

Persson Replies: In his Comment Borodich [1] has pointed out that not all self-affine fractal surfaces give a linear relation between the area of real contact and the load. However, the examples of rough surfaces he presents are very unphysical and never observed in real physical systems. The theory developed in Refs. [2,3] assumes randomly rough surfaces without sharp edges. The surface profiles presented by Borodich do not satisfy these conditions. In the last few years several (exact) numerical studies of contacts between randomly rough surfaces have been presented which clearly show that $A \sim F_N$ holds accurately (see below). Furthermore, the nearly universally observed linear relation between the sliding friction force F and the normal load is most naturally explained by assuming that $A \sim F_N$. Deviations from this “law” have been observed for soft elastic solids (e.g., rubber), where adhesion is important [4], and also in friction force microscopy studies, where a very sharp tip is slid on a substrate; however, in this latter case the contact area is too small (the diameter is usually below 100 Å) for the statistical contact theory to be valid.

It is possible to generate surface roughness profiles, which are very similar to experimentally observed surfaces profiles, as follows: The surface height profile $h(\mathbf{x})$ is written as [5]

$$h(\mathbf{x}) = \sum_{\mathbf{q}} B(\mathbf{q}) e^{i[\mathbf{q} \cdot \mathbf{x} + \phi(\mathbf{q})]}, \quad (1)$$

where, since $h(\mathbf{x})$ is real, $B(-\mathbf{q}) = B^*(\mathbf{q})$ and $\phi(-\mathbf{q}) = -\phi(\mathbf{q})$. In (1) $\phi(\mathbf{q})$ are independent random variables, uniformly distributed in the interval $[0, 2\pi]$. $B(\mathbf{q})$ is determined so as to reproduce the measured surface roughness power spectrum

$$C(q) = \frac{1}{(2\pi)^2} \int d^2x \langle h(\mathbf{x}) h(\mathbf{0}) \rangle e^{-i\mathbf{q} \cdot \mathbf{x}}.$$

Using (1) gives

$$C(q) = \frac{A_0}{(2\pi)^2} B(\mathbf{q}) B^*(\mathbf{q}),$$

where $A_0 = L^2$ is the nominal contact area. This equation is satisfied with $B(\mathbf{q}) = (2\pi/L) C^{1/2}(q)$. If $C(q)$ is chosen as $\sim 1/q^{2(H+1)}$, then surfaces generated from (1) will be self-affine fractal with the fractal dimension $D_f = 3 - H$. Greenwood [5] has shown that surfaces generated as described above are virtually indistinguishable from rough surfaces produced by, e.g., cleaving or sandblasting. Equation (1) shows that the amplitude of the different surface roughness wave vector components are uncorrelated, so that averaging over different $h(\mathbf{q})$ are independent processes, as assumed in Refs. [2,3]. As an example of contact between surfaces with roughness of the form (1), consider the results of Borri-Brunetto *et al.* [6].

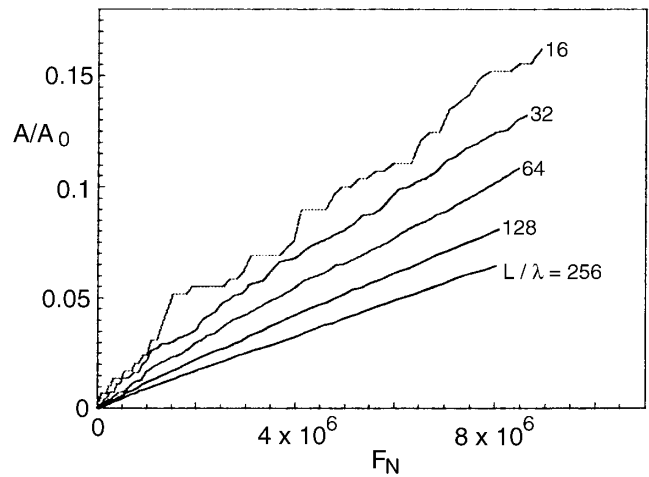


FIG. 1. The area of contact, A , as a function of the applied load F_N . Results are presented for five different magnifications L/λ . The fractal dimension $D_f = 2.3$. From Ref. [6].

Figure 1 shows the area of contact, A , as a function of the applied load F_N (from Ref. [6]). Results are presented for five different magnifications L/λ . Note that within the accuracy of the calculations, $A \sim F_N$. Note also that the slope of the curves decreases with increasing magnification; if we (arbitrarily) denote the slope for the magnification $L/\lambda = 256$ with unity, then the slopes for $L/\lambda = 128, 64, 32$, and 16 will be $1.24, 1.57, 1.93$, and 2.32 . This is in accordance with the theory presented in Ref. [2] where it was shown that the area of contact on the length scale λ is proportional to $A(\lambda) \sim (\lambda/L)^{1-H}$. Since $D_f = 2.3$ we have $1 - H = D_f - 2 = 0.3$ so that the theoretical slopes are $1.23, 1.52, 1.87$, and 2.30 , in excellent agreement with the numerical results.

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