TURBULENCE SUPPRESSION IN TRANSPORT BARRIERS

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ABSTRACT

Basic facts about transport barriers in toroidally confined plasmas are presented. The flow shear of poloidal plasma rotation is a common feature in the spontaneous and induced transport barriers as well and is considered as the basic cause of their formation. Basic ideas on the impact of flow shear on the radial transport are discussed. Different mechanisms for the generation of flow shears are considered and models for the transition to enhanced confinement states are sketched.

I. EXPERIMENTAL OBSERVATIONS OF TRANSPORT BARRIERS

Enhanced confinement operation is of vital importance for fusion devices and this has encouraged the discovery of numerous modes with improved confinement. In particular enhanced confinement operation in tokamaks have been studied extensively. In this device, transport barriers are either formed spontaneously or induced by external means.

A. Spontaneous edge transport barrier

The first observations of transport barriers were made nearly 20 years ago in divertor tokamak ASDEX.[1,2] It was observed that under certain conditions, e.g., when the power input exceeds a critical level, the radial variation of temperature and density steepens strongly in a narrow layer 1-2 cm thick at the plasma edge (see Fig.1). Simultaneously a sudden increase in the confinement time by a factor of 2 has been achieved. After this transition the luminosity of hydrogen neutral atoms generated by plasma recycling at the divertor plates or limiters drops promptly, indicating a significant decrease in losses of charged particles. The pre- and post-transition conditions were called L mode and H mode, for low and high confinement, respectively. Since then almost all tokamaks with the desirable X-point configuration and with sufficient heating power input from auxiliary sources (neutral beams, radio-frequency antennae) or even from ohmic dissipation have reproducibly demonstrated H-mode.[3,4]

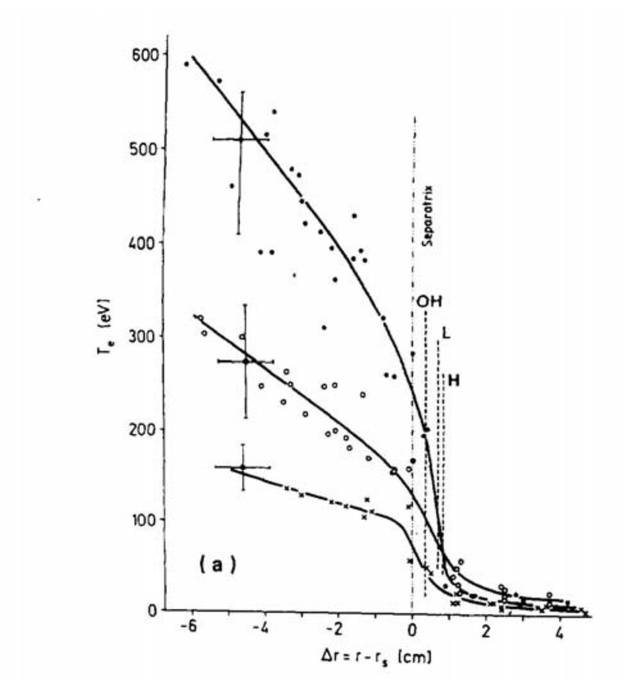

Figure 1: Radial profiles of T_e in the midplane in ASDEX, for ohmically and beam heated discharges in L and H mode.[2] Vertical lines give the separatrix position according different measurements.

B. Externally induced edge transport barrier

Several years after the discovery of H-mode scenarios it was recognized that the plasma drift motion especially produced by the radial electric field, i.e., $E \times B$ flow, plays a crucial role in the suppression of turbulence in the transport barrier. Therefore experiments were performed on several devices with the aim to establish and control such a field artificially. For this purpose a voltage was applied between the vessel wall and a surface inside the plasma[5] or an electrode was inserted into the plasma.[6,7] The transition to the H-mode occurs at a critical value of the current and leads

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to an abrupt increase of the effective radial plasma resistance, i.e., to a jump in the radial electric field. Also the particle confinement time increases significantly and fluctuations of the plasma density and potential are reduced strongly.

C. Internal transport barrier

In standard situations the plasma current density is peaked and the safety factor q, characterizing the helicity of the total magnetic field, approaches its minimum at the plasma axis. Internal transport barriers (ITBs) were created first in a configuration in which q as a function of the minor radius exhibits an off-axis minimum.[8–10] Such a situation with a region of negative magnetic shear (NMS) can arise transiently during the stage of current penetration into a hot plasma. Besides, a non-ohmic contribution from the so called bootstrap effect helps to realize the necessary profile of the current density. In ITBs, the turbulent heat transport can be reduced to a level even lower than that predicted by the standard neoclassical theory and established by collisional processes.

II. SUPPRESSION OF ANOMALOUS TRANSPORT BY FLOW SHEAR

Only a few years after the discovery of edge transport barriers it was observed experimentally that there is a fast plasma rotation perpendicular to the magnetic field (in poloidal direction) with a velocity strongly varying in the radial direction. Such a rotation should lead to suppression of turbulence and reduction of anomalous transport in the barrier. There is a wide consensus within the magnetic fusion research community that flow shear is responsible for the transport barriers observed in the experiment. There are at least three channels for the reduction of anomalous transport in the presence of flow shear namely (i) reduction of the linear growth rate of instabilities driving the turbulence, (ii) increase of the decorrelation of fluctuations and (iii) change of the phase shift between fluctuations of different parameters which determine turbulent fluxes.

A. Effect on linear growth rates

One of the first examples demonstrating that sheared plasma motion can lead to a reduction of the linear growth rate of an instability was the case of the flute mode which is typical for plasma conditions.[11] To elucidate the basic ideas we consider a plasma which is homogeneous in the direction z of the magnetic field $\mathbf{B}$ and assume a gradient of the plasma density in the radial direction x. Also it is assumed that in the x-direction a "gravitational" force $F_{gr} = m_i\, g\, n$ acts on the particles, where m_i is the ion mass and g the gravitational constant. For a plasma in fusion devices this "gravity" is usually the centrifugal force which arises due to particle motion along curved magnetic field. Consider now a small perturbation of the density in the poloidal direction y of the form: $\widetilde{n} \sim \exp(-i\omega t + iky)$, where k is the wavenumber and ω the (complex) frequency. As a result a perturbation in the gravitational force occurs: $\widetilde{F}_{gr} = m_i\, g\, \widetilde{n}$. This force leads to a drift motion of ions, and consequently to a perturbation of the electric current, in the y direction. Since the plasma remains quasi-neutral, i.e., $\boldsymbol{\nabla} \cdot \mathbf{j} = 0$, there is also a perturbation in the x-component of the current density. The Lorentz force produced by this component, $\widetilde{F}_y = -\widetilde{j}_x\, B$, leads to a new drift component in the direction of the density gradient. This motion increases the initial perturbation because it pushes more particles from the more denser plasma interim. Thus the flute instability arises.

Now imagine there is a macroscopic plasma motion in the y direction, which velocity V_y varies with x. Therefore the radial plasma motion caused by the perturbation will transfer also the y-component of the plasma momentum. This disturbs the force $\widetilde{F}_y$ and reduces the growth rate of the instability. A quantitative analysis can be done by using continuity and moment equations and leads to the expression for the linear growth rate:

$$\gamma_L \equiv \mathrm{Im}\{\omega\} = \sqrt{\frac{g}{L_n} - \left(\frac{S_V}{2L_n k}\right)^2} \qquad (1)$$

where

$$L_n = -\frac{1}{n}\frac{dn}{dx} \quad \text{and} \quad S_V = \frac{\partial V_y}{\partial x} \qquad (2)$$

Thus the flute instability is completely suppressed if the velocity shear is sufficiently strong, i.e., $S_V^2 > 4\, g\, L_n\, k^2$. However, there is no universal criterion for stabilization of linear instabilities by flow shear. In some cases flow shear, particularly a weak one, can be destabilizing.[4] Moreover the ideal of a collective linear mode, though convenient for analysis and interpretation, is not generally realized in plasmas. Numerical studies shows that there is a strong nonlinear coupling between fluctuations and flow shear which should be treated self-consistently. A particular surprising result of such a consideration applied to the ion temperature gradient (ITG) turbulence is a criterion for turbulence suppression in a very simple form:[12]

$$S_V^2 \sim \gamma_{max}^2 \qquad (3)$$

Here the subscript max means the maximum value of the growth rate over the whole k-spectrum.

B. Effect on decorrelation

A firm consideration of the effect of flow shear on anomalous transport requires an analysis of the influence on plasma eddies, which arise in the nonlinear phase of instabilities leading to turbulence. In such an eddy the plasma motion is coherent and can lead to effective transfer of plasma particles on a distance of the eddy diameter L. If there is a sheared flow with the velocity changing as $V_y = x\,S_V$ the eddy will be stretched during time t in the y direction to the length (see Fig.2)

$$L_l \approx (L^2 + (L\,S_V t)^2)^{1/2} = L\,(1 + (S_V\,t)^2)^{1/2} \quad (4)$$

Since the area of the fluid eddy is preserved by the stretching, the length of the minor axis is reduced as:

$$L_\perp = L\,(1 + (S_V\,t)^2)^{-1/2} \quad (5)$$

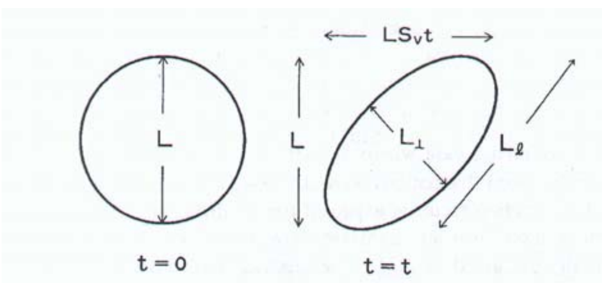

Figure 2: The circular element at $t = 0$ is stretched by the velocity shear after the elapse of time $t = t$. Here the flow is in the horizontal and the shear in the vertical direction.[3]

This relation can be regarded as an increase of the effective perpendicular wave number of fluctuations: $k_{eff} = k_\perp\,(1 + (S_V\,t)^2)^{1/2}$. If D is anomalous diffusivity due to turbulence, fluctuations become decorrelated in a time $\tau_{cor} \approx 1/(D\,k_{eff})^2$. Substituting $t = \tau_{cor}$ one gets an equation for the decorrelation time

$$\tau_{cor} \approx \frac{1}{D\,k_\perp^2\,(1 + S_V^2\,\tau_{cor}^2)} \quad (6)$$

Considering the limiting cases:

$$\begin{aligned} |S_V| \ll D\,k_\perp^2 \quad &: \quad \tau_{cor}^{-1} \approx D\,k_\perp^2 + S_V^2\,D^{-1}k_\perp^{-2} \\ |S_V| \gg D\,k_\perp^2 \quad &: \quad \tau_{cor}^{-1} \approx \left(D\,k_\perp^2\right)^{1/3} S_V^{2/3} \end{aligned} \quad (7)$$

one can see that flow shear increases the decorrelation rate of fluctuations. As a result the anomalous transport should be diminished. Indeed the decorrelation process reduces the effective nonlinear growth rate

$$\gamma_{NL} = \gamma_L - \tau_{cor}^{-1} = \gamma_L - D\,k_\perp^2\left(1 + S_V^2\tau_{cor}^2\right) \quad (8)$$

In stationary states one has $\gamma_{NL} = 0$ thus $\tau_{cor}^{-1} = \gamma_L$ and

$$D = \frac{\gamma_L}{k_\perp^2}\frac{\gamma_L^2}{\gamma_L^2 + S_V^2} \quad (9)$$

C. Effect on phase shifts

Besides the impact on the linear growth rate and reduction of the decorrelation time a flow shear changes the phase shift α between fluctuations of different parameters and leads to reduction of turbulent fluxes. For instance the fluctuation induced radial particle flux is given by $\Gamma_r = \langle \widetilde{n}\widetilde{v}_r \rangle$, where $\langle \ldots \rangle$ denotes a space-time average. For electrostatic instabilities the velocity fluctuation is the $E \times B$-drift:

$$\widetilde{V}_{r,k} = \frac{\widetilde{E}_{y,k}}{B} = -i\,k\,\frac{\widetilde{\varphi}_k}{B} \quad (10)$$

where $\widetilde{\varphi}$ is the perturbation of the electrostatic potential. Thus the sum over all wave numbers provides

$$\Gamma_r = -\frac{1}{B}\sum_k k\,|\widetilde{\varphi}_k||\widetilde{n}_k|\sin\alpha_k \quad (11)$$

As an example consider resistive interchange turbulence.[13] In this case a solution of nonlinear eigenmode problem allowed to estimate the cross phase in the weak shear limit:

$$\sin\alpha_k = -\left|1 - \frac{5k^2S_V^2\Delta_k^2}{4D^2}\right| \quad (12)$$

where Δ_k is the radial width of the eigenmode.

III. GENERATION OF SHEARED RADIAL ELECTRIC FIELDS

In the previous sections it has been pointed out that sheared poloidal flows can induce transport barriers by several mechanisms. It is now interesting, how such flow shears can develop in a magnetized plasma. The velocity of such a flow, $V_\perp$, can be found from the balance of forces applied to the ion plasma component in the direction r perpendicular to the magnetic surface. In a stationary state one has:

$$\frac{dP_i}{dr} = enE_r + en\,V_\perp\,B + F_{i,r} \quad (13)$$

where P_i is the ion pressure and $F_{i,r}$ includes all other forces, e.g., due to friction with neutrals by charge exchange. It is easy to find that

$$V_\perp = \frac{1}{enB}\left(\frac{dP_i}{dr} - F_{i,r}\right) - \frac{E_r}{B} \tag{14}$$

Although $V_\perp$ has several contributions, the last one, $V_{E\times B} = -E_r/B$, induced by the radial electric field is responsible for the observed suppression of turbulent fluctuations in plasmas. This statement is based on experimental observations and theoretical computations that show that the $E\times B$ component is the sole advectant of fluctuations in density, temperature, and flow. For example the first component, the diamagnetic drift provided by the pressure gradient, does not advect fluctuations because it is cancelled by a part of the viscous tensor.[14] Because of this unique status of $E\times B$ flow in plasmas, it is important to analyze the mechanisms of its generation.

A. Loss of particles

In order to find an equation governing the electric field we proceed from the Maxwell equation, $\varepsilon_0\,\nabla\cdot\mathbf{E} = e\,(n_i - n_e)$, where n_i and n_e are the densities of ions and electrons, respectively. Combining the time derivative of this equation with the continuity equations $\partial n_{i,e}/\partial t + \nabla\cdot\mathbf{\Gamma}_{i,e} = 0$ we get:

$$\varepsilon_0\,\frac{\partial E_r}{\partial t} = e\,(\Gamma_{er} - \Gamma_{ir}) \tag{15}$$

Thus, if the electron and ion fluxes are functions of the electric field, this equation allows to determine E_r. In particular, in stationary state the ambipolarity constraint $\Gamma_{er}(E_r) = \Gamma_{ir}(E_r)$ is fulfilled. In an axisymmetric tokamak plasma free from instabilities the particle fluxes arise only due to collisions between ions and electrons. The so called neoclassical transport theory predict that these fluxes are automatically ambipolar and do not prescribe E_r. The situation is, however, different at the plasma edge where charged particles can escape into the so called scrape-off layer (SOL) and be absorbed by divertor plates or limiters. The width of this edge region is of the distance Δ which charge particles can deviate from a magnetic surface. For the majority of particles this distance is of the order of their Larmor radius ρ_L. However, there is a relatively small population of the so called "trapped" particles for which Δ is much larger. Since the toroidal magnetic field is weaker at the outer side than at the inner one a charged particle moving along magnetic field feels a weak magnetic mirror. For particles with a small parallel velocity this mirror is, nevertheless, enough to trap them at the low field side. These particles drift from the magnetic surface due to the centrifugal force and the projection of their trajectories on the poloidal cross-section of tokamak recalls a banana. Therefore trapped particles are called also "banana" particles. The radial width of "banana" trajectory is of $\Delta_b \approx \rho_L q\sqrt{R/r}$, i.e., significantly larger than ρ_L. The loss of trapped particles from the edge region of the banana width Δ_b is balanced in the stationary state by the generation of such particles due to Coulomb collisions of non-trapped ones. The generation rate, which determines the ion flux Γ_{ir}, is estimated as $n\Delta_b F(E_r)/\tau_{ii,eff}$, where $\tau_{ii,eff} = \tau_{ii}\,r/R$ is the effective ion-ion collision time for trapped particles.[15] The function $F(E_r)$ mimics the fact that particles, for which $V_{E\times B}$ compensates the poloidal component of their parallel velocity, i.e., $V_{E\times B} + V_\parallel \frac{r}{qR} = 0$, are moveless and undergo collisions most easily. By taking into account that the percentage of such particles is $\exp\left(-m_i V_\parallel^2/2T\right)$ we get

$$\Gamma_{ir}(E_r) \approx \frac{nq\rho_i}{\tau_{ii}}\left(\frac{R}{r}\right)^{3/2}\exp\left(-X^2\right) \tag{16}$$

where

$$X = \frac{qR}{r}\frac{eE_r\rho_i}{T} \tag{17}$$

and ρ_i denotes the Larmor radius of the ions.

It is assumed that electrons, for which ρ_L and thus their banana orbit loss by a factor $\sqrt{m_e/m_i}$ smaller than for ions, can escape plasma along stochastic magnetic field lines.[16] This loss is determined by the density and temperature gradients and the radial electric field since E_r has a component along stochastic lines which confine electrons in the plasma interim:

$$\Gamma_{er}(E_r) = D_e\,n\,\left(\frac{d\ln n}{dr} + \alpha\,\frac{d\ln T}{dr} + \frac{eE_r}{T}\right) \tag{18}$$

where α is a constant of order unity and the electron diffusivity D_e is determined from models for transport in stochastic fields.[17] The ambipolarity condition $\Gamma_{er} = \Gamma_{ir}$ then gives an equation for E_r:

$$\exp\left(-X^2\right) = d\,(\lambda - X) \tag{19}$$

with

$$\begin{aligned} d &= \frac{D_e\tau_{ii}}{q^2\rho_i^2}\left(\frac{r}{R}\right)^{5/2} \\ \lambda &= \rho_i\frac{qR}{r}\left(\frac{d\ln n}{dr} + \frac{\alpha}{T}\frac{dT}{dr}\right) \end{aligned} \tag{20}$$

A graphic solution of this equation illustrated in Fig.3 and predicts that if λ exceeds a critical value a bifurcation occurs into a new state: the particle fluxes are strongly reduced and the radial electric field jumps to a large positive value.

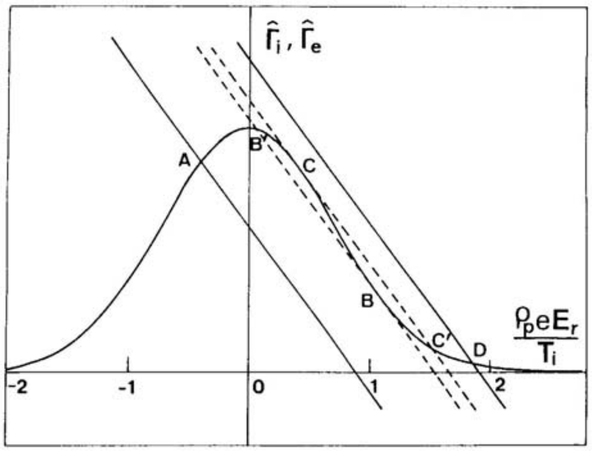

Figure 3: Normalized ion and electron fluxes vs the radial electric field. Dashed lines indicate the bifurcation condition.[16]

B. Loss of momentum

The results cited above are, however, in obvious contradiction to experimental observations: the radial electric field becomes more negative after the transition to the H-mode (Fig.4). This is understandable: not the field itself but its derivative determines the flow shear S_V. If we discard the idea that the anomalous transport of electrons is governed by stochastization of the magnetic field, it is reasonable to assume that ions, which have large larmor radii, tend to leave the plasma faster. In order to maintain quasi-neutrality the radial electric field should prevent this, i.e., be negative. At the plasma axis $E_r = 0$ and the more negative E_r at the edge the larger S_V.

To avoid the difficulty in using the ambipolarity constraint in the model above it was assumed[18] that the radial electric field is governed by the conservation of plasma momentum instead of the particle transport which itself is one of the most poorly understood topics in plasma physics. The loss of banana orbits means that in the edge region ions have a mean radial velocity $V_{ir} = \Gamma_{ir}/n$ and undergo a poloidal Lorentz force

$$F_L = e\Gamma_{ir}B \tag{21}$$

This force accelerates ions and in a stationary state it should be balanced by the ion viscosity due to Coulomb collisions. A viscous force F_V arises because the poloidal velocity V_θ is different at the high and low field sides due to the difference in toroidal circumferences by a factor of r/R:

$$F_V \approx \mu_\parallel \frac{r}{R}\frac{m_i n V_\theta}{(qR)^2} \tag{22}$$

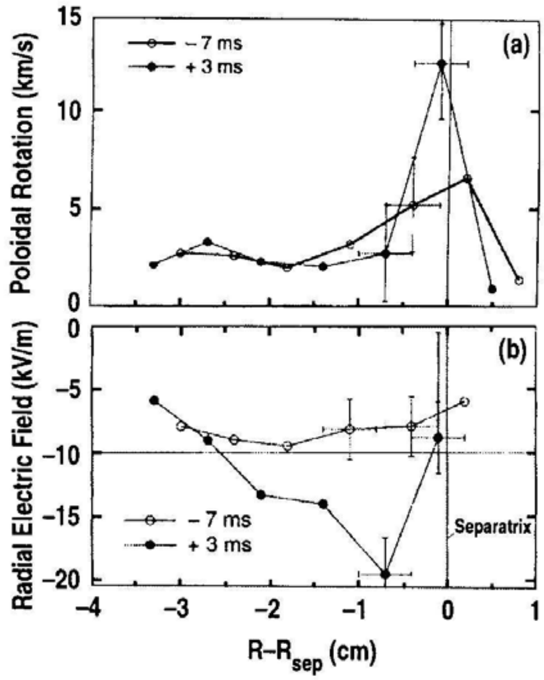

Figure 4: The main ion poloidal rotation (a) and radial electric field profile (b) computed using measured quantities for 7 ms before and 3 ms after L-H transition in DIII-D.[19]

where $\mu_\parallel$ is the parallel viscosity. When V_θ is small $\mu_\parallel \approx \tau_{ii}V_{th}^2$, where $V_{th}^2 = T/m_i$ is the thermal velocity and τ_{ii} is the ion-ion collision time; when V_θ approaches to V_{th} particles collide too rarely and viscosity decreases dramatically. Dependences of F_L and F_V on the poloidal velocity can be determined from kinetic calculations.[18] Fig.5 shows the behavior of normalized forces versus the parameter $Y = \frac{V_\theta}{V_{th}}\frac{B}{B_\theta} - \frac{\lambda}{2}$ for different magnitudes of the ion temperature (λ is defined in the previous section). At a relatively low temperature the plasma is in the state L with weak poloidal rotation but when T exceeds a critical level a transition into the state H with large V_θ takes place. This means a transition to a large negative $V_{E\times B}$ as it can be seen from Eq.(14) if the relation $V_\perp \approx V_\theta - V_\phi \frac{B_\theta}{B}$ is taken into account (ϕ is the toridal angle).

This model provides also a plausible explanation for edge transport barriers induced by electrode biasing. In this case the radial current from the electrode provides a Lorentz force, which induces a poloidal rotation and a radial electric field.

C. Revisited neoclassical theory

The models for bifurcation into the H-mode state discussed above assume that there should be a suffi-

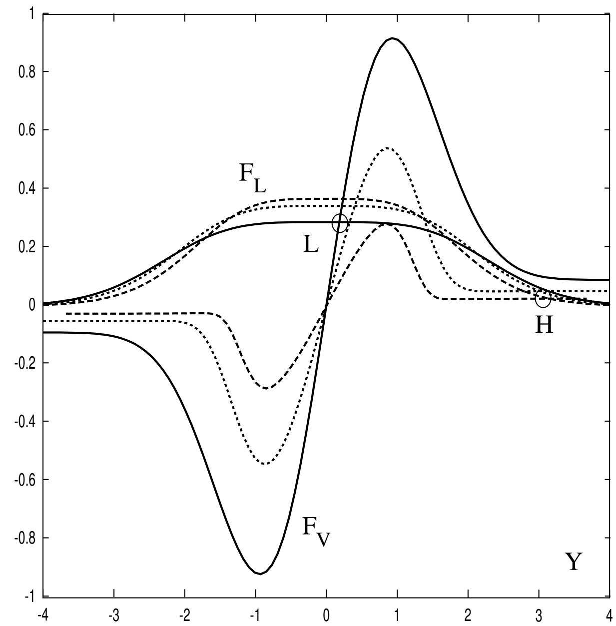

Figure 5: Normalized forces applied to the ions in the poloidal direction vs the parameter Y related to radial electric field. Ion temperature increases from solid through doted to dashed curves.[18]

cient amount of trapped particles at the plasma edge, i.e., the plasma collisionality should be relatively low. Measurements in several devices have shown that the L-H transition can occur even when collisionality is high and too few banana particles are lost from the confined plasma region. To explain these observations the so called revisited neo-classical theory (RNT) has been applied.[20] By taking into account effects of finite ion Larmor radius and ion inertia this theory generalizes the results of neoclassical theory for the case of sharp gradients of the plasma parameters at the edge. As a result the ion and electron fluxes are not automatically ambipolar in RNT and the ambipolarity constrain allows to determine the radial electric field. In agreement with measurements RNT predicts for stationary states a much stronger negative electric field under H-mode conditions compared to the L-mode. For further checks it is important to simulate the time dynamics of the L-H transition.

IV. TRANSITION TO ENHANCED CONFINEMENT STATES

In the previous sections the formation of transport barriers was considered as a combination of turbulence suppression by flow shear and the generation of flow shear by a variety of mechanisms. These processes were treated as independent, but in reality are coupled and must be treated selfconsistently. Without such a selfconsistent treatment it is not possible to understand the transition to an enhanced confinement state. The current state of transition modeling is rather heuristic and at present, no transition theory captures every detail of experimental transitions. As examples for transition models The first order and second order transition theory will be sketched. In analogy with the Landau theory of phase transitions the flow shear is the order parameter.

A. First order critical transition theory

In the simplest first order model[21] equilibrium balances for the ion momentum and the total momentum with a model for the heat flux that incorporates shear suppression heuristically. Employing standard neoclassical theory and assuming $V_\theta \approx V_{E\times B}$ one finds

$$S_V = \frac{\partial V_\theta}{\partial r} = -\frac{4\,\mu_1}{e\,B\,\nu_{*i}^2\,T}\left(\frac{\partial T}{\partial r}\right)^2 \tag{23}$$

with a constant μ_1 and the collisionality $\nu_{*i} = q\,R/(V_{th}\,\tau_{ii}\epsilon^{3/2})$, $\epsilon = r/R$. The thermal conductivity is then assumed to be the sum of the ion neoclassical contribution and a turbulent contribution that is modified by poloidal rotation shear

$$\kappa = \kappa_n + \frac{\kappa_{an}}{1+\gamma_{an}\,(\partial V_\theta/\partial r)^2} \tag{24}$$

and using this in the heat balance

$$-\kappa\,\frac{\partial T}{\partial r} = Q(r) \tag{25}$$

an equation of the form

$$\kappa_n\,g + \frac{\kappa_{an}\,g}{1+\lambda_{an}\,g^4} = Q(r) \tag{26}$$

is obtained, where $g = \partial T/\partial r$. To analyse this equation one considers a certain radius $r = a$ and prescribes the heat flux $Q(a)$ across the boundary. The function on the left hand side has a minimum and a maximum if $\kappa_{an}/\kappa_n > 16/9$ and is shown in Fig.6 for $\kappa_{an}/\kappa_n = 4$. At low power $Q(a)$ the thermal conductivity is $\kappa_n + \kappa_{an}$. The temperature gradient is small, since the poloidal rotation is not large enough to suppress the turbulence. This corresponds to L mode. As $Q(a)$ is increased, the temperature gradient increases continously up to g_1, but for larger $Q(a)$ the gradient has to jump to a larger value. The solutions then must have a gradient larger than g_3 and this corresponds to H mode. The transition is thus manifested by a discontinuity of the temperature gradient. This is analogous to the first order transition occurring, e.g., between the liquid and gas phases of a fluid. The order parameter can be either the temperature gradient or the flow shear. Characteristic of first order phase transitions there is hysteresis.[21]

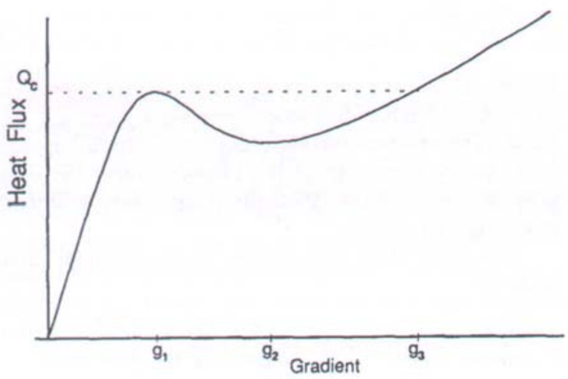

Figure 6: Local heat flux versus temperature gradient.[21]

B. Second order critical transition theory

A second order critical transition theory is possible when the so called Reynolds stress is larger than the ion pressure term in the ion momentum balance.[22] This Reynolds stress can drive a sheared poloidal flow and consequently damp fluctuations. Vice versa the magnitude of the Reynolds stress depends on both the fluctuation energy and the flow shear.

A dynamical model for the second order transition is derived from the poloidal momentum equation

$$\frac{\partial V_\theta}{\partial t} = -\frac{\partial}{\partial r}\left\langle \widetilde{V}_r \widetilde{V}_\theta \right\rangle - \mu_\theta V_\theta \tag{27}$$

where the last term represents the viscosity effect and the first term on rhs contains a part of the Reynolds stress tensor. Here $\langle \ldots \rangle$ denotes a flux surface average. It is assumed that the contributions from toroidal rotation and diamagnetic drift are small and again $V_\theta \approx V_{E\times B}$.

By taking the radial derivative the equation above is reduced to an equation for the flow shear S_V. By multiplication of this with S_V an equation for the value $U = S_V^2$ can be obtained.[22] The latter contains the term $\partial^2 \langle \widetilde{V}_r \widetilde{V}_\theta \rangle / \partial r^2$, which is proportional to the density fluctuation level $E \equiv |\widetilde{n}_k/n|^2$. Note that this Reynolds stress is non-zero only if the radial structure of the fluctuation is not symmetric with respect to the resonant magnetic surface. This is in particular the case for the eigenmodes of the drift wave equation in the presence of flow shear.[23] Finally the equation for U is as follows:

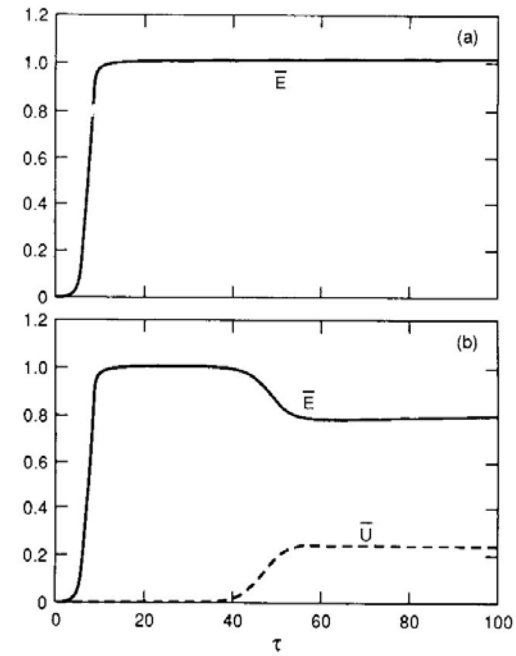

Figure 7: Time evolution[22] of fluctuation density E and square of flow shear U, for (a) $a = 0.5$ and (b) $a = 1.3$.

$$\frac{1}{2}\frac{\partial U}{\partial t} = \alpha_3\, E\, U - \mu\, U \tag{28}$$

To describe the evolution of E we start with an equation for the density fluctuation in the form $\partial \widetilde{n}/\partial t = \gamma_{NL}\widetilde{n}$. Here the nonlinear growth rate takes into account the trigger by linear instability and suppression by diffusion and flow shear: $\gamma_{NL} \approx \gamma_0 - \alpha_1 D - \alpha_2 U$. By multiplying the equation above with $\widetilde{n}$ and bearing in mind that for turbulent diffusion $D \sim E$ we get:

$$\frac{1}{2}\frac{\partial E}{\partial t} = \gamma_0\, E - \alpha_1\, E^2 - \alpha_2\, E\, U \tag{29}$$

The coefficients $\alpha_{1,2,3}$ for different instabilities are given in the literature.[22]

These two equations now describe the interplay between the fluctuation level and the flow shear via Reynolds stress. They are similar to the Verhulst population model,[24] with the flow shear being analogous to the predator species and the fluctuation level to the prey species.

Equations (17) and (18) have two stationary solutions: L-mode type with $E = \gamma_0/\alpha_1$ and $U = 0$ stable for $\gamma_0 \leq \alpha_1\mu/\alpha_3$ and H-mode type with $E = \mu/\alpha_3$ and $U = 1/\alpha_2\,(\gamma_0 - \mu\alpha_1/\alpha_3)$ stable for $\gamma_0 > \mu\alpha_1/\alpha_3$. Thus if the linear growth rate of instability increases, e.g., due to growth of the temperature gradient with increasing heat flux the system can spontaneously transit from the

L-mode behavior with strong fluctuations and no flow to the H-mode state with smaller turbulence and flow shear. This is demonstrated in Fig.7 where the solution of Eqs.(17) and (18) is plotted for two magnitudes of the parameter $a = \alpha_3/\alpha_1$ while $b = \mu/\gamma_0$ is set to 1. For $a = 0.5$ the fluctuation grows to the L-mode saturation level, $E = 1$, and no flow is generated. By increasing a to 1.3, the instability saturates first at the L-mode level while later on there is a smooth transition to the H mode, with generation of flow shear.

V. CONCLUSION

Very large progress has been done in understanding of mechanisms of turbulence suppression in transport barriers. Nevertheless a lot of work is ahead, which is needed to interpret the plasma properties in such structures and to optimize them in order to achieve the maximum benefit from regions with reduced transport in future fusion devices.

REFERENCES

1. F. WAGNER et al., *Phys. Rev. Lett.* **49**, 1408 (1982).
2. ASDEX TEAM, *Nucl. Fusion* **29**, 1959 (1989).
3. K. ITOH and S.-I. ITOH, *Plasma Phys. Control. Fusion***38**, 1 (1996).
4. P.W. TERRY, *Rev. Mod. Phys.* **72**, 109 (2000).
5. O. SAKAI et al., *Phys. Rev. Lett.* **70**, 4071 (1993).
6. R.J. TAYLOR et al., *Phys. Rev. Lett.* **63**, 2365 (1989).
7. R.R. WEYNANTS et al., *Plasma Phys. Control. Nucl. Fus. Res.,1990*, Vol.**1**, 473 (1991).
8. F.M. LEVINTON et al., *Phys. Rev. Lett.* **75**, 4417 (1995).
9. E.J. STRAIT et al., *Phys. Rev. Lett.* **75**, 4421 (1995).
10. R. WOLF, *Plasma Phys. Control. Fusion* **45**, R1 (2003).
11. B. LEHNERT, *Phys. Fluids* **9**, 1367 (1966).
12. R.E. WALTZ, et al., *Phys. Plasmas* **1**, 2229 (1994)
13. B.A. CARRERAS et al., *Phys. Plasmas* **2**, 2745 (1995).
14. Y.B. KIM et al., *Phys. Fluids B* **3**, 384 (1991).
15. F.L. HINTON et al., *Nucl. Fusion* **25**, 1457 (1985).
16. S.-I. ITOH and K. ITOH, *Phys. Rev. Lett.* **60**, 2276 (1988).
17. M.Z. TOKAR, *Phys. Plasmas* **6**, 2808 (1999)
18. K.C. SHAING and E.C. CRUME, Jr, *Phys. Rev. Lett.* **63**, 2369 (1989).
19. J. KIM et al., *Plasma Phys. Control. Fusion* **36**, A183 (1994).
20. A. ROGISTER, *Transac. Fusion. Technoology* **37**, No.2T, 287 (2000).
21. F.L. HINTON, *Phys. Fluids B* **3**, 696 (1991).
22. P.H. DIAMOND et al., *Phys. Rev. Lett.* **72**, 2565 (1994).
23. B.A. CARRERAS et al., *Phys. Fluids B* **4**, 3115 (1992).
24. P.F. VERHULST, *Nuov. Mem. Acad. Roy. Bruxelles* **18**, 1 (1845).