## Comment on "Solidification of a Supercooled Liquid in a Narrow Channel"

In a recent Letter [1], Sabouri-Ghomi *et al.* report on simulations of solidification in a channel geometry using a phase-field method. They conducted two series of simulations, one with very small surface tension (relative to the channel width), and the other at larger surface tension. This system has been studied extensively both analytically and numerically [2–7]. The authors appear to have overlooked the more recent of these studies [4–7], which can contribute to a clearer understanding of the simulations.

We first address the large surface tension case. The authors find a transition from a tip widening instability for low undercooling, $\Delta \lesssim 0.5$, to a stable finger for $\Delta \gtrsim 0.5$. The tip widening instability, as noted by [4], is a result of the lack of steady-state solutions for sufficiently low $\Delta$. This threshold in $\Delta$ is a function of surface tension $d_0$ and anisotropy $\epsilon$. Thus, for example, at $d_0 = 0.01$ (in units where the channel width $L = 2$), there are no steady-state solutions below $\Delta \approx 0.62$ for zero anisotropy and below $\Delta \approx 0.60$ at $\epsilon = 0.1$ [6]. (Note that we define $\epsilon$ as the anisotropy of the surface *tension* and not of the surface *energy,* as in Ref. [1], which is a factor of 15 smaller for 4-fold anisotropy.) This threshold decreases with $d_0$, going to 0 as $d_0 \to 0$, and also with increasing anisotropy. Above this threshold, there exists a bilaterally symmetric finger, which is the channel analog of the free dendrite. We have extended the calculation [6] of the threshold to the parameters of the large surface tension case of Ref. [1], namely (in our units), $d_0 = 0.005\,63$ and $\epsilon = 0.75$. The results are presented in Fig. 1, where we show the Péclet number (dimensionless velocity), $p$, as a function of $\Delta$. We see that the transition from the unstable Saffman-Taylor branch to the dendritic branch occurs at $\Delta = 0.49$, in good agreement with the onset of stable dendritic growth reported in Ref. [1]. The stable pattern seen above the threshold is thus indeed the channel analog of the dendrite [4,6], and not a new type of solution. Also, this solution exists and is, over some range of $\Delta$, stable [4,6], even with isotropic surface tension. Also, this discussion should make it clear that there is nothing special happening at $\Delta = 0.5$, which is of course the distinguished value in the Saffman-Taylor problem. We should also point out that at large $\Delta$, there also exist nonsymmetric fingers [4–7], which have been termed parity-breaking dendrites, or doublons. However, for such a large value of anisotropy, the minimum undercooling for these solutions is larger than those used in the simulations [6].

For small $d_0$, only fingers which slow down in time, and widen correspondingly, were seen in the simulations [1]. The surprise here is the absence of stable translating dendrites. These simulations were performed at very small $\Delta$, so it is possible that here, too, the parameters of the simulation were below the threshold for dendrites. The effective $d_0$ for this series of runs may in fact be much larger than the nominal value $d_0 = 1.6 \times 10^{-5}$ quoted, since it is extremely difficult to adequately resolve the interface over such small scales, even with adaptive gridding.

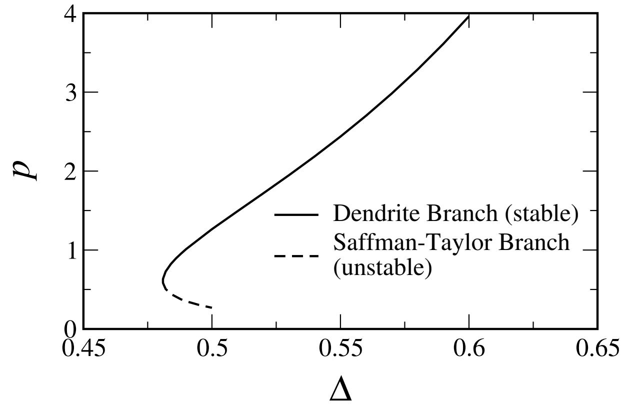


FIG. 1. Péclet number vs supersaturation $\Delta$ for symmetric solutions in a channel at $d_0 = 0.005\,63$ and $\epsilon = 0.75$.

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[1] M. Sabouri-Ghomi, N. Provatas, and M. Grant, Phys. Rev. Lett. **86**, 5084 (2001).
[2] D. A. Kessler, J. Koplik, and H. Levine, Phys. Rev. A **34**, 4980 (1986).
[3] E. A. Brener, M. Geilikman, and D. E. Temkin, Sov. Phys. JETP **67**, 1002 (1988).
[4] E. Brener, H. Müller-Krumbhaar, Y. Saito, and D. Temkin, Phys. Rev. E **47**, 1151 (1993).
[5] T. Ihle and H. Müller-Krumbhaar, Phys. Rev. Lett. **70**, 3083 (1993); Phys. Rev. E **49**, 2972 (1994).
[6] R. Kupferman, D. A. Kessler, and E. Ben-Jacob, Physica (Amsterdam) **213A**, 451 (1995).
[7] M. Ben Amar and E. Brener, Phys. Rev. Lett. **75**, 561 (1995); Physica (Amsterdam) **98D**, 128 (1996).

  