	2100	
E_3	2300		2400	

TABLE III. The excitation energy of the nucleon Roper resonance and of the Δ Roper resonance are listed with respect to the nucleon mass and Δ mass, respectively. Here $e^{-2}=8\epsilon^2$ and f_π are the parameters of the standard parametrization of the Skyrme model (see Ref. 1). The type of model is as follows: bag model with nonrelativistic vibrational kinetics (NRB), bag model with relativistic vibrational kinematic (RB), chiral bag with relativistic kinematics (RCB), Skyrme model with scaling (SS), Skyrme model in the semiclassical approximation for phase shifts (SPS), Skyrme model with rotation-vibrational coupling (SSRV), modified Skyrme Lagrangian with sixth-order terms (MSS), dielectric soliton model (DES), and projected nontopological chiral-soliton model (NTCS-P).

Authors	Type	Nucleon Roper (MeV)	Δ Roper (MeV)
Brown, Durso, and Johnson (Ref. 30)	NRB	398	518
Durso and Meissner (Ref. 31)	RB	444	292
Fiebig (Ref. 33)	RCR	328	351
Hajduk and Schwesinger (Ref. 15)			
$e^2=0.00421$, $f_\pi=64.5$ MeV	SS	231	167
$e^2=0.00552$, $f_\pi=9.30$ MeV		285	226
Hayashi and Holzworth (Ref. 14)			
$e=5.45$, $f_\pi=64.5$ MeV	SS	210	
Breit and Nappi (Ref. 17)			
$e=5.45$, $f_\pi=64.5$ MeV	SPS	330	330
Zahed, Meissner, and Kaulfuss (Ref. 18)			
$e=4.75$, $f_\pi=93$ MeV	SPS	270	270
Biedenharn, Dothan, and Tarlini (Ref. 16)			
$e=9.42$, $f_\pi=93$ MeV	SS-RV	388	292
Meissner and Zahed (Ref. 5)			
$\epsilon^2=0.005\,52$, $f_\pi=93$ MeV	SS-RV	390	290
Kaulfuss and Meissner (Ref. 38)			
$\epsilon^2=-0.004\,54$, $f_\pi=93$ MeV	MSS	476	
Broniowski, Cohen, and Banerjee (Ref. 40)	DES		
$M_n=1800$ MeV		340	340
$M_N=1085$ MeV		200	200
Present approach	NTCS-P	390	400
Experiment		502	468

through 90°, using the physical mass of the pion, but relate the energy for which $\delta=90°$ with the observed Roper resonances. Results of the two approaches are given in Table III. Rotation-vibrational coupling has been noted[5] to be very important in the phase-shift calculations and the effects on resonance energies can be gauged by comparing the results of Refs. 17 and 18 with those of Refs. 16 and 5 in Table III. We note that the full effect of rotation-vibrational coupling is automatically taken into account in the present GCM calculation. Together it appears that for the same parameter set in the Skyrme Lagrangian the deduced resonance energies from the phase-shift approach are about 50% greater than those from the scaling approximation and are closer in agreement with our own estimates from the GCM approach.

In a more complete review of phase shifts in the Skyrme model by the SLAC group[34,35] as reviewed by Karliner[36] and the Siegen group,[37] the Roper is not recognized as a resonance (nor the Δ1232 for that matter). The SLAC group surmises that the physical resonance arises perhaps from perturbation of poles and zeros of the S matrix away from the origin which, in the right scenario, could lead to the observed lowest-lying p-wave resonances including the Roper. There also seem to be some contributions from $K=1$ channels. Both the scaling approximation approach and the phase-shift approach have been applied by Kaulfuss and Meissner[38] to a modified Skyrme-Lagrangian by the addition of a quadratic factor of the baryon current. The two approaches are shown to yield equivalent energies if, in the semiclassical approximation, one considers the poles of the S matrix rather than the energy for which the phase shift is equal to the $\pi/2$ as suggested by the work of Walliser and Eckart.[39] The resulting energy for the Roper resonance is also shown in Table III and is the best of all approaches. We note, however, that the parameter e^2 of the conventional stabilizing term has the opposite sign to all other approaches.

There is actually a nontopological calculation of the Roper resonance by Broniowski, Cohen, and Banerjee[40] using a color-dielectric model which shows some solitonic solutions with confinement character. With only a scalar confining field their soliton corresponds to a mixture of nucleon and Δ. The authors evaluate the small-amplitude normal modes around the equilibrium solution and obtain an excitation energy of 340 MeV for the Roper. However, with their parameters set their soliton energy is 1785 MeV and no other properties of the nucleon or Δ were given. If they readjust their parameters in order to obtain an energy of 1085 MeV, the excitation energy of the Roper resonance changes to 200 keV.

If one summarizes the calculations reviewed in Table

III then one can conclude that all relativistic approaches estimate the Roper resonance at too low an excitation energy with a continuum of results ranging from 200 MeV until the experimental value of about 500 MeV. A similar statement holds for the Roper resonance of the Δ isobar. The present calculation gives excitation energy about 100 MeV smaller than the experimental one. We believe the final explanation of the structure of the Roper remains to be determined.

VI. SUMMARY AND CONCLUSIONS

In the present paper we applied the generator coordinate method (GCM) to the description of the Roper resonances of the nucleon and the Δ assuming them to be collective monopole vibration (breathing modes) of the ground states. To this end we constrained the quark wave function in the coherent-pair approximation (CPA) to yield solitons of different radii. These were then superimposed by GCM. The rotation-vibration coupling is fully taken into account in a quantum-mechanical manner since the CPA provides Fock states of the nucleon and Δ which are quantized and carry the proper angular momentum and isospin quantum numbers.

The results are encouraging since the masses of all known Roper resonances were reproduced within 150 MeV while qualitative agreement was simultaneously obtained for the nuclear observables, all with a single free parameter, the coupling constant g. (There is insensitivity to the mass of the σ.) Comparable results are found with calculations of the lowest vibrational energies using the Skyrme model, in particular with those that introduce some degree of rotation-vibration coupling.

The potential-energy surface evaluated in CPA allowed us to extract a nucleon compression modulus of $K_N \simeq 4$ GeV. Assuming harmonic-collective motion we also were able to evaluate the collective mass of about twice the nucleon mass associated with the nucleon monopole vibration.

The present approach has some problems with the polarization of the Dirac sea. For sufficiently large extensions of the soliton the quark eigenvalue approaches $\epsilon = -gf_\pi$ and the negative-energy continuum of the vacuum is strongly modified. Since the present model considered only valence quarks, Fock states having a strongly modified vacuum had to be excluded from the GCM superposition. Future considerations must take into account how the vacuum polarizations are to be treated within the framework of the chiral-soliton model so that more complete calculations can be done. As with all other approaches the present one also ignores the center-of-mass effects which may also be important for the properties of the Roper resonance. Widths of the Roper states should be calculated in order to check the method used, but in our view such a calculation should be left until details of the effects of the Dirac sea and center-of-mass have been formulated.

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