Son, Grassberger, and Paczuski Reply: In their Comment [1], Havlin and coworkers confirm our claim [2] that percolation transitions on interdependent networks do not show first order transitions, if these networks are diluted two-dimensional lattices. This was the main message in [2], since the opposite had previously been claimed *explicitly* by Havlin and coworkers in [3] and (implicitly) in numerous later papers. In their Comment [1], the authors now claim that our finding "is expected," and do not mention the previous claim of one of then to the opposite.

In addition, we claimed in [2] that the critical behavior of this model is not in the ordinary percolation (OP) universality class, and the same was found for interdependent diluted three-dimensional lattices. But this was not our "main conclusion" (as claimed in [1]), and it is unclear why the authors make the wrong statement that this was also "reflected in the title." Apart from that, it is not dilution per se that changes the universality class (or not, as suggested in [1])—it is the interdependence.

Figure 1 of [1] is essentially the same as our Fig. 3. But the fact that the curves in these figures are steeper than for OP has nothing to do with any universality class or with the question of whether the transition is first or second order.

It is true that the lattices simulated in [1] were bigger than those of [2], but (i) It is not clear that they had higher statistics. Indeed, the statistical fluctuations in Fig. 2a of [1] seem larger than those in Fig. 5 of [2]. (ii) The claim that D_f is closer to the OP value than to the estimate of [2] could only be substantiated by a careful estimate of p_c similar to the analysis in the Supplemental Material of [2] (see [4]), in particular using scaling plots like Figs. 2, 8, and 9 in the Supplemental Material.

Thus, although we agree that there is some evidence for D_f to be close to the OP value, this is not conclusive.

On the other hand, the claim of [1]—based on their Fig. 2b—that β agrees with the OP value $5/36 = 0.1389$ and disagrees with our estimate 0.172 ± 0.002 is clearly wrong. Figure 2b shows four curves, all three bent upwards, for $L = 512$, 750, 1024, and 3000. The fact that they appear roughly equidistant, although the gaps in L are very different, shows that the curve for $L = 3000$ is already asymptotic, and its curvature is not a finite-L effect. Thus, the slope for large $p - p_c$ is larger than the value ≈ 0.156 that can be read off Fig. 2b by plotting it on an enlarged scale. We should add that the question could have been clarified if the authors of [1] had simulated larger values of p_c. Unfortunately, they went only up to $p = 0.969 = p_c + 0.008$, while we had gone up to $p = 0.997 = p_c + 0.036$ (simulating at large values of p becomes very costly). In this way we had a much larger scaling region. Also, as seen from Fig. 2b of [1], there is virtually no L dependence for $p > 0.964$, whence our results for $L = 512$ should give the asymptotic ($L \to \infty$) estimate.

Finally, one might wonder whether we (and the authors of [1]) found a continuous transition instead of the discontinuous one claimed in [3], because many links in two diluted two-dimensional lattices are identical. It was shown in [5,6] that the transition can become continuous in similar situations. The results found in [2] for $d = 3$ (where there are much fewer overlapping links) indicate that this is not the case, and the effect is really due to the locality of the links.

In conclusion, the results of [1] support our main claim that the transition is continuous, in contrast to explicit claims by Havlin and coworkers in [3]). They also show that β is larger than the OP value $5/36$. The only valid new result of [1] could be that our estimate of the fractal dimension D_f was too low, but this requires more simulations—in particular for larger values of p, because only in this way would a precise estimate of p_c be possible.

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