

Simple operation sequences to couple and interchange quantum information between spin qubits of different kinds

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(Received 13 July 2015; revised manuscript received 7 September 2015; published 30 September 2015)

Efficient operation sequences to couple and interchange quantum information between quantum dot spin qubits of different kinds are derived using exchange interactions. In the qubit encoding of a single-spin qubit, a singlet-triplet qubit, and an exchange-only (triple-dot) qubit, some of the single-qubit interactions remain on during the entangling operation; this greatly simplifies the operation sequences that construct entangling operations. In the ideal setup, the gate operations use the intraqubit exchange interactions only once, and entangling operations with gate times similar to typical single-qubit operations are constructed. The limitations of the entangling sequences are discussed, and it is shown how quantum information can be converted between different kinds of quantum dot spin qubits. These gate sequences are useful for large-scale quantum computation because they show that different kinds of coded spin qubits can be combined easily, permitting the favorable physical properties of each to be employed.

DOI: [10.1103/PhysRevB.92.115448](https://doi.org/10.1103/PhysRevB.92.115448)

PACS number(s): 03.67.Lx, 03.67.Ac, 73.21.La, 85.35.Be

I. INTRODUCTION

Small arrays of singly occupied quantum dot (QD) qubits are now fabricated in GaAs and Si with great reliability [1,2]. These setups are of high interest for quantum computation because the electron spin can be used as a qubit [3]. Besides the single-spin qubit encoding, also more advanced qubit encodings have been suggested. Most promising are the singlet-triplet qubit (STQ) [4] and the exchange-only qubit [5]. These qubits encode quantum information in the $s_z = 0$ spin subspace of a two-electron double QD (DQD) or in two of the eight possible spin configurations of a three-electron triple QD (TQD).

For all the described qubits, single-qubit gates have been realized with high fidelities. Electric [6,7] or magnetic [8–10] field pulses can nowadays control single spins with very high fidelities. High-fidelity gates for STQs are also possible when the electron configuration of the DQD is modified, while the magnetic field across the DQD is inhomogeneous [11]. Experimentally, a preparation of the nuclear magnetic field [12,13] or a micromagnet [14] created such static magnetic field configurations. The three-electron TQD can be operated using exchange interactions alone [5,15,16]; more optimal qubit control has been realized if some of the exchange interactions are not reduced to zero [17,18]. Two-qubit gates have been proposed for all the qubit encodings using exchange couplings [3–5,19,20], while experiments have realized these gates only for single-spin qubits [21]. STQs or exchange-only qubits can be coupled indirectly via their charge sector, e.g., using Coulomb interactions [22] or couplings via cavity modes [17,23,24]. These approaches have not been successful yet due to a high amount of dephasing that is caused by charge noise [25–28].

There are pros and cons to either qubit encoding. The longest coherence times were measured for single-spin qubits [9,29,30]. While short-distance exchange couplings realized high-fidelity entangling gates [21], high-fidelity long-

distance couplings remain difficult. Single-qubit operations for STQs are arguably even easier than for the single-spin qubits because they use electric control pulses, while only a static magnetic field gradient across the DQDs needs to be prepared [11]. STQs can be tuned to operation points where the qubit states have different charge characters, which makes these STQs a more natural candidate for long-distance couplings (e.g., via cavities) than single-spin qubits. The exchange-only qubit is the natural generalization of a STQ: single-qubit and two-qubit gates can be controlled all-electrically (even without preparations of local magnetic fields) [5]. Modifications of their operation points also allows long-distance couplings with methods similar to for the STQs.

The present study assumes that universal qubit control is possible for the encoded qubits, while two QDs from different qubits are exchange coupled. Operation sequences for entanglement generation and qubit conversion are derived between QD qubits of different kinds. The operation sequences profit from always-on single qubit Hamiltonians during the entanglement sequences, as in earlier studies of TQDs [20,31]. For STQs, the magnetic fields at the QDs should be prepared independently. Their values need to differ anyway to realize single-qubit control. For the exchange-only qubit, a linear TQD arrangement is considered. Here, the exchange couplings between the neighboring pairs of QDs remain always at similar magnitudes. Such setups have been used in a previous experiment [17,18]. The TQD is operated in the (1,1,1) configuration (i.e., there is one electron at each QD), while virtual tunnelings of the electron at the middle QD to the outer QDs are strongly enhanced by increasing the chemical potential of the middle QD compared to the outer QDs [17,18].

The main findings of this paper are explicit operation sequences to entangle QD qubits of different kinds. The always-on single-qubit couplings greatly simplify the operation sequences because they reduce the possibility of leakage from the computational subspace. Effective Hamiltonians and entangling sequences are derived; the setups only require two operation sequences to entangle a single-spin qubit and a STQ (or an exchange-only qubit and a STQ), or four operation sequences

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to entangle a single-spin qubit and an exchange-only qubit. It is shown how the entanglement sequences can be used to swap quantum information between the qubits, and the limitations of the operation sequences are discussed. It should be emphasized that only the standard parameter regimes of current experiments are used to operate the spin qubits. Even though the interqubit exchange couplings are weak, still, the gate times of the entangling gates reach tens of nanoseconds. That is, they are comparable to the gate times for normal single-qubit gates.

The simplicity of the entangling operations shows that a large lattice of QD qubits does not necessarily need to contain identical types of coded qubits (e.g., the description of large scale quantum computation with STQs in Ref. [32]). One can easily convert and couple different QD qubits using the operation sequences derived in this paper. The standard fault-tolerant quantum computation approaches, like the surface code [33], permit combinations of different kinds of coded qubits. As a consequence, it is possible to use a qubit encoding just for the situation when it is most optimal. It is known that single-spin qubits have exceptionally long coherence times, which makes them an ideal quantum memory [8,34]. Encoded spin qubits, like the STQ or the exchange-only qubit, can be employed in their orbital sector, which makes them more ideal for readout or for long-distance couplings [1,22]. It is also possible to use the described operation sequences to couple QD spin qubits to other spin qubits, like, e.g., donor-bound spin qubits [35]. The electron spin bound to a donor atom is a well-known qubit candidate with many impressive experiments of coherent spin control in recent years [8,29,30,36]. Also tunnel couplings between donor-bound and gate-defined spin qubits were shown recently [37,38].

The organization of the paper is as follows. Section II introduces the mathematical descriptions of the single-spin qubit, the STQ, and the exchange-only qubit. Section III derives the operation sequences to entangle QD qubits of different qubit encodings. Section IV discusses the limitations of these operations and describes how quantum information is converted between different qubits. Finally, the results of the paper are summarized.

II. QUBIT DEFINITIONS

A. Single-spin qubit

A single spin defines a qubit using the states $|0\rangle = |\uparrow\rangle$ and $|1\rangle = |\downarrow\rangle$ [3]. Universal qubit control is realized when a magnetic field can be tilted to two different directions. The control mechanisms to manipulate spins are magnetic field pulses [9,39], moving spins in static magnetic fields with spin-orbit interactions [40], and driving spins through areas of different magnetic fields [6,41,42]. The standard operating schemes have in common that microwave control pulses enable Rabi-like gates [43]. Without further discussing the exact mechanism, it is assumed here that the magnetic field direction can be rotated to the z and x directions to generate rotations around the z and x axes of the Bloch sphere. These single-qubit gates are labeled $Z_\phi = e^{-i2\pi\frac{\phi}{2}\sigma_z}$ and $X_\phi = e^{-i2\pi\frac{\phi}{2}\sigma_x}$, where $\sigma_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ and $\sigma_x = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$ are the Pauli operators. The phase accumulation $\phi = E_z t/h$ ($\phi = E_x t/h$)

is caused by the Zeeman energy $E_z = g\mu_B B_z$ ($E_x = g\mu_B B_x$) of the magnetic field in the z direction (x direction).¹

B. Singlet-triplet qubit

STQs are coded using the $s_z = 0$ spin subspace of a two-electron DQD [4]. QD₁ and QD₂ label the individual QDs of the DQD. Ideally, the electrons are spatially separated, and each QD is occupied with one electron. The logical qubit states are defined by $|0\rangle = |\uparrow\downarrow\rangle$ and $|1\rangle = |\downarrow\uparrow\rangle$, where the first entry labels the electron at QD₁, and the second entry labels the electron at QD₂. Single-qubit control is realized using a magnetic field gradient between the QDs, corresponding to energy differences $(\Delta E_z/2)(\sigma_{z,1} - \sigma_{z,2})$, with $\Delta E_z = (E_{z,1} - E_{z,2})/2$, and the exchange interaction between the QD electrons $(J_{12}/4)(\sigma_1 \cdot \sigma_2 - \mathbf{1})$. $\sigma_i = (\sigma_{x,i}, \sigma_{y,i}, \sigma_{z,i})$ is the vector of Pauli matrices at QD_{*i*}.

ΔE_z is usually static in experiments, but J_{12} can be tuned within subnanoseconds by controlling the tunnel coupling or the potential difference of the QDs [44]. The magnetic field gradient generates rotations around the z axis of the Bloch sphere $Z_\phi = e^{-i2\pi\frac{\phi}{4}(\sigma_{z,1} - \sigma_{z,2})}$, with $\phi = 2\Delta E_z t/h$, and rotations around the x axis are caused by the exchange interaction $X_\phi = e^{-i2\pi\frac{\phi}{4}(\sigma_1 \cdot \sigma_2 - \mathbf{1})}$, with $\phi = J_{12} t/h$. To reduce the leakage probability, experiments are always done at global magnetic fields $(E_z/2)(\sigma_{z,1} + \sigma_{z,2})$, with $E_z = (E_{z,1} + E_{z,2})/2$, that lift the degeneracy between the leakage states $\{|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ and the computational subspace $\{|0\rangle, |1\rangle\}$.

C. Exchange-only qubit

The exchange-only qubit is coded using the $S = \frac{1}{2}$, $s_z = \frac{1}{2}$ subspace of three electrons [5]. The encoding of the exchange-only qubit in a subspace of the three-spin-1/2 Hilbert space (the subspace encoding) is strictly required for the operation sequences that are derived in this paper [45]. An alternative exchange-only qubit encoding (the subsystem encoding) equally permits the qubit initialization to the $S = \frac{1}{2}$, $s_z = \frac{1}{2}$, and $S = \frac{1}{2}$, $s_z = -\frac{1}{2}$ spin subspaces. In any case, working with the subspace encoding only requires a proper initialization routine. A strong, global magnetic field eases the state initialization because then the $S = \frac{1}{2}$, $s_z = \frac{1}{2}$, and $S = \frac{1}{2}$, $s_z = -\frac{1}{2}$ spin subspaces have different energies. Practically, one is always able to initialize a singlet for a doubly occupied QD, and the ground-state spin configuration for a singly-occupied QD.

The three singly-occupied QDs are labeled by QD₁, QD₂, and QD₃. The qubit states are defined by $|0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle)$ and $|1\rangle = \frac{1}{\sqrt{6}}(|\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\rangle) - \sqrt{\frac{2}{3}}|\uparrow\downarrow\uparrow\rangle$, with the spin labels $|\sigma_{QD_1}, \sigma_{QD_2}, \sigma_{QD_3}\rangle$. The sum of the exchange interactions $(J/4)[(\sigma_1 \cdot \sigma_2 - \mathbf{1}) + (\sigma_2 \cdot \sigma_3 - \mathbf{1})]$, with $J = (J_{12} + J_{23})/2$, and their difference $(\Delta J/4)[(\sigma_1 \cdot \sigma_2 - \mathbf{1}) - (\sigma_2 \cdot \sigma_3 - \mathbf{1})]$, with $\Delta J = (J_{12} - J_{23})/2$, provide universal control of the subspace $\{|0\rangle, |1\rangle\}$. J causes a rotation around the z axis of the Bloch sphere $Z_\phi =$

¹In contrast to Ref. [57], all the phase accumulations are given in multiples of 2π .

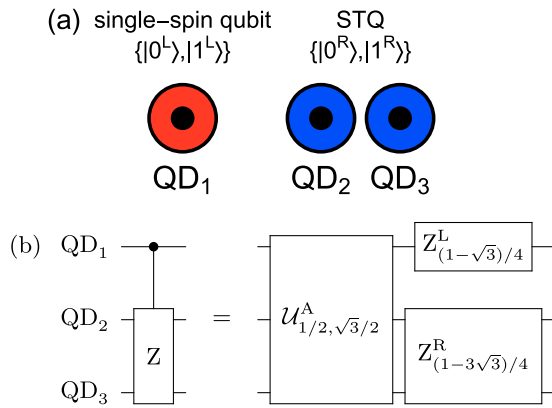


FIG. 1. (Color online) Entangling operation between a single-spin qubit and a STQ. (a) QD₁ defines a single-spin qubit with the qubit levels $\{|0^L\rangle, |1^L\rangle\}$; QD₂ and QD₃ define a STQ with the qubit levels $\{|0^R\rangle, |1^R\rangle\}$. A weak tunnel coupling between QD₁ and QD₂ couples the single-spin qubit and the STQ. (b) Sequence to create a CPHASE between a single-spin qubit (coded on QD₁) and a STQ (coded on QD₂ and QD₃). Z_{ϕ}^L and Z_{ψ}^R are the phase gates of the qubits L and R. $\mathcal{U}_{\phi, \psi}^A$ is defined in Eq. (2).

$e^{-i2\pi \frac{\phi}{4}[(\sigma_1 \cdot \sigma_2 - 1) + (\sigma_2 \cdot \sigma_3 - 1)]}$, with $\phi = Jt/h$, and ΔJ causes a rotation around the x axis $X_{\phi} = e^{-i2\pi \frac{\phi}{4\sqrt{3}}[(\sigma_1 \cdot \sigma_2 - 1) - (\sigma_2 \cdot \sigma_3 - 1)]}$, with $\phi = \sqrt{3}\Delta Jt/h$. In typical qubit manipulation protocols, J is constant and large, while ΔJ is rapidly driven around zero [17,18].

III. INTERFACES BETWEEN SPIN QUBITS

This section derives gate sequences that interconnect all the three kinds of coded qubits using interqubit exchange interactions. The normal parameters of experiments are used for gate-defined QDs in GaAs [1] and Si [2]: all manipulations are done with a global magnetic field E_z . Local magnetic field variations are only permitted for the QDs of STQs, and these variations are parallel to the global magnetic field. Also their magnitudes are much smaller than E_z . All exchange interactions can be tuned instantaneously, and the interqubit exchange interactions can be reduced to zero. These operation principles are idealized, and their limitations will be further discussed in Sec. IV. The exchange interactions between the QDs of the STQ are restricted to magnitudes of the order of the local magnetic field variations; otherwise, charge noise strongly couples to STQs. The TQD is always operated near its optimal operation point, where J is large and ΔJ is small.

A. Single-spin qubit and singlet-triplet qubit

Figure 1(a) shows a trio of singly-occupied QDs that encodes a single-spin qubit and a STQ. QD₁ defines the single-spin qubit, with the qubit levels $\{|0^L\rangle, |1^L\rangle\}$. QD₂ and QD₃ define the STQ, where the qubit levels are called $\{|0^R\rangle, |1^R\rangle\}$. A general Hamiltonian in this setup is

$$\mathcal{H}^A = \frac{J_{12}}{4}(\sigma_1 \cdot \sigma_2 - 1) + \frac{E_z}{2}(\sigma_{z,1} + \sigma_{z,2} + \sigma_{z,3}) + \frac{\tilde{E}_{z,2}}{2}\sigma_{z,2} + \frac{\tilde{E}_{z,3}}{2}\sigma_{z,3}. \quad (1)$$

QD₁ and QD₂ are coupled by the exchange coupling J_{12} that is described by the first term in Eq. (1). The second term describes the global magnetic field E_z , and the last two terms are the deviations of the local magnetic fields at QD₂ and QD₃ from E_z . The exchange interaction between QD₂ and QD₃, $(J_{23}/4)(\sigma_2 \cdot \sigma_3 - 1)$, is neglected in Eq. (1) because J_{23} is reduced to zero (or J_{23} is much smaller than $\tilde{E}_{z,2} - \tilde{E}_{z,3}$).

To construct entangling operations, $(E_z/2)(\sigma_{z,1} + \sigma_{z,2} + \sigma_{z,3})$ and $(\tilde{E}_{z,3}/2)\sigma_{z,3}$ can be neglected because these terms commute with the remaining parts of Eq. (1), and they generate only irrelevant phases. The relevant time evolution is described by

$$\mathcal{U}_{\phi, \psi}^A = e^{-i2\pi[\frac{\phi}{4}(\sigma_1 \cdot \sigma_2 - 1) + \frac{\psi}{2}\sigma_{z,2}]}, \quad (2)$$

with $\phi = J_{12}t/h$ and $\psi = \tilde{E}_{z,2}t/h$. For this gate operation (and for all the following entangling operations) the magnitude of the interqubit exchange interaction J_{12} can be prepared to a specific value, and the evolution time t can be adjusted properly. Even though $\tilde{E}_{z,2}$ is fixed at the beginning of an experiment, still all values of ϕ and ψ can be realized.

Only the states in the subspace $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |1^L 1^R\rangle, |\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\rangle\}$ are coupled in Eq. (2). There is no evolution from computational states to leakage states for $\sqrt{\phi^2 + \psi^2} = \mathbb{Z}$. An entangling operation that is, up to local unitaries, equivalent to the CPHASE operation is realized for $\phi = \mathbb{Z} + \frac{1}{2}$. One can use, e.g., $\mathcal{U}_{1/2, \sqrt{3}/2}^A$. A CPHASE in the basis $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |1^L 1^R\rangle\}$ is [see Fig. 1(b)]

$$Z_{(1-\sqrt{3})/4}^L Z_{(1-3\sqrt{3})/4}^R \mathcal{U}_{1/2, \sqrt{3}/2}^A = e^{-i\frac{\pi}{2}} \text{CPHASE}. \quad (3)$$

Another possible entangling gate is mentioned briefly. Only the states $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |1^L 1^R\rangle\}$ are coupled significantly in the parameter regime $E_z \gg \tilde{E}_{z,2} + \tilde{E}_{z,3} \gg J_{12}$. The leakage transitions to the states $\{|\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\rangle\}$ are very slow because these states are unfavored energetically. One can derive from Eq. (1) an effective interaction on the computational subspace

$$\begin{aligned} & \frac{E_z}{2} [|0^L\rangle\langle 0^L| - |1^L\rangle\langle 1^L|] \\ & + \frac{\tilde{E}_{z,2} - \tilde{E}_{z,3}}{2} [|0^R\rangle\langle 0^R| - |1^R\rangle\langle 1^R|] \\ & + \frac{J_{12}}{4} [|0^L\rangle\langle 0^L| - |1^L\rangle\langle 1^L|] [|0^R\rangle\langle 0^R| - |1^R\rangle\langle 1^R|] \end{aligned} \quad (4)$$

that is entangling. The problem is that the gate operation time is limited by the condition $\tilde{E}_{z,2} + \tilde{E}_{z,3} \gg J_{12}$, which will make such entangling gates too slow for high-fidelity operations.

B. Single-spin qubit and exchange-only qubit

Figure 2(a) shows a quartet of singly occupied QDs that encodes a single-spin qubit (QD₁; qubit states $\{|0^L\rangle, |1^L\rangle\}$) and an exchange-only qubit (QD₂-QD₄; qubit states $\{|0^R\rangle, |1^R\rangle\}$). A general interaction in this setup is

$$\mathcal{H}^B = \frac{J_{12}}{4}(\sigma_1 \cdot \sigma_2 - 1) + \frac{E_z}{2}(\sigma_{z,1} + \sigma_{z,2} + \sigma_{z,3} + \sigma_{z,4}) + \frac{J}{4}[(\sigma_2 \cdot \sigma_3 - 1) + (\sigma_3 \cdot \sigma_4 - 1)]. \quad (5)$$

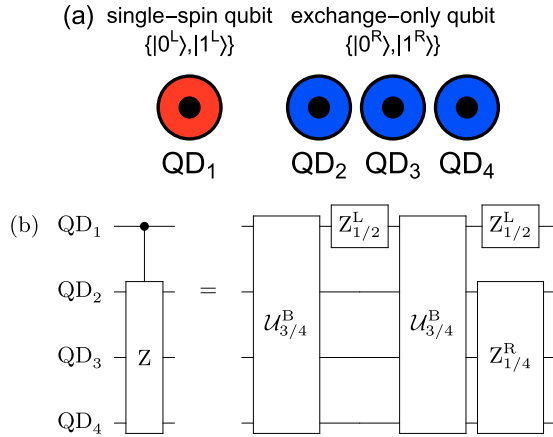


FIG. 2. (Color online) Entangling operation between a single-spin qubit and an exchange-only qubit. (a) QD₁ defines a single-spin qubit with the qubit levels $\{|0^L\rangle, |1^L\rangle\}$; QD₂-QD₄ define an exchange-only qubit with the qubit levels $\{|0^R\rangle, |1^R\rangle\}$. A weak tunnel coupling between QD₁ and QD₂ couples the single-spin qubit and the exchange-only qubit. (b) Sequence to create a CPHASE between a single-spin qubit (coded on QD₁) and an exchange-only qubit (coded on QD₂-QD₄). Z_ϕ^L and Z_ϕ^R are the phase gates of the qubits L and R. U_ϕ^B is defined in Eq. (8).

The first term in Eq. (5) is the exchange coupling between QD₁ and QD₂. The second term is the global magnetic field, and the third term describes the exchange couplings of the exchange-only qubit.

$$\mathcal{H}^B = \begin{pmatrix} E_z - \frac{J}{2} - \frac{J_{12}}{4} & \frac{J_{12}}{4\sqrt{3}} & & & & & -\frac{J_{12}}{\sqrt{6}} \\ \frac{J_{12}}{4\sqrt{3}} & E_z - \frac{3J}{2} - \frac{J_{12}}{12} & & & & & \frac{J_{12}}{3\sqrt{2}} \\ & & -\frac{J}{2} - \frac{J_{12}}{4} & 0 & & & \\ & & 0 & -\frac{J}{2} - \frac{J_{12}}{4} & & & \\ & & -\frac{J_{12}}{4\sqrt{3}} & \frac{J_{12}}{2\sqrt{3}} & & & \\ & & -\frac{J_{12}}{2\sqrt{3}} & \frac{J_{12}}{4\sqrt{3}} & & & \\ & & \frac{J_{12}}{2\sqrt{3}} & \frac{J_{12}}{2\sqrt{3}} & & & \\ -\frac{J_{12}}{\sqrt{6}} & \frac{J_{12}}{3\sqrt{2}} & & & & & E_z - \frac{2J_{12}}{3} \end{pmatrix}. \quad (7)$$

It is sufficient to consider the time evolution in the subspaces of equal energies that are defined by E_z and J . The borders in the matrix of Eq. (7) indicate these subspaces. J_{12} couples these subspaces, but for $E_z, J \gg J_{12}$ these processes can be neglected because the transition amplitudes are much smaller than the energy differences.

After neglecting all the entries outside of the marked subspaces in Eq. (7), also the time evolutions of E_z

$(E_z/2)(\sigma_{z,1} + \sigma_{z,2} + \sigma_{z,3} + \sigma_{z,4})$ commutes with the remaining parts of Eq. (5), and this term causes only an irrelevant time evolution of the single-spin qubit. The relevant time evolution through Eq. (5) is

$$\mathcal{U}_{\phi,\psi}^B = e^{-i2\pi\{\frac{\phi}{4}(\sigma_1 \cdot \sigma_2 - 1) + \frac{\psi}{4}[(\sigma_2 \cdot \sigma_3 - 1) + (\sigma_3 \cdot \sigma_4 - 1)]\}}, \quad (6)$$

with $\phi = J_{12}t/h$ and $\psi = Jt/h$. There are exact entangling operations between a single-spin qubit and an exchange-only qubit that use Eq. (6). However, these sequences are complicated and involve many operation steps.²

Simpler entangling operations can be constructed for $J \gg J_{12}$. The computational subspace is part of the four-spin subspaces $S = 0, s_z = 0$ and $S = 1, s_z = 1, 0$, which together have eight dimensions [46]. Because the Hamiltonian in Eq. (5) preserves the spin quantum numbers, it is sufficient to describe the time evolution only in the four-spin subspaces $S = 0, s_z = 0$ and $S = 1, s_z = 1, 0$ that are spanned by $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |l_1\rangle, |1^L 1^R\rangle, |l_2\rangle, |l_3\rangle, |l_4\rangle\}$, with $|l_1\rangle = |0^L\rangle|u_{-1/2}\rangle$, $|l_2\rangle = |0^L\rangle|v_{-1/2}\rangle$, $|l_3\rangle \propto |0^L\rangle|Q_{-1/2}\rangle - |1^L\rangle|Q_{1/2}\rangle$, and $|l_4\rangle \propto \sqrt{3}|1^L\rangle|Q_{3/2}\rangle - |0^L\rangle|Q_{1/2}\rangle$. The states $|u_{-1/2}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle)$ and $|v_{-1/2}\rangle = \frac{1}{\sqrt{6}}(|\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle) - \sqrt{\frac{2}{3}}|\downarrow\uparrow\downarrow\rangle$ span the $S = \frac{1}{2}, s_z = -\frac{1}{2}$ spin subspace of three electrons; $|Q_{3/2}\rangle = |\uparrow\uparrow\uparrow\rangle$, $|Q_{1/2}\rangle \propto |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle$, $|Q_{-1/2}\rangle \propto |\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle$, and $|Q_{-3/2}\rangle = |\downarrow\downarrow\downarrow\rangle$ are the $S = \frac{3}{2}$ quadruplet states of three spins. The spin labels correspond to $|\sigma_{QD_2}, \sigma_{QD_3}, \sigma_{QD_4}\rangle$ in these state definitions.

The projection of Eq. (5) to the given basis is

and J factor because they commute with the remaining entries. The global magnetic field $(E_z/2)(\sigma_{z,1} + \sigma_{z,2} + \sigma_{z,3} + \sigma_{z,4}) \simeq (E_z/2)(|0^L\rangle\langle 0^L| - |1^L\rangle\langle 1^L|)$ and the exchange interaction $(J/4)[(\sigma_2 \cdot \sigma_3 - 1) + (\sigma_3 \cdot \sigma_4 - 1)] \simeq (J/2)(|0^R\rangle\langle 0^R| - |1^R\rangle\langle 1^R|)$ cause single-qubit time evolutions that will be neglected in the following [note that these approximations require the previous assumptions where the terms outside of the marked regions in Eq. (7) are neglected]. Equation (6) can then be simplified on the subspace $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |l_1\rangle, |1^L 1^R\rangle, |l_2\rangle\}$ to

$$\mathcal{U}_\phi^B \approx e^{-i2\pi\phi \text{diag}\{-\frac{1}{4}, -\frac{1}{12}, -\frac{1}{4}, 0, -\frac{5}{12}, -\frac{1}{12}\}}. \quad (8)$$

$\text{diag}\{a, b, \dots\}$ describes the matrix with the diagonal entries a, b, \dots , and $\phi = J_{12}t/h$.

A single time evolution under Eq. (8) is never entangling because the criterion to prevent leakage only permits

²We found operation sequences to create entangling operations with a numerical search algorithm, similar to the description in Ref. [19]. An operation sequence that is equivalent to a CPHASE is

$$\mathcal{U}_{\phi_1, \phi_2}^B \mathbf{X}_{\phi_3}^R \mathcal{U}_{\phi_1, \phi_2}^B Z_{1/2}^L \mathbf{X}_{\phi_4}^R \mathcal{U}_{\phi_1, \phi_2}^B \mathbf{X}_{\phi_3}^R \mathcal{U}_{\phi_1, \phi_2}^B,$$

with $\phi_1 = 0.195613200942698$, $\phi_2 = 0.2178346646839128$, $\phi_3 = 0.7362256575556158$, and $\phi_4 = 0.735072280195903$.

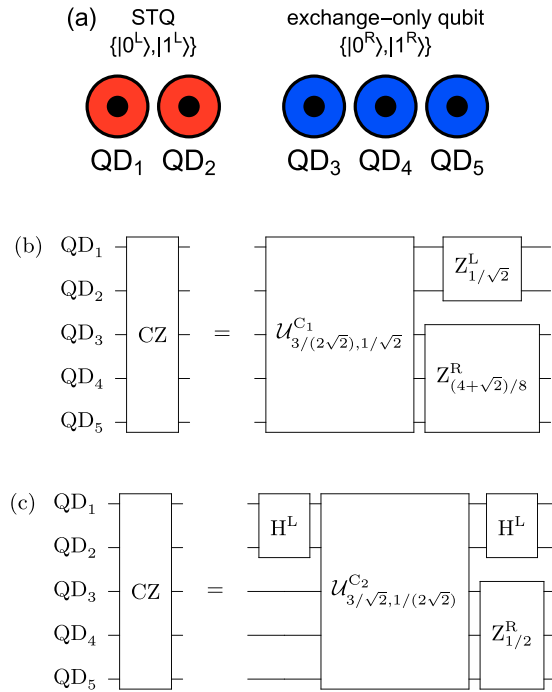


FIG. 3. (Color online) Entangling operations between a STQ and an exchange-only qubit. QD₁ and QD₂ define a STQ with the qubit levels $\{|0^L\rangle, |1^L\rangle\}$; QD₃-QD₅ define an exchange-only qubit with the qubit levels $\{|0^R\rangle, |1^R\rangle\}$. A weak tunnel coupling between QD₂ and QD₃ couples the STQ and the exchange-only qubit. (b) and (c) Sequences to create a CPHASE between a STQ (coded on QD₁ and QD₂) and an exchange-only qubit (coded on QD₃-QD₅). Z_ϕ^L and Z_ϕ^R are the phase gates of the qubits L and R. $\mathcal{U}_{\phi,\psi}^{C_1}$ and $\mathcal{U}_{\phi,\psi}^{C_2}$ are defined in Eqs. (12) and (16). The CPHASE gate is abbreviated as CZ, and H^L is the Hadamard gate for qubit L.

single-qubit gates. The two-step sequence $\mathcal{U}_\phi^B Z_{1/2}^L \mathcal{U}_\phi^B$ is equivalent to a CPHASE gate for $\phi = \frac{3}{4} + \frac{3}{2}\mathbb{Z}$. A CPHASE operation in the basis $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |1^L 1^R\rangle\}$ is created by [see Fig. 2(b)]:

$$Z_{1/2}^L Z_{1/4}^R \mathcal{U}_{3/4}^B Z_{1/2}^L \mathcal{U}_{3/4}^B = \text{CPHASE}. \quad (9)$$

Note that the implicit single-qubit phase evolutions through E_z and J , that are neglected in Eq. (8), need to be included in $Z_{1/2}^L$ and $Z_{1/4}^R$.

C. Singlet-triplet qubit and exchange-only qubit

A quintet of singly-occupied QDs, as shown in Fig. 3(a), defines a STQ (QD₁-QD₂; qubit states $\{|0^L\rangle, |1^L\rangle\}$) and an

exchange-only qubit (QD₃-QD₅; qubit states $\{|0^R\rangle, |1^R\rangle\}$). A possible interaction in this setup is

$$\begin{aligned} \mathcal{H}^{C_1} = & \frac{J}{4}[(\sigma_3 \cdot \sigma_4 - \mathbf{1}) + (\sigma_4 \cdot \sigma_5 - \mathbf{1})] \\ & + \frac{J_{23}}{4}(\sigma_2 \cdot \sigma_3 - \mathbf{1}) + \frac{\tilde{E}_{z,2}}{2}\sigma_{z,2} \\ & + \frac{E_z}{2}(\sigma_{z,1} + \sigma_{z,2} + \sigma_{z,3} + \sigma_{z,4} + \sigma_{z,5}). \end{aligned} \quad (10)$$

The first term in Eq. (10) describes the single-qubit interaction of the exchange-only qubit for $J_{34} = J_{45}$, with the abbreviation $J = (J_{34} + J_{45})/2$. The second term is the exchange interaction between QD₂ and QD₃. A global magnetic field across all five QDs, E_z , is represented by the last term. $\tilde{E}_{z,2}$ is a small deviation of the local magnetic field at QD₂ from the global magnetic field. Note that a possible deviation of the magnetic field at QD₁, $\tilde{E}_{z,1}$, is irrelevant when the exchange interaction between QD₁ and QD₂ is reduced to zero. $\tilde{E}_{z,1}$ would only cause single-qubit evolutions of the STQ. The exchange interaction between QD₁ and QD₂, J_{12} , is absent in Eq. (10) because it is reduced to zero or to values much smaller than the magnetic field difference between these QDs.

The time evolution under Eq. (10) can be used to construct an entangling operation between the STQ and the exchange-only qubit. Similar to the discussion in the previous section, E_z and J are much larger than $\tilde{E}_{z,2}$ and J_{23} . Therefore the qubit time evolution can be described using only the five-spin subspaces $S = \frac{1}{2}$, $s_z = \frac{1}{2}$ and $S = \frac{3}{2}$, $s_z = \frac{1}{2}$ that have together nine dimensions.

For $E_z, J \gg \tilde{E}_{z,2}, J_{23}$, only the states $|m_1\rangle = |T_+\rangle|u_{-1/2}\rangle$ and $|m_2\rangle = |T_+\rangle|v_{-1/2}\rangle$ coupled significantly to the computational subspace through Eq. (10). These states are eigenstates of $(J/4)[(\sigma_3 \cdot \sigma_4 - \mathbf{1}) + (\sigma_4 \cdot \sigma_5 - \mathbf{1})]$, and they have identical energies as the qubit states. $|u_{-1/2}\rangle$ and $|v_{-1/2}\rangle$ span the $S = \frac{1}{2}$, $s_z = -\frac{1}{2}$ subspace of the spins at QD₂-QD₄ (using the definitions from Sec. III B). $|m_3\rangle = \sqrt{\frac{1}{2}}|T_-\rangle|Q_{3/2}\rangle - \sqrt{\frac{1}{3}}|T_0\rangle|Q_{1/2}\rangle + \sqrt{\frac{1}{6}}|T_+\rangle|Q_{-1/2}\rangle$, $|m_4\rangle = \sqrt{\frac{2}{5}}|T_-\rangle|Q_{3/2}\rangle + \sqrt{\frac{1}{15}}|T_0\rangle|Q_{1/2}\rangle - \sqrt{\frac{8}{15}}|T_+\rangle|Q_{-1/2}\rangle$, and $|m_5\rangle = |S\rangle|Q_{3/2}\rangle$ have different energies, and therefore these states can be neglected. $|T_+\rangle = |\uparrow\uparrow\rangle$, $|T_0\rangle \propto |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, $|T_-\rangle = |\downarrow\downarrow\rangle$, and $|S_0\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ are the usual triplet and singlet states at QD₁-QD₂. Projecting Eq. (10) to $\{|0^L 0^R\rangle, |1^L 0^R\rangle, |m_1\rangle, |0^L 1^R\rangle, |1^L 1^R\rangle, |m_2\rangle\}$ gives

$$\mathcal{H}^{C_1} \approx \begin{pmatrix} \frac{E_z - J - \tilde{E}_{z,2}}{2} - \frac{J_{23}}{4} & 0 & 0 & -\frac{J_{23}}{4\sqrt{3}} & -\frac{J_{23}}{2\sqrt{3}} \\ 0 & \frac{E_z - J + \tilde{E}_{z,2}}{2} - \frac{J_{23}}{4} & 0 & \frac{J_{23}}{4\sqrt{3}} & -\frac{J_{23}}{2\sqrt{3}} \\ 0 & 0 & \frac{E_z - J + \tilde{E}_{z,2}}{2} - \frac{J_{23}}{4} & \frac{J_{23}}{2\sqrt{3}} & \frac{J_{23}}{4\sqrt{3}} \\ -\frac{J_{23}}{4\sqrt{3}} & \frac{J_{23}}{4\sqrt{3}} & \frac{J_{23}}{2\sqrt{3}} & \frac{E_z - 3J - \tilde{E}_{z,2}}{2} - \frac{5J_{23}}{12} & 0 \\ -\frac{J_{23}}{2\sqrt{3}} & \frac{J_{23}}{4\sqrt{3}} & \frac{J_{23}}{4\sqrt{3}} & 0 & \frac{E_z - 3J + \tilde{E}_{z,2}}{2} - \frac{J_{23}}{12} \end{pmatrix}. \quad (11)$$

Equation (11) contains two subspaces of virtually identical energies, as marked by the borders in the matrix. All the terms that couple these subspaces can be neglected.

After neglecting the block off diagonal entries in Eq. (11), also the time evolutions of E_z and J factor because they commute with the remaining entries. The time evolution through $(J/4)[(\sigma_3 \cdot \sigma_4 - 1) + (\sigma_4 \cdot \sigma_5 - 1)] \simeq (J/2)(|0^R\rangle\langle 0^R| - |1^R\rangle\langle 1^R|)$ causes only single-qubit time evolutions of the triple-QD qubit, and E_z causes global phase evolutions. The remaining time evolution is

$$\mathcal{U}_{\phi,\psi}^{C_1} \approx e^{-i2\pi(\phi m_1 + \frac{\psi}{2} m_2)},$$

$$m_1 = -\text{diag}\left\{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \begin{pmatrix} \frac{5}{12} & 0 \\ 0 & \frac{1}{12} \end{pmatrix}, \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix}, \frac{5}{12}\right\}, \quad (12)$$

$$m_2 = \text{diag}\{-1, 1, 1, -1, 1, 1\},$$

with $\phi = J_{23}t/h$ and $\psi = \tilde{E}_{z,2}t/h$.

Equation (12) causes no leakage for $\frac{1}{3}\sqrt{4\phi^2 + 9\psi^2} = 2\mathbb{Z} + 1$, and an entangling operation is realized for $\frac{1}{6}(2\phi - 3\psi) = \mathbb{Z}$. Alternatively, it is also possible to use $\frac{1}{3}\sqrt{4\phi^2 + 9\psi^2} = 2\mathbb{Z}$ and $\frac{1}{6}(2\phi - 3\psi) = \mathbb{Z} + \frac{1}{2}$. For example, the entangling operation $\mathcal{U}_{3/(2\sqrt{2}), 1/\sqrt{2}}^{C_1}$ gives a CPHASE in the basis $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |1^L 1^R\rangle\}$ using [see Fig. 3(b)]

$$Z_{1/\sqrt{2}}^L Z_{(4+\sqrt{2})/8}^R \mathcal{U}_{3/(2\sqrt{2}), 1/\sqrt{2}}^{C_1} = e^{i\pi \frac{\sqrt{2}-3}{2}} \text{CPHASE}. \quad (13)$$

$$\mathcal{H}^{C_2} \approx \begin{pmatrix} \frac{E_z - J}{2} - \frac{J_{23}}{4} & 0 & -\frac{J_{23}}{2\sqrt{6}} & -\frac{J_{23}}{4\sqrt{3}} \\ 0 & \frac{E_z - J}{2} + \Sigma E_z - \frac{J_{23}}{4} & \frac{J_{23}}{2\sqrt{6}} & \frac{J_{23}}{4\sqrt{3}} \\ -\frac{J_{23}}{2\sqrt{6}} & \frac{J_{23}}{4\sqrt{3}} & \frac{E_z - 3J}{2} - \frac{J_{23}}{4} & -\frac{J_{23}}{3\sqrt{2}} \\ -\frac{J_{23}}{4\sqrt{3}} & \frac{J_{23}}{2\sqrt{6}} & -\frac{J_{23}}{3\sqrt{2}} & \frac{E_z - 3J}{2} + \Sigma E_z - \frac{5J_{23}}{12} \end{pmatrix}.$$

All the terms in Eq. (15) outside of the marked subspaces are neglected for $E_z, J_{12}, J \gg \Sigma E_z, J_{23}$. Neglecting the contributions of E_z, J , and J_{12} [again these terms commute with the remaining entries in Eq. (15), and they cause either global phase evolutions, or single-qubit time evolutions] the effective time evolution is

$$\mathcal{U}_{\phi,\psi}^{C_2} \approx e^{-i2\pi(\phi m_1 + \psi m_2)},$$

$$m_1 = -\text{diag}\left\{\begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}, \begin{pmatrix} \frac{1}{4} & \frac{1}{3\sqrt{2}} \\ \frac{1}{3\sqrt{2}} & \frac{5}{12} \end{pmatrix}, \frac{1}{4}, \frac{1}{4}\right\}, \quad (16)$$

$$m_2 = \text{diag}\{0, 1, 0, 1, 0, 0\},$$

with $\phi = J_{23}t/h$ and $\psi = \Sigma E_z t/h$. The contributions of J_{12} and J are irrelevant in Eq. (15) because they dominantly cause single-qubit time evolutions for $E_z, J_{12}, J \gg \Sigma E_z, J_{23}$: $(J_{12}/4)(\sigma_1 \cdot \sigma_2 - 1) \simeq (J_{12}/2)(|0^L\rangle\langle 0^L| - |1^L\rangle\langle 1^L|)$ and $(J/4)[(\sigma_3 \cdot \sigma_4 - 1) + (\sigma_4 \cdot \sigma_5 - 1)] \simeq (J/2)(|0^R\rangle\langle 0^R| - |1^R\rangle\langle 1^R|)$. Also the phase evolution through E_z is neglected.

Note that in the construction of Eq. (13), it was assumed that J_{12} is turned to zero during the entangling operation. Small values of J_{12} can only be tolerated if they are much smaller than $\tilde{E}_{z,2}$. An alternative gate can be constructed for large J_{12} . This case is probably unfavored because the influence of charge noise increases with the exchange interactions. For completeness, we still discuss this parameter regime. In this case, Eq. (10) is modified to

$$\mathcal{H}^{C_2} = \frac{J_{12}}{4}(\sigma_1 \cdot \sigma_2 - 1) + \frac{J}{4}[(\sigma_3 \cdot \sigma_4 - 1) + (\sigma_4 \cdot \sigma_5 - 1)]$$

$$+ \frac{\Sigma E_z}{2}(\sigma_{z,1} + \sigma_{z,2}) + \frac{J_{23}}{4}(\sigma_2 \cdot \sigma_3 - 1)$$

$$+ \frac{E_z}{2}(\sigma_{z,1} + \sigma_{z,2} + \sigma_{z,3} + \sigma_{z,4} + \sigma_{z,5}). \quad (14)$$

Equation (14) contains the exchange interactions J_{12}, J_{23} , and J . Additionally, to a global magnetic field E_z , the sum of the magnetic field variations at QD₁ and QD₂ are important $\Sigma E_z = (\tilde{E}_{z,1} + \tilde{E}_{z,2})/2$. The magnetic field difference $\Delta E_z = (\tilde{E}_{z,1} - \tilde{E}_{z,2})/2$ can be neglected if it is much smaller than J_{12} . Using the equivalent arguments as before for $E_z, J_{12}, J \gg \Sigma E_z, J_{23}$, the qubit time evolution is restricted to the subspace $\{|T_0 0^R\rangle, |m_1\rangle, |T_0 1^R\rangle, |m_2\rangle, |S_0 0^R\rangle, |S_0 1^R\rangle\}$. Projecting Eq. (14) to this basis gives

The time evolution in Eq. (16) causes no leakage for $\frac{1}{2}\sqrt{\phi^2 - \frac{4\phi\psi}{3} + 4\psi^2} = 2\mathbb{Z} + 1$, and an entangling operation is realized for $\frac{1}{12}(\phi - 6\psi) = \mathbb{Z}$. Alternatively, it is also possible to use $\frac{1}{2}\sqrt{\phi^2 - \frac{4\phi\psi}{3} + 4\psi^2} = 2\mathbb{Z}$ and $\frac{1}{12}(\phi - 6\psi) = \mathbb{Z} + \frac{1}{2}$. For example, the entangling operation $\mathcal{U}_{3/\sqrt{2}, 1/(2\sqrt{2})}^{C_2}$ gives a CPHASE gate in the basis $\{|0^L 0^R\rangle, |0^L 1^R\rangle, |1^L 0^R\rangle, |1^L 1^R\rangle\}$ using [see Fig. 3(c)]

$$H^L Z_{1/2}^R \mathcal{U}_{3/\sqrt{2}, 1/(2\sqrt{2})}^{C_2} H^L = e^{i\pi \frac{3(\sqrt{2}-2)}{4}} \text{CPHASE}, \quad (17)$$

where H is the Hadamard gate.

IV. DISCUSSION AND CONCLUSION

It has been shown that the exchange interaction can be used to entangle a pair of QD qubits for all the distinct qubit encodings. Besides the single-qubit control, which has been experimentally realized for all the described spin qubits, only

exchange interactions between a pair of QDs of different QD qubits are needed. With the flexibility of the spin qubit setup, i.e., by keeping constant exchange interactions (for the STQ or the exchange-only qubit) or allowing local magnetic field variations (for the STQ), very short operation sequences can be constructed to entangle QD qubits. To entangle a STQ with a single-spin qubit or an exchange-only qubit, only one exchange interaction is needed between QDs of the different qubit types. To entangle a single-spin qubit and an exchange-only qubit, a sequence of two interqubit exchange interactions is needed.

The constructions of the entangling operations in Secs. III B and III C used a few approximations. It was assumed that the exchange interaction between the QDs of the STQ, or between the QDs of the exchange-only qubit are constantly turned on, while their magnitudes are much larger than the exchange interaction between the neighboring QDs of the different qubits. Figure 4 compares the time evolution of the Hamiltonians without any approximations to the ideal time evolutions. It is shown that the interqubit exchange interactions only need to be by one order of magnitude smaller than the exchange interactions within a qubit to reduce the effective gate errors below 1%. These gate errors are sufficient for quantum computation with standard quantum error correction protocols [33,47,48].

The advantage of exchange-based entangling operations is the controllability of the interaction mechanism. The exchange interaction depends on the tunnel coupling between distant QDs and their chemical potentials. It has been shown that exchange interactions can be tuned rapidly [44]. Even though the interqubit exchange interactions need to be weak, the time scales of the entangling gates can still reach tens of nanoseconds. The global magnetic field E_z can be large in experiments; the Zeeman energy $|g\mu_B B_z|$ reaches values of several μeV for external magnetic fields above 100mT in GaAs and Si (note that the absolute value of the g factor in Si is more than four times larger than in GaAs) [1,2]. The preparation of local magnetic field variations of the order of $\tilde{B}_{z,i} = 10$ mT are possible for STQs ($|g\mu_B \tilde{B}_{z,i}| \lesssim 1 \mu\text{eV}$) [12,13]. The exchange interactions can be tuned to several μeV , while high-fidelity operations require exchange interactions below $1 \mu\text{eV}$ due to charge noise [49,51]. An exception is the constant exchange interaction J of the exchange-only qubit that is typically by one order of magnitude larger because then the exchange-only qubit is still well protected from charge noise at an optimal operation point [17,18]. Our gates require magnitudes of the interqubit exchange interactions that are similar to the magnetic field gradient across the DQD of a STQ (see Sec. III A), or magnitudes of the interqubit exchange interactions that are by one order of magnitude smaller than E_z and J (see Fig. 4, Secs. III B and III C). This means that the entangling operations have limitations similar to the standard single-qubit gate operations, e.g., Rabi control for the single-spin qubit [6–10] or the exchange-only qubit [15,16] require a driving amplitude (that determines the gate time) that is much smaller than E_z or J . Using all these approximations also for the two-qubit operation times, the gate times are still comparable to the single-qubit operation times.

The limitations of the proposed entangling operations are similar to existing gate schemes. Local magnetic [50] and electric field [51] fluctuations are present in semiconductors.

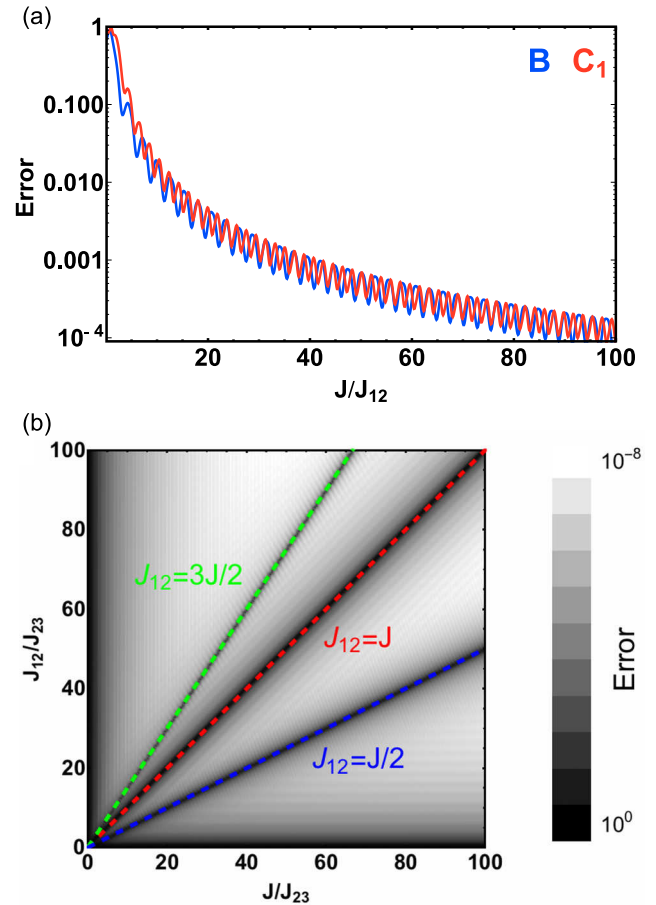


FIG. 4. (Color online) Gate errors for the operation sequences of Eqs. (9), (13), and (17). Only the operations $\mathcal{U}_{3/4}^B$, $\mathcal{U}_{3/(2\sqrt{2}),1/\sqrt{2}}^{C1}$, and $\mathcal{U}_{3/\sqrt{2},1/(2\sqrt{2})}^{C2}$ are analyzed. The gate errors are characterized by the deviation of the entanglement fidelity $F = \text{tr}(\rho^{RS} \mathcal{U}_{\text{ideal}}^{-1} \mathcal{U}_{\text{real}} \rho^{RS} \mathcal{U}_{\text{real}}^{-1} \mathcal{U}_{\text{ideal}})$ from 1. $\rho^{RS} = |\text{RS}\rangle\langle\text{RS}|$ is the maximally entangled state of two identical subspaces R and S, e.g., $|\text{RS}\rangle \propto |0000\rangle + |0110\rangle + |1001\rangle + |1111\rangle$, and the time evolutions $\mathcal{U}_{\text{ideal}}$ and $\mathcal{U}_{\text{real}}$ act only on S while R remains unchanged. (a) For $\mathcal{U}_{3/4}^B$ (blue curve) and $\mathcal{U}_{3/(2\sqrt{2}),1/\sqrt{2}}^{C1}$ (red curve), the exchange interaction of the exchange-only qubit J should be by more than one order of magnitude larger than J_{12} to reduce the gate error below 1%. (b) For $\mathcal{U}_{3/\sqrt{2},1/(2\sqrt{2})}^{C2}$, J_{12} , and J should be large. The gate errors increase for $J_{12} = 3J/2$, $J_{12} = J$, and $J_{12} = J/2$ (dashed lines) because of degeneracies in the level spectrum.

Both mechanism cause low-frequency fluctuations of the QD parameters. Since the magnetic field fluctuations are created mainly by the hyperfine spins of the host's nuclei, these fluctuations are suppressed for QDs in nuclear-spin free heterostructures. Natural Si contains already a substantially lower number of isotopes with nuclear spins compared to GaAs, and it is also possible to fabricate QDs in isotopically purified Si that contain almost no isotopes that have nuclear spins [52]. Charge noise is dominantly caused by impurities in the sample, and it couples to the electric dipole moments of qubits that are created by increasing the exchange interactions. To prevent a large influence of charge noise, one usually limits the magnitudes of the exchange interactions. Furthermore,

a small amount of low-frequency noise can be tolerated in experiments because it is possible to reduce its influence by refocusing protocols [53,54].

In the end, the proposed gate operations are idealized because they do not take into account finite rise times of the exchange pulses, or a residual exchange coupling that cannot be turned off. The modifications of exchange interactions on subnanosecond time scales are well established [44]. For some gates of the paper, the exchange interactions of STQs are reduced to zero, but it is sufficient to reduce them below the level of the magnetic field gradients across the DQDs. Being able to minimize the influence of interqubit exchange is very important. We are convinced that the recent experimental achievement of high amplitude control over the tunnel coupling between QDs is very useful [55]. Also preparation errors or misalignments of the local magnetic fields are present in real experiments. Further numerical studies can adjust the proposed gate sequences to the reality, and a procedure similar to Ref. [56] should be able to derive optimized gate sequences for high-fidelity gate operations. Spin-orbit interactions are weak in typical QD materials like, e.g., GaAs or Si [1,2], and they should have minor influence on the proposed operation sequences.

Besides entangling different kinds of spin qubits, it might also be useful to interchange quantum information between them. Figure 5 shows operation sequences for SWAP operations that only rely on CPHASE and Hadamard gates (cf. Ref. [57]). An unconditioned SWAP is realized using three CPHASE gates; only two CPHASE gates are needed if the state of a qubit should be transferred to another qubit that is initially in $|0\rangle$.

Altogether, very efficient operation sequences have been constructed to couple and interconvert different kinds of spin qubits. These operation sequences can couple all the standard qubit encodings in one, two, and three singly occupied QDs.

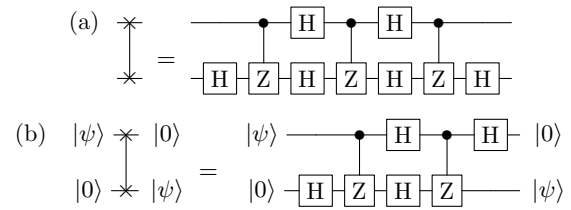


FIG. 5. Gate operations to interchange qubits using CPHASE gates. (a) The unconditioned SWAP operation requires three CPHASE gates together with Hadamard gates (H). (b) A simpler SWAP sequence can be realized if one of the qubits is initialized to a fixed state, e.g., $|0\rangle$. Then the SWAP operation with an arbitrary state $|\psi\rangle$ requires only two CPHASE gates.

Only the established single-qubit manipulation protocols are needed that have been successfully realized for all the qubit encodings. Different qubits are coupled using exchange interactions that are well controlled experimentally. With the current efforts to build larger arrays of tunnel-coupled QDs [58,59], the proposed operation sequences can be tested directly. The interconversion of different spin qubits allows to use all the advantages of the different QD setups in large arrays of QDs. For example, it is known that few-electron qubits couple stronger to cavities [17,23,24] or metallic gates [60], while single-spin qubits have extremely long coherence times [8,34]. Therefore the described operation sequences are another useful ingredient on the way towards quantum computation with large QD networks.

ACKNOWLEDGMENTS

We are grateful for support from the Alexander von Humboldt foundation.

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