A cluster version of the GGT sum rule

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Abstract

We discuss the derivation of a "cluster sum rule" from the Gellmann-Goldberger-Thirring (GGT) sum rule as an alternative to the Thomas-Reiche-Kuhn (TRK) sum rule, which was used as the basis up to now. We compare differences in the assumptions and approximations. Some applications of the sum rule for halo nuclei, as well as, nuclei with a pronounced cluster structure are discussed.

1 Introduction

Sum rules like the GGT and the TRK sum rule, have played an important role in the understanding of global properties of (well) bound system and their excitation spectrum. One of their major features is their universality, as they cannot only be applied to nuclei but also, e.g., to atoms or the substructure spectrum of the nucleus. For the derivation however one makes a number of assumptions; deviations of the experimental findings from the pure results are expected and found. Both the classical TRK and GGT sum rule were found experimentally for nuclei to overfulfil the theoretical result up to a factor of two.

More recently the TRK sum rules has been applied to the domain of very proton and very neutron rich nuclei or systems with a pronounced cluster structure. For these systems a so-called *"cluster sum rule"* was derived [1] and is has lead to some insight in the low-lying dipole strength (sometimes called the *" pigmy resonance"*) in such systems [2]. We want to show here that this cluster sum rule can be derived also from the Gellmann-Goldberger-Thirring (GGT) sum rule. Whereas this approach leads ultimately to the same mathematical expression as the TRK cluster sum rule (as it does already for the usual "non-cluster" sum rule), the approximations and assumptions are quite different here and therefore some further insight into the nature of the cluster sum rule can be gained. One advantage of the GGT cluster sum rule

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is, that it is not based on the long-wavelength limit and Siegert's theorem and that it is not restricted to the dipole excitation spectrum alone. We review briefly the foundations of the TRK cluster sum-rule and the approximations used before deriving the GGT version. We apply the result then to systems with a halo-structure and with two or more clusters.

2 Review of the TRK cluster sum rule

The classical TRK sum rule starts from the dipole operator strength for the transition of a nucleus from the ground state i to some excited state f

$$f_{fi} = \frac{2m_N e^2}{\hbar^2}(E_f - E_i)\left|\left\langle f|\vec{D}|i\right\rangle\right|^2. \tag{1}$$

with $\vec{D}$ the dipole operator $\frac{NZ}{A}\left(\vec{R}_p - \vec{R}_n\right)$, where $\vec{R}_p$ is the c.m. position of all protons $\frac{1}{Z}\sum_p \vec{r}_p$ and $\vec{R}_n$ of all neutrons $\frac{1}{N}\sum_n \vec{r}_n$ respectively. The dipole operator $\vec{D}$ is written in a way that the center of mass motion of the whole system is taken out. In order to sum this equation over all excited states f, one replaces E_f and E_i with the Hamilton operator operating either on $|i>$ or $|f>$ and makes use of closure to get

$$\sum_f f_{fi} = \frac{m_N e^2}{\hbar^2}\left\langle i\right|\left[\vec{D},\left[H,\vec{D}\right]\right]\left|i\right\rangle. \tag{2}$$

For a Hamiltonian, which consists of a nonrelativistic kinetic term together with a local (momentum independent) potential V, the double commutator in Eq. (2) is found to be $\hbar^2/m_N$. Using the relation between the dipole strength f_{fi} and the total photoabsorption cross section σ_γ based on Siegert's theorem (and therefore on the long wavelength approximation) one gets the TRK sum rule

$$\int_0^\infty \sigma_\gamma(\omega)d\omega = \frac{2\pi^2\hbar e^2}{m_N c}\frac{NZ}{A} = 60\frac{NZ}{A}\text{MeV mb}. \tag{3}$$

The important feature of this sum rule is the fact, that it only depends on the number of protons and neutrons and is completely independent of their arrangement within the nucleus.

Recently the same idea has been used in the case of a system composed of two clusters of nuclei with charge Z_a and Z_b ($Z = Z_a + Z_b$) and mass number

A_a and A_b ($A = A_a + A_b$) [1,2,3]. The idea is a decomposition of the dipole operator $\vec{D}$ into "external" and "internal" coordinates. Using

$$\vec{D} = \vec{D}_a + \vec{D}_b + \vec{D}_{(ab)} \tag{4}$$

where $\vec{D}_a$ and $\vec{D}_b$ are the dipole moments of each cluster a and b

$$\vec{D}_i = \sum_{p \in i} (\vec{r}_p - \vec{R}_i) \tag{5}$$

for $i = a, b$. The sum in Eq. (5) goes only over those protons being part of cluster i. $\vec{D}_{(ab)}$ is the relative dipole moment between a and b

$$D_{(ab)} = Z_a(\vec{R}_a - \vec{R}) + Z_b(\vec{R}_b - \vec{R}) = e\,[(Z_a A_b - Z_b A_a)/A]\,\vec{S}. \tag{6}$$

$\vec{R}$ is the position of the center of mass of the nucleus, R_a and R_b are the position of the centers of mass of a and b. Finally $\vec{S}$ denotes the vector connecting the center of mass of a and b, $\vec{S} = \vec{R}_a - \vec{R}_b$. Using this decomposition and looking at the part of the photo absorption cross section coming from $\vec{D}_{(ab)}$ one gets a "cluster version of the TRK sum rule"

$$\int_0^\infty \sigma_{(ab)}(\omega) d\omega = \frac{2\pi^2 \hbar e^2}{m_N c} \frac{(Z_a A_b - Z_b A_a)^2}{A A_a A_b}. \tag{7}$$

where the cross section $\sigma_{(ab)}(\omega)$ is connected with the relative dipole excitation due to $D_{(ab)}$. Let us emphasize at this point, that the derivation of this cluster sum rule makes no assumption, whether the decomposition of the system into two clusters a and b is also expected to be of a physical nature, that is, whether the nucleons in the nucleus are expected to be configured mainly into two subsystems a and b.

If one on the other hand is looking at the specific breakup channel, where only the "external" coordinates $\vec{R}_a$ and $\vec{R}_b$ are important, that is, the channel $(ab) \to a + b$, the integral

$$\int_0^\infty \sigma_{(\gamma + (ab) \to a + b)} d\omega \approx \frac{2\pi^2 \hbar e^2}{m_N c} \frac{(Z_a A_b - Z_b A_a)^2}{A A_a A_b}. \tag{8}$$

is a measure of how much a supposed cluster configuration contributes to the dipole sum, see also the comments in [1]. In this integral one assumes that the excitation to bound cluster configurations of a and b do not contribute considerably to the integral.

This sum rule plays an important role in connection with the question of the existence of low lying strength in exotic nuclei. If one assumes that at low excitation energies it is preferable that the two clusters (which are assumed to be loosely bound with respect to each other as compared to the internal binding) move against each other, the cluster sum rule gives a value for the strength, that is expected to lie at low energies [2,4]. Measurements especially of the neutron rich He isotopes and also O isotopes have confirmed the existence of this low lying strength, which is often also called the *"pigmy resonance"* in analogy to the giant dipole resonance, where most of the strength of the TRK sum rule is concentrated.

Let us review the assumptions that went into the derivation of this sum rule: It is based on the dipole operator $\vec{D}$ alone, therefore other multipole apart from the $E1$ transition are assumed to be small. It makes use of a Hamiltonian with a local potential, contributions coming especially from exchange currents are assumed to be small. In order to relate the dipole operator to the total photoabsorption cross section Siegert's theorem is used, therefore the sum rule is expected to be valid only at low excitation energies, where the long wavelength limit is valid.

Another problem, even though not relevant in the cases discussed here, is the nonrelativistic nature of the derivation. There is no known derivation in the relativistic case, that is, the TRK sum rule is a nonrelativistic sum rule with no relativistic analogon. Some of these problems are not present in the GGT sum rule, therefore we want to see whether the cluster sum rule can also be found in this approach.

3 Derivation of the cluster sum rule from the GGT sum rule

A different sum rule is the GGT sum rule [5,6,7]; this sum rule only makes assumptions about causality and analyticity of the forward elastic scattering amplitude $f(\omega)$ and uses a dispersion relation to relate the real and imaginary part of this amplitude. One gets the once subtracted dispersion relation [5]

$$Ref(\omega_0) - Ref(0) = \frac{2\omega_0^2}{\pi} P.P. \int_0^\infty d\omega \frac{Imf(\omega)}{\omega(\omega^2 - \omega_0^2)}. \tag{9}$$

Making use of the optical theorem, the imaginary part of the forward elastic scattering amplitude is related to the total cross section $\sigma(\omega)$ by

$$\sigma(\omega) = \frac{4\pi\hbar c}{\omega} Imf(\omega). \tag{10}$$

In addition the forward elastic scattering amplitude at $\omega = 0$ is real and given by the Thomson limit

$$f(0) = Ref(0) = -\frac{(Ze)^2}{Mc^2}, \tag{11}$$

where Ze is the total charge and $M = m_N A$ the total mass of the system. Let us assume that we have a system c composed of two subsystems ("clusters") a and b. The more general case with more than two subsystems will be discussed below. In the limit $\omega_0 \to \infty$ one gets a relation for $i = a, b, c$

$$Ref_i(\infty) + \frac{Z_i^2 e^2}{A_i m_N c^2} = -\frac{1}{2\pi^2 \hbar c} \int_0^\infty d\omega \; \sigma_i(\omega). \tag{12}$$

This relation is the basis of the usual GGT sum rule and also of a more general cluster sum rule. A first assumption is that the scattering amplitude $f(\omega)$ at infinity of the system c is just given by the sum of the scattering cross section of the two components a and b, that is,

$$Ref_c(\infty) - Ref_a(\infty) - Ref_b(\infty) \approx 0 \tag{13}$$

(which is not true strictly speaking due to hadronic components in the photon and shadowing corrections in the nucleus, see below) and therefore

$$\int_0^\infty d\omega \, [\sigma_c(\omega) - \sigma_a(\omega) - \sigma_b(\omega)] = \frac{2\pi^2 \hbar e^2}{m_N c} \left[\frac{Z_a^2}{A_a} + \frac{Z_b^2}{A_b} - \frac{Z_c^2}{A_c} \right] \tag{14}$$

$$= \frac{(Z_a A_b - Z_b A_a)^2}{A_a A_b A_c} 60 \text{ MeV mb}, \tag{15}$$

which is the main result. For the more general case of the decomposition of our system c into N different cluster the result is

$$\int_0^\infty d\omega \left(\sigma_c(\omega) - \sum_{i=1}^{N} \sigma_i(\omega) \right) = \frac{2\pi^2 \hbar e^2}{m_N c} \left[\sum_{i=1}^{N} \frac{Z_i^2}{A_i} - \frac{Z_c^2}{A_c} \right] \text{MeV mb}. \tag{16}$$

The usual GGT sum rule can be recovered from this expression by using as cluster each nucleon, that is $Z = Z_c$ protons and $N = N_c$ neutrons. With this we get the usual form of the GGT sum rule

$$\int_0^\infty d\omega \left[\sigma(\omega) - Z\sigma_p(\omega) - N\sigma_n(\omega)\right] = \frac{2\pi^2\hbar e^2}{m_N c}\frac{NZ}{A}, \tag{17}$$

which coincides with the expression for the TRK sum rule, as already did the expression for the cluster sum rule.

Let us decompose the total photoabsorption cross sections in Eq. (14) according to the final state the nucleus c goes to. We can write the cross section as

$$\sigma_c = \sigma_{c+\gamma\to a+b} + \sigma_{c+\gamma\to a+X_b} + \sigma_{c+\gamma\to X_a+b} + \sigma_{c+\gamma\to X_a+X_b} + \sigma_{c+\gamma\to c^*}, \tag{18}$$

where X_a and X_b denote "fragments" of a and b, that is, all final states not including a or b and c^* denotes excited bound states of c. Assuming that due to the clustering structure the breakup of a in the nucleus c is the same as the one of a alone (and a shift in energy due to the binding of a is not important in the integration), we have (*"spectator approximation"*)

$$\sigma_{c+\gamma\to a+X_b} \approx \sigma_{b+\gamma\to X_b}, \quad \sigma_{c+\gamma\to X_a+b} \approx \sigma_{a+\gamma\to X_a}, \quad \sigma_{c+\gamma\to X_a+X_b} \approx 0. \tag{19}$$

Also for the loosely bound system we are mainly looking at there is only a small number of excited states and we have

$$\sigma_{c+\gamma\to c^*} \approx 0 \tag{20}$$

(this approximation is not necessary, as $\sigma_{c+\gamma\to c^*}$ can easily be included on the left hand side of Eq. (21)) and find within the "spectator limit" of the GGT cluster sum rule

$$\int_0^\infty d\omega \sigma_{c+\gamma\to a+b}(\omega) \approx \frac{(Z_a A_b - Z_b A_a)^2}{A_a A_b A_c} 60 \text{ MeV mb}, \tag{21}$$

a relation, which can be measured in experiments and used as a test for the contribution of the clustering component to the cross section.

As was already the case for the GGT sum rule, where the excitation of the individual nucleons occur only above the pion production threshold, also here the excitation at lower energies is dominated by the relative excitation of a and b, especially if the subsystems a and b are well bound systems like α or even nucleons. But whereas the pion production threshold is very high compared to typical nuclear excitation energies, this is not the case here most of the time. Still the sum rules of Eq. (14) or Eq. (21) integrated up to infinity are valid.

In the case of more than two clusters the situation is more complex. Assuming that our system c consists of three clusters a, b, d, we have the added complication that the system can have bound states of a and b or any other binary system. In this case it is easy to show that the difference of the cross sections in the GGT cluster sum rule Eq. (16) in the spectator approximation corresponds to all channels with final states composed of bound or continuum states of the components a, b and d. This will be relevant for the study of ^{8}He below.

As in the TRK cluster sum rule no assumption about the validity of the clustering in the nuclear structure of c was made in the derivation of Eq. (14). On the other hand the last form in Eq. (21), based on the spectator approximation, of course strongly assumes that the major contribution of the excitation cross section comes from the excitation of the relative motion of a and b and therefore is only true if the system is dominated by this clustering structure. Deviations of the second form Eq. (21) can therefore be seen as a test of the clustering hypothesis.

As in the TRK sum rule also in the GGT sum rule there are deviations from the simple picture. Especially the assumption, that the difference $Ref_c(\infty) - Ref_a(\infty) - Ref_b(\infty)$ is zero is not really true. At high energies the hadronic component of the photon mainly interacts with the nucleons, making the nucleons black objects and therefore the nucleons are shadowing each other. This shadowing effects leads to an enhancement of up to a factor of two compared to the theoretical result. This is discussed extensively in [8,9]. On the other hand one expects that the deviation of the experimental results from the theoretical prediction of the GGT cluster sum rule is smaller as for the GGT sum rule itself. If the average distance between the two clusters a and b is large (ideally larger than the size of each cluster itself), shadowing corrections of a on b or b on a will be small, as the two clusters a and b do not block each other very much and shadowing within the clusters a or b do not lead to deviations.

Let us finally review the advantage of this derivation of the cluster sum rule: It is not based on any specific model of the system but only on general properties, like causality, of the forward elastic scattering amplitude. It is therefore valid for all multipole moments, not only for the dipole moment, and it is independent of the validity of the long wavelength limit, that is the validity of the Siegert's theorem. It is also a sum rule which is the same in the nonrelativistic as well as in the relativistic case. This is also the reason why the TRK sum rule is often found to hold also in relativistic models [10].

Table 1
The predictions of the GGT cluster sum rule are shown for different configurations of the cluster for ^{6}He and ^{8}He. The cross section differences on which the sum rule is based are shown together with the channels that this difference corresponds to in the limit of the spectator approximation. The last column gives the experimental results of [12].

σ difference	"spectator approx."	GGT result (MeV mb)	exp. result (MeV mb)
$\sigma(^8\mathrm{He}) - \sigma(^6\mathrm{He}) - 2\sigma(n)$	$\sigma(\gamma + {}^8\mathrm{He} \to {}^6\mathrm{He} + 2\mathrm{n})$	$1/6 \times 60 = 10$	7.5 ± 1.4
$\sigma(^8\mathrm{He}) - \sigma(^4\mathrm{He}) - 4\sigma(n)$	$\sigma(\gamma + {}^8\mathrm{He} \to {}^6\mathrm{He} + 2\mathrm{n})$ + $\sigma(\gamma + {}^8\mathrm{He} \to {}^4\mathrm{He} + 4\mathrm{n})$	$1/2 \times 60 = 30$	—
$\sigma(^6\mathrm{He}) - \sigma(^4\mathrm{He}) - 2\sigma(n)$	$\sigma(\gamma + {}^6\mathrm{He} \to {}^4\mathrm{He} + 2\mathrm{n})$	$1/3 \times 60 = 20$	26 ± 5

4 Application to nuclei

The TRK cluster sum rule has been applied already in some cases, e.g., for ^{11}Be, [11,2], the neutron rich helium isotopes ^{6}He and ^{8}He [18,12,13] and also for neutron rich oxygen isotopes [17]. We want to show how the analysis made in these cases can be taken further by using the more generalised versions of the cluster sum rule. In addition we will also look at cluster nuclei, especially ^{6}Li, ^{16}O and ^{9}Be.

A detailed analysis of the electromagnetic dissociation of both ^{6}He and ^{8}He was made at GSI [12]. Here the fragments after the Coulomb breakup reaction $^6\mathrm{He} \to {}^4\mathrm{He} + 2\mathrm{n}$ and $^8\mathrm{He} \to {}^6\mathrm{He} + 2\mathrm{n}$ where measured and from the invariant mass the excitation energy, that is, the photon energy was reconstructed. The cross section integrated over a range of photon energies (mainly limited by the experimental setup) was also calculated and compared with the prediction coming from the cluster sum rule under the assumption of a ^{4}He cluster together with the "halo-neutrons" making up the other cluster. Based on this analysis it was found that the cluster sum rule is almost exhausted in the case of ^{6}He.

We can find cluster sum rules also for other configurations. Therefore we want to reevaluate the findings especially of [12] in this light.

From Table 1 one can see, that within the spectator approximation of the GGT sum rule 1/3 of the dissociation cross section of ^{8}He to a final state including an ^{4}He nucleus goes into the channel ^{6}He + 2n, 2/3 is expected to go directly to ^{4}He+ 4n, the channel, which was not measured in the experiment, see [12]. On the other hand it was found that the dissociation cross section to ^{6}He+2n (in the energy range measured in the experiment, that is below 10 MeV) is about 1/3 of the one for the dissociation of ^{6}He to ^{4}He + 2n and exhausts the cluster sum rule based on a ^{4}He cluster and four neutrons to about 25%. This is close to the expected 33% from the analysis above. One expects a large cross section for the channel going to ^{4}He and four neutrons directly. In [18] it was already found that for ^{6}He to ^{4}He + 2n the cluster sum rule is exhausted to almost 100%.

As the cross section for a specific channel was measured in this case, this is a test of the validity of the spectator approximation and therefore a test of the cluster configuration of ^{8}He and ^{6}He. The experimental results are in agreement with the hypothesis that ^{8}He is predominantly in a structure with two neutrons building one cluster and ^{6}He the second. This doesn't mean that ^{8}He is a two-neutron halo system build around ^{6}He, but does indicate that two neutrons form a system that is more or less decoupled from the rest of the system. A configuration with a ^{4}He core together with two 2n cluster would fulfil the sum rule as well.

A second application can be made in the case of the neutron rich oxygen isotopes as measured in [13]. The electromagnetic dissociation was studied for $A = 17, 19, 20, 21, 22$ and it was found that for large neutron excess, that is large A, there is a tendency for the appearance of low lying dipole strength. The experiment measured only the cross section for the electromagnetic breakup with the emission of up to three neutrons, assuming that due to the large threshold for proton emission, this is to a good approximation identical to the total photoabsorption cross section. The authors of [13] only made a comparison with the prediction of the cluster sum rule based on the assumption of one cluster being the ^{16}O-core whereas all other neutrons are part of the second "cluster". Only the sum of xn neutron emission cross sections was published and no individual data for 1n, 2n or 3n.

We are analyzing their data under the assumption that the emission of up to three neutrons is to a good approximation already the full photoabsorption cross section, as was done in [17]. In Table 2 below we give the differences of the integrated cross sections extracted from Fig. 3 of [17] together with the sum rule prediction based on the differences of the integrated cross section for the A and $A-1$, $A-2$ or $A-3$ isotopes.

Please recall that by taking the difference of two integrated cross sections, we are comparing the experimental results with the prediction of Eq. (14), the cluster sum rule using the difference of the photoabsorption cross section. This sum rule doesn't use the spectator approximation and therefore should be fulfilled independent of the fact, whether the O isotope is clustered into a core and either one, two or three decoupled neutrons. The agreement with the theoretical expectation for the difference of the integrated cross sections for A and $A-1$ and $A-2$ is not too bad in some cases, but the agreement of the difference between A and $A-3$ is not good. The discrepancy can

Table 2
An analysis of the difference of the photoabsorption cross sections for ^{A}O and ${}^{(A-1)}$O, ${}^{(A-2)}$O and ${}^{(A-3)}$O from the GGT cluster sum rule. The theoretical expected values are compared with experimental results from [17]. Please note that the experimental results include only photon energies up to 20 MeV and reactions with up to 3 neutrons in the final state. Also shown is a comparison of the results by applying the spectator model, assuming that ^{A}O consists of a ${}^{(A-3)}O$ and a $3n$ cluster. The last column shows the threshold values for the reactions. See the text for details of the analysis.

cross section diff.	GGT result (MeV mb)	exp. result (MeV mb)	threshold (MeV)
$\sigma(^{22}\text{O})$-$\sigma(^{21}\text{O})$	8.34	10.1	6.849
$\sigma(^{21}\text{O})$-$\sigma(^{20}\text{O})$	9.12	-22.7	3.807
$\sigma(^{20}\text{O})$-$\sigma(^{19}\text{O})$	10.1	19.1	7.608
$\sigma(^{19}\text{O})$-$\sigma(^{18}\text{O})$	11.3	7.1	3.957
$\sigma(^{18}\text{O})$-$\sigma(^{17}\text{O})$	12.5	31	8.044
$\sigma(^{22}\text{O})$-$\sigma(^{20}\text{O})$	17.5	-12.6	10.656
$\sigma(^{21}\text{O})$-$\sigma(^{19}\text{O})$	19.2	-3.6	11.415
$\sigma(^{20}\text{O})$-$\sigma(^{18}\text{O})$	21.4	26.2	11.565
$\sigma(^{19}\text{O})$-$\sigma(^{17}\text{O})$	23.8	38.1	12.001
$\sigma(^{22}\text{O})$-$\sigma(^{19}\text{O})$	27.5	6.5	18.264
$\sigma(^{21}\text{O})$-$\sigma(^{18}\text{O})$	30.5	3.5	15.372
$\sigma(^{20}\text{O})$-$\sigma(^{17}\text{O})$	33.9	57.2	19.609
spectator model			
$\sigma(^{22}\text{O}\rightarrow {}^{19}\text{O} + 3\text{n})$	27.5	62.1	
$\sigma(^{21}\text{O}\rightarrow {}^{18}\text{O} + 3\text{n})$	30.5	52.7	
$\sigma(^{20}\text{O}\rightarrow {}^{17}\text{O} + 3\text{n})$	33.9	74.9	
$\sigma(^{19}\text{O}\rightarrow {}^{16}\text{O} + 3\text{n})$	38.9	55.7	

mainly be attributed to the limitation of the experimental results: The sum rule is calculated only for energies up to 20 MeV (whereas the data seem to be available up to 30 MeV) and therefore some contributions at higher energies are missing. Also mentioned in the table is the threshold for the emission of one, two or three neutrons. Especially for the difference $\sigma(A) - \sigma(A-3)$ the fact that only the emission of up to three neutrons was measured is clearly important in order to compare the difference of the total photoabsorption cross section. As the emission of up to three neutrons was measured, we can also test the cluster hypothesis based on the spectator approximation and Eq. (21).

This is a test of the (unrealistic) assumption that the oxygen isotope consist of a $A-3$-core together with three decoupled neutrons. The results of this are shown in the last rows of the table, where we take the experimental data as the cross section to be compared with the theoretical result. As expected the agreement is not good, stating that in none of the cases (with maybe the possible exception of ^{19}O, which can be thought to consist of the ^{16}O core and three neutrons) we expect a three neutron cluster. A comparison of the GGT cluster sum rule prediction with the results of the reaction channel $\sigma(\gamma+A \to (A-1)+n)$ and $\sigma(\gamma+A \to (A-2)+2n)$ would be more interesting. These results, even though measured in [17] are not quoted by the authors.

Finally let us also look at systems, which consist of more complex structures than the halo nuclei: Photodissociation of ^{6}Li and ^{16}O at low photon energies are of astrophysical interest [4] and several attempts have been made to determine these cross sections accurately. In the usual TRK approach only the dipole transition is taken into account and it is found that no dipole transition exists in the breakup of a system into two clusters with the same Z/A-ratio (as the effective charge is zero). Here we find that this result is also true in the GGT approach, where no assumption about the multipolarity of the absorbed photon is made. Therefore any contribution to the total integral of the cross section at low energies must either be compensated by a difference in the cross section at higher energies or must come from deviations of the sum rule. For the reactions $^6\mathrm{Li} + \gamma \to \alpha + d$ and $^{16}\mathrm{O} + \gamma \to ^{12} C + \alpha$ the spectator approximation also predicts a zero result. Here of course the cross section then mainly comes from configurations not clustered in this way.

As another example, let us look at the system ^{9}Be, which supposedly consists mainly of a two-alpha cluster configuration forming ^{8}Be with an additional neutron to bind the system. This nucleus and the reactions $^9\mathrm{Be}+\gamma \to \alpha+\alpha+n$ and its inverse are of interest for bridging the $A = 8$ gap in some astrophysical scenarios, e.g., in the high-entropy bubble in type II supernovae [14]. For the integrated cross section $^9\mathrm{Be} \to 2\ \alpha + \mathrm{n}$ we find

$$\int d\omega\sigma(^9\mathrm{Be} \to 2\ \alpha + \mathrm{n}\ ) \approx \int d\omega \left[\sigma(^9\mathrm{Be}) - 2\sigma(^4\mathrm{He}) - \sigma(n)\right] \tag{22}$$

$$= \left(2\frac{4}{4} - \frac{16}{9}\right) 60\mathrm{MeV\ mb} = \frac{2}{9} 60\mathrm{MeV\ mb} = 13.3\mathrm{MeV\ mb}. \tag{23}$$

(which is identical to the cross section difference one would expect going only through the unstable nucleus ^{8}Be alone). In this case the cross section at astrophysical energies is dominated by a transition $P_{3/2} \to S_{1/2}$. The experimental results at energies below 2.2 MeV ([14] and references therein) only give about 0.33 MeV mb, that is a very small fraction of this sum rule. On the other hand it is well known [20,21] that the cross section has a minimum at around 2.2 MeV before rising again due to the $P_{1/2} \to D_{5/2}$ transition.

On the other hand the cross section difference of more complex reactions can be studied in the same way. For both ^{6}Li and ^{16}O we can give results for the breakup to a $p+n$ final state, related to the breakup of the nuclei in the quasideuteron mode, which is a strong channel. We find a sum rule prediction of 1/3 of the total integrated photo cross section for the reaction $^6\mathrm{Li}+\gamma \rightarrow \alpha+p+n$ and 1/9 for the reaction $^6\mathrm{Li}+\gamma \rightarrow^3 He+t$. The integrated cross section of the "quasideuteron" reaction $^{16}\mathrm{O}+\gamma \rightarrow^{14}\mathrm{N}+p+n$ is 1/8 of the total integrated cross section, whereas the single neutron emission $^{16}\mathrm{O}+\gamma \rightarrow^{15}\mathrm{N}+n$ only 1/15. In this case and also the other following cases the cross section has contributions at much higher energies in contrast to the "soft dipole modes" studied here up to now. A comparison with experiments, which could be done for both the sum rule of Eq. (14) and the one from the spectator approximation Eq. (21) is difficult in these cases, as only a few results are available in the literature [22] and most of them also not for these exclusive channel.

Finally in [19] experimental and theoretical problems with 2p-radioactivity and three-body decay are discussed. They also mention a number of candidates for some genuine three-body decay. One could again apply the GGT cluster sum rule to the three-body decay in photoexcitation experiments. Most of the time one expects the contribution to the cross section integral to come not only from lower energies. But if a single resonance dominates the decay, one could again study the question whether this resonance saturates a major part of the cluster sum rule.

5 Conclusions

An alternative derivation of the so-called "cluster sum rule" was given, based on the GGT instead of the TRK sum rule. Whereas the final result is formally identical to the "usual" cluster sum-rule, the assumptions to derive this sum rule are different and the deviations of the measured from the theoretical results are expected to be smaller than in the case of the "usual" GGT sum rule. Whereas the sum rule of Eq. (14), expressed as the difference of two different total photoabsorption cross sections, is quite general, we have also looked at the case of a pronounced clustering structure, where one can make use of the "spectator approximation" to identify this difference as the cross section for the nucleus going into a certain final state, see Eq. (21). This can be seen as a test of the degree to which the clustered configuration contributes to this cross section channel. We have looked at applications of this sum rule for neutron-halo systems, as well as, for clustered nuclei and found some agreement with experimental findings. We think that this derivation gives some independent insight into the structure of exotic nuclei but also the foundations of the cluster sum rule for exotic nuclei.

We know from many cluster conferences [24] that the concept of a cluster is really quite a loose, qualitative concept. We know much more about nuclei from microscopic approaches, like the shell model. However, the cluster sum rules discussed in this paper give us a (semi)quantitative measure of the importance of certain cluster degrees of freedom. We may assume that such degrees of freedom become more and more relevant when going away from the valley of stability, which was found to lead in some cases to a decoupling of some nucleons (neutrons) from the rest of the system. One evidence of this is the well established low lying dipole strength (*"pigmy resonance"*) in nuclei away from the valley of stability.

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