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Universal asymptotics of poloidal spectra of magnetic perturbations of saddle coils in tokamaks

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Universal and generic features of poloidal spectra of external magnetic perturbations created by a set of saddle coils in tokamak plasmas are studied in a vacuum approximation. It is found that the poloidal mode spectra with high-accuracy are described by a linear combination of three universal asymptotical formulas which depend only on the safety factor of the equilibrium plasma and the geometry of perturbation coils. The validity of these formulas is confirmed by numerical calculations of the mode spectra in the DIII-D plasma [G. L. Jackson *et al.*, Europhys. Conf. Abstr. **27A**, P-4.47 (2007)] and a spherical tokamak, National Spherical Tokamak Experiment [D. A. Gates, *et al.*, Nucl. Fusion **49**, 104016 (2009)], plasmas perturbed by external control coils. © *2011 American Institute of Physics*. [doi:10.1063/1.3551756]

Many modern fusion devices, tokamaks are equipped with external coils to control magnetohydrodynamic instabilities in plasmas. For example, internal (I-) coils in the DIII-D tokamak are used to stabilize the resistive wall mode (RWM) and to suppress the large edge-localized modes (ELMs) in high-confinement (H-mode) regimes (see Refs. 1–4). To the similar purpose serves the error field coils (EFC) installed in the spherical tokamak, National Spherical Tokamak Experiment (NSTX).[5] Control coils are also planned to be installed in the ASDEX Upgrade tokamak and International Thermonuclear Experimental Reactor to control the RWM and to mitigate the ELMs (see Refs. 6 and 7).

The external coils are typically installed inside or outside of a plasma vacuum vessel as a set of toroidally distributed saddle coils as shown in Fig. 1. The resonant magnetic perturbations (RMPs) created by these coils are nonaxisymmetric along the poloidal and toroidal directions. The effect of the magnetic perturbations is mainly determined by their toroidal and poloidal spectra given by the (m,n)-components of a Fourier expansion of the RMPs in straight field line coordinates, ϑ and φ, i.e., poloidal and toroidal angles. Calculations of the spectra of RMPs are the important first step in studies of the effect of magnetic perturbations on various properties of plasmas in tokamaks (see Refs. 4, 6, and 8).

Although such calculations can be easily done for specific plasma and external coil configurations, from the theoretical and practical points of view, it is interesting to find generic features of the RMP's spectra for different plasmas and external coils. With the existence of such generic features one can expect from the generic behavior of magnetic field near the separatrix, for instance, generic asymptotics of the safety factor.

In the present work we shall show that the poloidal mode spectra of perturbation magnetic field generated by a set of saddle coils can be presented by a linear combination of three universal asymptotical formulas. They are determined only by the safety factor of plasmas and the geometry of the coils. The nature of this universality is related to the generic behavior of magnetic field lines due to the toroidicity of system.

To describe a magnetic field we will use the magnetic coordinates: the poloidal ψ_p and the toroidal ψ_t magnetic fluxes and the (intrinsic) poloidal and toroidal angles (ϑ,φ). In this coordinate system the vector potential $\mathbf{A}$ and the magnetic field $\mathbf{B}$ can be presented in the Clebsch form, $\mathbf{A}=\nabla g+\psi_t\nabla\vartheta-\psi_p\nabla\varphi$, $\mathbf{B}=\nabla\psi_t\times\nabla\vartheta-\nabla\psi_p\times\nabla\varphi$, where g is a scalar function.

The poloidal flux of the equilibrium field is given by $\psi_p=\psi_p^{(0)}(\psi_t)$, $g=0$, and the magnetic field lines lie on the magnetic surfaces $\psi_p=$const. The poloidal angle ϑ is a linear function of the toroidal angle φ: $\vartheta=(\varphi-\varphi_0)/q(\psi_p)+\vartheta_0$, where $q(\psi_p)$ is the safety factor defined as a ratio of the increment along the toroidal angle φ in one turn along poloidal angle ϑ.

Let $A_R^{(1)}$, $A_Z^{(1)}$, and $A_\varphi^{(1)}$ be the components of the vector potential of external magnetic field. We will choose such a gauge function g that the toroidal flux ψ_t and the poloidal angle ϑ are unchanged and the full poloidal flux ψ_p is determined by the sum of the unperturbed one $\psi_p^{(0)}$ and the perturbation part $\psi_p^{(\mathrm{per})}$, $\psi_p=\psi_p^{(0)}+\psi_p^{(\mathrm{per})}$. The latter is given by $\psi_p^{(\mathrm{per})}=-RA_\varphi^{(1)}+\partial g/\partial\varphi$, where the function g can be reduced to the integral taken on magnetic surface $\psi_t=$constant along poloidal angle ϑ:

$$g(R,Z,\varphi)=\int G(R(\vartheta,\psi_t),Z(\vartheta,\psi_t),\varphi)d\vartheta,$$
$$G(R,Z,\varphi)=A_R^{(1)}dR/d\vartheta+A_Z^{(1)}dZ/d\vartheta. \quad (1)$$

For calculations of poloidal spectra of magnetic perturbations we use the analytical equilibrium proposed in Ref. 9. This extended analytical solution of the Grad–Shafranov equation based on Solov'ev profiles describes several types of fusion devices, including standard tokamaks and spherical tokamaks. We shall consider the two type of plasmas with the large and small aspect ratios, namely, the DIII-D like plasma perturbed by the I-coils, and the NSTX-like plasma

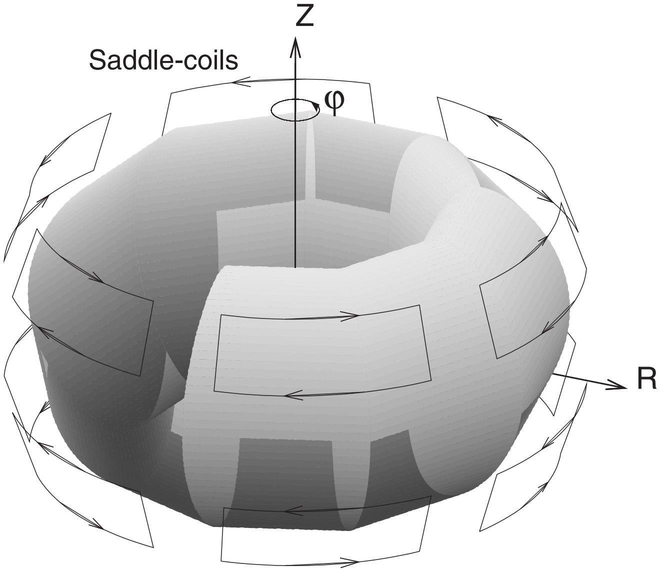

FIG. 1. Schematic view of the plasma configuration and saddle coils, the cylindrical coordinate system (R,Z,φ).

with the EFC. The configurations of these plasmas, the saddle coils, and their specific parameters are shown in Fig. 2 and listed in Table I.

Suppose that the set consists of N pairs of saddle coils whose the upper and lower horizontal segments lie on the circular loops of radii R_1 and R_2 located at the horizontal planes $Z=Z_1$ and $Z=Z_2$, respectively [Figs. 2(a) and 2(b)]. The values of (R_j,Z_j), $(j=1,2)$ for the DIII-D and the NSTX tokamaks are given in the caption of Fig. 2. Coil currents alternate between $\pm I_0$ for all coils of a set. Using the Biot–Savart law and some lengthy calculations one can show that the perturbation poloidal flux $\psi_p^{(j)}=-RA_\varphi^{(j)}(R,Z,\varphi)$ generated by the horizontal segments of the set of saddle coils can be presented by the Fourier series,

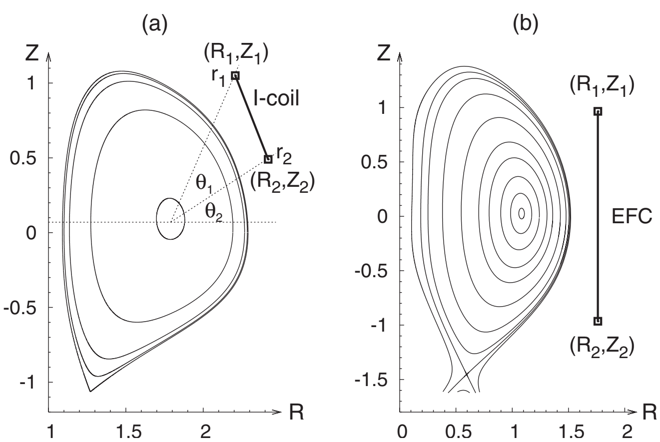

FIG. 2. Plasma configurations and RMP coils: (a) DIII-D-like plasma and I-coils; (b) NSTX-like plasma and EFC coils; The I-coil positions in the DIII-D tokamak are taken at $R_1=2.207$ m, $Z_1=1.045$ m, $R_2=2.42$ m, and $Z_2=0.49$ m. The EFC coil positions in the NSTX are taken at $R_1=R_2=1.761$ m, $Z_1=-Z_2=0.965$ m. (r_1,θ_1) and (r_2,θ_2) are the positions of the upper and lower segments of saddle coils in the quasitoroidal coordinate system (r,θ,φ).

TABLE I. Parameters of the DIII-D like and the NSTX-like plasmas.

Plasma parameters	DIII-D	NSTX
Plasma current I_p	1.55 MA	0.8 MA
Toroidal field B_t	1.85 T	0.45 T
Major/Minor radii R_0/a	1.69/0.6	0.85/0.663
Plasma β_{pol}	1.2	0.05
Elongation κ	1.8	2
Triangularity δ	0.36	0.35
The value of q_{95}	3.714	12.2

$$\psi_p^{(j)}(R,Z,\varphi)=\sum_{s=0}^{\infty}\psi_{n_s}^{(j)}(R,Z)\sin(n_s\varphi), \quad (2)$$

where

$$\psi_n^{(j)}(R,Z)=i_s\frac{\mu_o I_j R R_j}{\pi D_j}\int_0^{\pi/2}\frac{\cos(2n\phi)\cos(2\phi)d\phi}{\sqrt{1-k_j^2\sin^2\phi}},$$

$$k_j^2=4RR_j/D_j^2,\quad D_j=\sqrt{(R+R_j)^2+(Z-Z_j)^2}, \quad (3)$$

$$i_s=\frac{4}{\pi}\frac{\cos(n_s\varphi_d)}{2s+1},\quad n_s=(2s+1)N.$$

Here $I_j=(-1)^j I_0$, φ_d is the half angular distance between neighboring saddle coils along the toroidal distance. The toroidal component $\psi_n^{(j)}(R,Z)$ can be expanded in a Fourier series in ϑ,

$$\psi_n^{(j)}(R,Z)=\sum_{m=0}^{\infty}|H_{mn}^{(j)}(\psi)|\cos(m\vartheta+\chi_{mn}^{(j)}), \quad (4)$$

with the Fourier coefficients, $H_{mn}^{(j)}(\psi)=|H_{mn}^{(j)}(\psi)|e^{i\chi_{mn}^{(j)}}$. Furthermore, we will use the notations ψ and $\psi_N=\psi/\psi_S$ to label a magnetic surface ψ_t=const (or ψ_p=const) and the normalized flux, respectively, where ψ_S is the flux at the separatrix. Similarly, one can obtain the perturbation poloidal flux $\psi^{(3)}$ corresponding to the segments of saddle coils lying on the vertical plane (R,Z),

$$\psi_p^{(3)}=\sum_{s=0}^{\infty}\psi_{n_s}^{(3)}(\psi)\sin(n_s\varphi),$$

$$\psi_n^{(3)}=\sum_m|H_{mn}^{(3)}(\psi)|\cos(m\vartheta+\chi_{mn}^{(3)}), \quad (5)$$

$$H_{mn}^{(3)}(\psi)=-\frac{n}{m}G_{mn}(\psi),\quad \chi_{mn}^{(3)}=\phi_{mn}+\frac{\pi}{2},$$

where $G_{mn}(\psi)$ are the coefficients in the Fourier expansion of the function $G(R,Z,\varphi)$ in Eq. (1),

$$G(R,Z,\varphi)=\sum_{s=0}^{\infty}G_{n_s}(R,Z)\cos(n_s\varphi),$$

$$G_n(R,Z)=\sum_m|G_{mn}(\psi)|\cos(m\vartheta+\phi_{mn}). \quad (6)$$

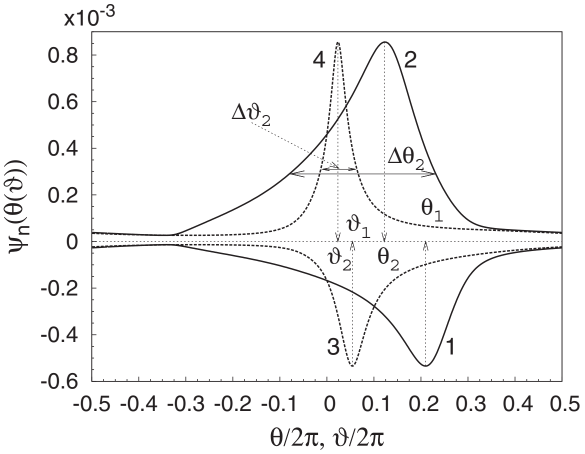

FIG. 3. Angular dependencies of the perturbation poloidal flux $\psi_n^{(j)}(R,Z)$ (3) generated by the horizontal segments of the set of saddle coils at the ψ_{95} magnetic surface for the DIII-D like plasma with I-coils: curves 1 and 2 correspond to $\psi_n^{(j)}(R,Z)$ vs θ, curves 3 and 4 correspond to $\psi_n^{(j)}(R,Z)$ vs ϑ. The plasma parameters and RMP coils positions are given in Fig. 2 and in Table I. Curves 1 and 3 correspond to the upper horizontal segment, curves 2 and 4 correspond to the lower horizontal segment.

From Eqs. (2), (3), and (5) it follows that the N pairs of saddle coils generate the magnetic perturbations with the primary toroidal mode $n=N$ and their harmonics $n_s=(2s+1)N$, $(s=1,2,\ldots)$. On the given magnetic surface ψ=const, each of the toroidal harmonics $\psi_n(R,Z)$ and $G_n(R,Z)$ are localized functions of angles θ and ϑ. For the case $n=3$ they are shown in Fig. 3 in the DIII-D like plasma with the $N=3$ pairs of saddle coils. The current amplitude I_0 is taken equal to $I_0=1$ kA. The quantities $\Delta\theta_j$ and $\Delta\vartheta_j$ stand for the width of peaks at the e-folding heights.

As seen from Fig. 3 the functions $\psi_n(R,Z)$ corresponding to the horizontal segments of saddle coils attain extremal values at θ_j (or ϑ_j) $(j=1,2)$ [see Fig. 2(a)]. The function $G_n(R,Z)$ corresponding to vertical segments of a set has a similar behavior with an extremal point at θ_3 (or ϑ_3). The widths $\Delta\vartheta_j$ of the perturbation functions $\psi_n(R,Z)$ and $G_n(R,Z)$ in ϑ strongly depend on the magnetic surfaces ψ. They can be approximated as $\Delta\vartheta_j \approx \Delta\theta_j d\vartheta/d\theta \sim \Delta\theta_j/(q(\psi)B_\theta)$, where B_θ is the value of the poloidal component of the equilibrium magnetic field $B_\theta(\psi,\theta)$ at $\theta=\theta_j$. One can see that $\Delta\vartheta_j$ is inversely proportional to the safety factor $q(\psi)$, and it decreases when one approaches the separatrix $\psi_N \to 1$.

One expects that the perturbation magnetic field of external coils decreases with the toroidal mode number n. Particularly, numerical calculations show that the function $\psi_n(R,Z)$ decays exponentially with n, $\max|\psi n(R,Z)| \sim \exp(-\alpha n)$ for $n>1$ with the constant α. The values of $\max|G_n(R,Z)|$ also decrease with n but much weaker than for the function $\psi_n(R,Z)$.

Using the arguments based on numerous numerical and analytical calculations (see Refs. 10–12 for details) one can establish possible asymptotical forms of poloidal spectra of magnetic perturbations for a given toroidal mode n. They are asymptotically presented by the following universal formulas

$$H_{mn}^{(j)}(\psi) = \frac{A_j(\psi)}{q(\psi)}\exp\left(-\frac{mC_j(\psi)}{q(\psi)}\right)e^{i\chi_{mn}^{(j)}}, \quad j=1,2,$$
$$H_{mn}^{(3)}(\psi) = \frac{nA_3(\psi,m)}{m}\exp\left(-\frac{mC_3(\psi)}{q(\psi)}\right)e^{i\chi_{mn}^{(3)}}, \tag{7}$$

where A_j, C_j, and χ_{mn}^j $(j=1,2,3)$ are functions of the magnetic flux ψ, with finite values at the separatrix $\psi_N=1$, $A_j(1)\neq 0$, $C_j(1)\neq 0$. The phases $\chi_{mn}^{(j)}(\psi)$ corresponding to the both horizontal and vertical segments are linear functions of the poloidal mode number m, $\chi_{mn}^{(j)}=\chi_n^{(j)}-m\vartheta_j$, where ϑ_j coincide with extremal points of the perturbation functions ψ_n, G_n. The numerical calculations show that the formulas in Eq. (7) indeed well describe the poloidal mode spectra.

The typical dependencies of the magnitudes $|H_{mn}(\psi)|$ on the poloidal mode number m are shown in Fig. 4 at the q_{95} magnetic surface for the toroidal mode $n=3$. As seen from Figs. 4(a) and 4(b), the dependencies of $|H_{mn}^j(\psi)|$ $(j=1,2)$ versus m corresponding to the horizontal segments of saddle coils are very closely described by the exponential law $\exp(-mc)$, c=const (curves 1 and 2), while the dependence $|H_{mn}^{(3)}(\psi)|$ on m corresponding to the vertical segments of saddle coils is close to the law $m^{-1}\exp(-mc)$. One can also notice that these dependencies are more pronounced for the NSTX-like plasma. It was also confirmed that the phases

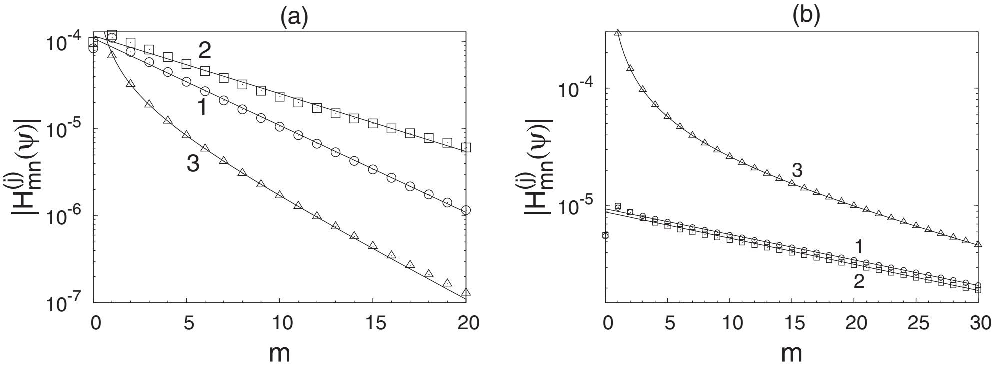

FIG. 4. Magnitudes of Fourier components $|H_{mn}(\psi)|$ vs the poloidal mode number m on the magnetic surface $\psi_{q_{95}}$: (a) the DIII-D like plasma; (b) the NSTX-like plasma. Symbols ○, □, and △ correspond to the upper horizontal, lower horizontal, and vertical segments of a set of coils, respectively. Solid curves 1, 2, and 3 describe to the corresponding asymptotical formulas (7) with the fitted parameters C_1, C_2, C_3, A_1, A_2, and a_0, a_1 in $A_3=a_0+ma_1$. The plasma and the RMP-coils parameters are the same as in Fig. 2 and the toroidal mode $n=3$.

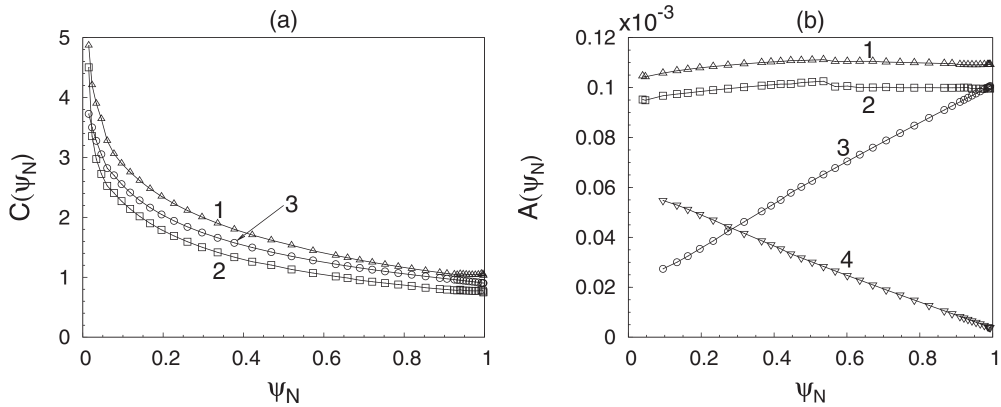

FIG. 5. (a) Radial profiles of fitting coefficients $C_j(\psi_N)$ in Eq. (7) for the DIII-D like plasma with the upper I-coils [see Fig. 2(b)]. Curve 1 corresponds to the upper horizontal segments, curve 2 corresponds to the lower horizontal segments, curves 3 correspond to the vertical segments of saddle coils. (b) Fitting coefficients $A_j(\psi_N)$ $(j=1,2,3)$ for the NSTX-like plasma with the EFC coils. Curve 1 corresponds A_1, curve 2 corresponds to A_2, curves 3 and 4 describes a_0 and a_1 in $A_3=a_0+ma_1$, respectively.

$\chi_{mn}^{(j)}(\psi)$, $(j=1,2,3)$ are linear functions of the poloidal mode number m as mentioned above.

In order to verify these dependencies quantitatively, we have fitted the numerically calculated values of $|H_{mn}^{(j)}(\psi)|$ $(j=1,2,3)$ with the asymptotical formulas given by Eq. (7). The calculations show that the asymptotical formulas well approximate the numerical values for $m>1$. The accuracy of the asymptotical formulas improves with increasing of the toroidal mode number n. Particularly, we have studied the dependence of the numerically calculated coefficients $C_j(\psi)$ on the poloidal mode m by fitting them with the function $C_{num}(\psi)=C_j(\psi)(1+mc_1)$. The coefficient c_1 has an order of $10^{-3}-10^{-4}$ for the DIII-D like plasma, and $10^{-3}-10^{-5}$ for the NSTX-like plasma, respectively. The $c_1(\psi)$ also decreases with approaching the separatrix, i.e., at $\psi_N\to 1$, as well as with increasing the toroidal mode number n.

Numerical calculations show that the functions $C_j(\psi)$ $(j=1,2,3)$ in the exponents of Eq. (7) monotonically grow inward from the finite, nonzero values at the separatrix, $C_j \to C_j(1)\neq 0$ at $\psi_N\to 1$ as shown in Fig. 5. The values of $C_j(1)$ do not depend on the perturbation current I_0, and are determined by the geometry of plasma shape and coils. Particularly, they depend on the angular positions of the horizontal segments of saddle coils, θ_j, and the corresponding widths, $\Delta\theta_j$, of the perturbation fields. The functions $C_j(\psi)$ grow when one approaches the magnetic axis $\psi_N=0$ according to the logarithmic law, $C_j\approx -\alpha q(\psi)\log\psi_N$. It corresponds to the following asymptotics of $|H_{mn}^{(j)}(\psi)|\sim\psi_N^{m\alpha}$. The numerical estimations show that $\alpha\approx 1/2$. The dependence of C_j $(j=1,2,3)$ of the toroidal mode n is a weak.

The pre-exponential functions A_j $(j=1,2)$ corresponding to the horizontal segments of the saddle coils, weakly changes along the radial coordinate ψ in the whole plasma region, while A_3 corresponding to their vertical segments can be presented by $A_3=a_0+ma_1$ with the coefficients a_0, a_1 depending on ψ. Particularly, at the limit $\psi_N\to 1$, i.e., near the separatrix, one has $a_0\to a_0(1)\neq 0$, $a_1\to a_1(1)=0$. One should also note that the dependence of A_j $(j=1,2,3)$ on the toroidal mode n is close to the exponential law $A_j\sim\exp(-Bn)$ for $n>1$, where B is a nonzero constant, $B\neq 0$.

It was also found that the angles ϑ_j and $\chi_n^{(j)}$ depend on ψ. Particularly, $\chi_n^{(j)}(\psi)$ and the product $\vartheta_j(\psi)q(\psi)$ tend to finite values at the limit $\psi_N\to 1$.

In conclusion we have established that in a vacuum approximation the magnetic perturbations in tokamak plasmas created by a typical set of saddle coils have a generic asymptotical behavior. It was shown that the poloidal spectra of perturbation magnetic field can be presented as a linear combination of three fields corresponding to the two horizontal segments and the vertical segments of a set of saddle coils. The poloidal spectra of each of these three magnetic fields can well be described by the universal asymptotical formulas, determined by the safety factor of the plasma and the geometry of coils. They are confirmed by numerical calculations of poloidal spectra for the two type of plasmas with the separatrix: the DIII-D like plasma with I-coils, and the spherical tokamak, NSTX-like plasma with the EFC coils. The accuracy of asymptotical formulas increases with increasing the toroidal mode number.

The universal asymptotical presentation of the RMPs spectra may be useful for the theoretical studies of the plasma response to external magnetic perturbations. They can be also employed in a technical design of saddle coils in modern tokamaks to obtain a desired spectrum of external magnetic perturbations.

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