

Charge symmetry violation in the electromagnetic form factors of the nucleon

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Experimental tests of QCD through its predictions for the strange-quark content of the proton have been drastically restricted by our lack of knowledge of the violation of charge symmetry (CSV). We find unexpectedly tiny CSV in the proton's electromagnetic form factors by performing the first extraction of these quantities based on an analysis of lattice QCD data. The resulting values are an order of magnitude smaller than current bounds on proton strangeness from parity-violating electron-proton scattering experiments. This result paves the way for a new generation of experimental measurements of the proton's strange form factors to challenge the predictions of QCD.

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Charge symmetry is the invariance of the strong interaction under an isospin rotation exchanging u and d quarks (i.e., exchanging the proton and neutron). The violation of this symmetry (CSV) is arguably small: the proton-neutron mass difference is one part in a thousand [1], and many nuclear reactions proceed identically if protons and neutrons are interchanged. The effects of this small CSV, however, may be hugely significant. For example, if the proton-neutron mass difference were reversed, protons could decay and atoms could not form. Charge symmetry violation also explains the discrepancy between the calculated and measured binding energy differences of mirror nuclei (Okamoto-Nolen-Schiffer anomaly) [2,3] and may play a role in precision tests of the Standard Model [4], including those at the LHC [5].

In the late 1980s it was suggested that one could use measurements of neutral weak current matrix elements by parity-violating electron scattering (PVES) [6–8] to determine the contribution of strange quark-antiquark pairs to the elastic electroweak form factors of the nucleon. These “strange form factors” have been the focus of intensive experimental and theoretical effort for the past two decades [9]. At present, the accuracy of theoretical calculations of the strange magnetic moment, in particular, [10–13] exceeds that of the best experimental values [14] by almost an order of magnitude—a remarkable exception in strong-interaction physics. The limiting factor in future

state-of-the-art PVES measurements at Mainz [15,16] and JLab [17–19] is theoretical, arising from the assumption that CSV in the proton's electromagnetic form factors is negligible.

Specifically, CSV form factors G_{CSV} , if not accounted for, mimic the strange-quark contribution $G_{E/M}^s$ in the combination of form factors accessed by experiment: the measured neutral weak current matrix elements $G_{E/M}^{p,Z}$ may be expressed as

$$G_{E/M}^{p,Z} = (1 - 4\sin^2\theta_W)G_{E/M}^{p,\gamma} - G_{E/M}^{n,\gamma} - G_{E/M}^s + G_{\text{CSV}}, \quad (1)$$

where the weak mixing angle, θ_W , and the total electromagnetic form factors, $G_{E/M}^{p/n,\gamma}$, are precisely determined from other experimental studies. With theoretical predictions of the size of G_{CSV} varying through several orders of magnitude [20–22], this uncertainty has halted experimental parity-violating electron scattering programs [19]. For future experiments which may be performed at values of Q^2 larger than about 0.3 GeV^2 , there is as yet no theoretical constraint on, or indication of the size of, CSV effects, leaving this uncertainty completely uncontrolled.

In this Letter we report the first determination of CSV in the proton's electromagnetic form factors based on an analysis of lattice QCD data. This work gives bounds on the

relevant CSV quantities for values of the momentum transfer Q^2 up to 1.4 GeV². In terms of individual u - and d -quark contributions to the Sachs electric and magnetic form factors of the proton and neutron (conventionally defined without the charge factors), the CSV form factors which we calculate are defined as

$$\delta_{E/M}^u = G_{E/M}^{p,u} - G_{E/M}^{n,d}, \quad \delta_{E/M}^d = G_{E/M}^{p,d} - G_{E/M}^{n,u}, \quad (2)$$

where we explicitly calculate $G_{E/M}^{p/n,u/d}$ and perform the subtractions indicated. The combination relevant to experimental determinations of nucleon strangeness using Eq. (1) is

$$G_{\text{CSV}}^{E/M} = \left(\frac{2}{3} \delta_{E/M}^d - \frac{1}{3} \delta_{E/M}^u \right). \quad (3)$$

The lattice results used here are an extension of those reported in Refs. [23,24]; we include two independent sets of 2 + 1-flavor simulations at different values of the finite lattice spacing a . Any discretization artifacts should appear at $\mathcal{O}(a^2)$. Each set consists of results for the individual connected quark contributions to the electromagnetic form factors of the entire outer-ring baryon octet at a range of pion masses down to 220 MeV and at six (set I) or seven (set II) fixed values of the momentum transfer Q^2 up to 1.4 GeV². These values of Q^2 are relevant to experimental studies of the strange nucleon form factors. The lattice volumes are $L^3 \times T = 32^3 \times 64$ and $48^3 \times 96$, and the lattice spacings are $a^2 = 0.0055(3) \text{ fm}^2$ and $0.0038(2) \text{ fm}^2$ (set using various singlet quantities [25,26]) for the two sets, respectively.

Our extraction of the CSV form factors from the lattice simulations is based on the extrapolation of those results to infinite volume and to the physical pseudoscalar masses using a formalism based on connected chiral perturbation theory [27,28]. The extrapolation procedure is detailed in

Refs. [23,24]. The small finite-volume corrections are model-independent and the chiral extrapolation is demonstrated to be under control—the fit includes lattice data at low meson masses within the convergence regime of the effective theory, and it reproduces the experimental form factors at the physical masses [23,24]. To determine the CSV terms we must extend that work to incorporate the breaking of the flavor-SU(2) symmetry, i.e., to allow for unequal light quark masses, $m_u \neq m_d$. This is a simple extension, and is performed precisely as in previous work where the same procedure was used to evaluate the mass splittings among members of baryon isospin multiplets [29], the CSV sigma terms [30], and the CSV parton distribution moments [31] from 2 + 1-flavor lattice simulation results. In brief, the low-energy parameters which appear in the SU(2)-breaking terms in the chiral extrapolation expressions for the CSV form factors also appear in the isospin-averaged expressions. These parameters are thus fixed by the fits to the $N_f = 2 + 1$ lattice QCD simulations on the baryon octet which are presented in Refs. [23,24].

In principle, the CSV form factors on an infinite volume and at the physical pseudoscalar masses may thus, given the extrapolations of Refs. [23,24], be evaluated simply by performing the subtractions shown in Eq. (2). This procedure, however, suffers from a significant systematic effect resulting from the omission of quark-line disconnected contributions in the simulations. To account for this omission we use the chiral extrapolation expressions to model the disconnected pieces of the loop integral expressions. This amounts to the replacement of the ‘connected’ extrapolation coefficients of Refs. [23,24] by the ‘full’ expressions, where the free parameters remain as fixed by the connected fits. The resulting expressions for the CSV electric and magnetic form factors (including disconnected quark-line contributions) as a function of meson masses can be written as

$$\begin{aligned} \delta_M^u = & \frac{1}{6} (2c_1^M - 3c_{10}^M - 3c_{12}^M - 4c_2^M - 2c_5^M - 5c_6^M - 54c_7^M + 3c_9^M) \mathcal{B}(m_d - m_u) \\ & + \frac{M_N}{16\pi^3 f_\pi^2} \frac{1}{9} [C^2(I_D^M(m_{K^0}) - I_D^M(m_{K^\pm})) - 12(D^2 + 3F^2)(I_O^M(m_{K^0}) - I_O^M(m_{K^\pm}))], \end{aligned} \quad (4)$$

$$\begin{aligned} \delta_M^d = & \frac{1}{6} (2c_1^M + 2c_{10}^M - 4c_{11}^M + 2c_{12}^M - 4c_2^M + 4c_5^M + c_6^M + 54c_7^M - c_9^M) \mathcal{B}(m_d - m_u) \\ & - \frac{M_N}{16\pi^3 f_\pi^2} \frac{2}{9} [C^2(I_D^M(m_{K^0}) - I_D^M(m_{K^\pm})) - 9(D - F)^2(I_O^M(m_{K^0}) - I_O^M(m_{K^\pm}))], \end{aligned} \quad (5)$$

$$\begin{aligned} \delta_E^u = & \frac{1}{6} (2c_1^E - 3c_{10}^E - 3c_{12}^E - 4c_2^E - 2c_5^E - 5c_6^E - 54c_7^E + 3c_9^E) Q^2 \mathcal{B}(m_d - m_u) \\ & - \frac{1}{16\pi^3 f_\pi^2} \frac{1}{9} [C^2(I_D^E(m_{K^0}) - I_D^E(m_{K^\pm})) + 6(D^2 + 3F^2)(I_O^E(m_{K^0}) - I_O^E(m_{K^\pm})) \\ & + 18(I_T^E(m_{K^0}) - I_T^E(m_{K^\pm}))], \end{aligned} \quad (6)$$

$$\delta_E^d = \frac{1}{6}(2c_1^E + 2c_{10}^E - 4c_{11}^E + 2c_{12}^E - 4c_2^E + 4c_5^E + c_6^E + 54c_7^E - c_9^E)Q^2\mathcal{B}(m_d - m_u) + \frac{1}{16\pi^3 f_\pi^2} \frac{1}{9} [2\mathcal{C}^2(I_D^E(m_{K^0}) - I_D^E(m_{K^\pm})) + 9(D - F)^2(I_O^E(m_{K^0}) - I_O^E(m_{K^\pm})) + 9(I_T^E(m_{K^0}) - I_T^E(m_{K^\pm}))], \quad (7)$$

where all symbols, including the low-energy constants $c_i^{E/M}$, are defined in Refs. [23,24]. The leading-order loop integral expressions include meson loops with octet-baryon (I_O) or decouplet-baryon (I_D) intermediate states, as well as tadpole loops (I_T). The Gell-Mann-Oakes-Renner relation suggests the definition

$$\mathcal{B}(m_d - m_u) = \frac{(1 - R)}{(1 + R)} m_\pi^2, \quad (8)$$

where R denotes the light-quark mass ratio $R = m_u/m_d$. We take $R = 0.553(43)$, determined by a fit to meson decay rates [32]. The shift in central value if we instead take the FLAG value $R = 0.46(2)(2)$ [33] is included as an uncertainty in our final results.

All of the low-energy parameters, other than $c_1^{E/M}$, $c_2^{E/M}$ and $c_7^{E/M}$, are determined from the chiral fits to the connected contribution to the isospin-averaged electromagnetic form factors which are described in Refs. [23,24]. As detailed in those references, a full error analysis is performed including correlations between all of the fit parameters (the resulting values of these parameters are given in the Appendixes of Refs. [23,24]). The corresponding uncertainties are propagated into our final results. While this procedure systematically includes some of the disconnected contribution to the CSV form factors, other disconnected terms—those which are linear in $\mathcal{B}(m_d - m_u)$ and not generated by chiral logarithms from meson loops—cannot be determined in this way. Specifically, the terms which are generated by the Lagrangian pieces with

coefficients $c_1^{E/M}$, $c_2^{E/M}$ and $c_7^{E/M}$ cannot be determined from the present lattice simulations. Physically, these terms arise from the diagrams illustrated and described in Fig. 1. These contributions are anticipated to be small based on the success of valence quark models in reproducing form factor data. This is also supported by the results of direct lattice QCD calculations of $G_{E/M}$ which find that the disconnected contributions at small finite momentum transfer are consistent with zero and are bounded at the 1% level [34]. In particular, there is no indication of any mass-dependence of the disconnected effects (which is the effect that would indicate disconnected contributions to the CSV). The terms corresponding to the low-energy parameters $c_1^{E/M}$, $c_2^{E/M}$ and $c_7^{E/M}$ are only part of the small contribution from disconnected loop terms.

We choose to set contributions from the unknown $c_1^{E/M}$, $c_2^{E/M}$ and $c_7^{E/M}$ terms to 0, with an uncertainty taken to be twice the magnitude of the corresponding contributions from meson loop diagrams, evaluated with a dipole cutoff regulator with mass scale $\Lambda = 0.8(2)$ GeV. We suggest that this error estimate is extremely conservative. The use of this method to evaluate the loops is justified by the well-established and successful use of this model to relate full and partially-quenched lattice QCD calculations [35]. The loop diagram used to estimate the $c_{1,2}^{E/M}$ terms is represented in Fig. 2(b), where only the “loop spectator” quark mass (i.e., the valence-quark part of the meson mass) is changed. For the $c_7^{E/M}$ term, represented in Fig. 2(a), only the sea-quark part of the loop meson mass is considered. These contributions are added in quadrature. The magnitude of this contribution to the total uncertainty varies with Q^2 ; it is largest at our lowest Q^2 values where it contributes 20%–60% of the quoted uncertainty on the final result

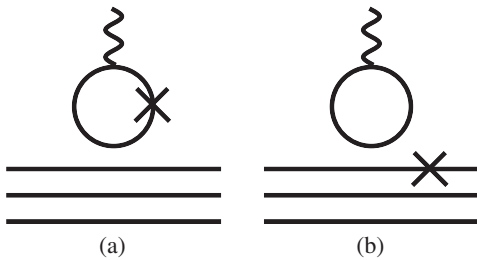


FIG. 1. Diagrammatic quark-line skeleton representation of omitted contributions to the CSV form factors. Solid and wavy lines represent quarks and photons, respectively. The crosses denote quark mass insertions, i.e., the figures represent the contribution from disconnected quark loops to CSV arising from the different (u and d quark) masses of (a) the struck sea quark and (b) spectator quarks. These contributions are proportional to $\mathcal{B}(m_d - m_u)$.

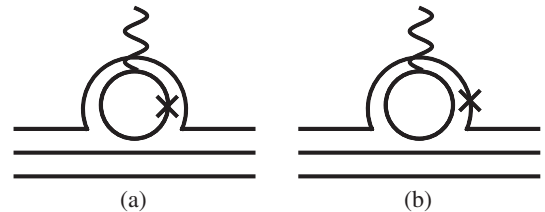


FIG. 2. Quark-line skeleton diagrams of the meson loops used to model the omitted contributions to the CSV form factors. Solid and wavy lines represent quarks and photons, respectively. The crosses denote quark mass insertions into (a) the struck sea quark in the meson loop and (b) the meson loop spectator quark.

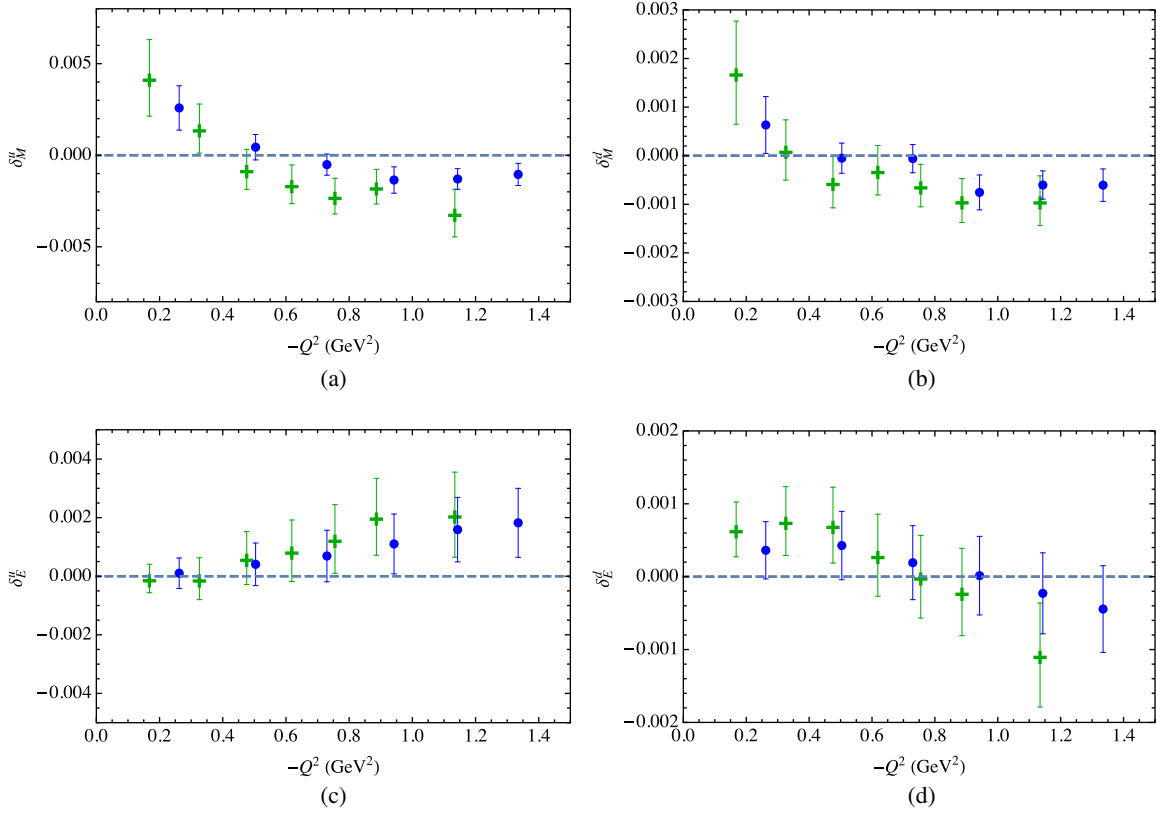


FIG. 3 (color online). Individual up and down quark contributions to the CSV form factors. These terms are combined to give the total CSV form factors $G_{\text{CSV}} = (\frac{2}{3}\delta_{E/M}^d - \frac{1}{3}\delta_{E/M}^u)$. Blue points and green crosses show the results of data sets I and II extrapolated to the physical point, with corrections applied to model the omitted disconnected terms.

(depending which of $\delta_{E/M}^{u/d}$ one is considering), while at larger values of Q^2 , consistent with the suppression of meson loops at high Q^2 , it contributes 1%–15%.

The results of this analysis for the individual u - and d -quark contributions to the CSV electric and magnetic form factors of the proton are shown in Fig. 3. The close agreement of the two sets of simulations (at different lattice

spacings a and on different simulation volumes) confirms that the finite-volume corrections and chiral extrapolations are under control and that any discretization effects resulting from the finite lattice spacing are small. The size of the CSV form factor combination, G_{CSV} , relevant to PVES experiments probing the strange electric and magnetic form factors of the nucleon by Eq. (1), is shown in Fig. 4, and a comparison of our results with previous determinations

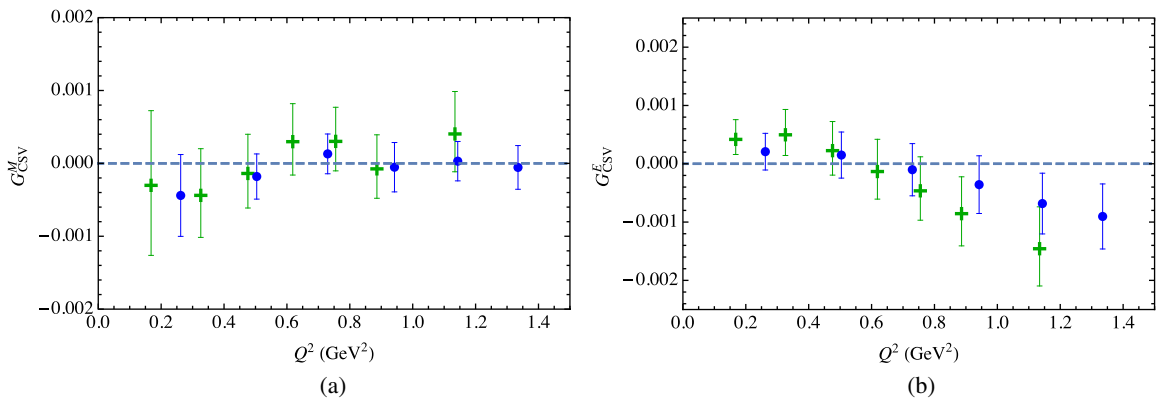


FIG. 4 (color online). Magnetic and electric CSV form factors as relevant to experimental determinations of nucleon strangeness. The blue circles and green crosses denote our results based on simulation sets I ($a^2 = 0.0055(3)$ fm²) and II ($a^2 = 0.0038(2)$ fm²), respectively.

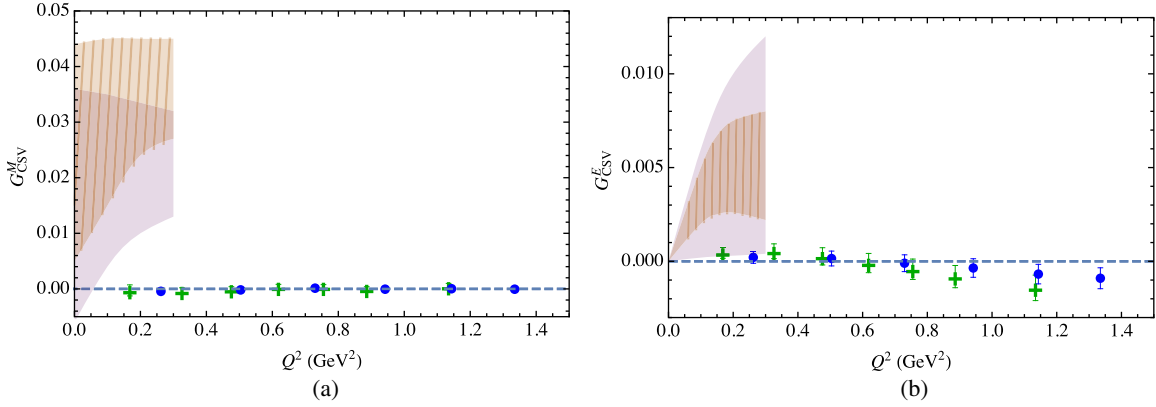


FIG. 5 (color online). Magnetic and electric CSV form factors as relevant to experimental determinations of nucleon strangeness, as in Fig. 4. The orange (striped) and purple (plain) bands show model calculations of these terms taken from figures in Refs. [21] and [20], respectively, (where in the latter case we have shown bands which span the full range of results given for various choices of the model parameters).

(Refs. [21] and [20]) is given in Fig. 5. This result gives quantitative confirmation that CSV effects in the electromagnetic form factors, for momentum transfers up to approximately 1.4 GeV², are bounded at the level of 1%, i.e., $\delta_{E/M}^{u/d}/G_{E/M}^{p,u/d} < 0.01$. This is an order of magnitude smaller than the precision of existing PVES studies. In particular, the most precise experimental determination of the strange nucleon form factors is the HAPPEX Collaboration result $G_E^s + 0.09G_M^s = 0.007(14)$ (assuming perfect charge symmetry) at $Q^2 = 0.109$ GeV² [19]. The additional uncertainty from previous bounds on CSV effects (from Ref. [21]) was 0.009 at this value of the momentum transfer; the same order as the experimental uncertainty. Our results constrain CSV effects to contribute less than 0.0009 at 1-sigma, a systematic effect which is an order of magnitude smaller. To put this in perspective, the level of CSV shown in Fig. 4(b) is equivalent to a CSV difference in charge radii of less than one attometer. These precise results open the door for a new generation of

experiments to probe the structure of the quantum vacuum through the strange quark form factors.

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