Strangeness Contribution to the Proton Spin from Lattice QCD

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We compute the strangeness and light-quark contributions Δs, Δu, and Δd to the proton spin in $n_f = 2$ lattice QCD at a pion mass of about 285 MeV and at a lattice spacing $a \approx 0.073$ fm, using the nonperturbatively improved Sheikholeslami-Wohlert Wilson action. We carry out the renormalization of these matrix elements, which involves mixing between contributions from different quark flavors. Our main result is the small negative value $\Delta s^{\overline{\rm MS}}(\sqrt{7.4}\text{ GeV}) = -0.020(10)(4)$ of the strangeness contribution to the nucleon spin. The second error is an estimate of the uncertainty, due to the missing extrapolation to the physical point.

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Introduction.—The proton spin can be split into a quark spin contribution $\Delta\Sigma$, a quark angular momentum contribution L_q, and a gluonic contribution ΔG (including spin and angular momentum) [1]:

$$\tfrac{1}{2} = \tfrac{1}{2}\Delta\Sigma + L_q + \Delta G. \quad (1)$$

In the naive nonrelativistic SU(6) quark model, $\Delta\Sigma = 1$, with vanishing L_q and ΔG. In this case, there will be no strangeness contribution, Δs to $\Delta\Sigma = \Delta u + \Delta d + \Delta s + \cdots$, where, in our notation, $\Delta q = \Delta\Sigma_q$ contains both the spin of the quarks q and of the antiquarks $\bar{q}$.

Experimentally, Δs is obtained by integrating the strangeness contribution $\Delta s(x)$ to the spin structure function g_1 over the momentum fraction x. The integral over the range in which data exist agrees with zero; see, e.g., new COMPASS data [2,3] for $x \geq 0.004$ or HERMES data [4] for $x \geq 0.02$, while global analyses give values [5–7] $\Delta s \approx -0.12$, suggesting a large negative $\Delta s(x)$ at very small x. Pioneering lattice simulations of disconnected matrix elements also indicated values [8,9] $\Delta s \approx -0.12$. However, the errors given in these studies are quite optimistic while the global fits rely on an extrapolation of the integrated experimental $\Delta\Sigma$ to small x and constrain the axial octet charge a_8 to a value obtained from hyperon β decays, assuming $\mathrm{SU}(3)_F$ flavor symmetry. Some time ago, employing heavy baryon chiral perturbation theory, Savage and Walden [10] pointed out that $SU(3)_F$ symmetry in weak baryonic decays may be violated by as much as 25% and hence $\Delta s(x)$ could remain close to zero also for $x < 0.001$; see also [11]. $\mathrm{SU}(3)_F$ symmetry is, however, supported by lattice simulations of hyperon axial couplings [12–15], albeit within non-negligible errors.

In this Letter, we directly compute the matrix elements that contribute to the Δq, including quark line disconnected diagrams. Preliminary results were presented at conferences [16–18].

Simulation details and methods.—We simulate $n_f = 2$ nonperturbatively improved Sheikholeslami-Wohlert fermions using the Wilson gauge action at $\beta = 5.29$ and $\kappa = \kappa_{ud} = 0.136\,32$. Setting the scale from the chirally extrapolated nucleon mass [19], we obtain the lattice spacing $a^{-1} = 2.71(2)(7)$ GeV, where the errors are statistical and from the extrapolation, respectively.

We realize two additional valence κ values, $\kappa_m = 0.136\,09$ and $\kappa_s = 0.135\,50$. The corresponding pion masses are $m_{\mathrm{PS},ud} = 285(3)(7)$ MeV, $m_{\mathrm{PS},m} = 449(3)(11)$ MeV, and $m_{\mathrm{PS},s} = 720(5)(18)$ MeV. κ_s was fixed so that the $m_{\mathrm{PS},s}$ value is close to the mass of a hypothetical strange-antistrange pseudoscalar meson: $(m_{K^\pm}^2 + m_{K^0}^2 - m_{\pi^\pm}^2)^{1/2} \approx 686.9$ MeV. We investigate volumes of $32^3 64$ and $40^3 64$ lattice points, i.e., $Lm_{\mathrm{PS},ud} = 3.36$ and 4.20, respectively, where the largest spatial lattice extent is $L \approx 2.91$ fm.

The quark polarizations are extracted from the large-time behavior of ratios of three-point over two-point functions. We create a polarized proton at a time $t_0 = 0$, probe it with an axial current at a time t, and destroy the zero momentum proton at $t_f > t > 0$. Quark line connected and disconnected terms contribute

$$R^{\text{con}}(t_f, t) = \frac{\langle \Gamma^{\alpha\beta}_{\text{pol}} C^{\beta\alpha}_{3\text{pt}}(t_f, t)\rangle}{\langle \Gamma^{\alpha\beta}_{\text{unpol}} C^{\beta\alpha}_{2\text{pt}}(t_f)\rangle},$$
$$R^{\text{dis}}(t_f, t) = -\frac{\langle \Gamma^{\alpha\beta}_{\text{pol}} C^{\beta\alpha}_{2\text{pt}}(t_f) \sum_{\mathbf{x}} \text{Tr}[\gamma_j\gamma_5 M^{-1}(\mathbf{x}, t; \mathbf{x}, t)]\rangle}{\langle \Gamma^{\alpha\beta}_{\text{unpol}} C^{\beta\alpha}_{2\text{pt}}(t_f)\rangle}. \tag{2}$$

Here M is the lattice Dirac operator, $\Gamma_{\text{unpol}} = \frac{1}{2}(\mathbb{1} + \gamma_4)$ is a parity projector, and $\Gamma_{\text{pol}} = i\gamma_j\gamma_5\Gamma_{\text{unpol}}$ projects out the difference between the two polarizations (in direction $\hat{\jmath}$). We average over $j = 1, 2, 3$ to increase statistics. For the up and down quark matrix elements we compute the sum of connected and disconnected terms while only R^{dis} contributes to Δs.

For disconnected contributions we fix the time distance between the source and the current insertion $t = 4a \approx 0.29$ fm and vary t_f. Both t and the distance between current and sink $t_f - t$ should be taken large, to suppress excited state contributions. Using the sink and source smearing described in [20], we find the asymptotic limit to be effectively reached for $t_f \simeq 6a$–$7a$; see Fig. 1 for an example. The saturation into a plateau at $t_f \leq 2t$ and the convergence of the point sink data toward the same value demonstrate that $t = 4a$ was reasonably chosen. To be on the safe side, we only fit the $t_f \geq 8a \approx 0.58$ fm smeared-smeared ratios. Building upon previous experience [21], the connected part, for which the statistical accuracy is less of an issue, is obtained at the larger, fixed value $t_f = 15a$, varying t.

The disconnected contribution is computed with the stochastic estimator methods described in [17,22], employing time partitioning, a second order hopping parameter expansion, and the truncated solver method. We compute the Green functions for four equidistant source times on each gauge configuration. We also construct backwardly propagating nucleons, replacing the positive parity projector $\frac{1}{2}(\mathbb{1} + \gamma_4)$ by $\frac{1}{2}(\mathbb{1} - \gamma_4)$, seeding the noise vectors on eight (4 times 2) time slices. In addition to the 48 (4 times spin times color) solves for smeared conventional sources, which are necessary to construct the two-point functions, we run the conjugate gradient algorithm on $N_1 = 730$ complex $\mathbb{Z}_2$ noise sources for $n_t = 40$ iterations. The bias from this truncation is corrected for [22] by $N_2 = 50$ BiCGstab solves that are run to convergence. We analyze a total of 2024 thermalized trajectories on each of the two volumes where we bin the data to eliminate autocorrelations.

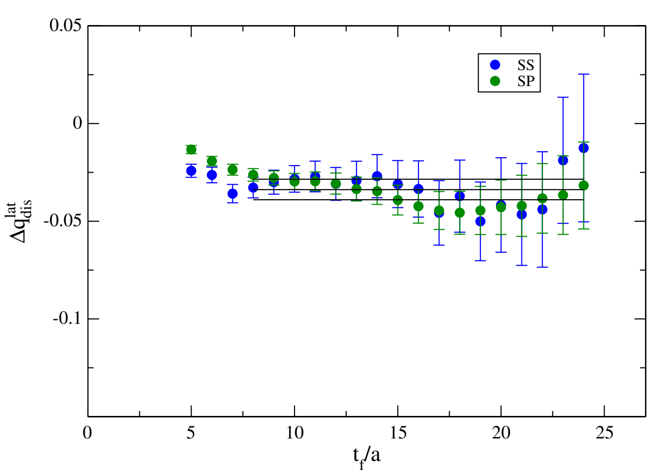

FIG. 1 (color online). The disconnected ratio R^{dis} versus t_f on the $40^3 64$ volume at $\kappa_{\text{val}} = \kappa_{\text{cur}} = \kappa_s$ for smeared-smeared (SS) and smeared-point (SP) source-sink combinations.

Renormalization.—Nonsinglet axial currents renormalize with a renormalization factor $Z^{\text{ns}}_A(a)$ that only depends on the lattice spacing. This was determined nonperturbatively for the action and lattice spacing in use [23]: $Z^{\text{ns}}_A = 0.764\,85(64)(73)$.

However, due to the axial anomaly, the renormalization constant of singlet currents, $Z^s_A(\mu, a)$, acquires an anomalous dimension. To first nontrivial order this reads [24,25] $\gamma^s_A(\alpha_s) = -6C_F n_f[\alpha_s/(4\pi)]^2$. Z^s_A deviates from Z^{ns}_A starting at $\mathcal{O}(\alpha_s^2)$ in perturbation theory. Both factors have been calculated to this order, with the result for the conversion into the $\overline{\text{MS}}$ scheme at a scale μ [26]

$$\begin{aligned} z(\mu, a) &= Z^s_A(\mu, a) - Z^{\text{ns}}_A(a) \\ &= C_F n_f[15.8380(8) - 6\ln(a^2\mu^2)]\left(\frac{\alpha_s}{4\pi}\right)^2, \end{aligned} \tag{3}$$

where we have set the Sheikholeslami-Wohlert parameter $c_{\text{SW}} = 1$ to be consistent to this order in perturbation theory. To this first nontrivial order, no scale enters the coupling parameter α_s. Since perturbation theory in terms of the bare lattice parameter $\alpha_0 = 6/(4\pi\beta)$ is known to converge poorly, we substitute α_s by a coupling defined from the measured average plaquette $\alpha_s = -3\ln\langle U_\Box\rangle/(4\pi) = 0.142\,78(5)$, where we have used the chirally extrapolated value [27] $\langle U_\Box\rangle = 0.549\,88(11)$.

No dimension-four operator can be constructed that mixes with the relevant forward matrix element of $\bar{q}\gamma_\mu\gamma_5 q$ and that cannot be removed, using the equations of motion [28]. This also holds for the singlet case [29], such that we only need to replace

$$Z^{\text{ns}}_A \mapsto Z^{\text{ns}}_A(1 + b_A am), \qquad Z^s_A \mapsto Z^s_A(1 + b^s_A am), \tag{4}$$

to achieve full $\mathcal{O}(a)$ improvement. The factor b_A is known to $\mathcal{O}(\alpha_s)$ [28]: $b_A = b^s_A + \mathcal{O}(\alpha_s^2) \approx 1 + 18.025\,39 C_F \frac{\alpha_s}{4\pi}$. We obtain the values

$$1 + b_A am = \begin{cases} 1.0324(3)(47) & (m_s, \kappa = 0.135\,50) \\ 1.0041(3)(5) & (m_{ud}, \kappa = 0.136\,32), \end{cases} \tag{5}$$

where the first error is due to the uncertainty in the quark mass and the second error corresponds to 50% of the one-loop correction. Considering the small size of this correction, it is unlikely that the (two-loop) difference between singlet and nonsinglet b_A factors will result in any noticeable effect, and, in particular, not at the light-quark mass m_{ud}, where it will be needed [see Eq. (11) below].

For $n_f = 2$ we get

$$z(\sqrt{7.4}\ \text{GeV}) = 0.0055(1)(27), \tag{6}$$

at the renormalization scale $\mu^2 = 7.4\ \text{GeV}^2 = 1.01(5)a^{-2}$. We again include a 50% systematic error to allow for higher order corrections. Because of the small anomalous dimension that only sets in at $\mathcal{O}(\alpha_s^2)$, the difference between singlet and nonsinglet renormalization constants remains small, also at other scales. For instance, we obtain $z(\sqrt{10}\ \text{GeV}) = 0.0049(25)$ and $z(2\ \text{GeV}) = 0.0082(41)$.

In the $n_f = 1 + 1 + 1$ theory the matrix elements renormalize as follows:

$$g_A = \Delta T_3 = (\Delta u - \Delta d)^{\overline{\text{MS}}} = Z_A^{\text{ns}}(a)(\Delta u - \Delta d)^{\text{lat}}(a), \tag{7}$$

$$\begin{aligned} a_8 = \Delta T_8 &= (\Delta u + \Delta d - 2\Delta s)^{\overline{\text{MS}}} \\ &= Z_A^{\text{ns}}(a)(\Delta u + \Delta d - 2\Delta s)^{\text{lat}}(a), \end{aligned} \tag{8}$$

$$\begin{aligned} a_0 = \Delta\Sigma^{\overline{\text{MS}}}(\mu) &= (\Delta u + \Delta d + \Delta s)^{\overline{\text{MS}}}(\mu) \\ &= Z_A^s(\mu, a)(\Delta u + \Delta d + \Delta s)^{\text{lat}}(a). \end{aligned} \tag{9}$$

We remark that for nonequal quark masses the nonsinglet combinations, Eqs. (7) and (8), also receive contributions from disconnected quark line diagrams.

We employ $n_f = 2$ sea quarks so that our singlet current is $\Delta u + \Delta d$ rather than the $\Delta\Sigma$ of Eq. (9). This modifies the renormalization pattern:

$$\begin{pmatrix} \Delta u(\mu) \\ \Delta d(\mu) \\ \Delta s(\mu) \end{pmatrix}^{\overline{\text{MS}}} = \begin{pmatrix} Z_A^{\text{ns}}(a) + \frac{z(\mu,a)}{2} & \frac{z(\mu,a)}{2} & 0 \\ \frac{z(\mu,a)}{2} & Z_A^{\text{ns}}(a) + \frac{z(\mu,a)}{2} & 0 \\ \frac{z(\mu,a)}{2} & \frac{z(\mu,a)}{2} & Z_A^{\text{ns}}(a) \end{pmatrix} \times \begin{pmatrix} \Delta u(a) \\ \Delta d(a) \\ \Delta s(a) \end{pmatrix}^{\text{lat}}. \tag{10}$$

$\Delta s^{\overline{\text{MS}}}$ receives light-quark contributions but the $\Delta u^{\overline{\text{MS}}}$ and $\Delta d^{\overline{\text{MS}}}$ remain unaffected by the (quenched) strange quark. Obviously, unitarity is violated, due to this quenching. The combination ΔT_8 still transforms with Z_A^{ns} [Eq. (8)] while Eq. (9) is violated, as it should be; instead, the $n_f = 2$ singlet operator $\Delta u + \Delta d$ renormalizes with Z_A^s. We remark that the above renormalization pattern is similar to that of the scalar matrix element in the $n_f = 2$ theory [20,30,31]. Note that in spite of the quenched strange quark, the mismatch between directly converting the result into the $\overline{\text{MS}}$ scheme at a scale μ, using $z(n_f = 2)/2$, and first converting into the $\overline{\text{MS}}$ scheme at another scale μ' and subsequently running within the $\overline{\text{MS}}$ scheme with $\ln(\mu/\mu')\gamma_A^s(n_f = 3)/3$ to the scale μ is tiny.

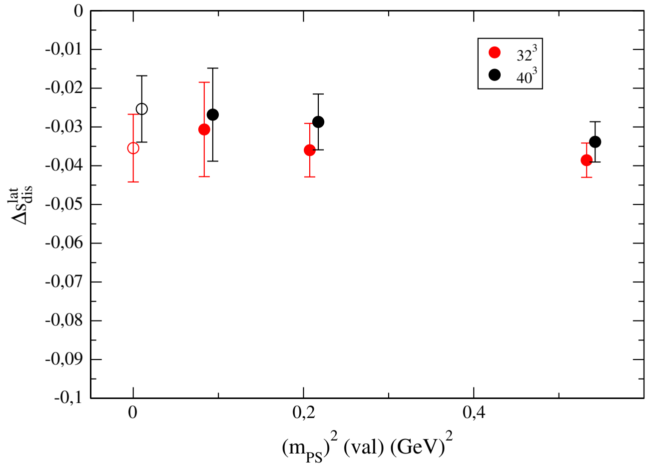

FIG. 2 (color online). Volume and valence quark mass dependence of the unrenormalized Δs^{lat}.

Results and systematics.—In Fig. 2 we display the volume and (light) valence quark mass dependence of our unrenormalized Δs^{lat}. There are no statistically significant finite size or mass effects.

Using Eqs. (10) and (4) we can renormalize

$$\Delta q^{\overline{\text{MS}}}(\mu) = Z_A^{\text{ns}}(1 + b_A a m_q)\Delta q^{\text{lat}} + \frac{z(\mu)}{2}(\Delta u + \Delta d)^{\text{lat}} \tag{11}$$

for $q \in \{u, d, s\}$. As discussed above, we omit the $\mathcal{O}(a)$ improvement factor $(b_A^s Z_A^s - b_A Z_A^{\text{ns}})am_{ud}$ of the $(\Delta u + \Delta d)^{\text{lat}}$ term. This is of $\mathcal{O}(\alpha_s^2 a m_{ud})$ and numerically negligible. We display the bare lattice numbers for the connected and disconnected contributions to the proton spin and the renormalized $\mathcal{O}(a)$ improved values in Table I for the two volumes. The $\Delta u^{\overline{\text{MS}}}$ and $\Delta d^{\overline{\text{MS}}}$ values are reduced by about 0.035, due to the sea quark contributions while $\Delta s^{\overline{\text{MS}}}$ increases by 0.002 ($< 10\%$), due to the mixing with light-quark flavors.

The uncertainties associated with the renormalization are much smaller than the statistical errors. Below we will only quote large volume results, with statistical and renormalization errors added in quadrature. Error sources that have so far not been accounted for are the missing continuum limit extrapolation, the quenching of the strange quark, and simulating at a light sea quark mass value that is 4 times bigger than the physical one. There are no indications of radical quark mass effects: the flavor mixing effects within the renormalization are small in spite

TABLE I. The connected and disconnected contributions to $\Delta q^{\rm lat}$ as well as the renormalized spin content at a scale $\mu = \sqrt{7.4}$ GeV. (The ΔT_i are scale independent.) The first error is statistical; the second is from the renormalization. In addition, an overall 20% systematic error needs to be added.

q	V, L	$\Delta q^{\rm lat}_{\rm con}$	$\Delta q^{\rm lat}_{\rm dis}$	$\Delta q^{\overline{\rm MS}}(\mu)$
u		1.065(22)	−0.034(16)	0.794(21)(2)
d		−0.344(14)	−0.034(16)	−0.289(16)(1)
s	$V = 32^3 64$	0	−0.031(12)	−0.023(10)(1)
T_3	$L \approx 2.33$ fm	1.409(24)	0	1.082(18)(2)
T_8		0.721(26)	−0.006(18)	0.550(24)(1)
Σ		0.721(26)	−0.098(42)	0.482(38)(2)
u		1.071(15)	−0.049(17)	0.787(18)(2)
d		−0.369(9)	−0.049(17)	−0.319(15)(1)
s	$V = 40^3 64$	0	−0.027(12)	−0.020(10)(1)
T_3	$L \approx 2.91$ fm	1.439(17)	0	1.105(13)(2)
T_8		0.702(18)	−0.044(19)	0.507(20)(1)
Σ		0.702(18)	−0.124(44)	0.448(37)(2)

of the comparatively large Δu and Δd values. The dependence on the valence quark mass is small too; see Fig. 2.

Nevertheless, having simulated only at one lattice spacing and sea quark mass, we cannot extrapolate our results to the physical point. Consequently, we underestimate the value [32] $g_A = 1.2670(35)$ from neutron β decays by 13% and find $\Delta T_3 = 1.105(13)$ instead. Our prediction $\Delta T_8 = 0.507(20)$ differs by the same 13% from the phenomenological estimate [32] $a_8 = 0.585(25)$. We take this as an indication of the size of the remaining systematics and add an additional 20% error to all our results.

Conclusions.—We determined the first moments of proton flavor singlet and nonsinglet polarized parton distributions from $n_f = 2$ lattice QCD, at a pion mass of 285 MeV, at a single lattice spacing $a \approx 0.073$ fm. We found $\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.45(4)(9)$ and a small negative $\Delta s = -0.020(10)(4)$, in the $\overline{\rm MS}$ scheme, at a scale $\mu = \sqrt{7.4}$ GeV. We underestimated both g_A and a_8 by similar factors ≈ 0.87 and this may suggest that some of the systematics cancel when considering ratios of matrix elements. Nevertheless, we emphasize that there is a considerable uncertainty in the a_8 value [10] and our $\Delta\Sigma$ is already relatively large, due to the small difference $\Delta T_8 - \Delta\Sigma = -3\Delta s = 0.059(29)(12)$.

Interestingly, our results are in remarkable agreement with the cloudy bag model prediction of [11]. The small (unrenormalized) $\Delta s^{\rm lat}$ value obtained recently in [31] is also consistent with our study. Our $\Delta\Sigma$ value is larger than previously expected; however, it is compatible with the latest COMPASS number [2] $a_0(\sqrt{3}\ {\rm GeV}) = 0.35(3)(5)$. The experimental number may increase further once smaller x values become accessible. We suggest relaxing the weak hyperon decay $SU(3)_F$ constraint on a_8 in determinations of polarized parton distribution functions [5–7], and including our Δs prediction instead.

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