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Does the shock wave in a highly ionized non-isothermal plasma really exist?

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Here, we study the structure of a highly ionizing shock wave in a gas of high atmospheric pressure. We take into account the gas ionization when the gas temperature reaches few orders above ionization potential. It is shown that after gasdynamic temperature-raising shock and formation of a highly-ionized nonisothermal collisionless plasma $T_e \gg T_i$, only the solitary ion-sound wave (soliton) can propagate in this plasma. In such a wave, the charge separation occurs: electrons and ions form the double electric layer with the electric field. The shock wave form, its amplitude, and front width are derived. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4930157]

I. INTRODUCTION

The shock wave in a medium is a propagating disturbance. It is characterized by an abrupt, nearly discontinuous change (shock) in thermodynamic parameters of the medium (for simplicity, we consider only the perfect gas). In a case of the plane shock wave, the gas parameters (pressure, temperature, density) before and after the shock are constant and homogeneous and for a strong wave, they can be interrelated like the following:[1]

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}M^2, \quad \frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}M^2, \quad \frac{n_2}{n_1} = \frac{\gamma+1}{\gamma-1}. \quad (1)$$

Here the subscripts 1 and 2 are referred, respectively, to the gas state before and after the shock front, M—Mach number is the ratio of speed of shock wave front and the speed of sound in a gas before the front, $\gamma = \frac{C_p}{C_v}$—adiabatic exponent, the ratio of the heat capacity at a constant pressure and the heat capacity at a constant volume (for one atomic gas $\gamma = 5/3$). These states are related to the non-isothermal shock. In a case of the isothermal shock when the thermal conductivity (for example, thermal radiation) in a medium is quite high, the relation (1) will transform into the following constraint:[1]

$$P_2 > P_{2min}, \quad P_{2min} = P_1\frac{\gamma+1}{3-\gamma}. \quad (2)$$

At $\gamma = 1$ $P_{2min} = P_1$, hence in such a medium, the shock wave does not exist because P_2 becomes equal to P_1. However, it does not mean that nonlinear waves cannot exist in such a medium. For example (we will demonstrate it further down), in a plasma, the propagation of a soliton is possible, whereas the plasma state before the wave front-side and behind the back-side of the front does not change, whereas in a gas, the states are different.

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Let us return to the shock waves and discuss the structure of the transition layer (wave front width) in a gas. There are a lot of works devoted to such a problem mainly related to the weak shocks ($M \simeq 1$). The waves with the arbitrary intensity were considered by Tamm in 1947. However, the work was first published in 1965.[2] In a case of the weak shock wave ($M \simeq 1$), the structure of the transition layer defined in a framework of hydrodynamics is given by the following formula:

$$d = \frac{P_2+P_1}{P_2-P_1}\delta, \quad \delta = 1.28l, \quad (3)$$

where l—mean free path of gas particles. From Eq. (3), it follows that the front width is greater than the free path and with an increase in P_2/P_1 - $d \to \delta$ limits to the mean free path.

In a case of the strong shock wave ($P_2/P_1 \gg 1$), the wave front width obtained in Ref. 2 is the following:

$$d = 0.503l_1 = 2.012l_2, \quad (4)$$

where l_1 and l_2 are the particles free paths in the front-side and back-side of the shock wave, respectively. The results obtained in the work[2] in the hydrodynamic approximation were found in a good agreement with those obtained in a frame of the Boltzmann approximation, i.e., in the kinetic approximation. However, a simplified approximation was used: Boltzmann equation with an account of the elastic collisions in a framework of the hard spheres.

II. STRONG IONIZING SHOCK WAVE

In this work, we consider the physical problem whether the non-isothermal fully ionized plasma, once formed, can support another shock wave by comparing with the similar problem of formation of a shock wave in a gas and by determining an ion-acoustic wave profile. The plasma is considered in a framework of the Vlasov collisionless model. Hence, in our case, the adiabatic exponent γ does not change.

In gases of high pressure (atmospheric and higher one), the structure of the strong shock waves described in Ref. 2 is too simplified. At Mach number greater than 10 due to the gas heating the gas ionization and generation of highly ionized plasma must occur what was not taken into consideration in Ref. 2. As a result, the structure of the strong ionizing wave must have few peculiarities. Namely, right behind the front-side of the shock wave at a free path length, the gas temperature gets increased to several electron-volts, leading to the gas ionization and formation of a strongly ionized non-isothermal plasma. The electron temperature in such a plasma is of order of a gas atom ionization potential, whereas the ion temperature is much less than that of electrons. At the same time according to the gas-phase approximation, Debye electron radius is less than electron free path. This phase transition occurs at an electron constant temperature of order of an atom ionization potential. It lasts for few orders of an inverse ionization frequencies until the whole gas gets ionized. The width of this domain amounts to few ionization free paths. As a result, the fully ionized non-isothermal plasma with the electron temperature of order of few atom ionization potentials gets formed behind the front in which the shock wave should travel.

We should ask ourselves: "Is this possible?".—"No, it is not!" The point is that the acoustic oscillation mode in non-isothermal highly ionized plasma exists only when $T_e \gg T_i$—the ion sound is an isothermal sound with $\gamma = 1$ ($T_e = \text{const}$ due to the high electron thermal conductivity).[3–6] According to Eq. (2), the isothermal shock in a neutral gas at $\gamma = 1$ is not possible. As it will be demonstrated below, in non-isothermal plasma when $T_e \gg T_i$, only the propagation of a solitary wave (soliton) with the half-width equal to the Debye electron radius is possible. This ion-acoustic wave represents the double electric layer in which the electrons (electron layer) overtake the ions (ion layer) and pull them on. The electric potential of the double layer can be derived using the following system of equations of motion of the plasma:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{e}{M}\frac{\partial \Phi}{\partial x}, \quad \frac{\partial n_i}{\partial t} + \frac{\partial (n_i v)}{\partial x} = 0$$
$$\frac{\partial^2 \Phi}{\partial x^2} = -4\pi e\left(n_i - n_0 e^{e\Phi/T_e}\right), \tag{5}$$

here v—plasma velocity, Φ—plasma electric potential, $n_e = n_0 \cdot \exp(e\Phi/T_e)$, n_i—electron and ion densities, respectively, n_0—density of the electrons and ions at $\Phi = 0$ (for simplicity $Z = 1$). Having solved the system, one can get the following equation for the electric potential described as a weakly nonideal wave:[4]

$$\frac{T_e r_{De}^2}{2e}\frac{d^2\Phi}{d\xi^2} - \frac{T_e}{e}\left(1 - \frac{V_s}{u}\right)\Phi + \frac{1}{2}\Phi^2 = 0, \tag{6}$$

here $\xi = x - ut$, $r_{De} = \sqrt{T_e/4\pi e^2 n_0}$—Debye electron radius, where $V_s = \sqrt{T_e/M}$—ion sound velocity, u—to be determined velocity of a soliton equal to the velocity of the shock wave.

Equation (6) known as the Kortewegde Vries equation (KdV equation for short) has the following explicit solution or one-dimensional ion-acoustic wave profile (see Fig. 1):

$$\Phi = \frac{\Phi_{max}}{ch^2(\xi/\Delta)}, \tag{7}$$

where

$$1 - \frac{V_s}{u} = \frac{e\Phi_{max}}{\pi T_e} \ll 1, \quad \frac{e\Phi_{max}}{6T_e} = \frac{r_{De}^2}{\Delta^2} \ll 1. \tag{8}$$

As one can see, $u \geqslant V_s$, i.e., the velocity of a solitary wave is almost equal to that of an ion sound, Δ—the wave full width at half maximum, $\Delta > r_{De}$ and decreases with increasing wave amplitude.

III. DISCUSSION OF THE RESULTS AND QUANTITATIVE ESTIMATIONS

Equations (7) and (8) are exact on implementation domain and describe exactly the solitary wave (7) in a plasma of the highly ionizing shock wave with temperature $T_e \sim 3I_i \sim 3 \times 10^5$ K, where $I_i \sim 10$ eV—gas atom ionization potential. At a atmospheric gas pressure $r_{De} \sim 7 \times 10^{-7}$ cm and the mean distance between plasma electrons $r_a \sim 3 \times 10^{-7}$ cm, i.e., the gas-phase approximation condition ($r_a^2 \ll r_{De}^2$) is satisfied. According to Eq. (8), potential amplitude increases with decreasing Δ and at $\Delta = 3r_{De}$ (the minimum magnitude at which Eqs. (6)–(8) are valid) becomes equal to

$$E_{max} \leq 10^7 \text{V/cm}. \tag{9}$$

There exists another constraint for the implementation domain of Eq. (7), namely, the small plasma disturbance by a soliton field when

$$E \leq \sqrt{4\pi n_e T_e} \sim 10^8 \text{V/cm}. \tag{10}$$

This field is one order higher than the field (9). The field $E = 10^7$ V/cm is high enough to create the electric discharge in a solid body (for example, in a glass). When the condition (9) is violated, then the plasma gets exploded, electrons will break apart of ions and get accelerated by the field. However, this stage is out of our consideration and requires a more detailed investigation.

For realization of the above process in atmosphere at normal conditions, the Mach number of the shock wave must be of order $M \sim \sqrt{T_e/T_0} \sim 33$. When the Mach number is quite high, then the plasma explosion occurs forming the electron gas where the field will exceed the magnitude (9) and the front width will increase with increasing field. This stage is out of our consideration here in this work.

In this way, the strong ionizing shock wave in a gas must have a very complex structure: right in the back-side of the front at a length of the mean free path of gas particles before the front, the temperature increases to few electron volts and the gas ionization occurs. In the formed plasma behind this temperature-raising, shock traveling only of the ion-sound soliton is possible. The soliton half-width is of order of hot electrons Debye radius. In a case of the strong shock wave, the ion-sound soliton gets transformed into the extended double layer with the following electric field:[6]

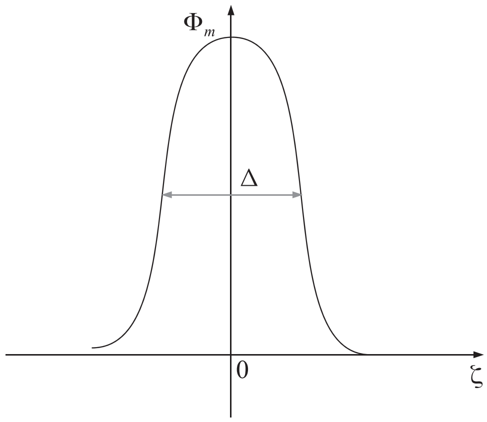

FIG. 1. A one-dimensional ion-acoustic wave profile equation (7).

$$E \sim \sqrt{4\pi n_e T_e} = \sqrt{4\pi n_e T_0 T_e / T_0} = M\sqrt{4\pi n_e T_0}, \quad (11)$$

where T_0—gas temperature in the front-side of the front, $M \gg 1$—Mach number, n_e—density of a plasma formed behind the front which is close to the neutral gas density. Relation (1) shows that compared to the normal gas where the pressure behind the front of the strong shock wave is M^2 higher than that in front-side, in a plasma, the plasma gets polarized behind the front of the shock wave producing the field with the same pressure.

According to the above description, in non-isothermal plasma formation, propagation of the ion-sound soliton (and not of the shock wave) is possible. From Eqs. (6) and (7), it follows that the soliton represents the blob of a electromagnetic field revealing not only the resonance formation of the soliton[7] but also that the soliton is a resonator itself having the electromagnetic field which can lead to the electric discharge at soliton disturbance, for example, at a collision with the hard target like glass. The latter, discharges inside the window glasses, were revealed after the Chelyabinsk meteor explosion that entered Earth's atmosphere over Russia on 15 February 2013 at about 09:20 YEKT.[8]

It is particularly interesting to discuss the concept of the shock waves in plasmas which got increased attention in the literature.[9–12] The short overview of these works is given in monography.[13] The authors discuss plasma and the long-range Coulomb interactions of charged plasma particles. However, they consider plasma as an one-atomic gas with the adiabatic constant $\gamma = 5/3$ and the ion temperature $T_i \geq T_e$. At the same time, it is well known that the Coulomb system of charged particles can be treated as a plasma only when the Langmuir plasma frequency is much greater than the electron collisions frequency

$$\omega_{Le}^2 \gg \nu_e^2. \quad (12)$$

We would like to note that the sound in a plasma is isothermal (hence $\gamma = 1$) and can exist in non-isothermal plasma only with $T_e \gg T_i$. Namely, in such a collisionless plasma, the density and temperature shocks are impossible, while only solitary waves can propagate therein.

If the inverse constraint (12) is satisfied, then the peculiar plasma properties cannot be revealed; plasma becomes similar to a gas of neutral particles with remaining charged particles playing no principle role. In our point of view, the authors of works[9–13] meant such plasma and the words (here citation): "In a plasma, an electron is strongly bound to the ion by the charge separation field and they move together as a whole unit" have nothing to do with the plasma.

The solution for the KdV-equation for a weakly nonideal case is exact. However, we are planning to run simulations for a strongly nonideal case.

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