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$B \to \gamma\gamma$ in an ACD model

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We present a full calculation of the amplitudes for $B_{d[s]} \to \gamma\gamma$ in a simple ACD model that extends an incomplete one in a previous paper. We find cancellations between the contributions from different KK towers and a small decrease relative to the SM predictions. It is conjectured that radiative QCD corrections might actually lead to an enhancement in the branching ratios and **CP** asymmetries, but no more than modest ones.

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1. Exciting times are ahead for fundamental physics, when the LHC experiments start taking data in 2008. Many in the community expect a new paradigm to emerge around the TeV scale, be it some variant of SUSY or of Technicolour or something even more radical, like extra (space) dimensions. Those novel structures can manifest themselves *directly* through the production of new quanta or the topology of events or *indirectly* by inducing forces that modify rare weak decays. Such indirect searches are not a luxury. We consider it likely that to differentiate between different scenarios of New Physics, one needs to analyze their impact on flavour dynamics.

In this note we address $B_{d[s]} \to \gamma\gamma$, which has been studied extensively in the Standard Model (SM) and several New Physics scenarios, namely those with non-minimal Higgs dynamics and/or SUSY. Within the SM one finds [1, 2, 3, 4]

$$\mathrm{BR}(B_s[B_d] \to \gamma\gamma) \simeq$$

$$10^{-7}\cdot\left(\frac{f_{B_s}}{240\ \mathrm{MeV}}\right)^2\left[10^{-9}\cdot\left(\frac{f_{B_d}}{220\ \mathrm{MeV}}\right)^2\right]. \qquad (1)$$

The exchange of charged scalars in the loop can enhance the branching ratios by an order of magnitude [5] . In a previous paper by some of us [1] $B_{d,s} \to \gamma\gamma$ has been treated in the ACD model with one extra dimension [6] by calculating the contributions from the Kaluza-Klein (KK) towers of charged scalar fields. A small enhancement of a few percent was found there. However there are other contributions as well due to the virtual KK towers of Goldstone and of gauge bosons in the loop. Those are computed in this note, which thus contains a full evaluation of $B_{d[s]} \to \gamma\gamma$ in a simple ACD model.

After listing the relevant features of the ACD model in a nutshell we calculate the full ACD contributions to the $B_{d[s]} \to \gamma\gamma$ amplitudes and their **CP** asymmetry. We then give numerical estimates before concluding with an outlook.

2. In the ACD model all particles move in the bulk, i.e. they are functions of all space-time dimensions. For the bosonic fields one simply replaces all derivatives and fields of the SM Lagrangian by their five-dimensional counterparts. The Higgs doublet is chosen to be even under parity operation in five dimensions and possesses a zero mode. Note that all zero modes remain massless before the Higgs mechanism becomes operative and that the fields receive additional masses $\sim n/R$ after dimensional reduction. After gauge fixing, one can diagonalize the kinetic terms for the bosons and derive their propagators. Compared to the SM, there are additional Kaluza-Klein (KK) mass terms. As they are common to all fields, their contributions to the gauge boson mass matrix is proportional to the unit matrix. Because of the KK contribution to the mass matrix, charged and neutral Higgs components with $n \neq 0$ no longer play the role of Goldstone bosons. Instead, they mix with the $W_5^\pm$ and Z_5 to form, in addition to the Goldstone modes $G^0_{(n)}$ and $G^\pm_{(n)}$, three additional physical states $a^0_{(n)}$ and $a^\pm_{(n)}$. The Lagrangian for the interaction of the $G^\pm_{(n)}, a^\pm_{(n)}$ and $W_{(n)}$ (the towers of W bosons) with ordinary down quarks reads

$$\begin{aligned}
\mathcal{L} \;=\;& \frac{g_2}{\sqrt{2}M_{W(n)}}\bar{Q}_{i(n)}\left(C_L^{(1)}P_L + C_R^{(1)}P_R\right)a^*_{(n)}d_j \\
&+ \frac{g_2}{\sqrt{2}M_{W(n)}}\bar{U}_{i(n)}\left(C_L^{(2)}P_L + C_R^{(2)}P_R\right)a^*_{(n)}d_j \\
&+ \frac{g_2}{\sqrt{2}M_{W(n)}}\bar{Q}_{i(n)}\left(C_L^{(3)}P_L + C_R^{(3)}P_R\right)G^*_{(n)}d_j \\
&+ \frac{g_2}{\sqrt{2}M_{W(n)}}\bar{U}_{i(n)}\left(C_L^{(4)}P_L + C_R^{(4)}P_R\right)G^*_{(n)}d_j \\
&+ \frac{g_2}{\sqrt{2}}\bar{Q}_{i(n)}\,\gamma_\mu C_L^{(5)}P_L\,W^\mu_{(n)}d_j \\
&+ \frac{g_2}{\sqrt{2}}\bar{U}_{i(n)}\gamma_\mu C_L^{(6)}P_L\,W^\mu_{(n)}d_j \qquad (2)
\end{aligned}$$

using the notation of Ref. [7]

$$C_L^{(1)} \;=\; -m_3^{(i)}V_{ij}\,, C_L^{(2)} = m_4^{(i)}V_{ij}\,,$$

$$
\begin{aligned}
C_R^{(1)} &= M_3^{(i,j)} V_{ij}\,, C_R^{(2)} = -M_4^{(i,j)} V_{ij}\,, \\
C_L^{(3)} &= -m_1^{(i)} V_{ij}\,, C_L^{(4)} = m_2^{(i)} V_{ij}\,, \\
C_R^{(3)} &= M_1^{(i,j)} V_{ij}\,, C_R^{(4)} = -M_2^{(i,j)} V_{ij}\,, \\
C_L^{(5)} &= c_{(i)n} V_{ij}\,, C_L^{(6)} = -s_{i(n)} V_{ij}\,, \\
M^2_{W(n)} &= m^2(a^*_{(n)}) = M^2(G^*_{(n)}) = M_W^2 + \frac{n^2}{R^2} \quad (3)
\end{aligned}
$$

with the V_{ij} elements of the CKM matrix. The mass parameters in Eq. (3) are defined as

$$
\begin{aligned}
m_1^{(i)} &= \frac{n}{R} c_{i(n)} + m_i s_{i(n)}\,,\ m_2^{(i)} = -\frac{n}{R} s_{i(n)} + m_i c_{i(n)}\,, \\
m_3^{(i)} &= -M_W c_{i(n)} + \frac{n}{R}\frac{m_i}{M_W} s_{i(n)}\,, \\
m_4^{(i)} &= M_W s_{i(n)} + \frac{n}{R}\frac{m_i}{M_W} c_{i(n)}\,, \\
M_1^{(i,j)} &= m_j c_{i(n)}\,, \quad M_2^{(i,j)} = m_j s_{i(n)} \\
M_3^{(i,j)} &= \frac{n}{R}\frac{m_j}{M_W} c_{i(n)}\,, \quad M_4^{(i,j)} = \frac{n}{R}\frac{m_j}{M_W} s_{i(n)}\,. \quad (4)
\end{aligned}
$$

Here, M_W and the masses of the up (down) quarks m_i (m_j) on the right-hand-side of Eq. (4) are zero mode masses and the $c_{i(n)}, s_{i(n)}$ denote the cosine and sinus of the fermion mixing angles, respectively:

$$
\tan 2\alpha_{f(n)} = \frac{m_f}{n/R}\,, n \geq 1\,. \quad (5)
$$

The masses for the fermions are given by

$$
m_{f(n)} = \sqrt{\frac{n^2}{R^2} + m_f^2}\,. \quad (6)
$$

In what follows, we will utilize the constraint $n/R \geq 250\,\mathrm{GeV}$ [8] and hence all fermionic angles except $\alpha_{t(n)}$ are practically zero.

3. The amplitude for the decay $B_{d[s]} \to \gamma\gamma$ has the form

$$
T(B \to \gamma\gamma) = \varepsilon_1^\mu(k_1)\varepsilon_2^\mu(k_2)\left[A\, g_{\mu\nu} + i\, B\, \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta\right], (7)
$$

where the scalar functions $A\,(B)$ are CP–even(odd). These are calculated from the 1PR diagrams shown in Fig. 1 with $a_{(n)}, G_{(n)}$ and $W_{(n)}$ particles running in the loops. The corresponding 1PI graphs are suppressed by $1/M_W^2$ and will be neglected in what follows (see Ref. [8] for more details). The resulting amplitudes $A_{\rm ACD}$ and $B_{\rm ACD}$ take the from

$$
\begin{aligned}
A_{\rm ACD} &= i\frac{\sqrt{2}}{32\pi^2}(eQ_d)^2 f_B G_F \frac{m_b^3}{m_{s[d]}}\frac{M_W^2}{M^2_{W(n)}} V_{ib} V^*_{is[d]} \\
&\times \left\{ C(x_{i(n)}) - 12\frac{m_i m_{i(n)}}{M_W^2} c_{(i)n} s_{(i)n} f_1(x_{i(n)}) \right. \\
&- \left. \frac{3}{2} f_2(x_{i(n)})\left(1 + \frac{m_i^2}{M_W^2} - 2\frac{m_b m_{s[d]}}{M^2_{W(n)}}\frac{n^2}{R^2 M_W^2}\right)\right\} \\
B_{\rm ACD} &= \frac{2}{m_b^2} A_{\rm ACD} \quad (8)
\end{aligned}
$$

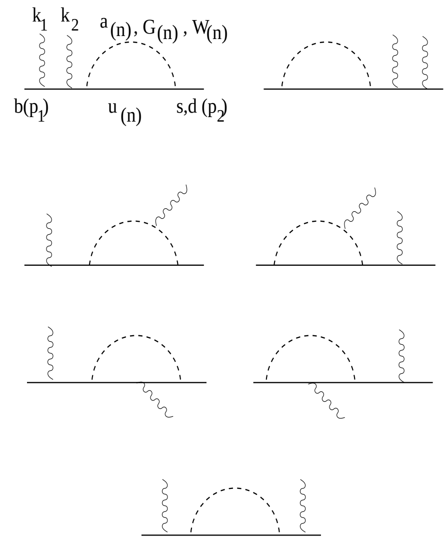

FIG. 1: 1PR diagrams for $B \to \gamma\gamma$ in the ACD model. The dashed lines denote the charged KK towers of $a_{(n)}, G_{(n)}$ and $W_{(n)}$, while the solid line inside the loops represent the up quark KK towers. Wiggly lines denote photons and the solid lines in the initial (final) state the b (d, s) quark. Crossed diagrams are not shown.

Where Q_d is charge of down quarks and

$$
\begin{aligned}
C(x) &= \frac{22x^3 - 153x^2 + 159x - 46}{6(1-x)^3} + \frac{3(2-3x)x^2 \ln x}{(1-x)^4}\,, \\
f_1(x) &= \frac{5x - 3}{6(1-x)^2} + \frac{3x - 2}{3(1-x)^3}\ln x\,, \\
f_2(x) &= \frac{2x^2 + 5x - 1}{6(1-x)^3} + \frac{x^2}{(1-x)^4}\ln x\,, \\
x_{i(n)} &= \frac{m^2_{i(n)}}{M^2_{W(n)}}\,. \quad (9)
\end{aligned}
$$

From Eq. (7) one can readily deduce the expression for the $B \to \gamma\gamma$ partial decay width Γ and the CP asymmetry parameter δ,

$$
\begin{aligned}
\Gamma(B \to \gamma\gamma) &= \frac{1}{32\pi M_B}\left(4|A|^2 + \frac{1}{2}M_B^4 |B|^2\right)\,, \\
\delta &= \frac{4|A|^2}{4|A|^2 + M_B^4|B|^2/2}\,, \quad (10)
\end{aligned}
$$

with M_B the corresponding B-meson mass. The asymmetry δ arises from the interference of the CP-even and CP-odd parts of the decay amplitude, see e.g. Refs. [4, 9]. The corresponding expressions for the SM amplitudes $A_{\rm SM}$ and $B_{\rm SM}$ can be taken from Refs. [2, 3]

$$
\begin{aligned}
A_{\rm SM} &= i\frac{\sqrt{2}m_b^3}{32\pi^2 m_{s[d]}} G_F f_B (eQ_d)^2 \lambda_t \left(C(x_t) + \frac{23}{3}\right)\,, \\
B_{\rm SM} &= i\frac{2\sqrt{2}m_b}{32\pi^2 m_{s[d]}} G_F f_B (eQ_d)^2 \lambda_t \left(C(x_t) + \frac{23}{3} + 16\frac{m_{s[d]}}{m_b}\right)\,, \quad (11)
\end{aligned}
$$

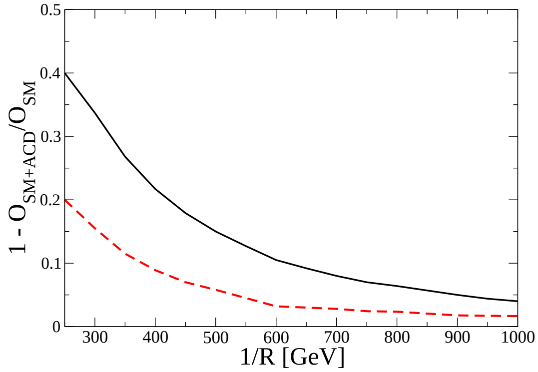

FIG. 2: B meson partial decay width (solid line) and asymmetry parameter in the ACD model compared to the SM as a function of the compactification scale $1/R$.

with G_F the Fermi constant, $x_t = m_t^2/M_W^2$ and $C(x)$ as defined in Eq. (9).

We define the relative contribution of the ACD model with respect to the SM as

$$R[\mathcal{O}] = 1 - \frac{\mathcal{O}_{\rm ACD+SM}}{\mathcal{O}_{\rm SM}} \ , \quad (\mathcal{O} = \Gamma, \delta) \ . \qquad (12)$$

The solid line in Fig. 2 shows this ratio for the width and for the CP-asymmetry parameter. For the smallest compactification radius, $1/R = 250\,\mathrm{GeV}$, we have $R[\Gamma] = 0.59$, but with decrasing compactification radius the ratio tends towards unity. A similar behaviour is observed for the asymmetry parameter δ, see the dashed line in Fig: 2.

4. It seems that $B_{d[s]} \to \gamma\gamma$ can realistically be observed only at a Super-B factory that for the B_s mode can operate also at the $\Upsilon(5S)$ resonance. The original incomplete evaluation of the ACD contributions showed a rather small enhancement. The full calculation presented here revealed a moderate to small decrease for the branching ratios and **CP** asymmetries. This is not necessarily the last word, though. The QCD corrections yield a substantial enhancement of 20 % [33 %] of $\Gamma(B_d[B_s] \to \mu^+\mu^-)$ [7, 8]. Radiative QCD corrections to $B_{d[s]} \to \gamma\gamma$, which likewise contains no hadrons in the final state, might be of similar size, yet not the same for the $a_{(n)}$, $G_{(n)}$ and $W_{(n)}$ KK towers. Hence we conjecture that the branching ratios of and the **CP** asymmetries in $B_{d[s]} \to \gamma\gamma$ can be modified by about 20 - 30 % in either direction, since they can vitiate the cancellations between the different KK contributions we have discussed in this note. Finding an effect of this size in the branching ratio might not be hopeless. Maybe the more relevant statement is that the simplest realization of an extra dimension model cannot affect $B_{d[s]} \to \gamma\gamma$ in a numerically large way. If a large deviation from the SM prediction were observed, one had to look elsewhere.

Acknowledgments

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