Neutral pion electroproduction off deuterium

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Threshold neutral pion electroproduction on the deuteron is studied in the framework of baryon chiral perturbation theory at next-to-leading order in the chiral expansion. To this order in small momenta, the amplitude is finite and a sum of two- and three-body interactions with no undetermined parameters. We calculate the *S*-wave multipoles for threshold production and the deuteron *S*-wave cross section as a function of the photon virtuality. We also discuss the sensitivity to the elementary neutron amplitudes.

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Chiral perturbation theory has been successfully applied to neutral pion photoproduction and electroproduction off the proton [1,2] as well as to π^0 photoproduction on the deuteron [3]. The scattering off deuterium is not only interesting *per se*, but also because this loosely bound two-nucleon system can be used as a neutron target. In particular, in Ref. [3] it was shown that one can indeed extract the elementary $\pi^0 n$ production amplitude from a precise measurement on the deuteron. This was vindicated by the experiment performed at SAL [4]. Furthermore, at MAMI experiments for neutral pion electroproduction off deuterium at small photon virtualities have been untertaken and are presently being analyzed [5]. In this Brief Report we wish to report on first results for this process obtained in chiral perturbation theory to third order. We use the methodology developed by Weinberg [6], which relates scattering processes involving a single nucleon to nuclear scattering processes. The nonperturbative effects responsible for nuclear binding are accounted for using phenomenological nuclear wave functions.[1] Although this clearly introduces an inevitable model dependence, one can compute matrix elements using a variety of wave functions in order to ascertain the theoretical error induced by the off-shell behavior of different wave functions. Here, we work to third order in the chiral expansion and consider only threshold kinematics (i.e., the pion is produced at rest) and thus calculate the pertinent transverse and longitudinal *S*-wave multipoles. While a third order computation is not sufficient for the normalization of the elementary amplitudes, the explicit calculations for π^0 electroproduction off the proton to fourth order let one expect that the momentum dependence of the *S*-wave cross section is sufficiently accurately described at the order we are working. This topic will be discussed in more detail below.

To third order $[O(q^3)$, where q denotes a small momentum or a pion mass$]$ in chiral perturbation theory, the *S*-wave neutral pion electroproduction amplitude off the deuteron can be decomposed as follows:

$$\mathcal{M}_d = \mathcal{M}_d^{ss} + \mathcal{M}_d^{tb} = 2\mathcal{M}_1 \vec{J}\cdot\vec{\epsilon} + 2i\mathcal{M}_2 \vec{J}\cdot\hat{k}\vec{J}\cdot(\vec{\epsilon}\times\hat{k}) + 2\mathcal{M}_3 \vec{J}\cdot\hat{k}\vec{\epsilon}\cdot\hat{k}, \tag{1}$$

where $\vec{J}$ is the deuteron angular momentum vector, and $\vec{\epsilon}$ and $\vec{k}$ are the polarization vector and three-momentum of the virtual photon, respectively. Note that in electron scattering $k^2 \leqslant 0$ there are three multipole amplitudes at threshold, which are of the electric, magnetic and longitudinal type. These can be mapped onto the notation of Ref. [9] via $\mathcal{M}_1 = E_{01}^1 + M_{01}^2$, $\mathcal{M}_2 = E_{01}^1$, and $\mathcal{M}_1 + \mathcal{M}_3 = L_{01}^1$. In that notation, the upper index gives the multipolarity, whereas the lower indices denote the orbital angular and the total angular momentum of the final deuteron-pion system, respectively. Effectively, however, one has only two combinations of these multipoles contributing at threshold, which only appear squared and are given by

$$|E_d|^2 \equiv |E_{01}^1|^2 + |M_{01}^2|^2, \quad |L_d|^2 \equiv |L_{01}^1|^2. \tag{2}$$

The electric dipole amplitude E_d characterizes the transverse response whereas L_d parametrizes the longitudinal response of the deuteron to the virtual photon. In general, the multipoles depend on the photon virtuality k^2 and the pion energy

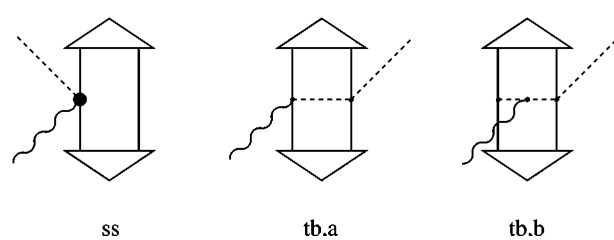

FIG. 1. Single scattering (SS) and three-body (TB) interactions which contribute to neutral pion electroproduction at threshold to order q^3 (in the Coulomb gauge). The solid, dashed, and wiggly lines denote nucleons, pions, and photons, in order. The deuteron wave function is symbolized by the triangle.

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[1]We are well aware of recent developments in chiral effective field theories which also provide fairly precise deuteron wave functions [7,8]. However, for our purpose the hybrid approach is suitable and we will comment on this issue later on.

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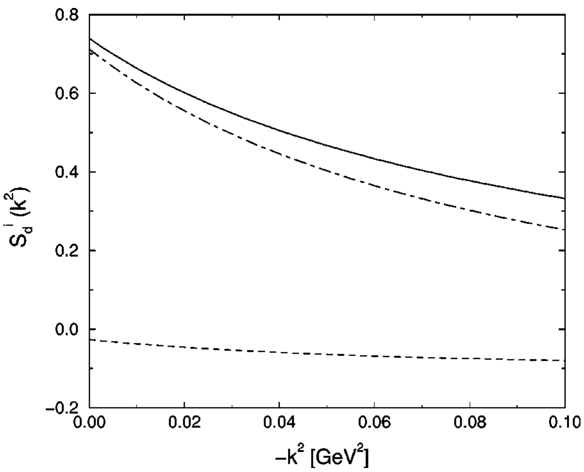

FIG. 2. Deuteron form factors. The solid, dashed, and dot-dashed line refers to $S_d^1(k^2)$, $S_d^2(k^2)$, and $S_d^3(k^2)$, respectively.

ω. Since we only consider the production threshold $\omega_{\rm thr}=M_{\pi^0}$, we will not further specify this energy dependence. In analogy to what is done in the single nucleon sector, we also consider the deuteron S-wave cross section a_{0d},

$$a_{0d}=|E_d|^2+\epsilon_L|L_d|^2, \tag{3}$$

with $\epsilon_L=-(k^2/k_0^2)\epsilon$ the longitudinal polarization for a photon with polarization ϵ.

As shown in Fig. 1, the amplitude obtains contributions from single scattering (SS) as well as the so-called three-body (TB) graphs. The single scattering contribution for the transverse and longitudinal multipoles at threshold takes the form

$$E_d^{\rm SS}=\frac{1+M_{\pi^0}/m_N}{1+M_{\pi^0}/m_d}\frac{1}{2}E_{0+}S_d^1(k^2),$$

$$L_d^{\rm SS}=\frac{1+M_{\pi^0}/m_N}{1+M_{\pi^0}/m_d}\frac{1}{2}\{E_{0+}[S_d^1(k^2)+S_d^2(k^2)-S_d^3(k^2)] +L_{0+}S_d^3(k^2)\}, \tag{4}$$

where M_π, m_N, and m_d denote the pion, nucleon, and deuteron mass, in order.[2] Furthermore,

$$X_{0+}=X_{0+}^{\pi^0p}+X_{0+}^{\pi^0n}, \quad X=\{E,L\}, \tag{5}$$

and the $S_d^i(k^2)$ are the pertinent deuteron form factors

$$\int d^3p\,\phi^*(\vec{p})\vec{S}\cdot\vec{\epsilon}\phi(\vec{p}-\vec{k}/2)=S_d^1(k^2)\vec{J}\cdot\vec{\epsilon}+S_d^2(k^2)\vec{J}\cdot\hat{k}\vec{\epsilon}\cdot\hat{k},$$

[2]Note that we work with $M_{\pi^0}=134.97$ MeV and $M_{\pi^+}=140.11$ MeV, to account for the neutron-proton mass difference in the rescattering diagrams. A detailed discussion of this point is given in Ref. [1].

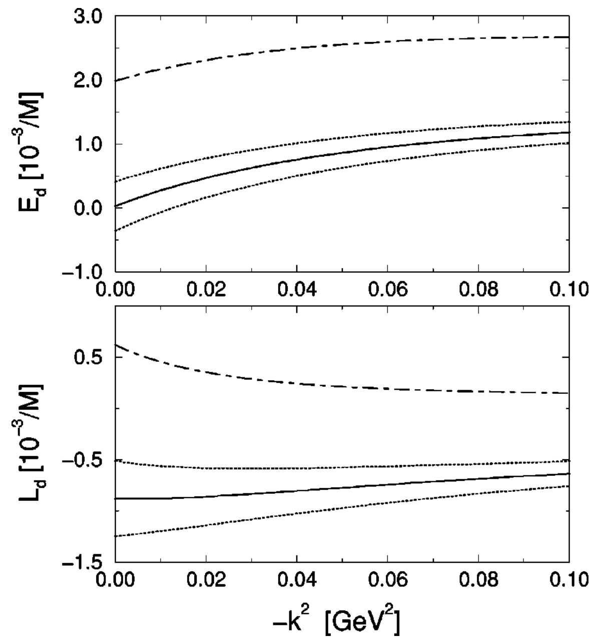

FIG. 3. S-wave multipoles of the deuteron. In the upper and lower panel, the electric dipole and the longitudinal (scalar) multipoles are shown, respectively, by the solid lines. The dot-dashed line is the single scattering contribution. The dotted lines are obtained by changing the elementary neutron amplitude by $\pm 1\times 10^{-3}/M_{\pi^+}$ (for all values of k^2).

$$\int d^3p\,\phi^*(\vec{p})\vec{S}\cdot\hat{k}\vec{\epsilon}\cdot\hat{k}\phi(\vec{p}-\vec{k}/2)=S_d^3(k^2)\vec{J}\cdot\hat{k}\vec{\epsilon}\cdot\hat{k}. \tag{6}$$

In the photoproduction case, one is of course only sensitive to S_d^1, see, e.g., Ref. [10]. $\phi(p)$ denotes the momentum space deuteron wave function. It is important to stress that E_d and L_d are defined with respect to the total angular momentum $\vec{J}=\vec{L}+\vec{S}$ and not only with respect to the deuteron spin vector $\vec{S}=(\vec{\sigma}_p+\vec{\sigma}_n)/2$. To properly extract the single scattering contribution, one has to look at the transverse and longitudinal response functions $R_{T,L}$ (the formalism is reviewed in Ref. [11]). The form factors $S_d^i(k^2)$ are shown in Fig. 2 for photon virtualities in the range from $k^2=0$ to $k^2=-0.1$ GeV^2. All of them exhibit a strong dependence on the photon virtuality, i.e., at $k^2\simeq -0.1$ GeV^2 they suppress the single scattering contribution by about a factor of 2.

In addition, there are two three-body contributions at third order (see also Ref. [10]). The corresponding amplitudes read

$$\mathcal{M}^{(\mathrm{TB},a)}=-2\frac{eg_Am_dM_{\pi^0}}{(2\pi)^3F_\pi^3}\left\langle\frac{\vec{S}\cdot\vec{\epsilon}}{\vec{q}^{\,\prime 2}}\right\rangle_{\rm wf},$$

$$\mathcal{M}^{(\mathrm{TB},b)}=2\frac{eg_Am_dM_{\pi^0}}{(2\pi)^3F_\pi^3}\left\langle\frac{\vec{S}\cdot(\vec{q}^{\,\prime}-\vec{k})(2\vec{q}^{\,\prime}-\vec{k})\cdot\vec{\epsilon}}{((\vec{q}^{\,\prime}-\vec{k})^2+M_{\pi^+}^2)\vec{q}^{\,\prime 2}}\right\rangle_{\rm wf}, \tag{7}$$

using standard kinematical notation [10,3]. Furthermore, $F_\pi=92.4$ MeV is the pion decay constant, $g_A=1.33$ the

TABLE I. S-wave cross section for the scaled single scattering amplitudes as explained in the text. The range is obtained by adding a constant shift of $\pm 1\times 10^{-3}/M_{\pi^+}$ to the elementary $\pi^0 n$ longitudinal multipole.

$-k^2$ [GeV^2]	0.02	0.04	0.06	0.08	0.10
a_{0d} [μb]	0.11	0.15	0.17	0.19	0.20
a_{0d} range [μb]	0.09–0.13	0.12–0.18	0.14–0.21	0.16–0.23	0.17–0.24

axial-vector coupling constant,[3] and $\langle \vartheta \rangle_{\rm wf}$ indicates that ϑ is sandwiched between deuteron wave functions. These matrix elements have been evaluated using a cornucopia of wave functions in coordinate and momentum space. This is a nontrivial check on our calculations. As a further check we mention that for $k^2=0$, we recover the corrected results of Ref. [10]. If not stated otherwise, all numbers quoted have been obtained using the Bonn potential. It is important to stress that the three-body corrections turn out to be quite independent of the wave function used. This implies that the chiral perturbation theory approach, which relies on the dominance of the pion exchange, is useful in this context.

We now discuss the results for the multipoles and the S-wave cross section. First, we consider the $\mathcal{O}(q^3)$ calculation, being aware of the fact that in particular the proton electric dipole amplitude at the photon point is not well described to that order. However, the k^2 dependence of the elementary amplitudes on the proton and the neutron is largely given by the third order contribution (see the results discussed in Ref. [2]). The $\mathcal{O}(q^3)$ results for E_d and L_d are shown in Fig. 3. In both cases, the three-body contribution is sizeable. As in the case of photoproduction, graph (TB,a) (see Fig. 1) totally dominates the electric dipole amplitude. This is different for the longitudinal response, where the contribution from graph (TB,b) is still smaller than the one of graph (TB,a) but of comparable magnitude. We observe that E_d varies more significantly with increasing $|k^2|$ than L_d. As illustrated by the dotted lines in Fig. 3, which have been obtained by a constant shift of the $\pi^0 n$ amplitudes by $\pm 1 \times 10^{-3}/M_{\pi^+}$, there is some sensitivity to the elementary scattering off the neutron. To get a more realistic estimate for the S-wave cross section, we have adjusted the values of $X_{0+}^{\pi^0 p}$ and $X_{0+}^{\pi^0 n}$ ($X=\{E,L\}$) at the photon point to the values obtained from the fourth order calculation. More precisely, the electric dipole amplitudes are taken from Ref. [1], the $L_{0+}^{\pi^0 p}$ from the best fit obtained in Ref. [2] and the $L_{0+}^{\pi^0 n}$ using the resonance saturation estimate with $g_3=-125.6$ and $X'=-0.23$, as detailed in Ref. [2] (see Ref. [12] for a more detailed discussion on this point). The so calculated S-wave cross section is collected in Table I for a photon polarization $\epsilon=0.67$. We also give a range, which is obtained by adding a constant shift of $\pm 1\times 10^{-3}/M_{\pi^+}$ to the elementary $\pi^0 n$ longitudinal multipole (but keeping the electric dipole amplitude at the same value as before). This serves to illustrate the sensitivity of the S-wave cross section to the so far unmeasured $L_{0+}^{\pi^0 n}$. For comparison, we note that the S-wave cross section measured on the proton [13,14] for $-0.10\leqslant k^2 \leqslant -0.04$ GeV^2 lies between 0.15 and 0.45 μb. We remark that for $k^2\simeq -0.1$ GeV^2, $a_{0d}\simeq a_0/2$, with a_0 the S-wave cross section for neutral pion production off the proton. However, the curvature of $a_{0d}(k^2)$ is rather different from the one for the proton case. This is partly due to the interference of the proton and neutron amplitudes and partly a kinematical effect, since for a given polarization ϵ and virtuality k^2, ϵ_L is larger for the proton than for the deuteron.

To summarize, we have considered neutral pion electroproduction off the deuteron at threshold using Weinberg's hybrid approach of nuclear effective field theory (i.e., using Weinberg's chiral counting for the interaction kernel and applying external realistic wave functions, taken here from the Bonn potential). To third order in the chiral expansion, we have worked out the S-wave multipoles as a function of the photon virtuality and discussed the sensitivity of these to the elementary $\pi^0 n$ amplitude. This calculation is free of undetermined parameters. We have also calculated the S-wave cross section and compared its behavior to the case of the proton. To get a better handle on the single scattering amplitudes, we have scaled the elementary amplitudes so that they reproduce the fairly precise fourth order result (at the photon point). The trends obtained in this short note should be substantiated by a full scale fourth order calculation, going also above threshold. In addition, with the precise deuteron wave functions obtained in Ref. [8], it will also be possible to perform the calculation entirely within the framework of effective field theory. Such an investigation is under way [12]. It would also be interesting to consider this process in the scheme of Ref. [7], although that approach has so far not been tested in reactions involving on-shell pions on the external legs.

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[3]This is the value determined using the Goldberger-Treiman relation with $g_{\pi N}=13.4$. This value was also used in the determination of the single scattering amplitudes, see, e.g., Ref. [1]. In any case, the difference between this value for g_A and its physical value of 1.26 is due to a dimension three πN operator, which does not occur at the order we are working.

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