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Key Points:
- Velocity-strengthening behavior might be a generic feature of friction
- Experimental data support the existence of velocity-strengthening friction
- Velocity-strengthening friction may be important for frictional phenomena

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On the velocity-strengthening behavior of dry friction

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Abstract The onset of frictional instabilities, e.g., earthquakes nucleation, is intimately related to velocity-weakening friction, in which the frictional resistance of interfaces decreases with increasing slip velocity. While this frictional response has been studied extensively, less attention has been given to steady state velocity-strengthening friction, in spite of its potential importance for various aspects of frictional phenomena such as the propagation speed of interfacial rupture fronts and the amount of stored energy released by them. In this note we suggest that a crossover from steady state velocity-weakening friction at small slip velocities to steady state velocity-strengthening friction at higher velocities might be a generic feature of dry friction. We further argue that while thermally activated rheology naturally gives rise to logarithmic steady state velocity-strengthening friction, a crossover to stronger-than-logarithmic strengthening might take place at higher slip velocities, possibly accompanied by a change in the dominant dissipation mechanism. We sketch a few physical mechanisms that may account for the crossover to stronger-than-logarithmic steady state velocity strengthening and compile a rather extensive set of experimental data available in the literature, lending support to these ideas.

1. Introduction

Understanding the constitutive behavior of dry frictional interfaces has far-reaching implications for a broad range of phenomena and scientific disciplines [*Armstrong-Hélouvry et al.*, 1994; *Scholz*, 1998, 2002; *Marone*, 1998a; *Olsson et al.*, 1998; *Persson*, 2000; *Urbakh et al.*, 2004; *Baumberger and Caroli*, 2006; *Ben-Zion*, 2008; *Kawamura et al.*, 2012; *Vanossi et al.*, 2013; *Ohnaka*, 2013]. It is well established that the onset of frictional instabilities, which might lead to interfacial failure (e.g., earthquakes), is intimately related to weakening effects, i.e., the reduction of frictional resistance with increasing slip displacement or slip velocity [*Rice and Ruina*, 1983]. In particular, when the slip velocity v is regarded as a basic frictional control variable, the variation of the steady state frictional resistance with v is of great importance. Naturally, steady state velocity-weakening (denoted hereafter as SVW) friction has been studied extensively. On the other hand, less attention has been given to steady state velocity-strengthening (denoted hereafter as SVS) friction, in which the steady state frictional resistance increases with increasing v. The existence of SVS might affect, for example, the propagation speed of rupture fronts, their propagation distance, and possibly the amount of stored energy released by them. Our goal in this note is to discuss SVS friction, its functional form and possible physical origins, and to point out direct evidence for its existence based on experimental data available in the literature.

The interface between two macroscopic bodies in dry frictional contact is typically composed of an ensemble of contact asperities whose total area A_r is orders of magnitude smaller than the nominal contact area A_n. The real contact area typically depends on the time elapsed since a contact was formed, i.e., on the contact's "age" (or "maturity") typically quantified by a state variable of time dimension ϕ [*Dieterich*, 1978; *Ruina*, 1983; *Dieterich and Kilgore*, 1994; *Rice et al.*, 2001; *Baumberger and Caroli*, 2006], an idea that dates back at least to *Rabinowicz* [1951]. The frictional stress (resistance) τ is proportional to $A_r(\phi)$ [*Bowden and Tabor*, 1964]. The proportionality factor depends on the slip velocity v and possibly on a set of internal state variables which we schematically denote by θ and can be interpreted as the shear strength $\sigma_s(\theta, v)$ (related to the plastic flow of contact asperities) [*Bowden and Tabor*, 1964; *Baumberger and Caroli*, 2006]. This contribution to the frictional resistance is rheological in nature. Putting the two together, one can write the frictional stress (resistance) as [*Bowden and Tabor*, 1964; *Baumberger and Caroli*, 2006]

$$\tau(\phi, \theta, v) = \frac{A_r(\phi)\,\sigma_s(\theta, v)}{A_n}\,. \tag{1}$$

During steady state sliding at a velocity v, the internal state variables attain unique values $\phi(v)$ and $\theta(v)$. Therefore, under steady state conditions the frictional stress $\tau^{ss}(v)$ takes the form

$$\tau^{ss}(v) = \frac{A_r[\phi(v)]\,\sigma_s[\theta(v), v]}{A_n} . \tag{2}$$

In this note we focus on the variation of $\tau^{ss}(v)$ with v, and in particular on the sign of $\partial_v \tau^{ss}$, its dependence on v, and its functional form. $\partial_v \tau^{ss} < 0$, i.e., SVW friction, is known to facilitate unstable accelerating slip and frictional instabilities [*Ruina*, 1983; *Scholz*, 1998]. On the other hand, $\partial_v \tau^{ss} > 0$, i.e., SVS friction, might promote stable slip, limit the propagation speed of interfacial rupture fronts and affect the magnitude of slip events [*Weeks*, 1993; *Kato*, 2003; *Shibazaki and Iio*, 2003; *Bouchbinder et al.*, 2011; *Bar Sinai et al.*, 2012; *Hawthorne and Rubin*, 2013; *Bar-Sinai et al.*, 2013], limit the seismogenic zone [*Marone and Scholz*, 1988], and affect earthquake afterslip and stress drops [*Marone et al.*, 1991]. In what follows we suggest that SVS friction, $\partial_v \tau^{ss} > 0$, generically emerges in dry friction over some range of slip velocities and discuss its possible physical origins and the available experimental evidence for its existence.

2. Real Contact Area Aging and Its Saturation During Sliding

Equation (2) suggests that the steady state frictional stress $\tau^{ss}(v)$ is a product of a steady state real contact area contribution $A_r^{ss}(v) \equiv A_r[\phi(v)]$ and a rheological contribution $\sigma_s^{ss}(v) \equiv \sigma_s[\theta(v), v]$. As the latter is assumed to be an increasing function of v, $\partial_v \sigma_s^{ss} > 0$ (to be discussed later), we focus first on $A_r^{ss}(v)$.

The physical argument we present has already appeared in the work of Baumberger, Caroli, and coworkers [*Baumberger and Berthoud*, 1999; *Bureau et al.*, 2002; *Baumberger and Caroli*, 2006], in a slightly different form in *Putelat et al.* [2011] and in *Bar Sinai et al.* [2012]. It is repeated here briefly for completeness. The starting point is the nonsteady behavior of $A_r(\phi)$ in the absence of slip, $v = 0$. In this case, ϕ grows linearly with the time t elapsed since the frictional interface was formed (when the bodies under consideration were brought into frictional contact) or since previous slip halted, $\phi = t$.

It is well established that under these conditions the real contact area undergoes logarithmic aging, i.e., $A_r(t)$ increases logarithmically with t, a behavior observed in many materials [*Dieterich*, 1972; *Ruina*, 1983; *Dieterich and Kilgore*, 1994; *Marone*, 1998a; *Berthoud et al.*, 1999; *Nakatani*, 2001; *Nakatani and Scholz*, 2006; *Baumberger and Caroli*, 2006; *Ben-David et al.*, 2010]. Specifically, the time evolution of $A_r(t)$ (normalized here by A_n) takes the following form

$$\frac{A_r(t)}{A_n} = \frac{\sigma}{\sigma_H}[1 + b\log(t/\phi^*)] , \tag{3}$$

where b and ϕ^* are positive constants [*Ruina*, 1983], σ is the normal stress, and σ_H is the material's hardness. This expression, however, cannot be valid for arbitrarily short times as it becomes singular for $t \to 0$, which is of course unphysical. This simply means that $A_r(t)$ actually takes the form

$$\frac{A_r(t)}{A_n} = \frac{\sigma}{\sigma_H}[1 + b\log(1 + t/\phi^*)] , \tag{4}$$

where ϕ^* can be interpreted as a typical cutoff timescale for the onset of logarithmic aging. Equation (3) provides a good approximation for equation (4) when $t \gg \phi^*$, but completely fails in the opposite limit, $t \ll \phi^*$. The last equation, and specifically the short-time cutoff, has been proposed by *Dieterich* [1978], has been verified experimentally [*Marone*, 1998b; *Nakatani and Scholz*, 2006; *Ben-David et al.*, 2010], and has been studied theoretically [*Estrin and Bréchet*, 1996; *Nakatani and Scholz*, 2006; *Putelat et al.*, 2011].

While this might appear as a somewhat academic discussion of a short time regularization of the logarithmic aging formula, and indeed it is almost always overlooked, this is not the case. To see the relevance of this short time regularization for our purposes here, we should consider steady sliding at a velocity v. Since ϕ quantifies the age of the real contact, it must be a decreasing function of v (the "lifetime" of a contact asperity is shorter the higher the slip velocity, i.e., "rejuvenation"). It is well established that under steady state conditions [*Dieterich*, 1978; *Baumberger and Berthoud*, 1999; *Baumberger and Caroli*, 2006]

$$\phi = D/v , \tag{5}$$

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where D is a typical slip distance (usually related to the contact asperities size) [*Marone*, 1998a; *Baumberger and Caroli*, 2006]. Therefore, under steady state sliding conditions the real contact area takes the form

$$\frac{A_r^{ss}(v)}{A_n} = \frac{\sigma}{\sigma_H}\left[1 + b\log\left(1 + \frac{D}{v\phi^*}\right)\right] . \tag{6}$$

This implies that $A_r^{ss}(v)$ decreases logarithmically with increasing v for $v \lesssim D/\phi^*$ [*Teufel and Logan*, 1978] and that it approaches a constant (saturates) for $v \gg D/\phi^*$. Therefore, if indeed the rheological contribution to the steady state frictional resistance increases with v, $\partial_v \sigma_s^{ss} > 0$, we conclude that *irrespective* of the precise form of $\sigma_s^{ss}(v)$ we expect $\partial_v \tau^{ss} > 0$ for $v \gg D/\phi^*$. This is an important observation.

3. Logarithmic Steady State Velocity-Strengthening Friction

The last section concluded with the observation that the steady state frictional stress (resistance) $\tau^{ss}(v)$ is expected to become velocity strengthening above a certain slip velocity $\sim D/\phi^*$ due to the saturation of the real contact area, assuming $\partial_v \sigma_s^{ss} > 0$. Our goal in this section, and the subsequent one, is to discuss the latter.

The standard approach to the velocity dependence of the rheological part of the frictional stress (shear strength) is to attribute it to thermal activation [*Baumberger and Berthoud*, 1999; *Rice et al.*, 2001; *Baumberger and Caroli*, 2006; *Putelat et al.*, 2011]. We briefly repeat the argument here as it sets the stage for what will follow. The starting point is to treat the real contact of area $A_r(\phi)$ as fixed, to neglect any rheological internal variables θ, and to assume that v is a result of a stress-biased thermally activated process such that

$$v = v_0\left(\exp\left[-\frac{\Delta(\tau)}{k_B T}\right] - \exp\left[-\frac{\Delta(-\tau)}{k_B T}\right]\right) . \tag{7}$$

Here v_0 is a reference velocity scale related to a basic attempt rate and an intrinsic length scale, $\Delta(\tau)$ is a stress-biased activation barrier, k_B is Boltzman's constant, and T is the temperature. The second exponential appears in order to account for backward transitions (implying a proper $\tau \to -\tau$ symmetry and consistency with the second law of thermodynamics).

The stress-biased activation barrier is assumed to take the form

$$\Delta(\tau) = E_0 - \Omega\,\tau^{loc}(\tau) , \tag{8}$$

where E_0 is the bare energy barrier, Ω is the activation volume (typically much larger than atomic volumes, i.e., corresponding to a collective multiatom process [*Rice et al.*, 2001; *Baumberger and Caroli*, 2006]) and $\tau^{loc}(\tau) = A_n\tau/A_r$ is the local stress at the asperity level. E_0 is the energy barrier in equilibrium, where forward and backward thermally activated transitions are equally likely and $v = 0$. The application of a stress τ^{loc} favors transitions in its direction over transitions in the opposite direction, giving rise to $v \neq 0$. Note that the local asperity stress τ^{loc} is significantly enhanced compared to the macroscopic stress τ by a large factor $A_n/A_r \gg 1$. Therefore, we can rewrite equation (7) as

$$v = 2\,v_0 \exp\left[-\frac{E_0}{k_B T}\right]\sinh\left(\frac{A_n\Omega\,\tau}{A_r k_B T}\right) , \tag{9}$$

which can be inverted in favor of the stress to read (putting back the ϕ dependence of A_r)

$$\tau(\phi, v) = \frac{k_B T A_r(\phi)}{\Omega\, A_n}\sinh^{-1}\left(\frac{v}{2\,v_0}\exp\left[\frac{E_0}{k_B T}\right]\right) . \tag{10}$$

Finally, since E_0 is typically much larger than $k_B T$, we can treat the argument of the inverse sinh function as large for all v values of interest and approximate $\sinh^{-1}(x) \simeq \log(2x)$, yielding

$$\tau(\phi, v) = \frac{A_r(\phi)}{A_n}\left[\frac{E_0}{\Omega} + \frac{k_B T}{\Omega}\log\left(\frac{v}{v_0}\right)\right] . \tag{11}$$

The frictional stress in equation (11) takes the form assumed in equation (1) and $\sigma_s(v)$ can be readily identified. Equation (11) predicts that the instantaneous response (i.e., faster than the typical evolution time of ϕ) of the frictional stress to slip velocity "jumps" would be logarithmic in the ratio between the final and the initial v's. This logarithmic "direct effect" [*Marone*, 1998a; *Baumberger and Caroli*, 2006] has been observed for many materials in a wide range of slip velocities v, but typically not larger than a few hundreds of μm/s.

What are the implications of equation (11) for the steady state frictional stress $\tau^{ss}(v)$? To answer this question one should substitute $A_r^{ss}(v)$ of equation (6) for $A_r(\phi)$ in equation (11) to obtain

$$\frac{\tau^{ss}(v)}{\sigma} \equiv \mu^{ss}(v) = f_0 + \alpha \log\left(\frac{v}{v_0}\right) + \beta \log\left(1 + \frac{D}{v\phi^*}\right) + \frac{\alpha\beta}{f_0} \log\left(\frac{v}{v_0}\right) \log\left(1 + \frac{D}{v\phi^*}\right) , \tag{12}$$

where

$$\alpha \equiv \frac{k_B T}{\sigma_H \Omega}, \quad \beta \equiv \frac{E_0 b}{\sigma_H \Omega}, \quad f_0 \equiv \frac{E_0}{\sigma_H \Omega} = \frac{\beta}{b} . \tag{13}$$

As typically the energy-scale $\sigma_H\Omega$ is much larger than k_BT, we expect $\alpha \ll 1$, which is confirmed by numerous experiments [for example, *Kilgore et al.*, 1993; *Heslot et al.*, 1994; *Nakatani*, 2001]. The aging coefficient b is typically much smaller than unity (for many materials it is of the order of 10^{-2} [*Baumberger and Caroli*, 2006]), and we expect $E_0 \lesssim \sigma_H\Omega$. This implies that f_0, which sets the overall magnitude of the friction coefficient, is roughly of order unity (as is widely observed) and that $\beta \ll 1$. These estimates suggest that the last term in equation (12), which is proportional to $\alpha\beta/f_0$, is small compared to the other v-dependent terms in this equation and hence will be neglected hereafter (unless otherwise stated).

Consider then relatively small slip velocities that satisfy $v \ll D/\phi^*$. In this case the v dependence of $\mu^{ss}(v)$ is logarithmic and we have

$$\frac{\partial \mu^{ss}(v)}{\partial \log v} = \alpha - \beta = \frac{k_B T}{\sigma_H \Omega}\left(1 - \frac{E_0 b}{k_B T}\right) . \tag{14}$$

The sign of the last expression, which is controlled by the relative magnitudes of b and k_BT/E_0 (or alternatively of α and β), determines whether friction is velocity weakening or velocity strengthening in this range of slip velocities. In particular, for $b > k_BT/E_0$ friction is velocity weakening and for $b < k_BT/E_0$ it is velocity strengthening. In the former case, steady state friction is velocity weakening for $v \ll D/\phi^*$ and then it crosses over to velocity-strengthening behavior for $v \gg D/\phi^*$, when $A_r(\phi)$ approaches a constant. In the latter case, friction is logarithmically velocity strengthening for slip velocities both below and above D/ϕ^*, but with different prefactors in each regime. Indeed, in some systems such as clay-rich fault gouge layers, velocity-strengthening friction is the rule rather than the exception [*Noda and Shimamoto*, 2009; *Ikari et al.*, 2009, 2013]. The most important implication then of equations (6) and (11), for our purposes here, is that they predict that steady state friction is logarithmically velocity strengthening for $v \gg D/\phi^*$. This is often overlooked in parts of the literature, but see *Bureau et al.* [2002] and *Baumberger and Caroli* [2006].

In Figures 1a and 1b we present examples from the available literature in which logarithmic SVW friction crosses over to logarithmic SVS behavior at some slip velocity v_m (where the curve reaches a minimum). The data in Figure 1c also exhibit logarithmic SVS.

4. Stronger-Than-Logarithmic Steady State Friction

As discussed above, a simple thermal activation model predicts the existence of logarithmic SVS friction above a slip velocity $\sim D/\phi^*$. This model assumes a single-activation barrier and a single-attempt rate; many of the materials of interest, however, are disordered, and hence, a distribution of activation barriers and timescales might be relevant. Moreover, the linear dependence of the activation barrier on τ in equation (8) might not be always valid. In spite of these simplifications and possible limitations, we adopt this framework and ask whether the logarithmic velocity-strengthening behavior might break down at some point.

Obviously, the simple thermal activation picture breaks down when the stress-biased barrier in equation (8) becomes comparable to k_BT. Alternatively, the breakdown occurs when the slip velocity v is not much smaller than v_0 in equation (7). There is, however, no easy way to independently estimate both v_0 and E_0, which are coupled in equation (11). Moreover, as E_0 appears in the exponential, small variations in it can be compensated by a huge variation in v_0, which suggests a large uncertainty in the latter. Nevertheless, some arguments for independently estimating v_0 have appeared in the literature. For example, *Rice et al.* [2001] estimated v_0 for rocks (quartzite and granite) to be in the mm/s range, which sets an upper bound for the validity of the thermal activation process and hence for logarithmic velocity strengthening (assuming, for the moment, that $v_0 > D/\phi^*$).

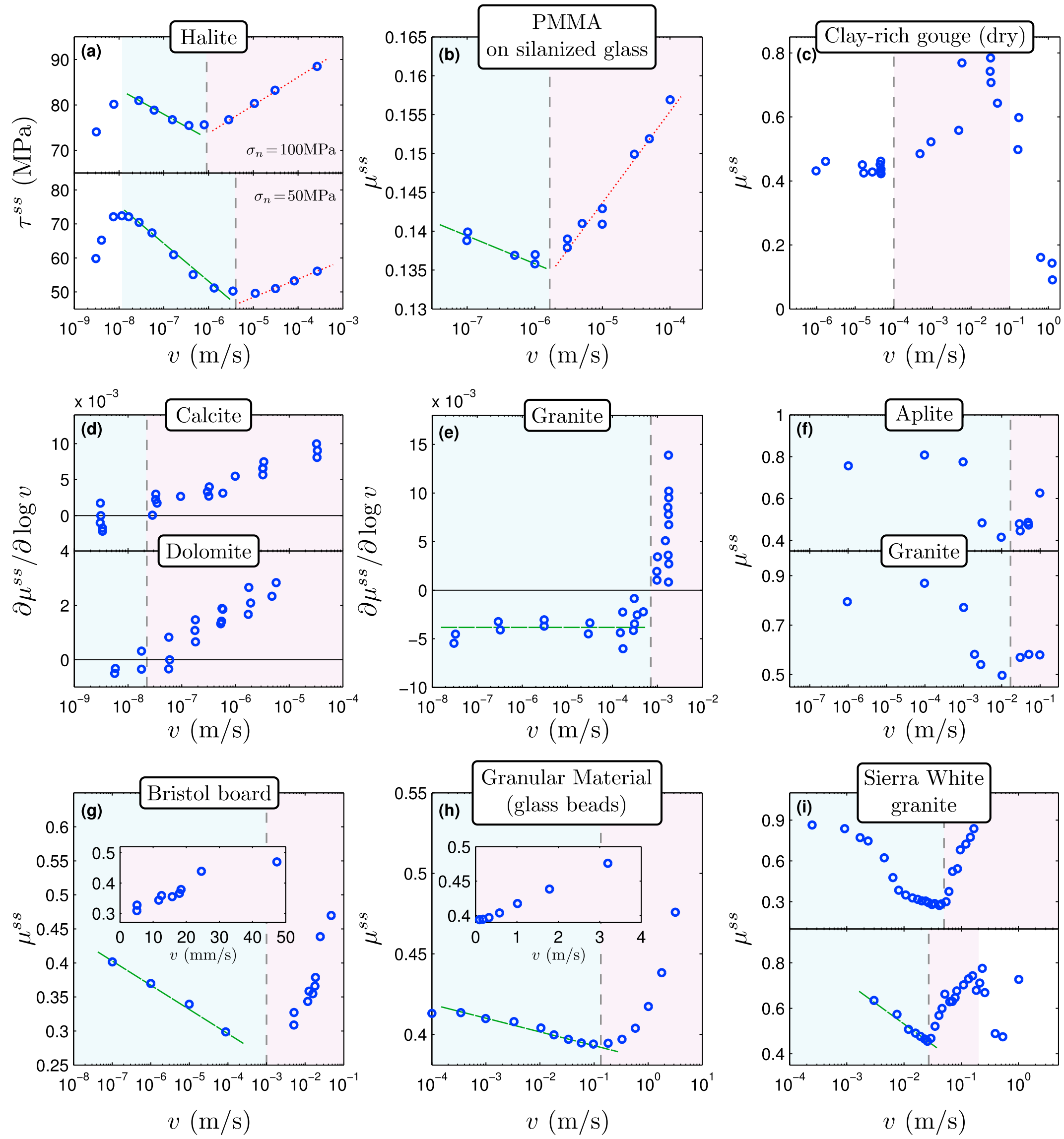

Figure 1. (a–i) Experimental observations of steady state velocity-strengthening friction in various materials. In each panel the region corresponding to velocity-weakening friction (light blue background) is separated by a vertical dashed line from the subsequent region corresponding to velocity-strengthening friction (light purple background). White background corresponds to either velocity-independent friction or a region that is not discussed in this paper. In all of the panels the horizontal axis is $\log v$ and the vertical panel is either τ^{ss} or μ^{ss}, with the exception of Figures 1d and 1e, where the vertical axis corresponds to the logarithmic derivative of the friction coefficient, $\partial\mu^{ss}/\partial \log v$. Green dashed lines mark logarithmic SVW, while red dotted lines mark logarithmic SVS. The figure is described in detail in section 5.

What happens for larger slip velocities, $v \gtrsim v_0$, when thermal activation breaks down? While we suspect that the answer might be material specific, we believe that rather generically steady state friction remains velocity strengthening in this regime (at least until thermal weakening possibly intervenes, see section 6), with a functional dependence which is typically *stronger than logarithmic*. Our basic argument is that the breakdown of the thermal activation process should also signal a change in the dominant energy dissipation mechanism associated with frictional dynamics. Since logarithmic velocity strengthening is usually intimately linked to thermal activation, we see no reason for other dissipative processes to give rise to such a weak (i.e., logarithmic) dependence on v and hence expect the dependence to be stronger than logarithmic.

While we do not aim here at developing a detailed model of the crossover to stronger-than-logarithmic steady state friction, we would like to sketch a few possible physical scenarios that might give rise to such a behavior. We first consider the possibility that logarithmic SVS friction crosses over to a linear behavior in which $\tau^{ss} \propto v$ (see, for example, Figure 1g and *Baumberger and Berthoud* [1999, Figure 15]). This viscous friction behavior might emerge as a standard viscous process obtained through linearization of a different thermally activated process, characterized by an activation volume significantly smaller than Ω. That is, if at high slip velocities (hence higher stresses) the physics of frictional dissipation changes such that the activation volume Ω decreases from a multiatom/supermolecular value to an atomic/molecular volume, the thermal activation formula of equation (10) remains valid, but now $\Omega\tau^{loc} \ll k_B T$ (recall that $\tau^{loc} = A_n \tau / A_r$) and linearization leading to $\tau \propto v$ is sensible (C. Caroli, private communication, 2013).

Another mechanism that may lead to a crossover to a linear viscous behavior is well known in the context of dislocation mechanics [*Hirth and Lothe*, 1967]. In this case, at relatively small mean dislocation velocities and applied stresses, dislocation motion is thermally activated with the barriers determined by local obstacles of various types and the Peierls lattice potential. At higher velocities and stresses, interactions with phonons and electrons control dislocation motion, leading to a linear drag-like relation between the stress and the velocity [*Kumar and Kumble*, 1969; *Zerilli and Armstrong*, 1992].

A crossover from a thermally activated regime at low slip velocities to a nonthermally activated regime at higher slip velocities has been briefly discussed in *Baumberger and Caroli* [2006]. The idea there was that plastic rearrangements at contact asperities give rise to mesoscopic stress fields that perturb nearby regions. The accumulated effect of these random spatiotemporal perturbations, originating from various plastic rearrangements taking place at different locations and times, can be regarded as a dynamical/mechanical noise, which acts in parallel to the ordinary thermal noise. This dynamical/mechanical noise becomes more intense as v increases and eventually takes over the thermal noise. While the precise functional form of the velocity-strengthening frictional response associated with the dynamical/mechanical noise-controlled regime has not been discussed, it was implied that it is stronger than the logarithmic dependence associated with the thermal noise-controlled regime [cf. *Baumberger and Caroli*, 2006, Figure 17].

Another physical scenario for the velocity-strengthening frictional response in the nonthermally activated regime might be based on applying Bagnold's scaling arguments, originally developed in the context of dense granular flows [*Bagnold*, 1956], to atomic/molecular systems [*Langer and Manning*, 2007]. The idea is that when thermal activation is irrelevant, the system has no characteristic energy scale, and flow rates are controlled by collisions between hardcore-like objects, where the detailed molecular interactions are not playing a central role. In this case, the frictional stress τ is proportional to the product of the momentum transfer per collision and the rate of collisions, both linear in v, leading to $\tau \propto v^2$. While the application of Bagnold's scaling arguments to atomic/molecular systems might be questionable, our goal here is just to highlight another known mechanism for a velocity-strengthening type of response.

In strictly athermal frictional interfaces, e.g., frictional interfaces composed of granular materials such as fault gouge, where the elementary units are macroscopic and no thermal motion takes place, we do not expect logarithmic velocity strengthening to emerge. In this case, one might expect logarithmic velocity-weakening friction at small slip velocities, due to logarithmic aging of the contacts between grains, to cross over to a stronger-than-logarithmic velocity-strengthening friction associated with nonlinear plastic rheology. This is precisely what has been observed and discussed very recently in *Kuwano et al.* [2013] (see also Figure 1h).

Finally, we note that there might exist additional strengthening mechanisms that go beyond the somewhat idealized multicontact interfaces picture discussed above, associated with wear and gouge accumulation. For example, the nonmonotonic steady state friction of Sierra White granite shown in Figure 1i, including both velocity-weakening and velocity-strengthening behaviors, has been suggested to be directly controlled by the rate of wear formation [*Reches and Lockner*, 2010]. The role of comminution as a dissipative mechanism that leads to frictional strengthening has been extensively discussed and demonstrated in *Spray* [2005, 2010].

This qualitative discussion of possible physical mechanisms that might give rise to stronger-than-logarithmic velocity-strengthening friction when thermal activation breaks down is only meant to show that such mechanisms are conceivable. The generic picture that emerges is that in thermal systems, when v_0 is sufficiently larger than D/ϕ^* and $b > k_B T/E_0$, we expect logarithmic SVW friction to cross over to logarithmic SVS friction at slip velocities $v \gtrsim D/\phi^*$, which in turn crosses over to stronger-than-logarithmic velocity-strengthening friction at slip velocities $v \gtrsim v_0$. When $v_0 < D/\phi^*$, we expect logarithmic SVS friction to cross over to stronger-than-logarithmic velocity-strengthening friction, not due to the saturation of the real contact area, but rather because stronger-than-logarithmic strengthening takes over logarithmic weakening. This is also the case for strictly athermal frictional interfaces.

To conclude this section, we note that while the discussion above—starting with equation (1)—has focussed primarily on frictional interfaces whose contact asperities deform plastically, a similar picture has been discussed by Byerlee in the context of frictional interfaces composed of geological materials governed by brittle fracture of asperities [*Byerlee*, 1967]. Moreover, we would like to draw the readers' attention to the work of *Estrin and Bréchet* [1996], who seem to discuss somewhat related ideas. Finally, it is important to mention that in the context of lubrication, i.e., frictional interfaces that contain fluids, the generic steady state dry friction curve $\tau^{ss}(v)$ discussed here is the standard known as the "Stribeck curve" [*Stribeck and Schröter*, 1903; *Olsson et al.*, 1998]. In this curve, solid contact dominates at small slip velocities and hydrodynamic viscous friction dominates at high slip velocities, with a mixed regime in between, where friction goes through a minimum.

5. Experimental Evidence

To test the physical picture described above, we have searched the available literature, looking for steady state friction experiments that go up to sufficiently high slip velocities. While these experiments are not easy to perform, and sometimes require to employ different experimental techniques in different ranges of slip velocities, we have been able to trace quite a few examples that lend support to the proposed picture. Figures 1a–1i, which span a rather wide range of materials, clearly exhibit SVS. Note that we classify a SVS behavior as "stronger-than-logarithmic" whenever it cannot be reasonably described by a $\log v$ variation, with a sensibly small prefactor. Here we provide additional information about the data presented in the figure (the list labels refer to the figure panels):

a. $\tau^{ss}(v)$ for a precut fault in halite in a triaxial apparatus, data extracted from *Shimamoto* [1986, Figure 3]. Figure 1a (bottom) corresponds to a normal stress of $\sigma = 50$ MPa and Figure 1a (top) to $\sigma = 100$ MPa. A crossover from logarithmic SVW (marked by a negative slope green dashed line) to logarithmic SVS (marked by a positive slope red dotted line) is observed. At yet higher normal stresses no SVW is observed (not shown).
b. $\mu^{ss}(v)$ for an interface between rough poly(methyl methacrylate) (PMMA, a glassy polymer) and smooth silanized glass, data extracted from *Bureau et al.* [2002, Figure 7]. A clear crossover from logarithmic SVW to logarithmic SVS is observed. Note that the smoothness of the substrate (a rigid silanized glass) implies that contact asperities are not continuously formed and destroyed during sliding and hence that a different mechanism for the crossover (as compared to the one discussed in the text) is involved.
c. $\mu^{ss}(v)$ for a dry (room humidity) clay-rich gouge layer, data extracted from *Ferri et al.* [2011, Figure 10]. The curve seems to be velocity independent at low velocities, followed by a quasi-logarithmic SVS and then a crossover to stronger-than-logarithmic velocity strengthening. Eventually, a sharp decrease in friction is observed at high velocities.
d. $\partial\mu^{ss}/\partial \log v$ for calcite (Figure 1d, top) and dolomite (Figure 1d, bottom), data sets extracted from *Weeks* [1993, Figure 2]. A crossover from SVW to SVS is observed ($\partial\mu^{ss}/\partial \log v < 0$ implies SVW, while $\partial\mu^{ss}/\partial \log v > 0$ implies SVS).
e. The same as Figure 1d, but for granite. Logarithmic SVW (marked by a horizontal line) and a crossover to stronger-than-logarithmic SVS are observed.
f. $\mu^{ss}(v)$ for aplite (Figure 1f, top) and granite (Figure 1f, bottom). Data courtesy of G. Di Toro, D. L. Goldsby and T. E. Tullis (unpublished data, 2013). A crossover from SVW to SVS is observed in both data sets.
g. $\mu^{ss}(v)$ for Bristol paper, data extracted from *Heslot et al.* [1994, Figure 4]. The curve shows logarithmic SVW at low velocities and a crossover to stronger-than-logarithmic SVS at higher velocities. The inset (linear v axis) shows that $\mu^{ss} \propto v$ in the SVS regime.
h. $\mu^{ss}(v)$ for a granular material composed of glass beads, under a very low normal stress of $\sigma = 30$ kPa; data was extracted from *Kuwano et al.* [2013, Figure 2]. A clear crossover from logarithmic SVW to a stronger-than-logarithmic SVS is observed. The inset (linear v axis) shows that $\mu^{ss} \propto v$ in the SVS regime.
i. $\mu^{ss}(v)$ for Sierra White granite in a rotary apparatus, data extracted from *Reches and Lockner* [2010, Figure 1]. The two curves correspond to two data sets that were selected out of many sets that appeared in the original figure. The lower curve exhibits logarithmic SVW at low velocities, a crossover

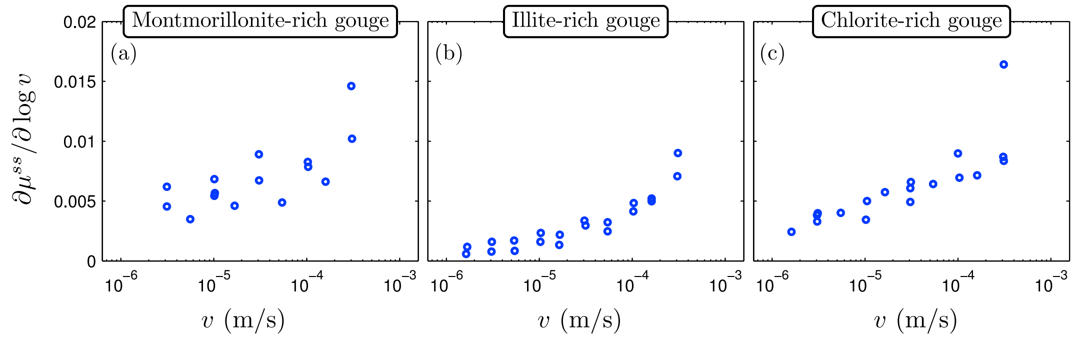

Figure 2. Logarithmic derivative of the steady state friction coefficient for various saturated clay-rich fault gouges. Data extracted from *Ikari et al.* [2009, Figure 9], where $\alpha - \beta$ (which is identical to $\partial\mu^{ss}/\partial \log v$) were reported. It is seen that these systems are velocity strengthening throughout the explored velocity range.

to stronger-than-logarithmic SVS at higher velocities, and eventually SVW at very high slip velocities. The upper curve exhibits a crossover from SVW to stronger-than-logarithmic SVS.

Some other works report on SVS and its variability with various control parameters such as the normal stress σ. For example, *Marone et al.* [1990] has observed SVS in experiments of simulated fault gouge. The magnitude of SVS varied inversely with the normal stress σ and directly with the gouge thickness and surface roughness. In this case, SVS has been associated with granular dilatancy within the gouge layer. In *Kilgore et al.* [1993], experiments on bare ground surfaces of Westerly granite have demonstrated a crossover from SVW to SVS at $v_m \simeq 10$ μm/s for a normal stress of $\sigma = 5$ MPa. For higher normal stresses, up to 150 MPa, SVS has not been observed in the range of measured slip velocities (up to 10^3 μm/s). It is not entirely clear whether SVS did not exist under these conditions or was simply shifted to higher slip velocities.

Velocity-strengthening friction has been also quite extensively discussed in the context of clay-rich fault gouge layers, which often exhibit only velocity-strengthening behavior [*Noda and Shimamoto*, 2009; *Ikari et al.*, 2009], cf. the data for dry clay-rich gouge shown in Figure 1c. Additional experimental data for $\partial\mu^{ss}(v)/\partial \log v$ in three different types of wet (saturated) clay-rich gouge layers are shown in Figure 2. All of these data sets exhibit stronger-than-logarithmic SVS, roughly consistent with $\partial\mu^{ss}(v)/\partial \log v \simeq c_1 + c_2 \log v$, where $c_1, c_2 > 0$ and c_2 is similar in magnitude to c_1 or larger. It is not entirely clear whether wet clay-rich gouge can be in fact properly described by the physical framework discussed above. On the one hand, the real contact area A_r may not be an adequate state variable in the context of wet clay-rich gouge, and on the other hand, additional state variables associated with compaction, permeability, and hydration may be required. Yet, constitutive frameworks similar to the one discussed above were invoked in the literature to phenomenologically interpret the frictional behavior of clay-rich gouge layers [*Ikari et al.*, 2009; *Noda and Shimamoto*, 2009; *den Hartog et al.*, 2012; *den Hartog and Spiers*, 2013]; hence, it might be somewhat useful to speculate about this issue here.

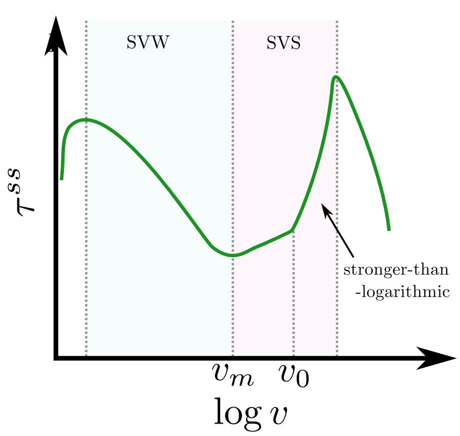

Figure 3. A schematic picture of the steady state friction law in the case $\alpha < \beta$ and $v_0 > v_m$. The color code follows Figure 1. At extremely low sliding velocities friction is velocity strengthening, governed by plastic creep (not discussed in this paper). At higher velocities, but below v_m, friction is logarithmically SVW. At $v = v_m$ friction becomes logarithmically SVS, and at $v = v_0$ it crosses over to stronger-than-logarithmic behavior. At even higher velocities, friction might decrease substantially due to thermal weakening effects (not discussed). If $\alpha > \beta$ the first peak will not exist, and if $v_0 < v_m$, the logarithmic strengthening regime will be absent.

First, we note that the relation $\partial\mu^{ss}(v)/\partial \log v \simeq c_1 + c_2 \log v$, which roughly characterizes the data in Figure 2, seems consistent with equation (12) for $v \ll D/\phi^*$, where $c_1 = \alpha - \beta$ and $c_2 = -\alpha\beta/f_0 = -\alpha\, b$. This, however, requires b (or equivalently β) to be negative, as was indeed suggested in *Ikari* *et al.* [2009]. This possibility has to be treated as a phenomenological reinterpretation of the aging coefficient b, which as such cannot possibly be negative. Furthermore, typical values of α and β are much smaller than unity, implying $c_2 \ll c_1$, which does not seem to be the case in the data of Figure 2. Another possible origin of the v dependence of $\partial\mu^{ss}(v)/\partial \log v$ can be that α is no longer v independent, in the spirit of the discussion in section 4.

Putting aside the question of the v dependence of $\partial\mu^{ss}(v)/\partial \log v$, one can speculate why $\partial\mu^{ss}(v)/\partial \log v$ is at all positive in clay-rich gouge layers, i.e., why $c_1 > 0$. The first possibility is that the real contact area A_r is not a relevant state variable for these systems in the range of slip velocities probed. This can happen either because $v \gg D/\phi^*$ in this range of v's, leading to $\log(1 + D/v\,\phi^*) \to 0$, or $b = 0$. Both of these possibilities were discussed to some extent in *Ikari et al.* [2009]. If the real contact of area is relevant, then equation (14) tells us that $\beta = f_0\, b$ should be sufficiently small. This of course can be achieved if b is small (the aforementioned limit $b \to 0$) or the background level of the friction coefficient f_0 is small. Interestingly, this is indeed the case for the clay-rich gouge systems presented in Figure 2, whose steady state coefficient of sliding friction was systematically smaller than 0.35. We finally note that while this discussion of clay-rich gouge systems is speculative, we hope it does shed some light on the variety of behaviors that can emerge from the proposed framework.

All in all, we believe that the diverse experimental data sets presented and discussed in this section imply that the physical picture depicted above should be seriously considered.

6. Summary and Discussion

In this brief note we argue that a steady state velocity-strengthening behavior might be a generic feature of dry friction over some range of slip velocities. We stress that the emergence of velocity strengthening is a natural consequence of an experimentally well-established phenomenological picture of dry friction at relatively low slip velocities. In this picture, logarithmic velocity-weakening friction (dominated by the "rejuvenation" of contact asperities) crosses over to logarithmic velocity-strengthening friction (dominated by thermally activated rheology) at a typical slip velocity $\sim D/\phi^*$ where the real contact area saturates. We further suggest that logarithmic steady state velocity-strengthening friction should cross over to a stronger-than-logarithmic velocity-strengthening behavior at a slip velocity v_0, typically accompanied by a change in the dominant frictional dissipation mechanism.

The above discussed scenario is expected to hold if $D/\phi^* < v_0$. However, as D/ϕ^* and v_0 correspond to different pieces of physics, one cannot exclude the possibility that $v_0 < D/\phi^*$. In this case we expect logarithmic velocity-weakening friction to cross over to stronger-than-logarithmic velocity-strengthening friction at $\sim v_0$. Some examples in Figure 1 seem to support this possibility. Moreover, this behavior is expected to be the generic case in athermal systems (e.g., granular materials), where no thermally activated rheology is relevant (cf. Figure 1h).

We compile a rather large number of experimental data sets available in the literature, directly demonstrating the existence of steady state velocity-strengthening friction (both logarithmic and stronger-than-logarithmic). These examples cover a rather wide range of materials, including various rocks (e.g., granite and halite), a glassy polymer (PMMA)—widely used in laboratory experiments—on smooth silanized glass, a granular material (glass beads), clay-rich gouge layers, and Bristol board. We suspect that this behavior is robust and will be observed in many other materials as long as careful steady state friction experiments cover a sufficiently large range of slip velocities.

We should mention two other aspects of steady state friction that were not discussed above but are observed in some of the data sets presented in Figure 1. First, at extremely small slip velocities one expects friction to be velocity strengthening due to creep-like plastic flow response [*Estrin and Bréchet*, 1996]. This is clearly observed in Figure 1a. At very large slip velocities thermal weakening effects might be operative, leading to significant (sometimes overwhelming) velocity-weakening friction [*Di Toro et al.*, 2004; *Rice*, 2006; *Goldsby and Tullis*, 2011]. This is clearly observed in Figures 1c and 1i. Combining these features with the previously discussed ones, an M-like friction curve emerges, as schematically shown in Figure 3. Note, however, that in some cases (e.g., the clay-rich gouge layers data in Figure 2) the first peak might be missing. Yet in all of the wide variety of examples presented, velocity-strengthening friction exists, which is our main point.

The existence of velocity-strengthening friction might have serious implications for various frictional phenomena. While these have not been studied extensively in the literature up to now, we would like to mention here the effect of velocity-strengthening friction on the upper cutoff in seismicity along well-developed faults [*Marone and Scholz*, 1988], its effect on earthquake afterslip and negative stress drops [*Marone et al.*, 1991], the role played by velocity-strengthening friction in the stability of homogeneous sliding between dissimilar materials [*Rice et al.*, 2001], in facilitating slow slip events [*Weeks*, 1993; *Kato*, 2003; *Shibazaki and Iio*, 2003; *Bouchbinder et al.*, 2011; *Bar Sinai et al.*, 2012; *Hawthorne and Rubin*, 2013; *Bar-Sinai et al.*, 2013], and in giving rise to steady state interfacial rupture fronts under stress-controlled boundary conditions [*Bar Sinai et al.*, 2012]. We hope that the present note will encourage further research in these, and other, directions.

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