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published in

NIC Symposium 2001, Proceedings,
Horst Rollnik, Dietrich Wolf (Editors),
John von Neumann Institute for Computing, Jülich,
NIC Series, Vol. **9**, ISBN 3-00-009055-X, pp. 37-50, 2002.

Supernova Explosions of Massive Stars

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Supernova explosions belong to the most energetic phenomena in the Universe. When a massive star is disrupted at the end of its life, a spectacular outburst of light is emitted which can reach the brightness of a whole galaxy. Heavy elements, produced by the star during millions of years of quiet nuclear burning, and radioactive nuclei, freshly created during the early moments of the explosion, are swept into the circumstellar space to form the seed of a new generation of stars and planets. As brilliant as it may be, such a cosmic catastrophy is only a weak side effect of an even more violent event: The iron core of the massive star collapses to a neutron star or a black hole. The gravitational binding energy released during this process is carried away by neutrinos, which are abundantly produced in reactions of energetic particles. These neutrinos play a crucial role for the dynamics of the stellar collapse and neutron star formation. Only a fraction of about one per cent of their energy is sufficient to cause the disruption of the star. The principle possibility of this neutrino-driven mechanism has been verified by analytic arguments and numerical experiments. Its viability, however, has not yet been convincingly demonstrated by self-consistent hydrodynamical models with a satisfactory treatment of all aspects of the relevant physics. In the project described here, we intend to perform such simulations by combining multi-dimensional hydrodynamics with an accurate handling of the neutrino transport and a state-of-the-art description of neutrino-matter interactions. These simulations will attempt to answer the fundamental question whether supernova explosions are caused by neutrino energy deposition behind the shock, aided by the effects of convection inside the nascent neutron star and by convective overturn in the neutrino-heating region.

1 Supernovae: The (Astro-)Physicist's Interests

Roughly every second one supernova explodes in any galaxy of the Universe. Something like a 100 million supernovae have enriched the gas of the Milky Way with heavy elements, the oxygen we breathe, the iron in our blood cells, the calcium in our bones and the silicon in the rocks beneath our feet. Their gigantic release of energy and momentum helped shaping the galaxies in the early Universe and affects the formation of new generations of stars. Supernova shocks, plowing through the interstellar medium, are considered to be responsible for the acceleration of high-energy cosmic rays, which continuously bombard the Earth. Moreover, explosions of massive stars are astrophysical sources of neutrinos and can produce gravitational waves which are potentially detectable by the new generation of laser interferometer experiments that are on the verge of starting data taking right now.

Due to their enormous brightness and unexpected appearance as "new stars" on the sky, supernovae have always fascinated human beings and have attracted the particular interest of astronomers. This has not changed until now, but in addition we have realized that a broad variety of physical phenomena are connected with supernova explosions. Extremely interesting physical processes play a role during the collapse and the explosion of a massive star and the associated formation of a neutron star or black hole. Supernova research therefore requires input from many other fields such as atomic physics, nuclear physics, and particle physics. Vice versa, a deeper understanding of supernova explosions

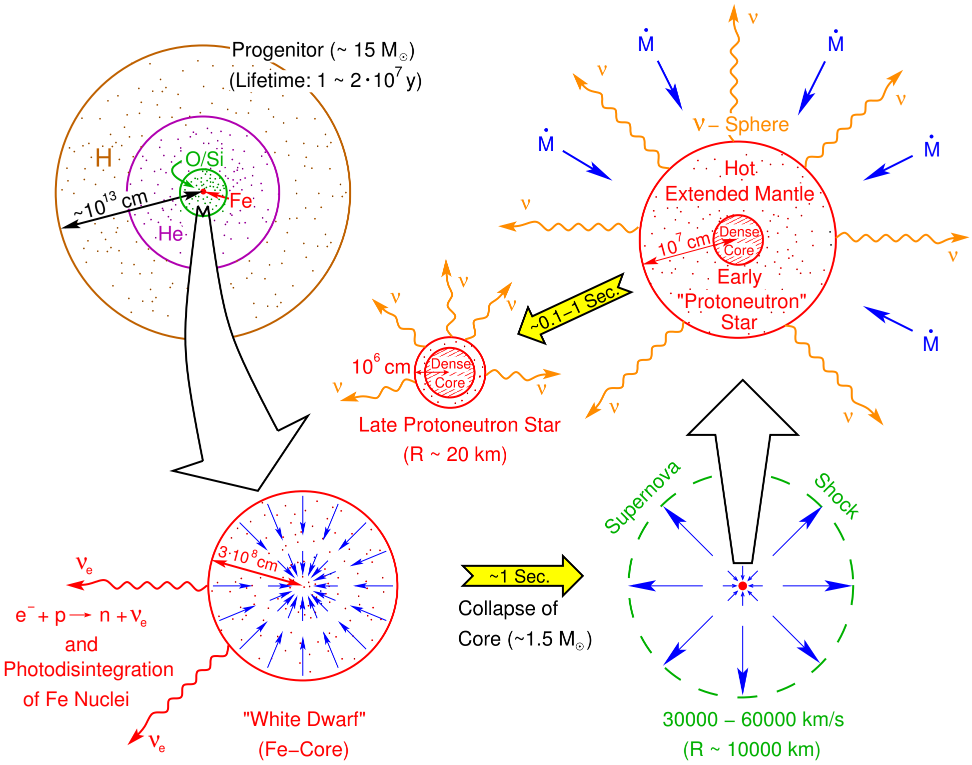

Figure 1. Evolution of a massive star from the onset of iron core collapse to a neutron star. The star has developed a typical onion-shell structure with layers of increasingly heavier elements surrounding the iron core at the center (upper left corner). This iron core (enlarged on the lower left side) collapses to a proto-neutron star within a fraction of a second. This gives rise to a strong shock wave which disrupts the star (lower right). The neutron star is initially very extended (enlarged in the upper right corner), and contracts to a more compact configuration while accreting more matter within the next second of its evolution. This phase as well as the subsequent cooling and neutronization of the remnant is driven by the emission of neutrinos of all flavors. (Figure adapted from Ref. [1].)

has important consequences not only for astrophysics and our knowledge of astronomical objects, but can also have far-reaching consequences for other areas of physics.

Perhaps the most fundamental problem of supernova research is the question about the cause of the explosion. What is the mechanism that accelerates the stellar debris up to a tenth of the speed of light? Besides being of fundamental interest, this question needs to be answered before we will be able to explain how the energy of the explosion varies with the mass of the progenitor star, how the production of radioactive nuclei like ^{56}Ni, ^{57}Ni or ^{44}Ti changes with the progenitor, when the compact remnant is a neutron star and when it is a black hole, and how the explosion can obtain a big anisotropy and the neutron stars the observed large kick velocities. Moreover, the question whether and how r-process elements can be created in supernovae is linked to a better understanding of the explosion mechanism and the early evolution of the newly formed neutron star. This, of course, is also true for reliable predictions of the neutrino signal which hopefully will be measured in very much detail in case of a Galactic supernova in the near future.

The historical detection of about 20 neutrinos from Supernova 1987A in three underground experiments in Japan, U.S.A. and Russia was in overall agreement with expectations from models and confirmed the theoretical perception that neutrinos play a crucial role during stellar core collapse and neutron star formation. Neutrinos dominate these events energetically and carry away the gravitational binding energy of the nascent neutron star. About one per cent of their energy, however, is already sufficient to account for the kinetic energy of a supernova (which again is 100 times larger than the energy emitted in electromagnetic radiation). Therefore neutrinos have been proposed to be responsible for the explosion of the star. Physical arguments as well as computer simulations support this idea[2], but explosions by the "neutrino-driven mechanism" have so far been obtained only in simulations with substantial and sometimes questionable approximations of the treatment of the input physics. Because of the extreme complexity of the problem, no finally convincing hydrodynamical simulations have been performed yet, although the idea of neutrino-driven explosions has been around for more than 30 years now, and despite of significant progress in supernova modelling has been achieved. A standard model for the explosion of massive stars therefore does not exist.

2 This Project: A Rigorous Approach

In the work described here, we attempt to perform the first numerical simulations which combine all ingredients that have been recognized as relevant for the explosion. The crucial neutrino physics, neutrino transport as well as neutrino-matter interactions, will be treated by the most reliable and accurate method applied to this problem so far. Convective mixing inside the neutron star and hydrodynamic instabilities in the supernova ejecta will be taken into account by performing the simulations in more than one spatial dimension. According to simplified models, these processes can decide between explosion or failure by boosting the neutrino emission of the neutron star on the one hand, and by increasing the efficiency of the energy transfer from neutrinos to the stellar matter on the other. Therefore multidimensional models are necessary to give an answer to the question whether supernova explosions can be obtained by the neutrino-driven mechanism.

Of course, a rigorously accurate and sophisticated approach has its price. Such simulations are very time consuming and require the use of top-end supercomputing facilities like those provided by the John von Neumann Institute for Computing (NIC).

The rest of this article is structured as follows: In Section 3, an overview of the evolution from stars to neutron stars will be given. Section 4 contains a more detailed discussion of the role of neutrinos in supernovae and their importance for the explosion mechanism. In Section 5, hydrodynamic instabilities will be described as an important phenomenon which seems to be essential for understanding supernova explosions and their observable properties. Section 6 will outline the objectives and the scientific progress that will be achieved by the computational project presented here.

3 From Massive Stars to Neutron Stars

After several 10 million years of quiet hydrostatic evolution, stars with initial masses of more than about 8 solar masses end their lives by supernova explosions. These stars have developed an "onion-shell" structure in a sequence of nuclear burning phases by which increasingly heavier elements were created. The heavier nuclei are contained in shells closer to the center of the star, while the lightest elements like helium and hydrogen form the outer layers of the stellar mantle and envelope (Fig. 1, upper left corner).

The core of the evolved star contains iron and iron-group elements, which are the most tightly bound atomic nuclei. No further energy release by nuclear fusion in the center of the star is possible at this phase of the evolution. Therefore the core cannot escape gravitational collapse once it has reached a mass near the critical Chandrasekhar limit.

During collapse the stellar matter experiences a dramatic compression, and protons begin to capture electrons to be converted to neutrons (Fig. 1, lower left corner). The collapse does not stop before the density of nuclear matter is reached at the center. This happens only a fraction of a second after the onset of the gravitational instability. At this moment repulsive nuclear forces stiffen the equation of state, and the core resists further compression. A strong shock wave is launched and starts propagating outward. Ultimately, it will disrupt the star in the supernova explosion and will eject most of the stellar gas into the circumstellar space (Fig. 1, lower right corner).

The newly formed neutron star which is left behind at the center is more massive than our Sun but will ultimately have a radius of only 10 km. A gigantic amount of gravitational energy was stored in its interior as thermal and degeneracy energy of neutrons, protons, and electrons during the collapse from the initial iron core with a radius of several 1000 km to the final, much more compact configuration. This energy is now released in the form of neutrinos within a period of several seconds (Fig. 1, upper right corner and center).

Ongoing electron captures on protons produce about 10^{57} electron neutrinos, which diffuse out of the star and thus drive its slowly progressing neutronization. In addition, the nascent neutron star is a very hot object, and roughly 10^{58} neutrinos and antineutrinos are created as pairs of all flavors in approximately equal numbers. The emission of these "thermal" neutrinos cools the star on its way to the final, degenerate remnant.

A small fraction of only one per cent of the energy that is carried away by the neutrinos is very well sufficient to power a typical supernova event. But how can this energy reservoir be tapped efficiently enough? How can the energy for the explosion be transferred from the dense neutron star to the more dilute stellar gas around it? Despite of more than 30 years of intense theoretical research, and despite of significant progress in our understanding of the physics going on in dying stars, a finally convincing answer to this fundamental question has not been found yet.

4 Neutrinos and the Explosion

In fact, neutrinos play a crucial role during all phases of stellar collapse, supernova explosion, and neutron star formation. Six characteristic stages can be discriminated during the evolution from the pre-collapse star to the neutrino-transparent, cool neutron star. Figure 2 visualizes these phases, showing the dynamical state, the corresponding nuclear composition of the stellar medium, and the basic character of the neutrino emission.

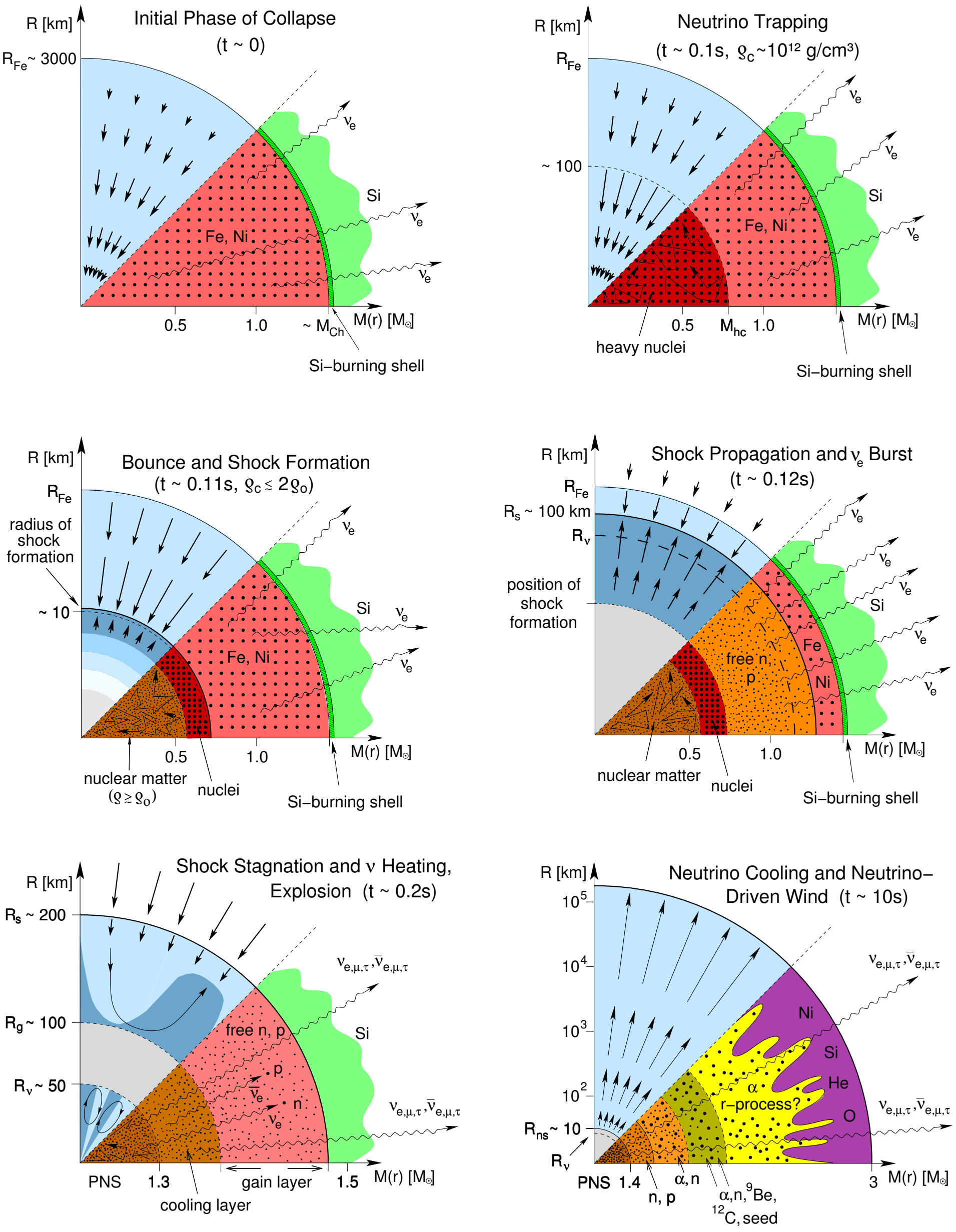

Figure 2. Schematic representation of the processes that occur in a collapsing stellar iron core on the way to the supernova explosion. The diagrams (from top left to bottom right) visualize the physical conditions at the onset of core collapse, neutrino trapping, shock formation, propagation of the prompt shock, shock stagnation and revival by neutrino heating, and r-process nucleosynthesis in the neutrino-driven wind of the newly formed neutron star, respectively, as suggested by current computer simulations. In the upper parts of the figures the dynamical state is shown, with arrows indicating the flow of the stellar fluid. The lower parts of the figures contain information about the nuclear composition of the stellar plasma and the role of neutrinos during the different phases.

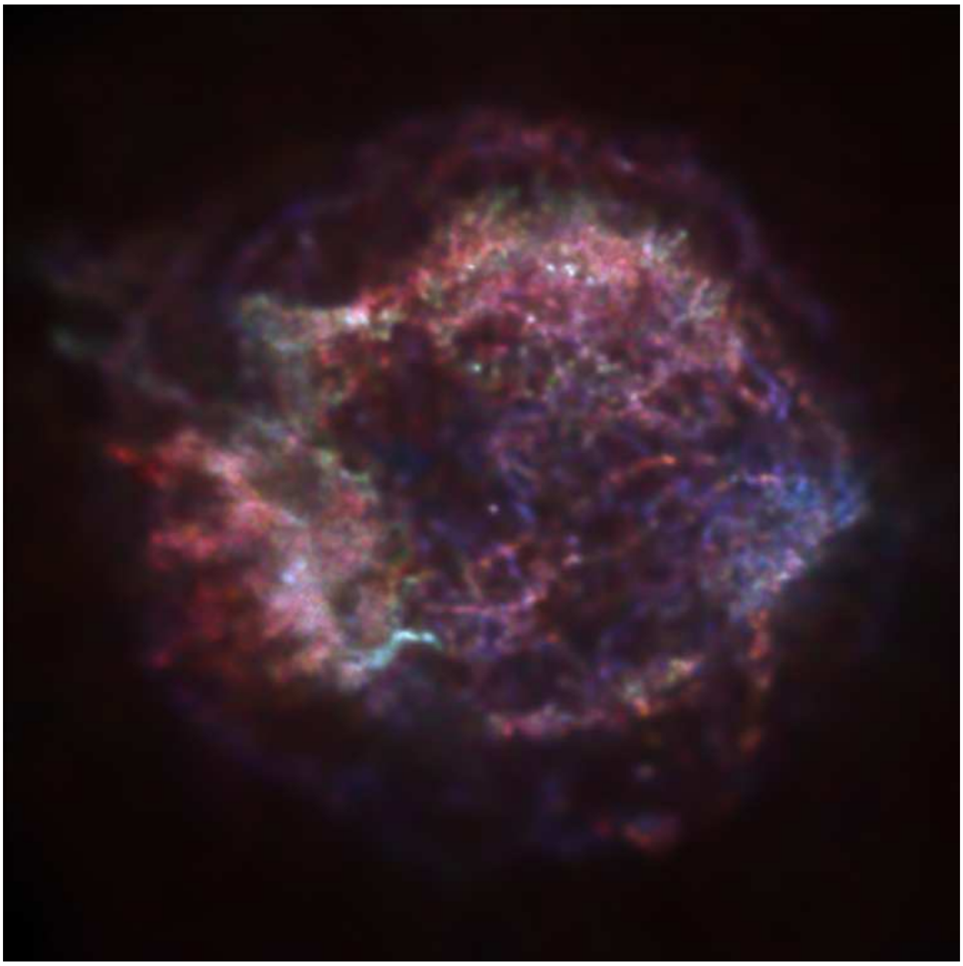

Figure 3. X-ray image of Cassiopeia A, the diffuse remnant of a supernova which exploded around the year 1680. The picture was taken by the *Chandra* satellite. Spectral information from three different energy bands was combined to show bright silicon-, sulfur- and calcium-rich filaments (green, white and blue) and fainter iron-rich features (red). Iron appears to have been flung farthest, although it was created deepest inside the exploding star. The compact remnant, probably a neutron star, can be identified as a faint point source near the geometrical center of the expanding, gaseous stellar debris. (Figure taken from Ref. [3].)

During core collapse electron neutrinos are produced by electron captures on protons and nuclei. Initially, these neutrinos escape essentially unhindered. When the central density increases to roughly a hundredth of the density of nuclear matter, however, neutrino scatterings off nuclei become so frequent that neutrino "trapping" sets in. From this moment on neutrinos are carried along with the infalling stellar plasma.

The production and escape of neutrinos reduces the electron fraction and the pressure in the collapsing stellar core. This accelerates the infall and decreases the size of the subsonic "inner core". The outer edge of the latter marks the position where a shock front forms shortly after nuclear matter density has been reached and the core bounces, resisting further compression. The sound waves that are created at this moment cannot travel into the supersonically falling outer layers but steepen to a hydrodynamical shock wave near the boundary of the inner core. This shock begins to propagate outward in mass and in radius.

Doing so, it suffers from severe energy losses because iron-group nuclei are disintegrated to free nucleons in the hot medium behind the shock. Free protons immediately capture electrons and produce electron neutrinos in large numbers. When the shock reaches a density so low that these neutrinos can stream faster than the shock is able to move, a very luminous burst of electron neutrinos is radiated. This moment occurs only a few milliseconds after shock formation and is called the "shock-breakout through the neutrinosphere". The additional neutrino losses are disastrous for the shock and lead to its stagnation before it has reached the outer edge of the iron core. All current simulations agree in the fact that the bounce shock is unable to directly cause the explosion of the star.

While during this very early post-bounce phase neutrinos drain energy from the layers behind the shock, the situation changes only fractions of a second later. As the shock is slowly pushed to larger radii by the hot stellar matter that is accreted through the shock and piles up on the forming neutron star, the density and temperature behind the shock begin to drop. At the same time, the flow of neutrinos and antineutrinos that diffuse out from deeper layers and the mean neutrino energies increase. This favors the formation of a "gain layer" behind the shock where energetic neutrinos start heating the stellar gas. This happens mainly by the absorption of electron neutrinos and antineutrinos on neutrons and protons, respectively. If the corresponding energy deposition is strong enough, it can revive the stalled supernova shock and can provide the energy for the explosion of the star. Since the timescale for this to happen is a tenth of a second or more, which is much longer than the propagation of the prompt hydrodynamic shock, neutrino-driven explosions were named "delayed" explosions.

The neutron star that is formed at the center of the explosion neutronizes and cools by the emission of neutrinos over a period of several seconds. Neutrinos can diffuse out of the dense and hot inner regions of the compact remnant only slowly. Also at late times, they still heat the surface-near layers of the star and thus cause a continuous outflow of baryonic matter, the so-called "neutrino-driven wind". This wind can become a very neutron-rich environment and can therefore provide favorable conditions for the production of heavy elements such as gold, lead or uranium through rapid neutron-capture reactions by seed nuclei. Whether such an r-process can occur or not is currently a matter of vivid debate. It requires a suitable combination of a number of properties which characterize the neutrino-driven wind, e.g., its neutron excess, entropy and expansion velocity. These parameters depend on the neutron star mass and radius and are mainly determined by the interaction of the outflowing matter with the intense flux of neutrinos from the forming neutron star.

Because of the central role of neutrinos for the supernova problem, their transport and interactions with matter have to be described with particular care and the highest possible accuracy. This important aspect is therefore treated with high priority in the described project.

Figure 4. X-ray image of the Vela nebula obtained with the X-ray satellite *Rosat*. Vela is the remnant of a supernova that occurred in a distance of about 1500 lightyears at least 10.000 years ago. At the edge of the remnant fragments have overtaken the supernova shock and have formed Mach cones due to their supersonic motion. These hot clumps can be traced back to a common origin near the center of the gaseous remnant, which indicates their possible formation already during the supernova explosion. (Figure provided by B. Aschenbach, cf. Ref [4].)

5 Convection and Mixing in Supernovae

Supernova 1987A was a true milestone of supernova research. Exploding in the Large Magellanic Cloud, a small satellite galaxy of the Milky Way, and thus in our immediate cosmic neighbourhood, this supernova offered us the unique possibility of a relatively close view of the events that accompany the death of a massive star. Due to its proximity and the advantages of modern observational technology, Supernova 1987A provided us with an unprecedented wealth of data from nearly the first moment of the explosion to even several years later.

Spectral information could be obtained in very much detail and provided us with new insights into the dynamical processes during the explosion of a star. In particular, Supernova 1987A showed an unexpectedly early emission of X-rays and gamma-rays, at a time when the expanding star was still very opaque and radiation could only escape from its hydrogen envelope. This meant that radioactive nuclei must have been transported from the site of their creation near the newly formed neutron star into the hydrogen envelope of the exploding star. Indeed, Doppler features of iron lines indicated that iron was distributed over a large range of velocities. Some of the iron was expanding with a speed of up to 4000 km/s, much faster than predicted by spherically symmetric models where the heavier elements are found deeper inside the star and move with smaller velocities. The observations could only mean that the onion-shell structure of the progenitor star had been destroyed during the explosion.

More recently, advanced observational instruments, especially high-resolution X-ray telescopes and gamma-ray detectors, are used to study the morphology and chemical composition of the gaseous remnants of supernovae which exploded hundreds or thousands of years ago. Data obtained by the X-ray satellite *Chandra* in three different spectral bands show a spatially inhomogeneous and anisotropic distribution of nucleosynthesis products in the supernova remnant Cas A[3] (Figure 3). Iron-rich filaments seem to be located at the outer edge of the remnant, although iron was created deepest inside the supernova. X-ray images of the Vela supernova remnant reveal fast-moving fragments of hot gas that have overtaken the shock front and form Mach cones by their supersonic propagation through the surrounding medium[4] (Fig. 4). Reconstruction of the directions of their motion points to a common origin near the center of the remnant, consistent with the assumption that these clumps might have been formed during the stellar explosion. Such findings are in conflict with expectations from spherically symmetric models. They are therefore interpreted as strong indications that large-scale mixing processes and macroscopic anisotropies seem to be a generic feature of exploding stars.

On grounds of theoretical considerations convection was long thought to be of potential importance in stellar explosions[5]. However, the large radial extent of the mixing as inferred from properties of the lightcurve and the spectral lines for the first time in case of Supernova 1987A and meanwhile found in a number of other supernovae, too, was unexpected. Radioactive nickel is produced in the deep interior of the supernova, in the immediate vicinity of the nascent neutron star. This nickel is accelerated to very high velocities during the first second of the explosion. It seems to be able to retain much of its initial velocity, thus penetrating even into the helium and hydrogen layers of the star. This requires that multi-dimensional phenomena are present already during the very early stages of the explosion.

Computer simulations in two and three dimensions have indeed shown that the region of neutrino energy deposition behind the shock is convectively unstable[6–9]. Rising mushrooms transport heated gas closer to the shock while downflows of cold matter replace the hot gas near the gain radius, where the heating is strongest[6–9] (Fig. 5). This increases the efficiency of the neutrino-driven mechanism and helps pushing the shock farther out. Convective processes were discovered to be present also inside the nascent neutron star and speed up its deleptonization and cooling[10] (Fig. 5). The corresponding boost of the neutrino luminosity also strengthens the neutrino heating and thus supports the shock revival.

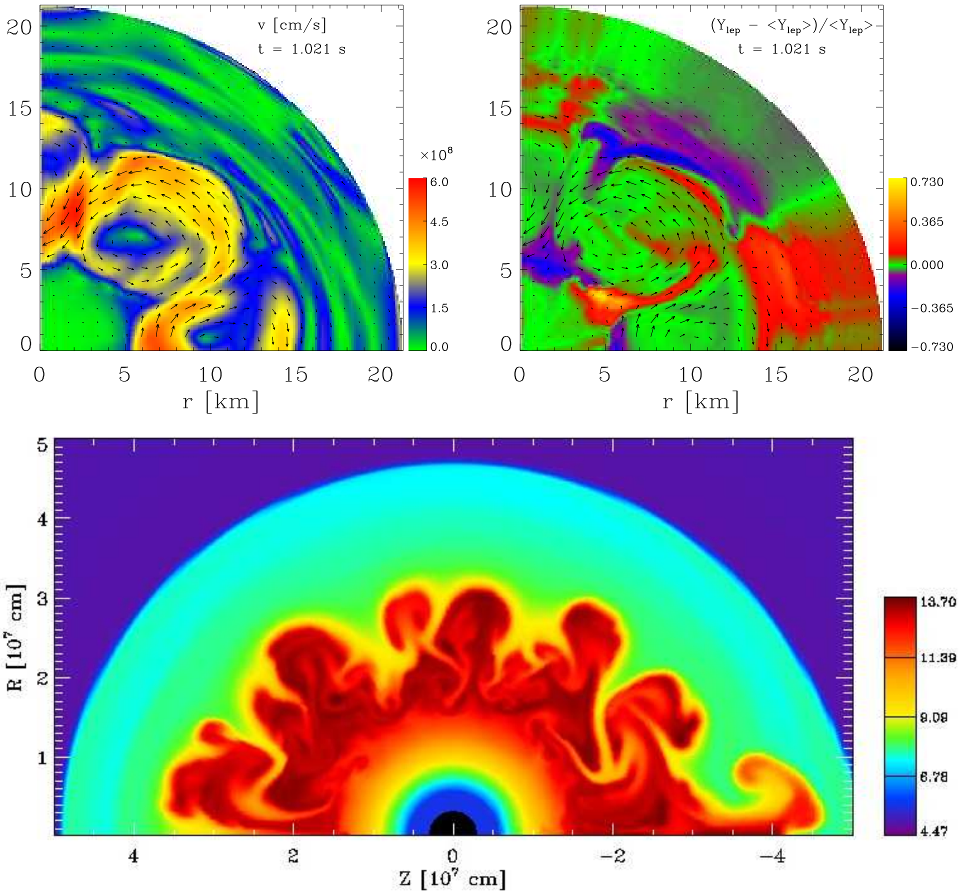

Figure 5. Convection inside the neutron star (top) and in the neutrino-heating region behind the supernova shock. The pictures show the velocity field (top left) and the fluctuations of the lepton-to-baryon ratio (top right) in a two-dimensional (axially symmetric), hydrodynamic simulation one second after neutron star formation. Neutron-rich gas sinks in while proton-rich matter rises due to buoyancy forces. The convective motions reach velocities of several 1000 km/s. In the lower figure, the entropy distribution outside the neutron star is displayed at 0.1 seconds after shock formation. The shock has reached a radius of about 500 km at this time. Neutrino-heated plasma appears in deep red, the neutron star is visible as a black circle at the center.

When the shock finally moves out through the onion-shell structure of the progenitor star, hydrodynamic instabilities begin to grow after the shock passage at the boundaries between layers of different chemical composition. The anisotropies that were created in the neutrino-heating region during the first second after bounce act as seed perturbations for these later instabilities. As a consequence of the large seed amplitudes, heavy elements are carried outward with high velocities and helium is mixed inward very efficiently (Fig. 6). The convective processes around the neutron star during the shock-revival phase can therefore naturally explain the inhomogeneities and the mixing observed in stellar explosions.

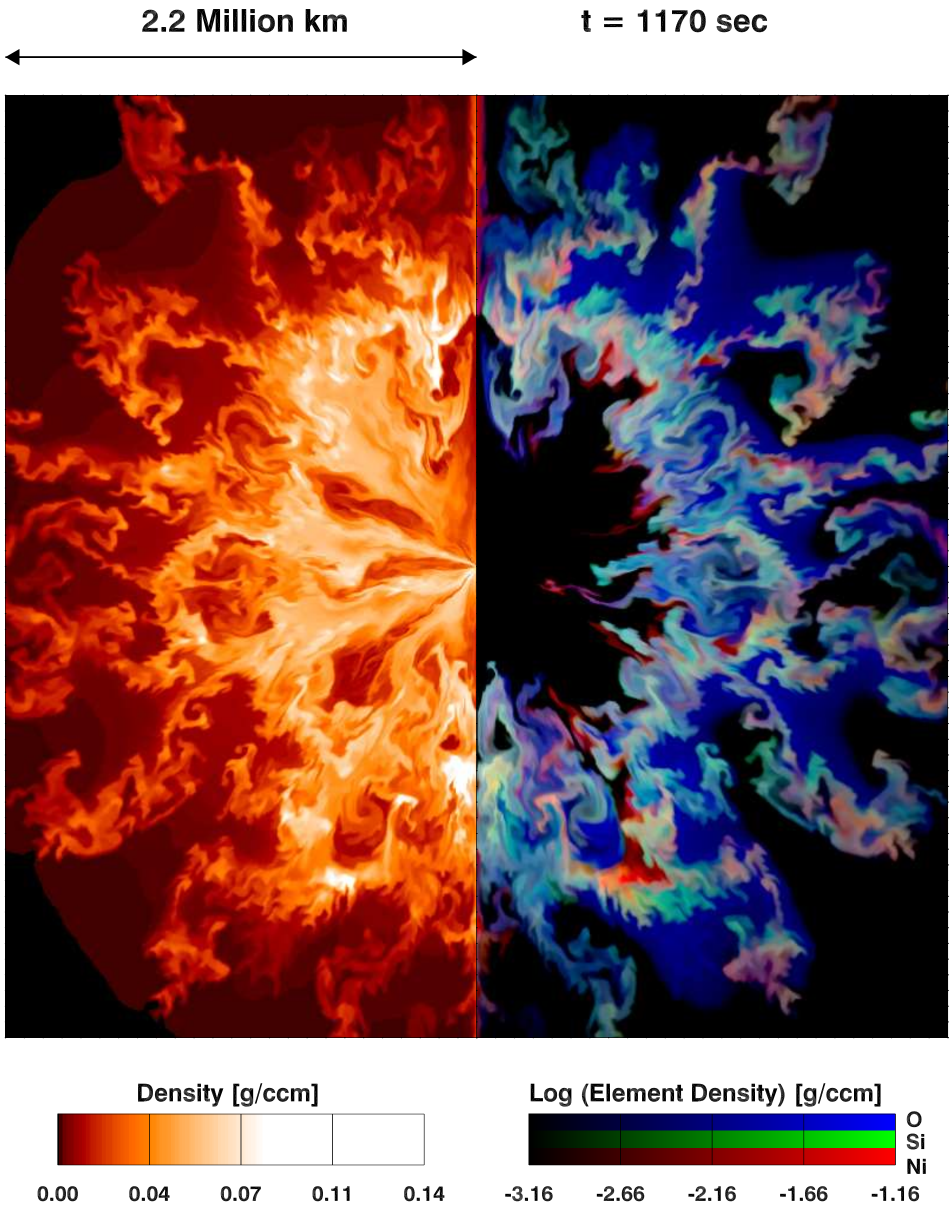

Figure 6. Snapshot of a two-dimensional supernova simulation about 20 minutes after the formation of the supernova shock in a 15 solar-mass star. The shock has already left the displayed volume, which has a radius of more than two million kilometers, and is now propagating through the hydrogen layer. On the left side, the density distribution is color coded with white marking the highest densities. On the right side, the partial densities of radioactive nickel (red and purple), silicon (green) and oxygen (blue) are largest in the dense filaments and clumps that expand with very high velocities into the more dilute gas of the helium shell.

The nickel velocities found in the calculations are in good agreement with measurements for some supernovae[11].

6 Towards a Standard Model of Supernova Explosions

Spherically symmetric models with a state-of-the-art description of the input physics do not yield explosions. This result was confirmed by simulations with the most advanced description of the neutrino-matter interactions applied so far, where the neutrino transport was handled by solving the Boltzmann equation. Simulations of this kind have become possible and the corresponding codes have been developed only recently[12–14]. This new generation of supernova models has reached an unprecedented level of sophistication in the treatment of neutrino effects. For the first time, the numerical inaccuracies are now smaller than the uncertainties associated with the approximations of the microphysics.

On the other hand, multi-dimensional simulations demonstrate the potentially crucial role of convective processes for the success of the neutrino-heating mechanism. Models which do not show explosions in spherical symmetry have been found to explode when convection was taken into account[8]. The neutrino physics in such comparative studies and in all multi-dimensional simulations performed so far[6, 9, 15–17], however, was described by making serious simplifications and partly by using questionable approximations. Although these simulations are enlightening, they do not yield an answer to the question whether supernova explosions can be explained by the neutrino-heating mechanism, when the microphysics in the supernova core is described according to our best current knowledge.

The current project attempts to take a major step towards a solution of this important question. In a new computer code we have combined our highly accurate Boltzmann neutrino transport method with a multi-dimensional treatment of the hydrodynamics. Also for the first time, the neutrino-matter interactions include a detailed description of the reaction kinematics and the correlation effects of nucleons in the dense neutron star medium. In all previous simulations these effects have been ignored, thereby accepting errors of the order of several ten per cent around the neutrinosphere up to factors of a few at several times nuclear matter density. First calculations with the new implementation of the neutrino-nucleon interactions show significant and interesting differences.

The multi-dimensional neutrino-hydrodynamics problem has been coined in a form such that it can be handled on current supercomputers with an acceptable amount of processor time, although the requirements are appreciable. The numerical algorithms employed for solving the set of complex equations allow for an optimal performance on shared memory vector-parallel computers like the Cray T90 of the NIC.

In fact, a shared memory architecture offers significant advantages for dealing with the considered problem. On the one hand, it was possible to generalize the chosen methods easily from spherical symmetry to more than one dimension, with rather limited requirements of manpower for the necessary adaptations to a multi-processor shared-memory environment. On the other hand, the neutrino transport, which is done with an implicit time integration, implies solving big matrix problems. Breaking up the latter into pieces which do not need intense communication and at the same time guarantee a high computational efficiency, is very hard to achieve. In contrast, algorithms are known and have already been developed, which exhibit a nearly ideal scaling behavior on shared-memory platforms.

Acknowledgments

It is a great pleasure for us to acknowledge the input from K. Takahashi and C. Horowitz for upgrading our treatment of the neutrino-nucleon interactions. We are very grateful for conversations with J. Beacom, A. Burrows, M. Liebendörfer, A. Mezzacappa, G. Raffelt, S. Reddy, J. Pons, and T. Thompson. We also thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for support during a visit of the Summer Program on Neutron Stars. HTJ thanks Patrick Inderst for preparing the graphics of Figs. 1 and 2. This work was also supported by the Sonderforschungsbereich 375 on "Astroparticle Physics" of the Deutsche Forschungsgemeinschaft. The majority of the computations with the new Boltzmann solver for the neutrino transport was only possible because of a grant of computer time on the Cray T90 of the NIC in Jülich.

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