

Evidence that the $a_0(980)$ and $f_0(980)$ are not elementary particles

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Abstract

We study the interesting problem of whether it is possible to distinguish composite from elementary particles. In particular we generalize a model-independent approach of S. Weinberg to the case of unstable particles. This allows us to apply our formalism to the case of the $a_0(980)$ and $f_0(980)$ resonances and to address the question whether these particles are predominantly genuine, confined quark states (of $\bar{q}q$ or $qq\bar{q}\bar{q}$ structure) or governed by mesonic components.

1 Introduction

In the mid-sixties S. Weinberg suggested an elegant way to decide whether a given particle is composite or elementary [1]. The idea was applied to the case of the deuteron, and it was shown that the physical deuteron is not an elementary particle. In order to prove it the field renormalization factor Z , $0 \leq Z \leq 1$, was used, which is the probability of finding the physical deuteron $|d\rangle$ in a bare elementary-particle state $|d_0\rangle$, $Z = |\langle d_0|d\rangle|^2$.

If the deuteron is purely elementary, then $Z = 1$. On the contrary, for a purely composite particle made of a proton and a neutron, $Z = 0$. The way to determine the value of Z from hadronic observables is to express the scattering length, a , and the effective range, r_e , in terms of Z as [1]

$$a = \frac{2(1-Z)}{2-Z}R + O(1/\beta), \quad r_e = -\frac{Z}{1-Z}R + O(1/\beta), \quad (1)$$

where $R = 1/\sqrt{m_N \epsilon}$, ϵ is the deuteron binding energy and $1/\beta$ is the range of forces. The relations (1) are valid in case of a loosely bound state with small binding energy, so that $R \gg 1/\beta$, and they are model-independent in this limit.

If the deuteron is composite, then $Z = 0$, $a = R$, $r_e \approx O(m_\pi^{-1})$, with $r_e > 0$. The limit $Z \rightarrow 0$ is in agreement with the experimental values for a and r_e : $a = +5.41$ fm, $r_e = +1.75$ fm. It means that the deuteron is indeed mostly a composite system made of a proton and a neutron [1]. If the deuteron had considerable admixture of elementary-particle component, then r_e , in accordance with Eqs. (1) would be large and negative!

Three requirements are needed for the analysis to be applicable [1]: i) the particle must couple to a two-body channel with threshold close to the nominal mass; ii) this two-body channel must have zero orbital momentum; iii) the particle must be stable, otherwise the analysis cannot be performed in terms of a real Z , and the probabilistic interpretation is lost. At present one can find particles for which (i) and (ii) are satisfied but, usually, (iii) is not satisfied, and one tends to agree with S. Weinberg that “nature is doing her best to keep us from learning whether the elementary particles deserve that title”.

In the present paper we adapt Weinberg’s idea for the case of unstable particles. In particular, we consider the spectral density $w(E)$ which is the probability for finding a bare elementary state in the continuum introduced in [2]. We show that the integral of this quantity over the resonance region serves as a natural generalization of Weinberg’s variable Z . We also consider the near-threshold singularities of the scattering amplitude. The concept of “pole counting” as a tool for resonance classification was formulated by D. Morgan in [3]. We demonstrate that the “pole counting” scheme is directly related to Weinberg’s analysis in terms of Z (or its continuum counterpart $w(E)$). Thus the notion of pole counting can be put on a much more quantitative basis.

The most obvious case to apply our approach to are the $a_0(980)/f_0(980)$ resonances, the most controversial objects of meson spectroscopy. Quark models [4] predict 1^3P_0 $q\bar{q}$ states made of light quarks to exist at about 1 GeV (see also recent developments in [5]), and the $f_0(980)$ and $a_0(980)$ are natural candidates for such states. However, as these states are rather close to the $K\bar{K}$ threshold, a significant $qq\bar{q}\bar{q}$ affinity is expected from a phenomenological point of view [6], either in the form of compact $qq\bar{q}\bar{q}$ states [7] or in the form of loosely bound $K\bar{K}$ states.

A central question of the ongoing debate regarding the nature of those scalar resonances is, whether there are sufficiently strong t -channel forces so that $K\bar{K}$ molecules are formed, as advocated in Refs. [8,9,10], or whether the meson-meson interaction is dominated by s -channel states. In the former case the

$f_0(980)$ and $a_0(980)$ would be composite particles whereas in the latter case they would be elementary states linked to quark-gluon dynamics. (For a compact presentation of the discussion we refer to Ref. [11].) To complicate matters further, there is no consensus on the manifestation of the s -channel state. There are claims that this s -channel state (of $q\bar{q}$ [5,12] or of four quark nature [7]) manifests itself as genuine confined quark state, while in Refs. [13,14,15] it is found that a strong coupling to the $K\bar{K}$ channel causes unitarity effects which lead to large hadronic component in the wave function of the a_0/f_0 mesons. More recent works stress the relevance of chiral symmetry for the interactions of the lightest pseudo-scalars as well as for the formation of corresponding bound states [16]. Up to date the observed features of the a_0 and f_0 states could be explained by both the existence of bare confined states strongly coupled to mesonic channels and/or by potential-type interactions. Thus the question arises whether it is possible to distinguish, at a quantitative level, between different assignments for a_0/f_0 . In the present paper we show, that this could be indeed the case.

The decays $f_0 \rightarrow \pi\pi$ and $a_0 \rightarrow \pi\eta$ are known to be the main source of the width for these mesons. The results of data analyses are often presented in terms of the Flatté parameters [17]. We argue that the Flatté parametrization conveniently offers the possibility to connect the singularities of the scattering amplitude with the manifestations of bare states and to calculate the spectral density and admixture of bare states in the near-threshold region. However, we also show the limitations of the Flatté parametrization and point to situations where its use is doomed to fail.

The paper is structured in the following way: In the next section we outline our formalism. In section 3 we discuss the connection of the field renormalization factor Z and the spectral density $w(E)$ with the pole counting scheme proposed by D. Morgan and also with the effective range parameters. In section 4 we apply our formalism to the case of the $a_0(980)$ and $f_0(980)$ resonances. The paper ends with a short summary.

2 Dynamics of coupled channels

First we briefly review the dynamics in a coupled channel system, and show that the small binding limit of it yields, in the elastic case, the relations (1). It is assumed that the hadronic state is represented symbolically as

$$|\Psi\rangle = \begin{pmatrix} \sum_{\alpha} c_{\alpha} |\psi_{\alpha}\rangle \\ \sum_i \chi_i |M_1(i)M_2(i)\rangle \end{pmatrix}, \quad (2)$$

where the index α labels bare confined states $|\psi_\alpha\rangle$ with the probability amplitude c_α , and χ_i is the wave function in the i -th two-meson channel $|M_1(i)M_2(i)\rangle$. In coupled-channel models like in Refs. [13,14,10] these bare states are taken to be of $q\bar{q}$ nature, but, for example, totally confined $qq\bar{q}\bar{q}$ states [7] may be considered as well. One should only ensure the orthogonality condition $\langle\psi_\alpha|M_1(i)M_2(i)\rangle = 0$ to be fulfilled. The wave function $|\Psi\rangle$ obeys the equation

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}_{MM} \end{pmatrix}, \quad (3)$$

where $\hat{H}_c$ defines the discrete spectrum of bare states, $\hat{H}_c|\psi_\alpha\rangle = E_\alpha|\psi_\alpha\rangle$, $\hat{H}_{MM}$ includes the free-meson part as well as direct meson-meson interaction (e.g., due to t - or u -channel exchange forces), and the term $\hat{V}$ is responsible for dressing the bare states.

Let us start from the simple two-channel case of the coupled-channel equation (3). Namely, we consider only one bare state $|\psi_0\rangle$ and only one hadronic channel ($|K\bar{K}\rangle$). In addition, we assume proper field redefinitions to be performed, such that the residual $\bar{K}K$ interaction can be treated perturbatively [1]. The interaction part is specified by the transition form factor $f_K(\mathbf{p})$,

$$\langle\psi_0|\hat{V}|K\bar{K}\rangle = f_K(\mathbf{p}), \quad (4)$$

where $\mathbf{p}$ is the relative momentum in the mesonic system. For the mesonic channel in the relative S -wave the form factor depends on the modulus of p . We require the function f_K to decrease with p , with some range β whose scale is set by the range of forces or—speaking in quark language—by the internal size scales of the quark wave functions. Both scenarios lead to the estimate for β to be of the order of a few hundred MeV. One immediately arrives at a system of coupled equations for $c_0(E)$ and $\chi_E(\mathbf{p})$:

$$\begin{cases} c_0(E)E_0 + \int f_K(p)\chi_E(\mathbf{p})d^3p = c_0(E)E, \\ \frac{p^2}{m}\chi_E(\mathbf{p}) + c_0(E)f_K(p) = E\chi_E(\mathbf{p}). \end{cases} \quad (5)$$

Here m is the meson mass and $2m + E_0$ is the mass of the bare state. Throughout the paper we take $m = (m_{K^+} + m_{K^0})/2 = 495.7$ MeV. In what follows we will be interested in the near-threshold phenomena in the S -wave $K\bar{K}$ system, so that nonrelativistic kinematics employed in Eq. (5) is justified. The system of Eqs. (5) can be easily solved yielding for the $K\bar{K}$ scattering amplitude the

form

$$F_{K\bar{K}}(k, k; E) = -\frac{2\pi^2 m f_K^2(k)}{E - E_0 + g_K(E)}, \quad k = \sqrt{mE}, \quad \frac{d\sigma}{d\Omega} = |F_{K\bar{K}}|^2, \quad (6)$$

where

$$g_K(E) = \int \frac{f_K^2(p)}{\frac{p^2}{m} - E - i0} d^3p. \quad (7)$$

Let the system possess a bound state with the energy $-\epsilon$, $\epsilon > 0$. The binding energy ϵ fulfills the equation

$$-\epsilon - E_0 + g_K(-\epsilon) = 0. \quad (8)$$

The wave function $|\Psi_B\rangle$ of this bound state takes the form

$$|\Psi_B\rangle = \begin{pmatrix} \cos\theta |\psi_0\rangle \\ \sin\theta \chi_B(\mathbf{p}) |K\bar{K}\rangle \end{pmatrix}, \quad \langle\Psi_B|\Psi_B\rangle = 1, \quad \cos\theta = \langle\psi_0|\Psi_B\rangle, \quad (9)$$

where $\chi_B(\mathbf{p})$ is normalized to unity, and $\cos\theta$ defines the admixture of the bare elementary state in the physical state $|\Psi_B\rangle$. Clearly, $\cos^2\theta$ equals the field renormalization Z already discussed in the beginning. The angle θ is given by

$$\tan^2\theta = \int \frac{f_K^2(p) d^3p}{(\frac{p^2}{m} + \epsilon)^2}. \quad (10)$$

Let us now demonstrate that in the small binding limit $\sqrt{m\epsilon} \ll \beta$ it is possible to express the effective range parameters in terms of the binding energy ϵ and angle θ in a model-independent way. In the region $\sqrt{m|E|} \ll \beta$ the integral $g_K(E)$ can be written as

$$g_K(E) = \bar{E}_K + 2\pi^2 i m f_{K0}^2 \sqrt{mE} + O(m^2 E / \beta^2), \quad (11)$$

where $\bar{E}_K = 4\pi m \int_0^\infty f_K^2(p) dp$ and $f_{K0} = f_K(0)$. The integral $\bar{E}_K$ depends on the explicit form of the transition form factor and its actual value is thus renormalization scheme dependent. However, only the difference $E_0 - \bar{E}_K$ enters the expressions for observables and the scheme dependence of $\bar{E}_K$ is absorbed in E_0 . In the small binding limit the expression Eq. (10) takes the form $\tan^2\theta = \pi^2 m^2 f_{K0}^2 / \sqrt{m\epsilon}$. Now one easily reads from Eq. (6) the result given

in Eq. (1), and these expressions indeed do not depend on the explicit form of the form factor f .

The factor Z is the cornerstone of the analysis [1]. Obviously, the same information is also contained in its continuum counterpart $w(E)$, the spectral density of the bare state [2], given by the expression

$$w(E) = 2\pi m k |c_0(E)|^2, \quad (12)$$

where $c_0(E)$ is found from the system of Eqs. (5),

$$c_0(E) = \frac{f_K(k)}{E - E_0 + g_K(E)}, \quad (13)$$

and which is defined for energies $E > 0$. $w(E)$ defines the probability to find the bare state in the continuum wave function $|\Psi_E\rangle$ and, as we will see, can be easily generalized to a situation where inelastic channels are present. The normalization condition for the distribution $w(E)$ follows from the completeness relation for the total wave function (2) projected onto bare state channel. It reads

$$\int_0^\infty w(E) dE = 1 - Z \quad \text{or} \quad 1, \quad (14)$$

depending on whether there is a bound state or not. Rewriting Eq. (12) as

$$w(E) = \frac{1}{2\pi i} \left(\frac{1}{E - E_0 + g_K^*(E)} - \frac{1}{E - E_0 + g_K(E)} \right) \quad (15)$$

the integral (14) can be easily calculated (for details see Ref. [2]). One immediately sees that, for the case of a bound state, all the information on the factor Z is encoded, due to Eq. (14), in $w(E)$ too. On the other hand, $w(E)$ can be applied for the case of resonance states as well.

The generalization of the formulae above to the multichannel case is straightforward. To this end one should introduce, in addition to (4), a transition form factor $f_P(\mathbf{q})$ which couples the bare state to light pseudo-scalars ($\pi\pi$ or $\pi\eta$) with relative momentum $\mathbf{q}$. The $K\bar{K}$ scattering amplitude is then given again by Eq. (6) with the obvious replacement

$$g_K(E) \rightarrow g_K(E) + g_P(E), \quad g_P(E) = - \int \frac{f_P^2(p)}{E_{tot} - E_P(p) + i0} d^3p, \quad (16)$$

where $E_P(p)$ denotes the (relativistic) energy of the two light pseudo-scalars in the intermediate state in the center of mass system and $E_{tot} = E_P(q) = E + 2m$. The same replacement should be done in Eq. (15) for the spectral density. Thus, the spectral density is now defined below as well as above the $K\bar{K}$ threshold, and, if the exotic possibility of the existence of $\pi\pi$ or $\pi\eta$ bound states is ignored, is normalized to unity with a lower limit of integration in Eq. (14) that corresponds to the threshold of the light pseudo-scalar channel. It is assumed here that there is no direct interaction in all mesonic channels.

Since we are interested only in the phenomena near the $K\bar{K}$ threshold one can introduce some simplifications. Specifically, one can make use of the smooth dependence of the integral $g_P(q)$ on the momentum q and replace $g_P(q)$ by $\bar{E}_P + \frac{i}{2}\Gamma_P$. The quantities $\bar{E}_P$ and $\frac{1}{2}\Gamma_P$ are the real and imaginary parts of $g_P(\bar{q})$, where $\bar{q}$ is the momentum in the light pseudo-scalar channel averaged over the $K\bar{K}$ near-threshold region. This simplification allows also to avoid problems arising from the necessity of a relativistic treatment of the light pseudo-scalar channel. We come back to these corrections later in the manuscript.

In the near-threshold region the expression for the $K\bar{K}$ scattering amplitude with inelasticity can be written as

$$F_{K\bar{K}} = -\frac{1}{2k} \frac{\Gamma_K}{E - E_f + i\frac{\Gamma_K}{2} + i\frac{\Gamma_P}{2} + O(m^2 E/\beta^2)}, \quad (17)$$

where

$$E_f = E_0 - \bar{E}_K - \bar{E}_P, \quad \Gamma_K = \bar{g}_{K\bar{K}}\sqrt{mE}, \quad \bar{g}_{K\bar{K}} = 4\pi^2 m f_{K0}^2.$$

If the terms $O(m^2 E/\beta^2)$ in Eq. (17) are omitted, one immediately recognizes in this form the Flatté parametrization [17] of the near-threshold $K\bar{K}$ amplitude.¹ It is convenient to define the mass of the resonance M_R as the mass at which the real part of the denominator of Eq. (17) is zero:

$$M_R = 2m + E_R, \quad E_R - E_f - \frac{1}{2}\bar{g}_{K\bar{K}}\sqrt{-mE_R}\Theta(-E_R) = 0. \quad (18)$$

In the same way we obtain the near-threshold expression for the spectral density:

$$w(E) = \frac{1}{2\pi} \frac{\Gamma_P + \bar{g}_{K\bar{K}}\sqrt{mE}\Theta(E)}{(E - E_f - \frac{1}{2}\bar{g}_{K\bar{K}}\sqrt{-mE}\Theta(-E))^2 + \frac{1}{4}(\Gamma_P + \bar{g}_{K\bar{K}}\sqrt{mE}\Theta(E))^2}. \quad (19)$$

¹ Our dimensionless coupling constant $\bar{g}_{K\bar{K}}$ is related to the dimensional coupling constant $g_{K\bar{K}}$ commonly used in the literature as $\bar{g}_{K\bar{K}} = \frac{g_{K\bar{K}}^2}{8\pi M_R^2}$, where M_R is the mass of the resonance.

Eq. (19) expresses the spectral density $w(E)$ in terms of hadronic observables (Flatté parameters), just in the same way as Weinberg's factor Z is expressed in terms of hadronic observables (effective range parameters) via Eqs. (1). Thus, Eq. (19) generalizes Weinberg's result to the case of unstable particles.

Note that formula (17) can be easily rewritten as effective range expansion, and the scattering length a and the effective range r_e are then given in terms of the Flatté parameters by

$$a = -\frac{\bar{g}_{K\bar{K}}}{2(E_f - i\frac{\Gamma_P}{2})}, \quad r_e = -\frac{4}{m\bar{g}_{K\bar{K}}}. \quad (20)$$

Therefore, in the Flatté approximation with inelasticity the scattering length becomes complex, and r_e remains real and is negative. In this context it is worth mentioning that r_e is also negative in Weinberg's case, once the corrections of order $O(1/\beta)$ are omitted in Eq. (1).

3 Spectral density and pole counting

The singularities of the $K\bar{K}$ scattering amplitude (Eq. (17)) are given by the zeros of the denominator. The pole positions can be expressed in terms of the effective range parameters (20):

$$k_{1,2} = \frac{i}{r_e} \pm \sqrt{-\frac{1}{r_e^2} + \frac{2}{ar_e}} \quad (21)$$

In the purely elastic case and in the presence of a bound state, one has $E_f < 0$ and therefore $a > 0$, cf. Eq. (20). The poles are then located at the imaginary axis of the k -plane, one in the upper and the other one in the lower half plane. In terms of Weinberg's variable Z the pole positions are given by

$$k_1 = i\sqrt{m\epsilon}, \quad k_2 = -i\sqrt{m\epsilon}\frac{2-Z}{Z}. \quad (22)$$

The first pole in (22) is located in the near-threshold region. (Recall that we consider the small binding energy limit.) For a deuteron-like situation, i.e. for $Z \ll 1$, the second pole is far from the threshold and even moves to infinity in the limit $Z \rightarrow 0$. On the other hand, if Z is close to one, i.e. if there is considerable admixture of an elementary state in the wave function of the bound state, both poles are near threshold. In the limiting case $Z \rightarrow 1$ (pure bare state) the poles are located equidistant from the point $k = 0$. Obviously there is a one-to-one correspondence between Z and the "pole

counting” arguments of [3]: a bound state with large admixture of a bare state (large Z) manifests itself as two near-threshold pole singularities, while a deuteron-like state corresponds to a small Z and gives rise to only one nearby pole.

Let us now consider the case $E_f > 0$ and go over to the spectral density $w(E)$. Since the singularities of $c_0(E)$ (13) coincide with the ones of the amplitude, the behaviour of $w(E)$ is also governed by the poles (21). Specifically, one expects the spectral density to be enhanced in the vicinity of poles. Thus, if both poles (21) are located in the near-threshold region, the spectral density in this region would be large – and indicates thereby that the bare state admixture in the near-threshold resonance is large. If, on the contrary, there is only one near-threshold pole, a considerable part of the spectral density is smeared over a much wider energy interval, which is a signal that the bare state admixture in the near-threshold resonance is small.

As pointed out in the Introduction, in case of a weakly-bound state there is also a unique relation between Z and the effective range parameters. Specifically, r_e is large and negative if there is a large admixture of the bare state in the physical bound state wave function. It is interesting to see what happens in the case of a near-threshold resonance state, i.e. when $E_f > 0$. First we see from Eq. (20) that then the scattering length a is negative. For small values of $|r_e|$ which fulfil the relation $|a| > 2|r_e|$ both poles are located on the negative imaginary axis, cf. Eq. (21). When r_e is larger so that $|a| < 2|r_e|$ and also negative then the pole positions acquire a real part. For very large and negative r_e , $|a| \ll 2|r_e|$, the real part of both poles positions is much larger than the imaginary one – which corresponds to the case of a well-pronounced narrow resonance at the energy E_f . Thus, a large and negative r_e corresponds to a spectral density that is strongly enhanced around the resonance region and, consequently, to the case of a large admixture of the bare state. On the other hand, for small (positive or negative) r_e there is only one near-threshold pole, the spectral density is not strongly enhanced in the near-threshold region and, accordingly, the resonance contains a large mesonic component.

The inclusion of an inelastic channel (in our case $\pi\pi$ or $\pi\eta$) does not change the picture qualitatively. Specifically, if the poles of the $K\bar{K}$ amplitude are again close to each other and to the threshold then the effective range should be negative and large. This will be directly reflected in the Flatté parameter $\bar{g}_{K\bar{K}}$, which then should be small (see Eq. (20)).

Table 1

Parameters and results for the a_0 meson. The values M_R , $\Gamma_{\pi\eta}$ and E_f are given in MeV, r_e and a in fm, and k_1 and k_2 in MeV/c.

Ref.	M_R	$\Gamma_{\pi\eta}$	$\bar{g}_{K\bar{K}}$	E_f	r_e	a	k_1	k_2	W_{a_0}
[18]	1001	70	0.224	9.6	-7.1	-0.16-i0.59	-104+i55	104-i111	0.49
[19]	999	146	0.516	7.6	-3.1	-0.07-i0.69	-134+i71	134-i199	0.29
[20]	1003	153	0.834	11.6	-1.9	-0.16-i1.05	-129+i44	129-i250	0.24
[20]	992	145.3	0.56	0.6	-2.8	-0.01-i0.76	-126+i73	126-i212	0.29
[21]	984.8	121.5	0.41	-18.0	-3.9	0.18-i0.61	-102+i97	102-i199	0.36

4 Application to the a_0 and f_0 mesons

Let us now apply our formalism to the a_0 and f_0 mesons. We start from some recently published Flatté and Flatté-type representations of the corresponding $\pi\eta$ and $\pi\pi$ spectra. For convenience we have summarized the employed Flatté parameters of the a_0 and f_0 mesons (together with the references from where the values are taken) in the first four columns of Tables 1 and 2.

After the discussion in the previous section it should be clear that it is rather instructive to study the behaviour of $w(E)$ over the region of interest, i.e. over the energy region containing the $K\bar{K}$ threshold. Thus, it is useful to introduce an integrated quantity by

$$W_{a_0(f_0)} = \int_{-50\text{MeV}}^{50\text{MeV}} w_{a_0(f_0)}(E) dE . \quad (23)$$

Obviously, $W_{a_0(f_0)}$ is the probability for finding the bare state in the specified energy interval – which we have chosen to be roughly twice as large as the peak width of the a_0 and f_0 mesons. Recall that $w(E)$ is normalized in such a way that the integral over the whole energy range amounts to unity. For a well-pronounced “pure” resonance of Breit-Wigner type, i.e. with a negligibly small coupling $\bar{g}_{K\bar{K}}$ in Eq. (19), one gets for the integral W a value of $\frac{2}{\pi} \arctan 2 \approx 0.70$ for the considered energy interval. Therefore, the deviation from this value provides a direct measure for the admixture of mesonic components in the a_0/f_0 mesons.

The results for the a_0 meson – the positions of the poles $k_{1,2}$ in the complex plane k , W_{a_0} , and the $K\bar{K}$ scattering lengths and effective ranges calculated via Eq. (20) – are given in the last six columns of Table 1, while those for the f_0 meson can be found in Table 2.

Table 2

Parameters and results for the f_0 meson. The values M_R , $\Gamma_{\pi\pi}$ and E_f are given in MeV, r_e and a in fm, and k_1 and k_2 in MeV/c.

Ref.	M_R	$\Gamma_{\pi\pi}$	$\bar{g}_{K\bar{K}}$	E_f	r_e	a	k_1	k_2	W_{f_0}
[22]	969.8	196	2.51	-151.5	-0.63	1.15-i0.74	-58+i107	58-i729	0.17
[23]	975	149	1.51	-84.3	-1.05	0.99-i0.88	-65+i97	65-i477	0.23
[21]	973	253	2.84	-154	-0.56	1.09-i0.89	-69+i100	69-i804	0.14
[24]	996	128.8	1.31	+4.6	-1.22	-0.14-i 1.99	-84+i17	84-i351	0.21

First, let us point out that the couplings $\bar{g}_{K\bar{K}}$ for the a_0 and f_0 mesons differ drastically. In case of the a_0 it is, in average, significantly smaller than for the f_0 . Accordingly, the effective range r_e is fairly large for the a_0 case and much smaller for the f_0 case, as can be seen from the Tables.

The pole positions in the complex k plane, cf. Tables 1 and 2, are shown graphically in Fig. 1. For the a_0 meson the poles look roughly equidistant, and both poles are still close to the real axis. Thus, it is expected that both singularities influence the behaviour of the near-threshold amplitude. For the f_0 meson there is only one pole close to the physical region.

Let us now examine the spectral densities shown in Fig. 2. The a_0 meson appears as an above-threshold phenomenon (with the exception of the fit of Ref. [21]), with a spectral density peaked at the $K\bar{K}$ threshold. The density $w_{a_0}(E)$ for various Flatté representations look rather similar, with exception of the curve for the result of Ref. [18]. Here the coupling $\bar{g}_{K\bar{K}}$ is very small. This fit leads to almost equidistant positions of the poles, and the near-threshold fraction of $w(E)$ is sizable, as reflected in the large value of W_{a_0} . Still, even for this fit W_{a_0} does not exceed 50%, while the average for all considered Flatté parametrizations is about 30%. Clearly, this means that the a_0 (980) should not be considered as a pure quark state, but definitely has a sizeable admixture of mesonic components.

The f_0 meson appears predominantly as a sub-threshold resonance (again with one exception [24]), with the spectral density being peaked a few MeV below the $K\bar{K}$ threshold. Above the $K\bar{K}$ threshold $w(E)$ is very small. The pole positions are not equidistant at all as can be seen in Fig. 1b. Correspondingly, the f_0 meson resembles a pure 1^3P_0 $q\bar{q}$ state even less than the a_0 meson.

As a side note we want to emphasize that the Flatté parametrizations leading to a small $|r_e|$ should be treated with caution. This can be seen from the relation (20) between the Flatté parameters and the effective range parameters. With $r_e \rightarrow 0$ all the Flatté parameters tend to infinity. Obviously, should the physical $K\bar{K}$ effective range r_e indeed be small then the determination of

E_r , Γ_P and $\bar{g}_{K\bar{K}}$ from a Flatté fit becomes unstable. Moreover, it is possible that an equal or even better quality fit may be achieved with positive effective range. As mentioned above, a Flatté analysis automatically implies that $r_e < 0$ whereas a small but positive value of r_e is typical for the deuteron-like situation, i.e. for a dynamically generated bound state. In any case, it would be desirable to use more general parametrizations of resonance data that also allow for a positive effective range and hence can also be applied to potential-type resonances. One option would be to employ directly an effective range expansion to fit the data. Another possibility is to include the relativistic corrections to the loop integral $g_K(E)$ (see [24]) as well as the corrections $O(m^2 E/\beta^2)$ that appear in our formalism (but were omitted so far).

5 Summary

In the present paper we have investigated the interesting problem of whether it is possible to distinguish composite from elementary particles. In particular we have generalized the model-independent approach of S. Weinberg that is based on the probabilistic interpretation of the field renormalization Z to the case of unstable particles (resonances). This could be achieved by introducing a suitably defined spectral density $w(E)$ which allows to analyse not only situations where the physical states lie below the threshold but also in the continuum. Both Z as well as $w(E)$ provide a measure for the admixture of the bare state in the physical state. Within this formalism it is possible to address the long-standing question whether the $a_0(980)$ and $f_0(980)$ mesons are genuine quark states or whether they contain a dominant admixture of mesonic components.

Furthermore, we have shown that there is a one-to-one correspondence between the value of Z (or the behaviour of $w(E)$) and the pole counting scheme suggested by Morgan [3]. Specifically, in case of values of Z close to one (or a strongly enhanced spectral density near threshold) there are always two nearly equidistant poles located close to the threshold. This situation corresponds to a large admixture of a bare ($q\bar{q}$ like) state in the physical bound state or resonance and it manifests itself also by a weak coupling to the bare state to the $K\bar{K}$ channel and by a large and negative effective range r_e . In the opposite case where Z is almost zero or where $w(E)$ is smeared out over the whole energy range there is only one stable pole near the threshold. This situation corresponds to the case of a dynamically generated bound state or resonance and is characterized by a strong coupling to the $K\bar{K}$ channel and by an effective range that is small or even positive.

The spectral density $w(E)$ of bare states can be constructed from hadronic “observables” like the Flatté parameters in a similar way the field renormaliza-

tion factor Z can be extracted from the effective range parameters. In order to exemplify the potential of the proposed method we evaluated $w(E)$, for energies near the $K\bar{K}$ threshold, for several recently published Flatté parametrizations of measured $\pi\pi$ and $\pi\eta$ spectra. Thereby we found that the (near-threshold) probability for finding the a_0 meson in a bare states is only about 25 to 50%, indicating that the a_0 should have a significant mesonic component. As for the f_0 meson, the employed Flatté parametrizations yielded even smaller values for this probability, namely of the order of 20% or less, providing evidence that its mesonic component should be indeed rather large. Therefore, we conclude that a simple 1^3P_0 $q\bar{q}$ or four quark assignment for the $a_0(980)$ should be considered with caution and it is certainly questionable for the $f_0(980)$.

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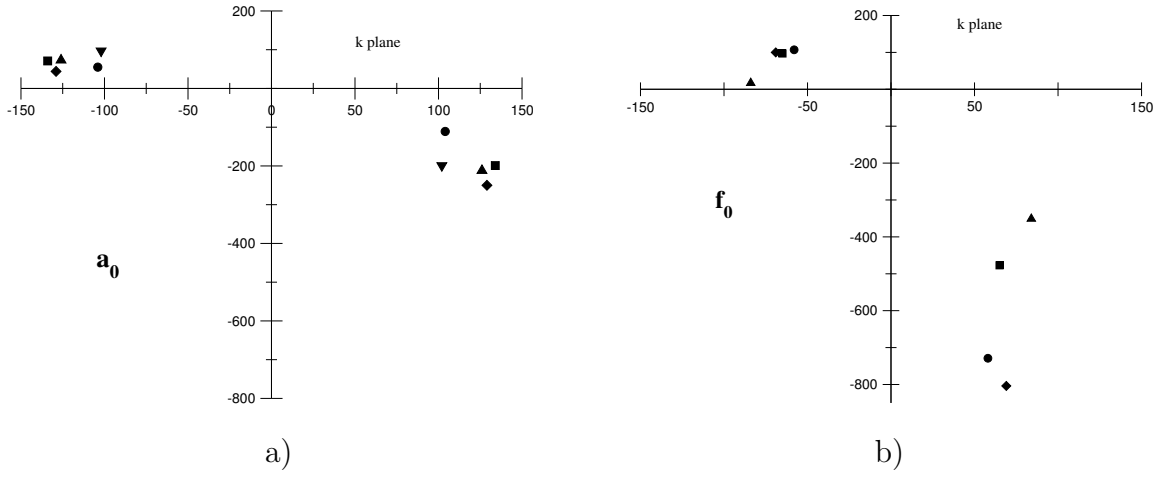


Fig. 1. a) Pole positions for the a_0 meson in the complex k plane [in MeV/c] based on the Flatté parameters taken from Ref. [18] (circles), Ref. [19] (squares), Ref. [20] (diamonds), Ref. [20] (triangles), and Ref. [21] (reversed triangles). b) Pole positions for the f_0 meson in the complex k plane [in MeV/c] based on the Flatté parameters taken from Ref. [22] (circles), Ref. [23] (squares), Ref. [21] (diamonds), and Ref. [24] (triangles).

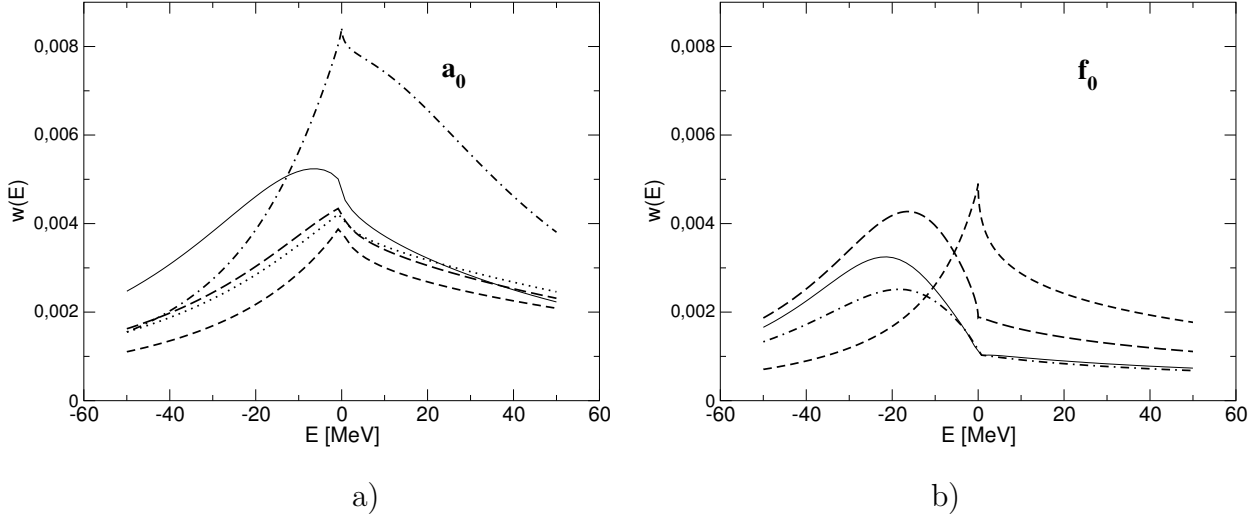


Fig. 2. a) Spectral densities $w(E)$ for the a_0 meson based on the Flatté parameters taken from Ref. [18] (dashed-dotted line), Ref. [19] (dotted line), Ref. [20] (dashed line), Ref. [20] (long dashed line), and Ref. [21] (solid line). b) Spectral densities $w(E)$ for the f_0 meson based on the Flatté parameters taken from Ref. [22] (solid line), Ref. [23] (long-dashed line), Ref. [21] (dashed-dotted line), and Ref. [24] (dotted line).