

Reply to “Comment on ‘Lattice determination of Σ - Λ mixing’”

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In this Reply, we respond to the Comment [Phys. Rev. D **92**, 018501 (2015)]. Our computation [Phys. Rev. D **91**, 074512 (2015)] only took into account pure QCD effects, arising from quark mass differences, so it is not surprising that there are discrepancies in isospin splittings and in the Σ - Λ mixing angle. We expect that these discrepancies will be smaller in a full calculation incorporating QED effects.

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We agree with Gal [1] that lattice numbers for the mixing angle are not complete until numbers for the isospin splittings are correctly accounted for. In our paper [2], the calculation only includes QCD effects arising from quark mass differences. For a complete result, a full calculation with QED effects added is also needed.

Before addressing the main issue, we would first like to mention that it is not surprising that our results satisfy the Coleman-Glashow [3] and Dalitz-von Hippel (Eq. (3) of [4]) relations. Our fit function has all the $SU(3)$ symmetry constraints built in, so it automatically obeys every symmetry relation to the appropriate order. In this case, violations of the Coleman-Glashow relation are at $O(\delta m^3)$, while violations of the Dalitz-von Hippel relation are at $O(\delta m^2)$.

We believe that lattice QCD gives reliable numbers for the part of the isospin splitting due to quark mass differences, and that the differences between the values in [2] and the experiment are mainly due to electromagnetic effects. Our reasons to believe that lattice numbers for the purely QCD contribution are accurate are the following: first, the lattice gives good values for the splittings between the multiplets (N , Λ , Σ , Ξ), and it is hard to see how there could be a systematic error that would spoil the isospin splittings without also showing up in splittings between multiplets. Second, at leading order, pure QCD relates the isospin splittings to the splittings between the multiplets through the relations

$$\begin{aligned} M_n^2 - M_p^2 &\approx \frac{m_d - m_u}{m_s - m_u} (M_{\Xi^0}^2 - M_{\Sigma^+}^2) \\ M_{\Sigma^-}^2 - M_{\Sigma^+}^2 &\approx \frac{m_d - m_u}{m_s - m_u} (M_{\Xi^-}^2 - M_p^2) \\ M_{\Xi^-}^2 - M_{\Xi^0}^2 &\approx \frac{m_d - m_u}{m_s - m_u} (M_{\Sigma^-}^2 - M_n^2) \end{aligned} \quad (1)$$

(see Fig. 2 of [5]). Our simulations show that this leading-order formula only has minor corrections from higher-order effects, and the above relations hold reasonably well, with a value ≈ 0.022 for the quark mass ratio $(m_d - m_u)/(m_s - m_u)$. The main systematic uncertainty in this mass ratio is due to the difficulty of correcting for electromagnetic effects in the pseudoscalar meson sector. In Fig. 1 we show the splitting values. The squares and circles are consistent with each other, but do not reproduce the full result without adding a non-QCD force (namely QED).

The relations (1) do not hold in the real world, which we take this as evidence that the QED corrections to the isospin splittings are substantial. Simulations with QED included [6] show that QED effects on the isospin splittings are comparable with the effects of the $(m_d - m_u)$ difference, and that the combined simulation reproduces the physical numbers very closely. Unfortunately, we do not have lattice results for the QED shift in the Σ^0 mass, so we cannot estimate the QED effect in the mixing matrix element.

In the paper by Isgur [7] there are electrostatic (Coulomb) contributions to the isospin splittings, but they cancel completely for the combination $[(M_{\Sigma^0} - M_{\Sigma^+}) - (M_n - M_p)]$, which, by the Dalitz-von Hippel relation, is

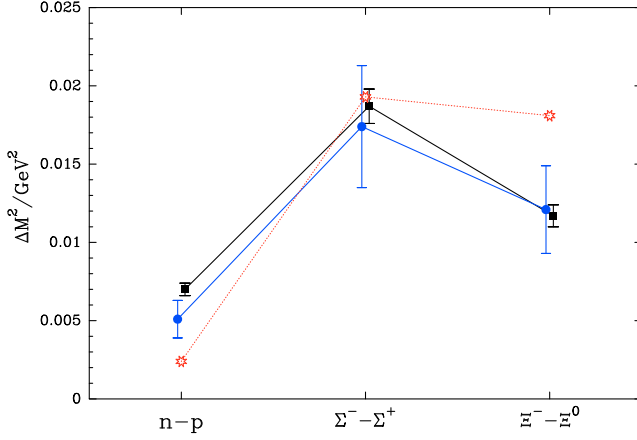


FIG. 1 (color online). Baryon octet (mass splittings)². Black squares are from Eq. (1), while blue circles the lattice QCD results. Both use the pure QCD numbers from Table 5 of [2]. Red stars use the physical values (i.e. for QCD and QED).

proportional to the Σ - Λ mixing angle. If this holds, we would expect to see some shifts in the isospin splittings, but no Coulomb contribution to the mixing angle. However, this exact cancellation is model dependent. The Coulomb contributions in [7] are calculated for a particular Gaussian wave function, which has more symmetry than required by QCD. For equal-mass quarks, the Isgur spatial wave function is completely symmetric under exchange of any quark pair. Since the octet is a mixed symmetry multiplet, we do not expect complete symmetry. In the proton (uud) there is no theorem that says $\langle 1/r_{uu} \rangle = \langle 1/r_{ud} \rangle$.

We have applied our flavor analysis to QED effects too. We find that there are five allowed coefficients for the electromagnetic effects, one of which just shifts all masses by the same amount, making no contribution to splitting or mixing. In terms of these coefficients, we find (assuming small mixing angle)

$$\begin{aligned}
 M_n^2 &= \frac{1}{3} [2B_0^{\text{EM}} + 2B_1^{\text{EM}} + B_2^{\text{EM}} + 3B_3^{\text{EM}}] e^2 \\
 M_p^2 &= \frac{1}{3} [2B_0^{\text{EM}} + 3B_1^{\text{EM}} - B_2^{\text{EM}} + 3B_3^{\text{EM}}] e^2 \\
 M_{\Sigma^-}^2 &= \frac{1}{3} [2B_0^{\text{EM}} + B_1^{\text{EM}}] e^2 \\
 M_{\Sigma^0}^2 &= \frac{1}{12} [8B_0^{\text{EM}} + 8B_1^{\text{EM}} - 2B_2^{\text{EM}} + 3B_3^{\text{EM}} + 9B_4^{\text{EM}}] e^2 \\
 M_{\Lambda^0}^2 &= \frac{1}{12} [8B_0^{\text{EM}} + 8B_1^{\text{EM}} + 2B_2^{\text{EM}} + 9B_3^{\text{EM}} + 3B_4^{\text{EM}}] e^2 \\
 M_{\Sigma^+}^2 &= \frac{1}{3} [2B_0^{\text{EM}} + 3B_1^{\text{EM}} - B_2^{\text{EM}} + 3B_3^{\text{EM}}] e^2 \\
 M_{\Xi^-}^2 &= \frac{1}{3} [2B_0^{\text{EM}} + B_1^{\text{EM}}] e^2 \\
 M_{\Xi^0}^2 &= \frac{1}{3} [2B_0^{\text{EM}} + 2B_1^{\text{EM}} + B_2^{\text{EM}} + 3B_3^{\text{EM}}] e^2
 \end{aligned} \tag{2}$$

as the general expression for the electromagnetic contribution to the masses and

$$\langle \Sigma^0 | M_{\text{EM}}^2 | \Lambda \rangle = \frac{\sqrt{3}}{12} [-2B_2^{\text{EM}} - 3B_3^{\text{EM}} + 3B_4^{\text{EM}}] e^2 \tag{3}$$

for the mixing matrix element. (We fit to the squares of the masses—the form of the expansion is, of course, the same if a fit to the masses themselves is made.) It may readily be checked that these electromagnetic contributions satisfy the Coleman-Glashow and Dalitz–von Hippel relations for any value of the B^{EM} coefficients.

In the Isgur model, these coefficients are not all independent. The Coulomb terms of the Isgur model follow the pattern

$$B_3^{\text{EM}} = B_4^{\text{EM}} = -B_1^{\text{EM}}, \quad B_2^{\text{EM}} = 0, \tag{4}$$

which in turn ensures that there is no Coulomb contribution to the mixing; see Eq. (3). However, these interrelations are model dependent, and we need lattice calculations to see how well they hold.

We are currently working on a combined simulation with QED effects included. Preliminary results suggest that QED effects do account for much of the difference between the current QCD-only results and the experimental values. In the Isgur model, it is expected that although the individual splittings will be changed by QED effects, the QED contribution to the Σ - Λ mixing angle will cancel. This is, however, a model-dependent statement, with other choices for the spatial wave function the cancellation would not be complete. Our joint QED and QCD results are not yet at the point where we can comment on how much the mixing angle is changed by QED, but we are grateful to Gal for bringing the issue to our attention and will certainly include a discussion of the question when we have our final results.

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- [1] A. Gal, preceding Comment, [Phys. Rev. D **92**, 018501 \(2015\)](#).
- [2] R. Horsley, J. Najjar, Y. Nakamura, H. Perlt, D. Pleiter, P. E. L. Rakow, G. Schierholz, A. Schiller, H. Stüben, and J. M. Zanotti (QCDSF and UKQCD Collaborations), [Phys. Rev. D **91**, 074512 \(2015\)](#).
- [3] S. Coleman and S. L. Glashow, [Phys. Rev. Lett. **6**, 423 \(1961\)](#).
- [4] R. H. Dalitz and F. von Hippel, [Phys. Lett. **10**, 153 \(1964\)](#).
- [5] R. Horsley, J. Najjar, Y. Nakamura, D. Pleiter, P. E. L. Rakow, G. Schierholz, and J. M. Zanotti (QCDSF and UKQCD Collaborations), [Phys. Rev. D **86**, 114511 \(2012\)](#).
- [6] Sz. Borsanyi, S. Dürr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, L. Lellouch, T. Lippert, A. Portelli, K. K. Szabo, and B. C. Toth, [Science **347**, 1452 \(2015\)](#).
- [7] N. Isgur, [Phys. Rev. D **21**, 779 \(1980\)](#); [23, 817 \(1981\)](#).
- [8] Y. Nakamura and H. Stüben, Proc. Sci. LATTICE2010 (**2010**) 040 [[arXiv:1011.0199](#)].
- [9] R. G. Edwards and B. Joó, [Nucl. Phys. B, Proc. Suppl. **140**, 832 \(2005\)](#).