

SPECTRUM OF THE EXCITED N^* AND Δ^* BARYONS IN A RELATIVISTIC CHIRAL QUARK MODEL*

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The spectrum of the $SU(2)$ flavor baryons is studied in the frame of a relativistic chiral quark potential model based on the one-pion and one-gluon exchange mechanisms. All the N^* and Δ^* resonances appearing in the πN scattering data are identified with the orbital configurations $(1S_{1/2})^2(nlj)$ with a single excited valence quark. The obtained selection rules allow one to construct a schematic periodic table of baryon resonances, consistent with the experimental data and yielding no "missing resonances". A new original method for the center of mass correction problem of the zero-order three-quark core energy values of the excited baryon resonances is suggested, which is based on the separation of the three-quark Dirac Hamiltonian into the parts, corresponding to the Jacobi coordinates. The numerical estimations for the energy positions of the Nucleon and Delta baryons, obtained within the field-theoretical framework, yield an overall good description of the experimental data at a level of the relativized Constituent quark model.

Keywords: Excited baryon spectroscopy; chiral quark model; missing resonances problem.

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1. Introduction

The Constituent Quark Models have been extensively used for baryon spectroscopy.¹⁻⁴ From very beginning these models have met a number of very serious problems, such as "missing resonances problem", "three-body spin-orbit puzzle",

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etc. The aim of present contribution is the application of a relativistic chiral quark model,^{5–8} which respect the chiral symmetry, to the excitation spectrum of the nucleon and delta. Numerical calculations are done at one loop or at order of accuracy $o(1/f_\pi^2, \alpha_s)$.

2. Model

The effective Lagrangian of the model $\mathcal{L}(x)$ (see Ref. 10) contains the quark core part $\mathcal{L}_Q(x)$, the quark-pion $\mathcal{L}_I^{(q\pi)}(x)$ and the quark-gluon $\mathcal{L}_I^{(qg)}(x)$ interaction terms, and the kinetic parts for the pion $\mathcal{L}_\pi(x)$ and gluon $\mathcal{L}_g(x)$ fields:

$$\begin{aligned} \mathcal{L}(x) &= \mathcal{L}_Q(x) + \mathcal{L}_I^{(q\pi)}(x) + \mathcal{L}_I^{(qg)}(x) + \mathcal{L}_\pi(x) + \mathcal{L}_g(x) \\ &= \bar{\psi}(x)[i \not{\partial} - S(r) - \gamma^0 V(r)]\psi(x) - 1/f_\pi \bar{\psi}[S(r)i\gamma^5 \tau^i \phi_i]\psi \\ &\quad - g_s \bar{\psi} A_\mu^a \gamma^\mu \frac{\lambda^a}{2} \psi + \frac{1}{2}(\partial_\mu \phi_i)^2 - \frac{1}{2}m_\pi^2 \phi_i^2 - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}. \end{aligned} \quad (1)$$

Here, $\psi(x)$, ϕ_i , $i = 1, 2, 3$ and A_μ^a are the quark, pion and gluon fields, respectively. The matrices τ^i ($i = 1, 2, 3$) and λ^a ($a = 1, \dots, 8$) are the isospin and color matrices, correspondingly. The pion decay constant $f_\pi = 93$ MeV.

The strength parameter $c = 0.16$ GeV² of the confinement potential $S(r) = cr + m$ is defined from the Lattice QCD,¹¹ while m can be fitted to reproduce the axial charge g_A of the proton ($m = 60$ MeV).⁶ The parameter $\alpha = \pi/12$ of the Coulomb type vector potential $V(r) = -\alpha/r$ is defined from the QCD flux-tube study.¹²

We fix the excited baryon orbital configuration as $(1S_{1/2})^2(nlj)$. The corresponding quark core energy is evaluated as:

$$E_Q = 2E(1S_{1/2}) + E(nlj) \quad (2)$$

For the center of mass (CM) correction in E_Q for the ground state nucleon and delta baryons we use the development of the Ref. 13, where approximations on the basis of the Moshinsky transformation have been used.

For the excited nucleon and delta states, the Moshinsky transformation is not applicable. A new method is based on the separation of the total three-quark core Dirac Hamiltonian with the scalar and vector mean field potentials

$$\hat{H} = \sum_{i=1}^3 [\vec{\alpha}_i \vec{p}_i + S(\vec{r}_i - \vec{R})\beta_i + V(\vec{r}_i - \vec{R})] \quad (3)$$

into three parts corresponding to the Jacobi coordinates \vec{r} , $\vec{\rho}$ and \vec{R} . At the zero order, the energy values of all baryon states with fixed orbital configuration, degenerate. This means that we can fix the two S-quarks in the 1S_0 singlet scalar diquark state.

With above conditions the kinetic energy term can be separated exactly into the three parts, corresponding to the Jacobi coordinates as:

$$\begin{aligned} \hat{H}_0 &= \hat{H}_{R,0} + \hat{H}_{r,0} + \hat{H}_{\rho,0}, & \hat{H}_{R,0} &= \frac{\vec{\alpha}_1 + \vec{\alpha}_2 + \vec{\alpha}_3}{3} \vec{P}_R, \\ \hat{H}_{r,0} &= (\vec{\alpha}_1 - \vec{\alpha}_2) \vec{P}_r, & \hat{H}_{\rho,0} &= \left(\frac{\vec{\alpha}_1 + \vec{\alpha}_2}{2} - \vec{\alpha}_3 \right) \vec{P}_\rho = -\vec{\alpha}_3 \vec{P}_\rho. \end{aligned} \quad (4)$$

However, the separation of the interaction Hamiltonian on the potentials V_r and V_ρ , can be done only approximately by using the expansion over multipole terms (see Ref. 8 for details). By solving a two-body bound state (scalar diquark) Dirac equation with the Hamiltonian \hat{H}_r and a single quark Dirac equation with \hat{H}_ρ , one is able to estimate the CM corrected quark-core energy value of baryon resonances with the fixed orbital structure $(1S_{1/2})^2(nlj)$.

The second order perturbative corrections to the energy spectrum of the SU(2) baryons due to the pion ($\Delta E^{(\pi)}$) and gluon ($\Delta E^{(g)}$) fields are calculated on the basis of the Gell-Mann and Low theorem. The one-pion exchange mechanism between the ground $1S$ and excited (nlj) valence quarks yields a selection rule

$$L_\pi = l' = l \pm 1. \quad (5)$$

The second rule comes from the strong coupling of the excited baryon state to the $N(939) + \pi$ channel:

$$\vec{L}_\pi + 1\vec{1}/2 = \vec{J}. \quad (6)$$

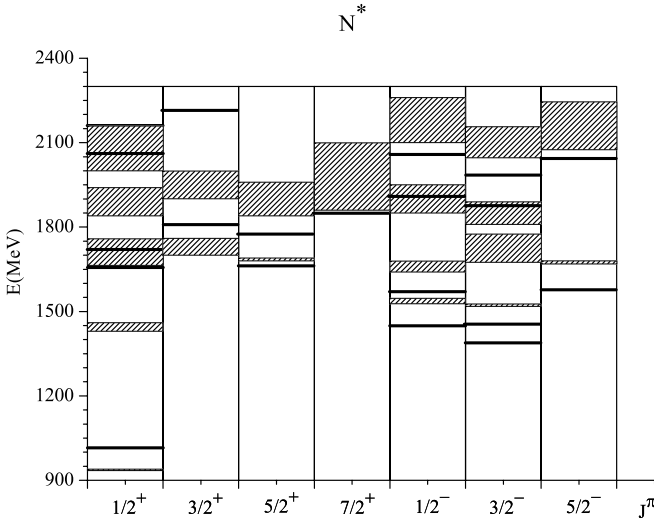


Fig. 1. Spectrum of the Nucleon states. Theoretical estimations (solid lines) in comparison with experimental data (boxes)

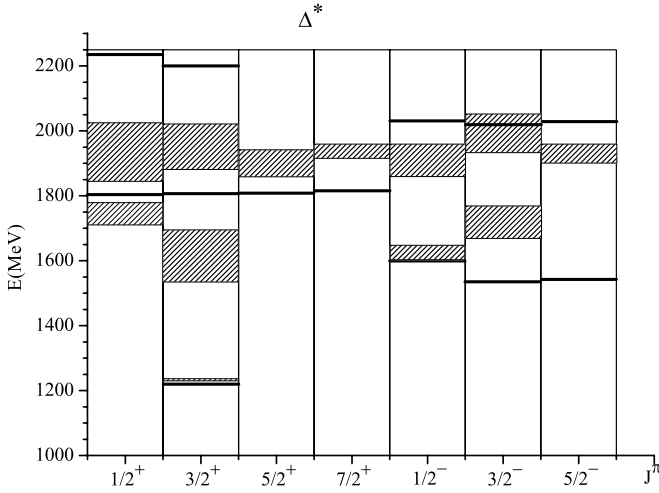


Fig. 2. Spectrum of the Delta states (notations are the same as in Fig. 1)

An important consequence of the obtained selection rules is that all the N^* and Δ^* resonances appearing in the πN scattering data are identified with the orbital configurations $(1S_{1/2})^2(nlj)$. Additionally, the selection rules allow to decrease the number of baryon resonances, up to the value, consistent with the data.

3. Numerical results

In Fig. 1 and Fig. 2 we give the theoretical estimations of the nucleon and delta states in comparison with the experimental data from Ref. 14. As can be seen from the figures, the mass spectrum of the N^* and Δ^* is described reasonably well at the level of the relativized CQM.³ Further development of the model is in progress.

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