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Elementary Particle Physics: Lattice Gauge Theories - Broader and Deeper

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The approximation of a finite space-time volume by a regular hypercubic lattice enables quantum field theories in imaginary time to be simulated on a computer as if they were described by equilibrium statistical mechanics. In principle, provided the resulting lattice action is real so that the Boltzmann weight is positive, Monte Carlo methods are very effective at computing expectation values of products of fields. Then, with sufficient computer power, a quantum field theory may be simulated for any choice of its input parameters (masses and couplings for the elementary fields) and the lattice approximation can be controlled systematically by adjusting the lattice spacing while holding the volume fixed and large enough. This approach has given particle physicists a tool for understanding quantum field theories from first principles, without having to rely on perturbation theory, and a laboratory for exploring different parameter values from those in the real world.

The introduction of the lattice breaks some of the symmetries of the continuum quantum field theory. Therefore, it is crucial to demonstrate that the correct symmetries are recovered in the continuum limit, which is reached in practice by extrapolating results from simulations at several lattice spacings. In the language of critical phenomena, this limit is achieved at a point where the lattice theory has a continuous phase transition and physical length scales diverge in units of the lattice spacing. The lattice theory should lie in the same universality class as the continuum theory. Happily, Lorentz symmetry is an accidental symmetry of the lattice theory, i.e., it is only broken by irrelevant operators, and so is automatically recovered in the continuum limit. Ken Wilson's breakthrough in 1974[1] was to discover how local gauge symmetries could be maintained exactly on a lattice (at each of the sites), thereby, opening up the possibility that the confinement of quarks in Quantum Chomodynamics (QCD), which is believed to be due to non-abelian gauge invariance, might be studied and even understood using lattice QCD. Combined with Monte Carlo methods introduced by Michael Creutz in 1980[2], this launched a world-wide programme of QCD simulations.

One symmetry proved problematic – the chiral symmetry of massless quarks, which is spontaneously broken resulting in massless Goldstone pions. In the real world, u and d quarks have small masses which generate correspondingly small pion masses described by chiral perturbation theory. Early lattice formulations of QCD could not avoid explicitly breaking chiral symmetry while preserving the correct flavour content. Now we understand how to realise the full chiral and flavour symmetries at non-zero lattice spacing, but at a higher computational cost, which arises from the fact that chiral symmetry is naturally realised non-perturbatively in one extra space dimension. Having all the internal symmetries of continuum QCD means that operator mixing and renormalisation are no more complicated than in continuum perturbation theory and this has extended the range of quantities that can be computed reliably. In addition to this theoretical progress, numerical algorithm improvements have delivered at least as great a speed-up as Moore's Law over the past three decades.

Consequently, lattice field theory is currently undergoing both a broadening of its range of application beyond QCD and a deepening of its penetration into QCD. The twisted mass formulation of lattice QCD[3] is a compromise between preserving continuum symmetries and speed that is enabling simulations to go deeper into the regime where they can be matched to chiral perturbation theory. It is then possible to extract low energy constants of this effective theory from which a range of physical quantities may be computed. Algorithmic improvements are enabling simulations of SU(3) Yang-Mills theory with one fermion flavour[4], which has a non-positive definite Boltzmann weight and a very different realisation of chiral symmetry from QCD. These are beginning to test conjectures about the relationship to supersymmetric Yang-Mills theories in the limit of a large number of colours. The overall advance in simulation capability means that lattice QCD is now providing detailed information about the internal structure of the nucleon[5]. Finally, the full power of the chirally-symmetric formulation gives us hope that the 30 year old puzzle of the $\Delta I = 1/2$ Rule can be solved[6]. These results show that lattice gauge theories have become an indispensable tool for a detailed understanding the Standard Model and for insight into the physics that may lie beyond.

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