Collision-free speed model for pedestrian dynamics

Antoine Tordeux, Mohcine Chraibi, and Armin Seyfried

Abstract We propose in this paper a minimal speed-based pedestrian model for which particle dynamics are intrinsically collision-free. The speed model is an optimal velocity function depending on the agent length (i.e. particle diameter), maximum speed and time gap parameters. The direction model is a weighted sum of exponential repulsion from the neighbors, calibrated by the repulsion rate and distance. The model's main features like the reproduction of empirical phenomena are analysed by simulation. We point out that phenomena of self-organisation observable in force-based models and field studies can be reproduced by the collision-free model with low computational effort.

1 Introduction

Modelling of pedestrian dynamics have been strongly developed since the 1990's [4, 23, 8]. Microscopic models describe the movement of individuals in two-dimensional representation of space. They are used for theoretical purposes [14, 13], as well as for applications e.g. design and conception of escape routes in buildings [24, 25] or optimal organization of mass events or public transport facilities

Antoine Tordeux
Jülich Supercomputing Centre, Forschungszentrum Jülich GmbH, Germany and Computer Simulation for Fire Safety and Pedestrian Traffic, Bergische Universität Wuppertal, Germany, e-mail: a.tordeux@fz-juelich.de

Mohcine Chraibi
Jülich Supercomputing Centre, Forschungszentrum Jülich GmbH, Germany e-mail: m.chraibi@fz-juelich.de

Armin Seyfried
Jülich Supercomputing Centre, Forschungszentrum Jülich GmbH, Germany and Computer Simulation for Fire Safety and Pedestrian Traffic, Bergische Universität Wuppertal, Germany, e-mail: a.seyfried@fz-juelich.de

(VISWalk [22], Legion [3], ...). In the microscopic class of models, pedestrians are represented as autonomous entities (Lagrangian representation) with local interactions. Complex collective phenomena of self-organisation emerge from the interactions. Examples are the lane formation, clogging at bottlenecks, zipper effect or intermittent flow at bottlenecks, stop-and-go waves, herding, strip formation or circular flows (see [4, 12] and references therein). Even simple microscopic models can yield in rich dynamics [15, 6]. Yet, the relations between the microscopic model parameters and the emergence of phenomena of self-organisation are not straightforward. In most of the cases, they have to be analysed by simulation.

Microscopic pedestrian models could be defined in continuous or discrete time, space and state variables (see [23, Chapter 5]). One of the most investigated class is the class of *force-based* (or acceleration) models [15, 6, 5]. They use an analogy between pedestrian movement and Newtonian dynamics. Force-based approaches allow to describe a large variety of pedestrian dynamics [15, 6]. Yet, this model class describes particles with inertia and does not exclude particle collision and overlapping. This is especially problematic at high densities [5]. Moreover, the force-based approach may lead to numerical difficulties resulting in small time steps and high computational complexity, or use of mollifies [16].

Pedestrian behaviors result from repulsive and attractive forces with the acceleration models. They are based on the visual perception of distances or obstacle speeds resulting in instantaneous changing of the speed or the direction within the speed models. Also, this model class is generally called *vision-based*. One example is the *synthetic-vision-based steering approach* that notably allows to describe complex collective structures avoiding gridlocks [20]. Also the *velocity obstacle models* or *reciprocal velocity obstacle model* borrowed from robotics exist [10, 2]. These models are defined in discrete time and are driven by collision avoidance. They are by construction collision-free if the time step is smaller than a horizon time of anticipation. In the evacuation model by Venel, the pedestrians move as fast as possible to the desired destination with no overlapping [17]. There exits some variants of the model with different interaction strategies [26]. Note that there exists also rule based multi-agent models aiming to describe pedestrian psychology (see for instance [21, 11]) or mixed models, see for instance the *gradient navigation model* where the direction model is defined at first order while the speed is of second order [9]. In most of cases, these models need a large number of parameters with inherent calibration difficulties and, as for force-based models, high computational efforts.

In this paper, we aim to develop a minimal model for which the dynamics are by construction collision-free (i.e. overlapping-free). The model belongs to Maury and Venel mathematical framework [17]. We show by simulation that it allows to describe some expected phenomena of self-organisation observed in field studies or in simulations with forced based models. The model is defined in section 2 while the simulation results are presented in section 3. Conclusion and working perspective are given in section 4.

2 Collision-free speed-based pedestrian model

A continuous speed model is a derivative equation for the velocity. Typical examples are

$$\dot{\mathbf{x}}_i = \mathbf{v}(\mathbf{x}_i, \mathbf{x}_j, \ldots) \qquad \text{or} \qquad \dot{\mathbf{x}}_i = V(\mathbf{x}_i, \mathbf{x}_j, \ldots) \times \mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \ldots), \tag{1}$$

with $\mathbf{x}_i$ the pedestrian position and $\dot{\mathbf{x}}_i$ the velocity of pedestrian i (see figure 1). The velocity in regulated in one function for the first equality while the speed V and the direction $\mathbf{e}_i$ (unit vector) are regulated separately in the second approach.

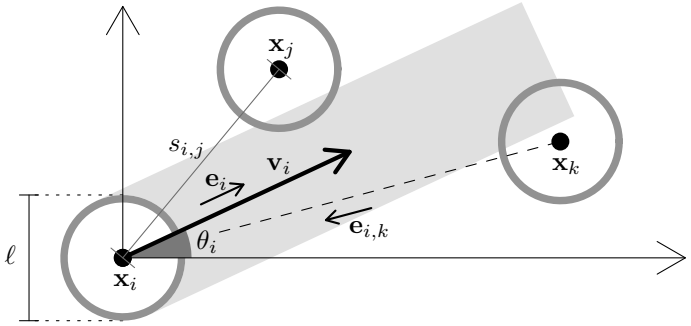

Fig. 1 Notations used. $\mathbf{x}_i$, $\mathbf{v}_i$ and θ_i are the position, velocity and direction of the pedestrian i; ℓ is the pedestrian size; $\mathbf{e}_{i,j}$ is the unit vector from $\mathbf{x}_j$ to $\mathbf{x}_i$; $\mathbf{e}_i = (\cos\theta_i, \sin\theta_i)$; $s_{i,j} = ||\mathbf{x}_i - \mathbf{x}_j||$.

2.1 Definition of the model

The speed model is the optimal speed (OV) function depending on the minimal spacing in front. The approach is borrowed from road traffic model [1]. The OV approach has been already developed with a force-based model [19]. Here we use the OV function at the first order with the minimal spacing in front.

For a given pedestrian i, the set of the pedestrians in front is defined by

$$J_i = \left\{ j,\ \mathbf{e}_i \cdot \mathbf{e}_{i,j} \leq 0 \ \text{ and } \ |\mathbf{e}_i^{\perp} \cdot \mathbf{e}_{i,j}| \leq \ell / s_{i,j} \right\}. \tag{2}$$

The pedestrians in front are the pedestrians overlapping the grey area in figure 1. The minimum distance in front s_i is

$$s_i = \min_{j \in J_i} s_{i,j}. \tag{3}$$

The model is

$$\dot{\mathbf{x}}_i = V\big(s_i(\mathbf{x}_i,\mathbf{x}_j,\ldots)\big) \times \mathbf{e}_i(\mathbf{x}_i,\mathbf{x}_j,\ldots), \tag{4}$$

with $V(\cdot)$ the OV function and $\mathbf{e}_i(\mathbf{x}_i,\mathbf{x}_j,\ldots)$ the direction model to define. As shown below, such model is by construction collision-free if

$$V(s) \geq 0 \quad \text{for all } s \qquad \text{and} \qquad V(s) = 0 \quad \text{for all } s \leq \ell. \tag{5}$$

In the following, the OV function is the piecewise linear $V(s) = \min\{v_0, \max\{0, (s-\ell)/T\}\}$, with v_0 the desired speed and T the time gap in following situations (ℓ is the pedestrian diameter, see figure 1). This OV function satisfies the collision-free assumption (5). The direction model is a simplified version of the additive form of the *gradient navigation model* [9]. It is based on a repulsion function depending on the distances $(s_{i,j})$ with the neighbours

$$\mathbf{e}_i(\mathbf{x}_i,\mathbf{x}_j,\ldots) = \tfrac{1}{N}\left(\mathbf{e}_0 + \textstyle\sum_j R(s_{i,j})\,\mathbf{e}_{i,j}\right), \tag{6}$$

with $\mathbf{e}_0$ the desired direction given by a strategic model, N a normalization constant such that $\|\mathbf{e}_i\| = 1$ and $R(s) = a\exp\big((\ell - s)/D\big)$ the repulsion function, calibrated by the coefficient $a > 0$ and distance $D > 0$. The parameter values used in the simulation are presented in figure 2.

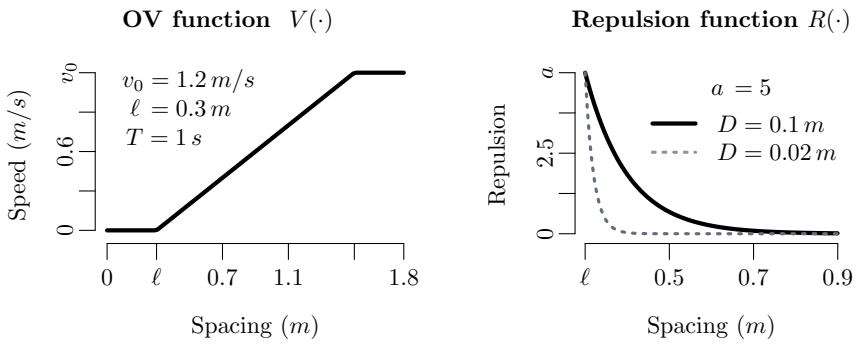

Fig. 2 Functions and associated parameters for the model: The OV function (3 parameters, left panel), and the repulsion function (2 parameters, right panel).

2.2 Collision-free property

Oppositely to the force-based models, the presence of collision and overlapping can be controlled by construction with the speed-based models (non-overlapping constraint). If pedestrians are considered as discs with diameter ℓ, the set of collision-

free configurations is for a given pedestrian i

$$Q_i = \left\{ \mathbf{x}_i \in \mathbb{R}^2,\ s_{i,j} \geq \ell\ \ \forall j \right\}. \tag{7}$$

The set of collision-free velocities

$$C_{\mathbf{x}_i} = \left\{ \mathbf{v} \in \mathbb{R}^4,\ s_{i,j} = \ell\ \Rightarrow\ \mathbf{e}_{i,j} \cdot \mathbf{v}_i \geq 0\ \text{ and }\ \mathbf{e}_{j,i} \cdot \mathbf{v}_j \geq 0 \right\} \tag{8}$$

is such that the speeds are nil or in opposite direction for a pedestrian in contact with an other (see [17] for more general conditions). Therefore, if initially $\mathbf{x}_i(0) \in Q_i$, then $\mathbf{x}_i$ remains in Q_i for any dynamics in $C_{\mathbf{x}_i}$. In these conditions Q_i is an invariant set for $\mathbf{x}_i$, i.e. the dynamics are collision-free (see also [18]). It is easy to see that the model (4) belongs to this class if assumption (5) is satisfied. Consider $s_{i,j} = \ell$ then either $\mathbf{e}_i \cdot \mathbf{e}_{i,j} \leq 0$ and then $j \in J_i$, i.e. $s_i \leq s_{i,j} = \ell$ and $V(s_i) = 0$, or neither $\mathbf{e}_i \cdot \mathbf{e}_{i,j} \geq 0$ and then $V(s_i) \geq 0$ since $V(\cdot) \geq 0$. Therefore $\mathbf{v}_i \cdot \mathbf{e}_{i,j} = V(s_i) \times \mathbf{e}_i \cdot \mathbf{e}_{i,j} \geq 0$ and the velocity belongs to $C_{\mathbf{x}_i}$. The arguments are valid for any direction model $\mathbf{e}_i$.

3 Model features

We describe in this section by simulation some characteristics of the model with uni- and bi-directional flows. The parameter settings are given in figure 2. The simulations are done on rectangular systems with length $L = 9$ m and width $W = 3$ m from random initial configurations and by using explicit Euler numerical scheme with time step $dt = 0.01$ s.

3.1 Counter flows and the lane formation

We observed with the model the formation of lanes by direction for counter flows (figure 3, left panels). Such phenomena frequently occurs in real data (see for instance [27]). The system needs an organization time for that the lanes emerge (figure 3, top right panel), where the mean flow to the desired direction for counter flows is compared to uni-directional ones). The formation of lanes is observed with the model for some density levels up to $\rho = 6$ ped/m^2 (figure 3, bottom right panel). As expected, the density threshold value for that the lanes appear depends on the pedestrian size ℓ (here $\ell = 0.3$ m). Note that the lane formation phenomenon disappears when a noise is introduced in the model (freezing by heating phenomenon, see [13] and in figure 3, thin dotted line in bottom right panel where a Brownian noise with standard deviation $\sigma = 0.1$ m/s is added to the model – the lane formation breaks as soon as $\rho \geq 2$ ped/m^2).

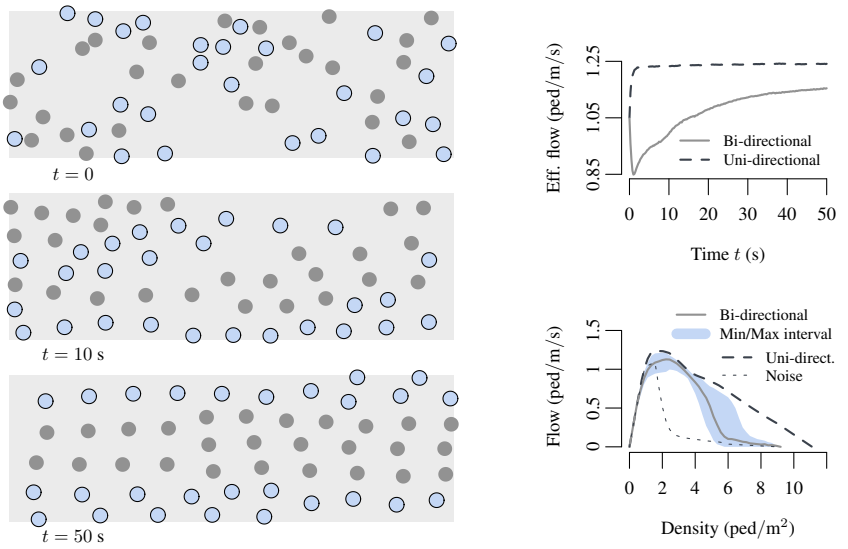

Fig. 3 Counter flows. Left panels, snapshots of the system at time $t = 0$, 10 and 20 s from random initial conditions ($\rho = 2$ ped/m^2). Right panels, the mean flow sequence to the desired direction and the fundamental diagram.

3.2 Intermittent bottleneck flows

Oscillating phenomena for counter flows in bottlenecks are observed with both real data and models ([15, 12, 7]). Such phenomena are related as intermittent bottleneck flows in the literature [14]. We observe that the speed-based model is able to reproduce them (see figure 4, left and top right panels). The phenomenon occurs even at relatively high density levels (see figure 4, bottom right panel). Yet it induces frictions and the flow volumes obtained for counter flows are less than the ones of uni-direction. As expected, the frictions tend to increase as the density increases. Some simulation results not presented here show that the intermittent flow phenomenon subsists for high density levels when D is sufficiently high and that the frequency of the flows oscillations tend to increase as the density increases.

4 Conclusion and working perspective

A new speed-based model is proposed for pedestrian dynamics in two dimensions. Oppositely to classical force-based approaches, the model is intrinsically collision-free and no overlapping phenomena occur, for any density level. The model has five parameters. Three of them concern the optimal speed function. They are the

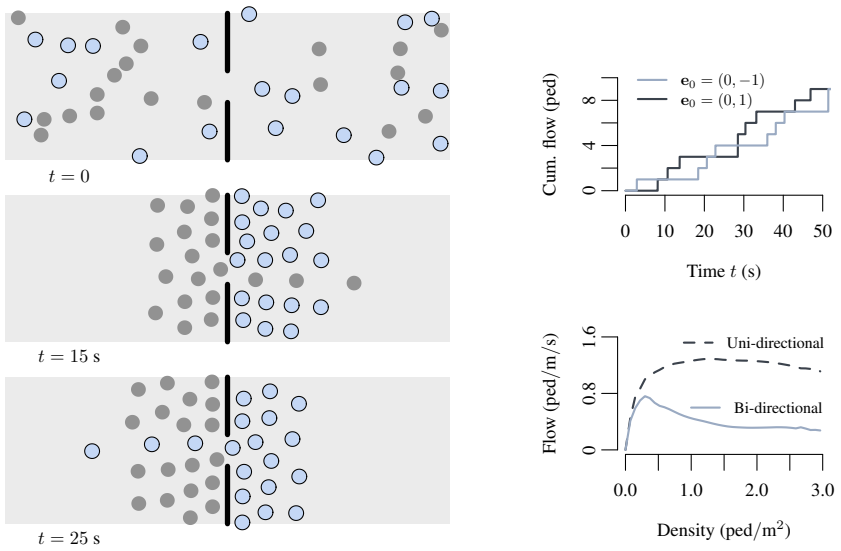

Fig. 4 Counter flows with bottleneck. Left panels, snapshots of the system at time $t = 0$, 10 and 20 s from random initial conditions ($\rho = 1.4$ ped/m^2 and $\omega = 0.6$ m). Right panels, the corresponding flow sequences by direction and the fundamental diagram.

pedestrian length, desired speed and time gap with the predecessor. The two others calibrate the direction model. They are the repulsion rate and repulsion distance.

The model main properties are described by simulation. A large range of dynamics observed in real data and force-based models are reproduced. For instance, linear increase of flow with the bottleneck width, lane formation for counter flows (with the freezing by heating effect) or intermittent flows, are obtained with identical setting of the parameters. However, other well-known characteristic such that stop-and-go phenomena can not be described. Further mechanisms (and parameters) remain to be introduced to the model.

References

1. Bando, M., Hasebe, K., Nakayama, A., Shibata, A., Sugiyama, Y.: Dynamical model of traffic congestion and numerical simulation. Phys. Rev. E **51**(2), 1035–1042 (1995)
2. van den Berg, J., Lin, M., Manocha, D.: Reciprocal velocity obstacles for real-time multi-agent navigation. In: Robotics and Automation, 2008. ICRA 2008. IEEE International Conference on. 2008 IEEE International Conference on Robotics and Automation Pasadena, CA, USA, May 19-23, 2008 (2008)
3. Berrou, J., Beecham, J., Quaglia, P., Kagarlis, M., Gerodimos, A.: Calibration and validation of the legion simulation model using empirical data. In: N. Waldau, P. Gattermann,

H. Knoflacher, M. Schreckenberg (eds.) Pedestrian and Evacuation Dynamics 2005, pp. 167–181. Springer Berlin Heidelberg (2007).
4. Chowdhury, D., Santen, L., Schadschneider, A.: Statistical physics of vehicular traffic and some related systems. Phys. Rep. **329**(4–6), 199–329 (2000).
5. Chraibi, M., Kemloh, U., Seyfried, A., Schadschneider, A.: Force-based models of pedestrian dynamics. Netw. Heterog. Media **6**(3), 425–442 (2011)
6. Chraibi, M., Seyfried, A., Schadschneider, A.: Generalized centrifugal force model for pedestrian dynamics. Phys. Rev. E **82**, 046,111 (2010).
7. Corradi, O., Hjorth, P., Starke, J.: Equation-free detection and continuation of a hopf bifurcation point in a particle model of pedestrian flow. SIAM J. Appl. Dyn. Syst. **11**(3), 1007–1032 (2012)
8. Degond, P., Appert-Rolland, C., Moussaid, M., Petté, J., Theraulaz, G.: A hierarchy of heuristic-based models of crowd dynamics. J. Stat. Phys. **152**(6), 1033–1068 (2013)
9. Dietrich, F., Köster, G.: Gradient navigation model for pedestrian dynamics. Phys. Rev. E **89**, 062,801 (2014)
10. Fiorini, P., Shiller, Z.: Motion planning in dynamic environments using velocity obstacles. Int. J. Robot. Res. **17**(7), 760–772 (1998).
11. Guo, R., Wong, S.C., Huang, H., Lam, W.H.K.: A microscopic pedestrian-simulation model and its application to intersecting flows. Phys. A **389**(3), 515–526 (2010)
12. Helbing, D.: Traffic and related self-driven many-particle systems. Rev. Mod. Phys. **73**, 1067–1141 (2001).
13. Helbing, D., Farkas, I., Vicsek, T.: Freezing by heating in a driven mesoscopic system. Phys. Rev. Lett. **84**, 1240–1243 (2000)
14. Helbing, D., Johansson, A., Mathiesen, J., Jensen, M.H., Hansen, A.: Analytical approach to continuous and intermittent bottleneck flows. Phys. Rev. Lett. **97**, 168,001 (2006).
15. Helbing, D., Molnár, P.: Social force model for pedestrian dynamics. Phys. Rev. E **51**, 4282–4286 (1995).
16. Köster, G., Treml, F., Gödel, M.: Avoiding numerical pitfalls in social force models. Phys. Rev. E **87** (2013)
17. Maury, B., Venel, J.: Un modèle de mouvement de foule. ESSAIM: Proc. **18**, 143–152 (2007)
18. Monneau, R., Roussignol, M., Tordeux, A.: Invariance and homogenization of an adaptive time gap car-following model. Nonlinear Differ. Equ. Appli. **21**(4), 491–517 (2014)
19. Nakayama, A., Hasebe, K., Sugiyama, Y.: Instability of pedestrian flow and phase structure in a two-dimensional optimal velocity model. Phys. Rev. E **71**, 036,121 (2005)
20. Ondřej, J., Pettré, J., Olivier, A.H., Donikian, S.: A synthetic-vision-based steering approach for crowd simulation. In: ACM Trans. Graph., vol. 29, pp. 123:1 – 123:9. ACM, New York, NY, USA (2010)
21. Pelechano, N., O'Brien, K., Silverman, B., Badler, N.: Crowd simulation incorporating agent psychological models, roles and communication. In: First International Workshop on Crowd Simulation, vol. 2, pp. 21–30. Lausanne (2005)
22. PTV AG: PTV Vissim 7.0 – User Manual. PTV Group, Haid-und-Neu-Str. 15, D-76131 Karlsruhe, Germany (2014)
23. Schadschneider, A., Chowdhury, D., Nishinari, K.: Stochastic Transport in Complex Systems. From Molecules to Vehicles. Elsevier Science Publishing Co Inc. (2010)
24. Schneider, V., Könnecke, R.: Simulating evacuation processes with aseri. In: Pedestrian and Evacuation Dynamics, pp. 303–314. Springer (2002)
25. TraffGo HT GmbH: Handbuch PedGo 2, PedGo Editor 2 (2005). Www.evacuation-simulation.com
26. Venel, J.: Integrating strategies in numerical modelling of crowd motion. In: Pedestrian and Evacuation Dynamics 2008 (2010)
27. Zhang, J., Klingsch, W., Schadschneider, A., Seyfried, A.: Ordering in bidirectional pedestrian flows and its influence on the fundamental diagram. J. Stat. Mech. Theor. Exp. **2012**(02), P02,002 (2012).