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Extended Theory of Finite Fermi Systems:
Collective Vibrations in Closed Shell Nuclei

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Contents

1 Introduction **2**
1.1 Aims of the review 2
1.2 Brief review of the standard theory 5
1.2.1 The Landau-Migdal interaction 6
1.2.2 General theoretical background 9
1.2.3 Microscopic derivation of the basic eqns of the TFFS . 11
1.2.4 Coordinate representation 14
1.2.5 Main results of the GMR description within the TFFS 16
1.3 Physical arguments for extending the standard approach . . . 19
1.3.1 Giant resonances in nuclei: the width problem 20
1.3.2 Nuclei far from the stability line 23
1.3.3 Implications of experiments for and status of the theory 24
1.4 Qualitative discussion of the extension 26

2 Framework of the extended theory **29**
2.1 General description of collective excitations 29

2.1.1 General relations 29
2.1.2 Renormalization of the general equations 30
2.2 Approx. for the mass operator and the irreducible amplitude . 34
2.3 Constructing the generalized propagator 36
2.3.1 The basic idea. 36
2.3.2 Formulation of the method 38
2.3.3 The generalized propagator in the energy representation 41
2.3.4 Ground state correl. induced by 1p1h⊗phonon configs. 43
2.4 Inclusion of the single-particle continuum 45

3 Application to giant resonances 46
3.1 Scheme of the calculations 46
3.1.1 Electric sum rules 47
3.1.2 Input quantities for the ETFFS 49
3.1.3 Numerical details 50
3.2 Electric and magnetic resonances in magic nuclei 51
3.2.1 Photoabsorption cross sections; E1 excitations. 51
3.2.2 E0 and E2 resonances 53
3.2.3 M1 resonances 57
3.2.4 Effects of GSC_{phon} 59
3.2.5 The pygmy resonance and low-lying structures 60
3.3 Giant resonances in unstable magic nuclei 62

4 Microscopic transition densities and cross sections 63
4.1 Comparison of microscopic and phenomen. transition densities 63
4.2 Microscopic analyses of inelastic electron and alpha scattering 65
4.2.1 Electron scattering 65
4.2.2 α particle scattering 66
4.3 Calculation of the ^{58}Ni and ^{40}Ca $(\alpha, \alpha\prime)$ cross sections 67
4.3.1 ^{58}Ni results 68
4.3.2 ^{40}Ca results 70

5 Conclusion 73

References 78

Appendix 94

Abstract

We review an extension of Migdal's Theory of Finite Fermi Systems which has been developed and applied to collective vibrations in closed shell nuclei in the past ten years. This microscopic approach is based on a consistent use of the Green function method. Here one considers in a consistent way more complex 1p1h⊗phonon configurations beyond the RPA correlations. Moreover, these configurations are not only included in the excited states but also explicitly in the ground states of nuclei. The method has been applied to the calculation of the strength distribution and transition densities of giant electric and magnetic resonances in stable and unstable magic nuclei. Using these microscopic transition densities, cross sections for inelastic electron and alpha scattering have been calculated and compared with the available experimental data. The method also allows one to extract in a consistent way the magnitude of the strength of the various multipoles in the energy regions in which several multipoles overlap. We compare the microscopic transition densities, the strength distributions and the various multipole strengths with their values extracted phenomenologically.

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1 Introduction

1.1 Aims of the review

The atomic nucleus is a complicated quantum mechanical many-body system with a rich excitation spectrum. The experimental investigations of nuclei during the past 50 years have provided us with an overwhelming amount of excellent data which have to be understood in the framework of a quantum mechanical many-body theory. Moreover, we expect new data from the coming radioactive beam facilities for atomic nuclei far from the stability line, which will give us information about, for example, nuclei with a large neutron-to-proton ratio. Some of the features of the nuclei, such as their level fluctuations, can be described in terms of random matrix theory, which is now interpreted as the manifestation of chaotic motion. This stochastic nature of the levels is, however, only one aspect of the spectrum of the atomic nucleus which shows, on the other hand, many regular features, such as single-particle structure and collective modes. This regular behavior is a consequence of the Pauli principle and the special structure of the nucleon-nucleon interaction that leads to the concept of a mean field. As a consequence of the mean field, one is able to define quasiparticles in the sense of Landau and to apply Landau's theory of Fermi liquids [1] to the nuclear many body problem, as has been done by Migdal in his theory of finite Fermi systems (TFFS) [2].

In this article we focus on the question of how nuclei respond to a weak external field. This corresponds in lowest order to linear response and Migdal's equations look formally very similar to it. As we will show in the next section, however, the range of validity is much larger than one would expect from the conventional derivation of the linear response equations. Migdal's equations are derived within the many body Green functions (GF) theory. Using this powerful formulation of the many-body problem one is able to obtain equations which in principle are "exact". This is due to a renormalization procedure similar to the one used in quantum field theory. In the original version of the TFFS one considers explicitly only the propagation of particle-hole pairs in the nuclear medium. All the other configurations, such as the two particle-two holes ones, are renormalized into an effective two-body interaction and effective operators. The final equations have the same form as the corresponding equations of the conventional linear response theory, or random phase approximation (RPA).

The important observation by Migdal was that the effective interaction

and effective operators so defined depend only weakly on the mass number and the energy. Therefore, as in Landau's original theory, these effective quantities can be parameterized and the corresponding parameters should be the same for all nuclei except for the lightest ones. Moreover, these parameters have been chosen to be density–dependent, as the interaction inside and outside of nuclei may be quite different. These assumptions have been shown to be correct in numerous applications and they were and still are the most important features of TFFS. One can hope that the same parameters are also applicable to nuclei far from stability where we shall next apply the theory.

It is well known that pairing correlations are important in non-magic nuclei. In this case on has to define quasiparticles analogously to the BCS theory, which include the pairing gap, and in the linear response equation one has to consider as well the change of the pairing gaps [2, 3] in a consistent way. The equations of the extended theory look formally identical to the quasiparticle RPA (QRPA) [4].

The standard TFFS is not a self-consistent theory in the sense that one starts with an effective interaction from which one obtains simultaneously the mean-field quantities such as single-particle energies, single-particle wave functions and the particle-hole interaction for the linear response. Instead, one has independent parameters for the single-particle model and the particle-hole interaction. Of the various extensions of Migdal's original theory that have been made over the years, the first were directed at taking self-consistency effects into account [5, 6, 7].

It has been shown that the TFFS can also be applied to strongly deformed nuclei [8]. In that case the appropriate quasiparticles can be deduced from the deformed shell model of Nilsson. The extension of TFFS to a second order response theory has been developed [9] and the formalism for the calculation of electric and magnetic moments of excited states in even-even nuclei, transitions between excited states, as well as isomer shifts of rotational states in deformed nuclei, can be found in refs. [10, 11].

In addition, the Δ-isobar has been included in the space of quasiparticles [12] and, in the effective interaction, the effect of the one pion exchange, which turned out to be crucial for the existence of pion condensation and the properties of spin modes [7, 13, 14].

As mentioned earlier, both the linear response theory and (effectively) the TFFS are based on the RPA, which is restricted to a configuration space which includes one particle–one hole (1p1h) configurations only. If applied

to collective modes in nuclei it gives values of, for example, the centroid energies and total strengths of giant resonances in good agreement with the data. The widths of the resonances and their fine structure, however, are not. These can only be obtained if one includes in the conventional RPA and its related approaches the coupling of the 1p1h excitations to more complex configurations. Therefore, if one wants to apply TFFS to a realistic theoretical interpretation of giant resonance experiments one has to extend the formalism to include explicitly in the theory more complex configurations than 1p1h pairs.

The main aim of this article is therefore to discuss such extensions of Migdal's theory and their application to giant resonances in nuclei. The essential feature—that which determines the necessity and the title of our approach—is of course the explicit taking of complex configurations into account. After a short review of the standard TFFS we discuss in a qualitative way possible extensions of the theory. Here we will give physical arguments for why the coupling of the most collective low-energy phonons to the single-particle and single-hole propagators is the most important and which higher configurations are necessary for a quantitative description of the widths of giant resonances.

In section 2 we derive the basic equations of the extended TFFS (ETFFS) for closed shell nuclei. As in the original TFFS, the GF method is used, and for convenience we restrict ourselves to doubly closed shell nuclei, where pairing correlations can be neglected. As we shall see, there exist several stages of sophistication of the extension, and we will discuss these in some detail. As a first approximation we include the complex configurations in the excited states only. This already gives rise to the fragmentation of the multipole strength, but the total strength remains unchanged. In the next step we include also the more complex configurations in the ground state of the nuclei. This gives a further fragmentation but, in addition, gives changes in the magnitudes of the electric and magnetic transition strengths, which turns out to be important for a quantitative comparison of the theoretical results with the data. The final formulas of the extended theory are given in **r**-space, which is especially appropriate for including effects of the single-particle continuum.

In section 3 we apply the various versions of our theory to giant multipole resonances (GMR) in medium and heavy mass nuclei. The comparison with experiment demonstrates the power of our new theoretical framework and, simultaneously, the importance of the different steps of our approach.

In section 4 the calculations of transition densities within our approach are reviewed. With this information one is able to derive cross sections that can be compared directly with electron and alpha scattering experiments. One major difference between the microscopically calculated transition densities and the phenomenological ones is an energy dependence of the microscopic ones. In this connection we discuss also the problems that arise in the analysis of giant resonances in nuclei in which the resonances are very broad and where the various multipolarities overlap.

Finally, we summarize our review and discuss possible extensions that may be important for a theoretical understanding of nuclei far from the stability line.

1.2 Brief review of the standard theory

Migdal [2] has applied Landau's theory of interacting Fermi systems to atomic nuclei. Here one has first to deal with two kinds of fermions—protons and neutrons—and second with a relatively small number of particles. In the past 30 years Migdal's theory has been applied successfully by many groups to various nuclear structure problems. (See, for example, the reviews [5, 6, 12, 14, 15, 16, 17] and the book [7].)

Landau's theory deals with infinite systems of interacting fermions, such as liquid ^{3}He or nuclear matter. If there were no interaction, the system would simply be a collection of independent particles, each characterized by its spin and a wave number $\mathbf{k}$. Landau's basic assumption is that the interacting system can be obtained from the non-interacting one by an adiabatic switching on of the interaction. In particular, there should be a one-to-one correspondence between the single-particle states of non-interacting systems and the so-called quasiparticle states in interacting systems. These quasiparticles behave in the correlated system like real particles in a non-interacting system. They obey Fermi-Dirac statistics and occupy, like the non-interacting particles, corresponding quasiparticle states up to the Fermi energy. In order to define such quasiparticles Landau considers the total energy of an interacting system as a functional of the occupation function $n(\mathbf{k})$ of the quasiparticle states $\mathbf{k}$. The quasiparticle energies are then given as first functional derivatives of the total energy with respect to $n(\mathbf{k})$ and the interaction between the quasiparticles is defined as its second functional derivative. Using this approach, one can calculate properties of the excited system of a Fermi liquid, such as the zero sound mode in ^{3}He. In infinite

systems a quasiparticle differs from a real particle essentially by its mass, because both can be described by a plane wave. The effective mass of the quasiparticle is, in general, momentum–dependent and is deduced from experiments. The renormalized quasiparticle interaction depends on spin and momenta and is expanded at the Fermi surface in terms of Legendre polynomials with free parameters—the well-known Landau parameters—that are also determined from experiments.

Migdal extended these ideas to finite Fermi systems and applied his TFFS to atomic nuclei. Here the quasiparticles are the single-particle states of the nuclear shell model, which can be obtained experimentally from the neighboring odd-mass nuclei of closed shell nuclei. The quasiparticle interaction here is defined in the same way but it is isospin dependent. As in the infinite system, one expands the interaction at the Fermi surface in terms of Legendre polynomials and the parameters of the expansion, the famous Landau-Migdal parameters, are considered universal. They are also determined from experiment.

The TFFS is for that reason a semi-phenomenological microscopic theory. It is a microscopic theory because all its fundamental equations are derived rigorously from first principles, however it also contains phenomenological aspects such as the above-mentioned quasiparticle energies and the quasiparticle interaction. All the parameters are well-defined microscopically and in principle could be calculated starting from the bare nucleon-nucleon interaction. Such calculations, however, are very involved and in actual calculations one has to make severe approximations so that we can not expect to obtain full quantitative agreement with the phenomenological parameters. Nevertheless the calculated interaction parameters are in surprisingly good agreement with the phenomenological ones [17].

1.2.1 The Landau-Migdal interaction

As mentioned above, the energy E of an interacting system may be considered to be a functional of the occupation functions $n(\mathbf{k})$ of the quasiparticles $E = E(n(\mathbf{k}))$. If one excites the system, one basically changes the occupation functions by an amount $\delta n(\mathbf{k})$. The corresponding change of the energy is

$$\begin{aligned} \delta E = \textstyle\sum_{\mathbf{k}} \epsilon^0(\mathbf{k})\delta n(\mathbf{k}) + \sum_{\mathbf{k},\mathbf{k}'} f(\mathbf{k},\mathbf{k}')\delta n(\mathbf{k})\delta n(\mathbf{k}') = \\ \textstyle\sum_{\mathbf{k}}[\epsilon^0(\mathbf{k}) + \sum_{\mathbf{k}'} f(\mathbf{k},\mathbf{k}')\delta n(\mathbf{k}')]\delta n(\mathbf{k}) = \sum_{\mathbf{k}} \epsilon(\mathbf{k})\delta n(\mathbf{k}), \end{aligned} \tag{1.1}$$

where the $\epsilon^0(\mathbf{k})$ are the equilibrium energies of the quasiparticles.

The quasiparticle energies $\epsilon(\mathbf{k})$ and the interaction between a quasiparticle and a quasi-hole $f(\mathbf{k},\mathbf{k}')$ defined in this way are the first and second derivatives of the energy functional with respect to the occupation functions:

$$\epsilon(\mathbf{k}) = \frac{\delta E}{\delta n(\mathbf{k})}, \quad f(\mathbf{k},\mathbf{k}') = \frac{\delta^2 E}{\delta n(\mathbf{k})\delta n(\mathbf{k}')}. \tag{1.2}$$

In nuclear matter we have an explicit spin and isospin dependence:

$$f(\mathbf{k},\mathbf{k}') = F(\mathbf{k},\mathbf{k}') + F'(\mathbf{k},\mathbf{k}')\boldsymbol{\tau}\cdot\boldsymbol{\tau}' + [G(\mathbf{k},\mathbf{k}') + G'(\mathbf{k},\mathbf{k}')\boldsymbol{\tau}\cdot\boldsymbol{\tau}']\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'. \tag{1.3}$$

In addition to the central interaction one has in principle also to consider tensor and spin-orbit forces which, however, have been neglected in most of the actual calculations. If one denotes the momenta before the collision by $\mathbf{k}_1 = \mathbf{k}$ and $\mathbf{k}_2 = \mathbf{k}'$ then, by translational invariance, the momenta after the collision are given by $\mathbf{k}_3 = \mathbf{k} + \mathbf{q}$ and $\mathbf{k}_4 = \mathbf{k}' - \mathbf{q}$, where $\mathbf{q}$ is the momentum transfer. In the third article of ref. [1] Landau showed that the main contribution (singularity) to the full scattering amplitude should come from small $\mathbf{q}$. He also renormalized the integral equation for this amplitude in such a way that it contained a microscopic analog of the function $f(\mathbf{k},\mathbf{k}')$ and an integration over a small region near the Fermi surface. In this case the interaction depends only on the angle θ between $\mathbf{k}$ and $\mathbf{k}'$. This suggests an expansion in Legendre polynomials:

$$F(\mathbf{k},\mathbf{k}') = \sum_l F_l P_l(\cos\theta). \tag{1.4}$$

The constants F_l are the famous Landau-Migdal parameters. One introduces dimensionless parameters by defining

$$F_l = C_0 f_l, \tag{1.5}$$

where C_0 is the inverse density of states at the Fermi surface:

$$C_0 = (\frac{dn}{d\epsilon})^{-1}|_{\epsilon=\epsilon_F}. \tag{1.6}$$

Likewise, one may also expand the other components of the interaction (1.3) and define the parameters f_l', g_l, g_l' for the various terms.

Some of the Landau parameters can be related to bulk properties of the nucleus, such as the compression modulus K,

$$K = k_F^2 \frac{d^2(E/A)}{dk_F^2} = 6\frac{\hbar^2 k_F^2}{2m^*}(1 + 2f_0), \tag{1.7}$$

the symmetry energy β,

$$\beta = \frac{1}{3}\ \frac{\hbar^2 k_F^2}{2m^*}(1 + 2f_0') \tag{1.8}$$

and the effective mass m^*

$$m*/m = 1 + \frac{2}{3}f_1. \tag{1.9}$$

The various parameters have to be determined from experiments and can then be used to predict other experimental quantities. In particular, these parameters enter into the equation for $\delta n(\mathbf{k}, \omega)$, the solution of which determines small amplitude excitation of Bose type in Fermi systems. In nuclei these excitations correspond to the giant multipole resonances that we are going to investigate.

If one restricts the expansion in Eq. (1.4) to the lowest order, $l = 0$, then the interaction Eq. (1.3) is a constant in momentum space, which corresponds in $\mathbf{r}$-space to a delta function in $(\mathbf{r} - \mathbf{r}')$. The next order, $l = 1$, is the derivative of a delta function, which introduces a momentum dependence into the particle-hole (ph) interaction. In the application of TFFS to nuclei nearly all calculations have been performed with only the $l = 0$ component of the interaction, which corresponds in $\mathbf{r}$-space to the following form of the Landau-Migdal interaction:

$$F(\mathbf{r}, \mathbf{r}') = C_0[f(r) + f'(r)\boldsymbol{\tau_1} \cdot \boldsymbol{\tau_2} + (g + g'\boldsymbol{\tau_1} \cdot \boldsymbol{\tau_2})\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}]\delta(\mathbf{r} - \mathbf{r}'), \tag{1.10}$$

In *finite* nuclei, one has to introduce density-dependent parameters because it is obvious that the interaction inside a nucleus is different from the interaction in the outer region of the nucleus. The most often used ansatz is

$$f(r) = f_{ex} + (f_{in} - f_{ex})\rho_0(r)/\rho_0(0) \tag{1.11}$$

and similarly for the other parameters. Here $\rho_0(r)$ is the density distribution in the ground state of the nucleus under consideration and f_{in} and f_{ex} are

the parameters inside and outside of the nucleus. In actual calculations it turned out that g and g' depend only weakly on the density, so that one uses the same parameters inside and outside. This density dependence of the Landau-Migdal interaction is the basic reason for its success and universal applicability.

1.2.2 General theoretical background

Most of the results discussed so far were obtained within the phenomenological version of the theory of Fermi liquids. In the third article of ref. [1] Landau gave a microscopic justification of his original phenomenological theory using the GF technique. There he derived in a fully microscopic way the basic equations of his theory. He showed, in particular, that the scattering amplitude of two quasiparticles is connected with the response function R (and two-particle GF) in the ph channel and that the quantity $f(\mathbf{k}, \mathbf{k}')$ is connected with the forward scattering amplitude of two quasiparticles at the Fermi surface.

The starting point of the TFFS are the equations for the energies and transition amplitudes of excited states in even-even nuclei and the equations for moments and transitions in odd mass nuclei. As in Landau's theory, these equations can be also derived using the GF technique.

The one-particle GF is defined by

$$G_{12}(t_1, t_2) = -\imath\langle A0|Ta_1(t_1)a_2^+(t_2)|A0\rangle \tag{1.12}$$

and the two-particle GF by

$$K_{1234}(t_1, t_2; t_3, t_4) = \langle A0|Ta_1(t_1)a_2(t_2)a_3^+(t_3)a_4^+(t_4)|A0\rangle. \tag{1.13}$$

Here a and a^+ denote time-dependent annihilation and creation operators, $|A0\rangle$ is the exact eigenfunction of the ground state of an A-particle system and T is the time-ordering operator. In the following we use the single-particle states $\varphi_\lambda(\mathbf{r}, s)$ of a nuclear shell model as basis states; therefore the subscripts 1...4 stand for a set of single-particle quantum numbers.

We also need the response function R, which is defined as

$$R(12, 34) = K(23, 14) - G(21)G(34), \tag{1.14}$$

where the indices denote space and time coordinates.

The calculation of the excitation spectra of even-even nuclei and their transition probabilities can often be reduced to the calculation of the strength function, which describes the distribution of the transition strength in a nucleus induced by an external field $V^0(\omega)$

$$S(\omega) = \sum_{n\neq 0} |\langle An|V^0|A0\rangle|^2 \delta(\omega - \omega_n), \tag{1.15}$$

where $\omega_n = E_n - E_0$ is the excitation energy, while $|An\rangle$ and $|A0\rangle$ refer to the excited and ground states of a nucleus with mass number A, respectively. As the response function $R(\omega)$ in the energy representation has the spectral expansion

$$R_{12,34}(\omega) = \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi\imath} R_{12,34}(\omega,\epsilon) = \sum_n \left(\frac{\phi_{21}^{n0*}\phi_{43}^{n0}}{\omega + \omega_n - \imath\delta} - \frac{\phi_{12}^{n0}\phi_{34}^{n0*}}{\omega - \omega_n + \imath\delta} \right), \tag{1.16}$$

where

$$\phi_{12}^{n0} = \langle An|a_1^+ a_2|A0\rangle \tag{1.17}$$

is the transition amplitude between the ground state and the excited state n, the strength function $S(\omega)$ is completely determined by the response function $R(\omega)$:

$$S(\omega) = \frac{1}{\pi} \lim_{\Delta\to+0} \mathrm{Im} \sum_{1234} V_{21}^{0*} R_{12,34}(\omega + \imath\Delta) V_{43}^0. \tag{1.18}$$

This response function is defined by the Bethe–Salpeter equation in the ph channel [2]:

$$R_{12,34}(\omega,\epsilon) = -G_{31}(\epsilon+\omega)G_{24}(\epsilon) + \sum_{5678} G_{51}(\epsilon+\omega)G_{26}(\epsilon) \int_{-\infty}^{\infty} \frac{d\epsilon'}{2\pi\imath} U_{56,78}(\omega,\epsilon,\epsilon') R_{78,34}(\omega,\epsilon'), \tag{1.19}$$

which is shown graphically in Fig. 1.1.

In Eq. (1.19) G is the exact one-particle GF, and U is the irreducible amplitude in the ph channel, which is the unrenormalized ph interaction. (For details see ref. [18].) The one-particle GF G and the response R are

related by a system of nonlinear equations [2, 19]. In particular, this system of equations includes the relation

$$U(12,34) = \imath \frac{\delta \Sigma(3,4)}{\delta G(1,2)}, \tag{1.20}$$

where Σ is the so-called mass operator related to the one-particle GF by the Dyson equation

$$G_{12}(\epsilon) = G^0_{12}(\epsilon) + \sum_{34} G^0_{13}(\epsilon)\Sigma_{34}(\epsilon)G_{42}(\epsilon) \tag{1.21}$$

and $\mathrm{G}^0_{12}(\epsilon) = ((\epsilon - \mathbf{p}^2/2m)^{-1})_{12}$ is the one-particle GF of a free particle.

1.2.3 Microscopic derivation of the basic equations of the TFFS

Here we will briefly describe the derivation of the basic equations of the standard TFFS in such a way as to clarify the explicit inclusion of complex configurations to be discussed in section 2. For a more detailed derivation see refs. [2, 15].

As mentioned before, the GF G and the response function R are determined self-consistently by a system of non-linear equations. This is in principle an exact formulation of the (non-relativistic) A-particle problem, but is of little use for practical applications.

In order to arrive at solvable equations, one applies Landau's quasiparticle concept and his renormalization procedure, which he developed for the microscopic theory of Fermi liquids [1]. For the nuclear many-body problem, it was done by Migdal [2] in his TFFS.

As a first step, one splits the one-particle GF into a quasiparticle pole part which is diagonal in the shell model basis and a remainder,

$$G_{12}(\epsilon) = \delta_{12} a_1 \left(\frac{1-n_1}{\epsilon - \epsilon_1 + \imath\delta} + \frac{n_1}{\epsilon - \epsilon_1 - \imath\delta}\right) + G^r_{12}(\epsilon). \tag{1.22}$$

Here a_1 denotes the single–particle strength of the shell model pole, ϵ_1 is the single-particle energy, n is the quasiparticle occupation number (1 or 0) and $G^r(\epsilon)$ is that part of the exact one-particle GF that remains when the shell model pole has been removed. It is assumed in Migdal's TFFS that $G^r(\epsilon)$ is a smooth function of ϵ in the vicinity of ϵ_F. The dominance of the first term in Eq. (1.22) with respect to the ϵ–dependence of the one-particle GF

is one of the basic assumptions on which the TFFS rests. The validity of this assumption is crucial for the reliability of the results of all TFFS calculations. As we will see in the following, there might be additional pole terms in the expansion of Eq. (1.22) that originate from more complex configurations and which have to be considered in addition to the simple shell model poles. This extension of the theory will be the main topic of this review.

The main goal in the following is to obtain an equation for the response function that can be solved in practice because $R(\omega)$ contains all the information we need. In Eq. (1.19) the product of two one-particle Green functions enters. Here it is important to realize that with the quasiparticle ansatz in Eq. (1.22) this product can be separated at low transferred energies into a part A, which depends strongly on the energy ω and has a δ–function maximum with respect to ϵ for ϵ_1 approaching ϵ_2 (that is, near ϵ_F) and a weakly energy–dependent part B,

$$-G(\epsilon+\omega)G(\epsilon) = A(\epsilon,\omega) + B(\epsilon,\omega). \tag{1.23}$$

A is given by

$$A_{12,34}(\epsilon,\omega) = 2\pi\imath a_1 a_2 \delta_{13}\delta_{24}\frac{n_2-n_1}{\epsilon_1-\epsilon_2-\omega}\delta(\epsilon-\frac{\epsilon_1+\epsilon_2}{2}) \tag{1.24}$$

whereas B, which contains all the rest, does not give rise to a pronounced ω–dependence.

We now insert Eq. (1.24) into the equation for the response function and integrate over ϵ to obtain in compact notation

$$R(\omega) = (A+B) - (A+B)UR(\omega), \tag{1.25}$$

where A is the shell-model ph-propagator:

$$A_{12,34}(\omega) = \int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi\imath}A_{12,34}(\epsilon,\omega) = a_1 a_2 \delta_{13}\delta_{24}\frac{n_2-n_1}{\epsilon_1-\epsilon_2-\omega}. \tag{1.26}$$

With the help of Landau's renormalization procedure one can rewrite Eq. (1.25) in such a way that only the known A appears explicitly in the equation, whereas the unknown B changes U into the renormalized ph interaction F and gives rise to effective charges for the external fields. We introduce a renormalized response function $\tilde{R}(\omega)$, which is connected with the original response function $R(\omega)$ by the relation

$$R = e_q\tilde{R}e_q + Be_q, \tag{1.27}$$

where

$$e_q = 1 - FB \tag{1.28}$$

and the renormalized ph interaction F satisfies the integral equation

$$F = U - UBF. \tag{1.29}$$

A detailed investigation of these equations shows that F depends smoothly on the energy variables. For that reason the energy dependence is neglected when F is parameterized. The equation for $\tilde{R}(\omega)$ is given by

$$\tilde{R} = A - AF\tilde{R} \tag{1.30}$$

and the explicit form is

$$\tilde{R}_{12,34}(\omega) = \frac{n_2 - n_1}{\epsilon_1 - \epsilon_2 - \omega}\delta_{13}\delta_{24} - \frac{n_2 - n_1}{\epsilon_1 - \epsilon_2 - \omega}\sum_{56} F_{12,56}\tilde{R}_{56,34}. \tag{1.31}$$

This is the basic equation of the TFFS.

Due to the renormalization procedure only the 1p1h configurations appear explicitly in the final equation, whereas all the more complex configurations give rise to renormalized quantities: the ph-interaction F and the effective charges e_q. These quantities are not calculated within the theoretical framework but are parameterized in the way that was discussed before for the ph-interaction. Due to conservation laws the electric proton and neutron operators have $e_q^p = 1$, $e_q^n = 0$, respectively, and only spin-dependent operators have to be renormalized. In all practical cases the second term on the right side of Eq. (1.27) does not contribute, so it is sufficient to solve Eq. (1.31). Here all quantities are known. The single-particle energies are given by the nuclear shell model or taken, as far as possible, from experiment and the parameters of the interaction and the effective operators have been deduced from experimental data.

Equation (1.31) has exactly the same form as the conventional RPA equation that, however, has been derived using approximations from the outset. In the present derivation, Eqs. (1.27) and (1.28) are still exact. For that reason one is able to obtain relations (the Ward identities) between the effective operators and the effective interaction. In cases where conservation laws exist these relations determine the effective operators completely. In addition, the present derivation shows more clearly the range of validity of the theory, which naturally also applies to the conventional RPA equation.

Moreover, the GF formalism provides a natural basis for an extension of the theory, as we will see in the next section.

For practical reasons one solves not Eq. (1.31), but the related equation for the change of the density matrix $\rho_{12}(\omega)$ in the external field $V^0(\omega)$, which is defined as

$$\rho_{12}(\omega) = -\sum_{34} \tilde{R}_{12,34}(\omega) e_q V^0_{43}(\omega). \tag{1.32}$$

The equation for ρ_{12} follows from Eq. (1.31) and has the form

$$\rho_{12}(\omega) = -\sum_{34} A_{12,34} e_q V^0_{43} - \sum_{3456} A_{12,34} F_{34,56} \rho_{56}(\omega). \tag{1.33}$$

The expression for the strength function is then given by

$$S(\omega, \Delta) = -\frac{1}{\pi} \text{ Im } \sum_{12} e_q V^{0*}_{21} \rho_{12}(\omega + \imath\Delta), \tag{1.34}$$

where Δ is a (finite) smearing parameter which simulates a finite experimental resolution and at the same time phenomenologically can include configurations not dealt with explicitly in the approach under consideration.

Equations (1.33) and (1.34) are the main equations that are used in the calculations within the TFFS.

1.2.4 Coordinate representation

Most of the calculation that we will present here have been performed in **r**-space and not in the configuration space of a shell model basis. For the cases of RPA and TFFS the method was suggested in refs. [20, 21] and was included by us in the ETFFS [78]. The main reason for this choice is that the **r**-space representation is much more appropriate for the treatment of the single-particle continuum, as first pointed out in refs. [20, 21]. Therefore we give here some relevant equations in the coordinate representation.

Equation(1.33) has the form

$$\rho(\mathbf{r}, \omega) = -\int A(\mathbf{r}, \mathbf{r}', \omega) e_q V^0(\mathbf{r}', \omega) d^3r' - \int A(\mathbf{r}, \mathbf{r_1}, \omega) F(\mathbf{r_1}, \mathbf{r_2}) \rho(\mathbf{r_2}, \omega) d^3r_1 d^3r_2. \tag{1.35}$$

The ph propagator A, given by

$$A(\mathbf{r},\mathbf{r}',\omega)=\sum_{12}\frac{n_2-n_1}{\epsilon_1-\epsilon_2-\omega}\varphi_1^*(\mathbf{r})\varphi_2(\mathbf{r}')\varphi_2^*(\mathbf{r})\varphi_1(\mathbf{r}'), \tag{1.36}$$

can be rewritten as

$$A(\mathbf{r},\mathbf{r}',\omega)=-\sum_{1}n_1\varphi_1^*(\mathbf{r})\varphi_1(\mathbf{r}')[G(\mathbf{r}',\mathbf{r};\epsilon_1+\omega)+G(\mathbf{r}',\mathbf{r};\epsilon_1-\omega)] \tag{1.37}$$

using the formula for the one-particle GF

$$G(\mathbf{r},\mathbf{r}';\epsilon)=\sum_{2}\frac{\varphi_2(\mathbf{r})\varphi_2^*(\mathbf{r}')}{\epsilon-\epsilon_2}, \tag{1.38}$$

where $\varphi_2(\mathbf{r})$ are the single-particle wave functions calculated in a mean-field potential.

The summation in Eq. (1.37) is over states *below* the Fermi surface, i.e. the single-particle continuum is already contained in Eq. (1.38). On the other hand, the coordinate part of this GF $G_{lj}=g_{lj}/rr'$ can be expressed in closed form in terms of the regular $y_{lj}^{(1)}$ and irregular $y_{lj}^{(2)}$ solutions of the one-dimensional Schrödinger equation as

$$g_{lj}(r,r';\epsilon)=\frac{2m}{\hbar^2}y_{lj}^{(1)}(r_<;\epsilon)y_{lj}^{(2)}(r_>;\epsilon)/W_{lj}(\epsilon), \tag{1.39}$$

where $r_<$ and $r_>$ denote the lesser and the greater of r and r', respectively and W is the Wronskian of the two solutions. The irregular solution $y_{lj}^{(2)}$ is determined by the boundary conditions at ∞; e.g., for neutrons

$$y_{lj}^{(2)}(r\to\infty)\sim exp(-kr) \tag{1.40}$$

for negative energies $\epsilon<0$ and

$$y_{lj}^{(2)}(r\to\infty)\sim exp[\imath(kr-\frac{\pi l}{2}+\delta_{lj})] \tag{1.41}$$

for positive energies $\epsilon>0$, where $k=\sqrt{2m|\epsilon|}/\hbar$ and δ_{lj} is the scattering phase for the mean nuclear potential considered.

Thus, the functions $y_{lj}^{(1,2)}$ are calculated numerically if the mean potential is known. For $\epsilon<0$ the functions g_{lj} have no imaginary part; that is, the 1p1h states have automatically no width if the smearing parameter $\Delta=0$.

Inclusion of the single-particle continuum makes it possible to obtain a physical envelope of the resonance without using a smearing parameter, that is, to obtain directly the escape width $\Gamma^{\uparrow}$.

In the complex configuration problem under consideration, using the representation of the single-particle wave functions (λ-representation) gives matrices of a very large rank especially, in the case of treatment of the ground state correlations caused by complex configurations. Using the coordinate representation affords a big numerical advantage in this problem because the rank of matrices is determined not by the number of configurations but by the number of mesh points used in solving the corresponding integral equation.

1.2.5 Main results of the GMR description within the TFFS

In order to distinguish GMR from other collective excitations, one can define them as follows:

1. The form and the width Γ of the resonance depend rather weakly on A; as a rule, the dependence $\Gamma \sim A^{-2/3}$ is used.

2. The resonance mean energy E also depends weakly on A; usually one uses $E \sim A^{-1/3}$.

3. The resonance width is small compared with its excitation energy.

4. The resonance exhausts a large fraction of its energy weighted sum rule (EWSR)—usually more than 50 percent.

The last of these is the most quantitative characteristic of the resonance and justifies its name "giant".

In 1971-72 in inelastic electron [22, 23] and proton [24] scattering, giant multipole resonances (GMR) were detected that were different from the well-known isovector electric dipole resonance. That was the starting point of a period, sometimes called a renaissance of giant resonance physics, of very rapid and intensive development. A large amount of experimental data on the GMR in stable nuclei—principally their energies, total strengths, widths and resonance envelopes—has been accumulated and discussed since the mid-seventies. Currently there exists experimental information on more than 20 different types of GMR that were detected in a large number of nuclei in a broad range of excitation energies. A detailed review, with experimental

results up to the end of the eighties, is presented in ref. [25]. Further information can be found in the proceedings of the last four international conferences devoted to giant resonances [26, 27, 28, 29], and in a recent monograph [30] that gives an excellent review of the present experimental situation. Conventional theoretical methods such as the RPA and QRPA, and their comparison with the data, have been the subject of many review articles and books (e.g., refs. [30, 65, 25, 88] and [67, 90, 12, 89, 66, 15, 16]). For nuclei with $A > 40$, the experimental situation is essentially settled. There is no longer any major controversy over the centroid energies and the total strengths of GMR, and the theoretical interpretation of these data within the framework of the conventional theoretical methods is also clear. These methods, however, do not allow description of the widths and the fine structure of the resonances, nor do they offer any possibility to analyze complex spectra with overlapping resonances. This is the main subject of our review and will be presented in the following sections.

In the following we illustrate and briefly discuss typical results that have been obtained within the continuum TFFS (CTFFS), which is, as mentioned before, formally identical to the continuum RPA (CRPA). We show in Figs. 1.2-1.4 and Table 1.1 the hadron strength functions for the E2 isoscalar (IS) and isovector (IV) resonances in ^{208}Pb and the E2 IS resonance in ^{40}Ca and some of their integral characteristics. The calculations have been performed in the coordinate representation within the CTFFS, i.e., using Eqs. (1.35), (1.37) and (1.39). In order to simulate the finite experimental resolution, we introduced a smearing parameter Δ with a value of 250 keV. As the TFFF is not self–consistent, as discuss in the beginning, we have to determine the parameters of the effective interaction, Eqs. (1.10) and (1.11), from experiment by fitting some specific theoretical results to experimental data. In our calculations we always used, with the exception of f_{ex}, the following Landau-Migdal interaction parameters, which were adjusted previously to various experimental quantities [101, 102]:

$$\begin{aligned} f_{in} &= -0.002 \\ f'_{ex} &= 2.30 \\ f'_{in} &= 0.76 \\ g &= 0.05 \\ g' &= 0.96 \\ C_0 &= 300 \text{ MeVfm}^3. \end{aligned} \tag{1.42}$$

For the nuclear density $\rho_0(r)$ in the interpolation formula (1.11) we chose the theoretical ground state density distribution of the corresponding nucleus,

$$\rho_0(r) = \sum_{\epsilon_i \leq \epsilon_F} \frac{1}{4\pi}(2j_i + 1)R_i^2(r), \tag{1.43}$$

where $R_i(r)$ are the single-particle radial wave functions of the particular Woods-Saxon potential used. For other details of the calculations, including the definitions used in Table 1.1, see section 3.1.3

For the parameter f_{ex} we have used the values $f_{ex} = -1.9$ and -2.2 for ^{208}Pb and ^{40}Ca, respectively. These parameters were adjusted to reproduce the energies of the first excited 2_1^+ level in ^{208}Pb and 3_1^- level in ^{40}Ca. We will see in sections 3 and 4 that the same parameters (1.42) can also be used if more complex configurations are considered explicitly. Even the parameter f_{ex} changes only slightly in the latter case. We have used these parameters in all our calculations for stable and unstable closed shell nuclei from ^{16}O to ^{208}Pb, where we investigated many different types of GMR. In our opinion, the reasonable agreement with experiment we obtained confirms the assumed universality [2] of the parameters of the Landau-Migdal interaction, Eq. (1.10).

In Figs. 1.2-1.4 the CTFFS results are given together with the results of the calculations without taking the effective interaction into account ("free response"). As the parameters f_{in}, f_{ex} (f'_{in}, f'_{ex}) for the isoscalar (isovector) resonance are negative (positive), the IS (IV) resonances are shifted to lower (higher) energies, when the interaction is included, compared with the free response. For the latter we have the shell model estimate for ^{208}Pb $E \simeq 2 \times 41 A^{-1/3} = 13.8$ MeV while, according to Table 1.1, $\overline{E}_{is} = 8.1$ MeV, $\overline{E}_{iv} = 18.2$ MeV or, if one uses another definition for $\overline{E} = E_{2,0}$ (see section 3.1.3), $\overline{E}_{is} = 10.2$ MeV, $\overline{E}_{iv} = 19.3$ MeV. In both cases—with and without interaction—reasonable depletion values (90–100 percent for large energy intervals) of the corresponding EWSR have been obtained (see Table 1.1). The depletion is in satisfactory agreement with the corresponding experimental values of the EWSR. (See sections 3 and 4 and, in particular, Tables 3.3 and 3.7.) In Table 1.1 we give also the experimental values of the mean energies. It should be noted, however, that the experimental data were obtained, as a rule, for intervals which are much smaller than those given in Table 1.1, not to mention that the experimental mean energies may be determined in different ways that may be important for unstable nuclei for which taking the single-particle continuum into account is necessary. Therefore the comparison with

experiment should be made more carefully, and this will be done in section 3, where we will also discuss the width question.

Thus, we have obtained a relatively good description of the mean energies and total strengths of the GMR under consideration. These are typical results of the CRPA.

In Figs. 1.2-1.4 we have chosen the smearing parameter noticeably larger than the experimental resolution in order to simulate at least some of the decay width not included in the present approach. Nevertheless, one cannot see in Figs. 1.2-1.4 any resemblance to an observed resonance because the smallest width among the three resonances under discussion is the one for the E2 IS resonance in ^{208}Pb, which has an experimental width of 3.1 ± 0.3 MeV [35]. In other words, the widths of resonances, which are among the most important characteristics, are not reproduced within the CRPA. Even in medium–mass nuclei, where the role of the continuum (escape width)is much larger, the theoretical widths are still in disagreement with the experimental ones. The reason is well known: the one particle–one hole configurations describe only the escape widths, which—in general—are only a small fraction of the total widths. For a realistic description one has to include the spreading widths, which are due to the coupling of the one particle–one hole configurations to more complicated configurations. One possible solution to this problem will be discussed next.

1.3 Physical arguments for extending the standard approach

We have seen that the Landau-Migdal theory is based an a microscopic many–body theory with, however, important elements taken from experiment. For that reason it is quite natural that an extension of that successful theoretical frame work is also based on experimental facts. It is well known from nuclear spectroscopy that in odd–mass nuclei that differ from a magic nucleus by one nucleon (or hole), the coupling of the low–lying phonons of the even nucleus to the odd particle (or hole) play an important role. It gives rise to a strong fragmentation of the corresponding single-particle (hole) strength over a range of the phonon energy. This observation gives us the possibility to include in the standard TFFS the coupling of the 1p1h states to more complex configurations. We will show that with this coupling to the low–lying phonons one includes those 2p2h configurations that give rise to the strongest

fragmentation of the giant resonances, which—together with the coupling to the single-particle continuum—makes it possible to calculate quantitatively the strength distribution of the GMR.

1.3.1 Giant resonances in nuclei: the width problem

It is clear now that GMR are a universal property of nuclei. The investigations of GMR are not only important for a detailed understanding of the structure of nuclei, but they are also an important tool for a better understanding of nuclear reaction mechanisms involved in the excitation of the different types of these resonances. Moreover, we obtain from the investigation of GMR additional information on the Landau parameters. The most important of these is f_{in}, which is connected with the compressibility of nuclear matter and is therefore of crucial importance in astro-physics. The isoscalar electric monopole resonance (breathing mode), on the other hand, is closely related to this parameter and therefore one would need to know this resonance in nuclei far from stability in order to obtain the dependence of the compression modulus on the number of protons and neutrons. The magnetic resonances are related to the parameters g and g' of the spin- and isospin-dependent parts of the forces. The latter is related to pions in nuclei and is of special interest in connection with the possibility of pion condensation. Phenomenologically, GMR inform us about the nuclear shape (splitting of the E1, E2and E0 resonances in deformed nuclei), and about volume, surface and other kinds of vibrations.

The understanding of the widths of GMR is obviously connected with the damping of small amplitude vibrations in finite systems, as we shall soon discuss. Thus the general problem of how energy from highly ordered excitations is dissipated in nuclei, including the question of transition from order to chaos, can be clarified through GMR studies [31].

Many ideas from GMR physics have been used in other applications, such as the recent investigations of metallic clusters [49] and the fullerene molecules [50]. See also ref. [30], chapt. 11.

Necessity of inclusion of complex configurations and single-particle continuum

As discussed in section 1.2.5, the standard continuum TFFS or the continuum RPA in closed shell nuclei are able to describe only two integral characteristics of GMR: their mean energies and total strengths. The quantity

that is of equal importance, the strength distribution of the GMR (i.e., their widths) cannot be reproduced within this approximation. The reason for this failure has been already been indicated above: the coupling of the 1p1h configurations to more complex configurations, which gives rise to the spreading width, has so far been neglected. The escape width, which is included in the present continuum approaches, represents in the giant resonance region only the lesser part of the total width. There are several reasons why one would like do describe theoretically the widths of the GMR quantitatively:

1. the intellectual challenge to develop a microscopic theory that gives a quantitative explanation for the collective motion in strongly interacting finite Fermi systems;

2. the new insight into the fine structure of the GMR due to the rapid improvement of the experimental resolution to $\Delta E < 10$ keV, which needs to be understood (see, for example, ref. [51]);

3. the need for microscopically derived strength distributions that quantitatively reproduce the date on resonances in the medium mass region, where the various multipole resonances overlap in energy. The conventional analyses with phenomenological transition densities are no longer applicable because they introduce strong uncertainties, as we shall discuss in the following sections.

The inclusion of the single–particle continuum gives the physical envelope for processes at excitation energies higher than the nucleon separation energy. For giant resonances this gives the escape width $\Gamma^{\uparrow}$, the magnitude of which depends significantly on the mass number, the excitation energy, the multipolarity, etc., so that it is not justified, especially in the calculations of such delicate properties as fine structure and decay characteristics, to simulate the role of the continuum by a constant smearing parameter. The exact microscopic treatment of the continuum is therefore crucial to a realistic theory of giant resonances.

In addition to this, a realistic microscopic theory of collective motion in nuclei has also to consider more complex configurations than those included in the RPA. There are new data, e.g. [144, 148, 145], and the largely unsatisfactory explanation of the older results concerning the low-lying structures in cross sections in a wide excitation energy range around the nucleon binding energy [53, 54, 55, 56, 111, 57], see also section 3.2.5. An extended theory will

also have implications for the interpretation of experimental data obtained with modern germanium detectors and gamma spectrometers such as EUROBALL cluster, EUROBALL and others [58, 59]. Unprecedentedly high resolution and high efficiency of detecting gamma rays with energies up to 20 MeV have already given new and very precise information, not only on deformed nuclei, but also on low-lying levels in odd and even-even spherical nuclei. In fact, these detectors give direct information about the low-lying complex configurations containing phonons [58, 59, 60, 61, 62], which may be seen again in the fine structure of the giant resonances. At last, in order to explain the large amount of available data on the decay properties of GMR's gained in experiments with coincidences of secondary particles [52, 30] it is also necessary to take complex configurations into account (see, for example, refs. [74, 76]).

It is clear that in the immediate future the number of such data will increase rapidly and that these results require improved microscopic approaches for their interpretation.

Ground state correlations caused by complex configurations

The ground state correlations (GSC) problem has a long history. (See, for example, references in [63, 68] and also the article [69].) It started with the Hartree-Fock approximation (HF), where the effects of the Pauli principle was included in the calculation of the ground state of fermion systems. Some specific ground state correlations are taken into account if one calculates excited states within the RPA. The most important consequence of these ground state correlations is that the energy-weighted sum rule for the transition probabilities is conserved. This is not so in the Tamm-Dankoff approximation, which starts from the uncorrelated HF ground state. During the past ten years, in connection with the development of the extended TFFS, where configurations beyond the 1p1h states are included in the excited states, the question arose as to how far one has also to consider the same configurations in the ground state. [68, 84, 85] There is a fundamental difference between the effects of GSC in the RPA and their effects in models where more complex configurations are included explicitly. The RPA GSC do not lead to the appearance of new transitions compared with the TDA, but only shift the energies and redistribute the transition strengths.

The GSC induced by the more complex configurations, on the other hand, lead not only to a redistribution of strength but also to new transitions,

which give rise to a change of the EWSR [96, 85]. Thus these GSC are at least as important and physically interesting as the RPA GSC. Actually, their consequences are much richer and far-reaching. The present approach is an extension of the previously developed Extended Second RPA (see, for example refs.[96, 89]), in which uncorrelated 2p2h GSC have been considered.

We will see in the following that these effects play a noticeable—sometimes decisive—role in the theoretical description of the experimental data. The most striking example obtained within the GF approach is the explanation [93, 95] of the observed M1 excitations in ^{40}Ca and ^{16}O with energies of about 10 MeV and 16 MeV, respectively, solely as a result of ground state correlations.

1.3.2 Nuclei far from the stability line

There is increasing interest in the structure of nuclei far from the stability line. The study of these exotic nuclei, is of importance not only in itself [42, 43, 51], but also for its relevance to astrophysics [42, 43, 44, 45, 51]. The ETFFS we are discussing here may play an important role in the analysis of the experiments done at the proposed radioactive beam facilities. As mentioned earlier, one of the crucial quantities one wants to know is the breathing mode in nuclei with very different numbers of protons and neutrons. This will give us the compression modulus as a function of the proton and neutron number, which is needed for the extrapolation to nuclear matter. We may suppose that in nuclei far from stability, even with closed shells, the high-lying spectra may be as complicated as in the medium mass nuclei, where the various multipole resonances overlap and a microscopic theory is necessary for the analysis in order to obtain reliable nuclear structure information.

If we extrapolate our present knowledge of unstable nuclei to nuclei far away from the stability line we may expect two characteristic features: (i) there will be very low-lying collective states and (ii) the nucleon separation energy may also be relatively low. For these reasons a realistic theory has to treat the continuum in an exact way, and the phonon coupling is not only important for the analysis of GMR but also for a quantitative understanding of the low-lying spectrum. In connection with the application of the present approach, one has to investigate the extent to which the Landau-Migdal parameters are dependent on the numbers of protons and neutrons. Our extensive experience indicates that this dependence may be quite weak, so we can at least use the present parameters as a good starting point. The

second important input into the theory concerns the single–particle spectrum and the single–particle wave functions. Here one may use self-consistent approaches, for example, those with the density functional, in order to obtain a reliable quasiparticle basis. Investigations in these directions are in progress.

It should be emphasized that only a reliable inclusion of the single-particle continuum can make it possible to do calculations for nuclei with the separation energy near zero.This is important for understanding drip-line nuclei and for astrophysical sdudies. For neutron-reach nuclei with the separation energy near zero, this is of prime interest because of the absence of the Coulomb barrier. The CRPA calculations in ^{28}O have shown that the strength distributions of the E2 [46] and isovector E1 [47] resonances are very different from those for ^{16}O; the resonances are more spread out, shifted down and have a noticeable low-lying strengths. The effect of complex configurations is also noticeable, at least for the isoscalar E2 resonance in ^{28}O [48]. It should be pointed out, however, that except for ref. [48] and the calculations we shall present in section 3.3, there exists almost no theoretical information about the role of complex configurations in unstable nuclei [30].

1.3.3 Implications of experiments for and current status of the microscopic theory

We can summarize our discussion so far by asserting that a microscopic theory that is able to describe quantitatively the structure of collective excitations in nuclei and which is based on the mean-field approximation has to include four major effects:

1. the 1p1h RPA, which creates collectivity out of the uncorrelated ph states, as a starting point;

2. complex configurations beyond the 1p1h states, which give rise to a fragmentation of the collective states derived from the RPA;

3. the single-particle continuum;

4. ground state correlations induced by the complex configurations under consideration.

In addition, one should not use separable forces, because one then must use different forces for each mulipolarity, which strongly reduces the predictive power of the theory. Indeed one needs an interaction that is universal for

the whole periodic table, or at least that changes only very little with the mass number, and which should be adjusted to quantities other than those that one is going to calculate. As we shall see, the GF approach that we are going to discuss in the what follows allows the inclusion of all these effects simultaneously.

It is obvious that, compared with the simple 1p1h configuration problem, the present task is much more difficult—both theoretically and numerically. In addition we shall develop and apply various stages of sophistication of our theory to the nuclear structure problem in order to clarify the different effects. At present there exist several other approaches that have considered some of the effects mentioned above. These are reviewed in refs. [41, 89] ("pure" 2p2h configurations) and [65, 66, 90] (configurations with phonons).

In the past, microscopic theories of GMR have been developed using two different approaches: RPA + continuum on the one hand and RPA + complex configurations on the other. As we have seen, however, both extensions of the RPA are need to explain the data. The first of these can now be considered solved, and there exist several numerical methods for it. One, which was mentioned in section 1.2.4, uses the GF method. There one considers the one-particle continuum exactly (for a contact interaction) by transforming the RPA equation into the coordinate representation. Other methods for solving this problem have also been developed that even admit the use of nonlocal forces [71, 70]. As for the problem of including complex configurations, the most advanced approach is the quasiparticle-phonon model for magic and non-magic nuclei by Soloviev and his co-workers [64, 65]. These authors, however, used separable forces in order to reduce the numerical difficulties of the problem, and they leave out the single-particle continuum. In addition, the ground state correlations are included only partially, that is, mainly on the RPA or QRPA level.

The microscopic theory for GMR that satisfies the requirements mentioned above turned out to be quite difficult to formulate and especially to realize numerically if one uses non-separable forces, as shall will do.

There have been some successful developments in this direction in the past ten years. The first attempts of this kind, which simultaneously consider RPA configurations, the single-particle continuum (escape width $\Gamma\uparrow$) and more complex (2p2h [72, 73] or 1p1h$\otimes$phonon [75, 74]) configurations (spreading width $\Gamma\downarrow$) using non-separable forces were made in refs. [72, 73, 75, 74] for some closed shell nuclei. These authors investigated various types of GMR using, of course, different approximations and methods.

The model developed in ref. [74] used only 1p1h⊗phonon configurations and it considered only a particle-phonon interaction. It is based on the Bohr-Mottelson model for the strength function of the phonons. This model was also successfully used [76] to calculate partial branching coefficients of the proton decay of the isobar–analog and Gamow–Teller resonances in ^{208}Bi. The papers [74, 76] and [72] were the first articles in which all three (that is, the above-mentioned items 1, 2 and 3) microscopic mechanisms of GMR formation were used to explain such delicate properties as the decay characteristics of the GMR. It was also shown in refs. [74, 76] that the complex 1p1h⊗phonon configurations noticeably improve the description of the decay characteristics. The advantage of this method [74, 76] is a self-consistency (on the RPA level), that is, the phonons that are used in the extended theory have been obtained in RPA using the calculated interaction. In this development, however, ground state correlations due to complex configurations have been ignored.

The most extensive investigations of GMR, which include the above-mentioned effects, were performed within our ETFFS approach, where calculations for stable and unstable closed shell nuclei have been made. The theory is based on the consistent use of the GF method [78, 79, 80, 81, 82, 83, 84, 85, 86, 87]. The ETFFS simultaneously takes into account 1p1h, complex 1p1h⊗phonon configurations, the single-particle continuum and ground state correlations both of the RPA type and of those caused by the complex configurations under consideration. In addition, in its final equations it includes explicitly both the effective particle-hole interaction and the quasiparticle-phonon interaction.

1.4 Qualitative discussion of the extension

A theoretical approach that takes into account the 2p2h configuration space including the full 2p2h interaction is numerically hardly solvable if one also uses a realistically large configuration space. For that reason the main approximation in ETFFS concerns the selection of the 2p2h configurations. In our approach, guided by experimental observations, we include the most important correlations in the 2p2h space by coupling phonons (correlated 1p1h states) to a one-particle and one-hole state. With this procedure we obtain effectively 1p⊗phonon and 1h⊗phonon configurations. If we then couple an additional hole and particle, respectively, to the previous configurations we obtain 1p1h⊗phonon configurations where part of the 2p2h interaction is

included. As one can see from the applications, these configurations are indeed the most important ones for the understanding of the spreading width of GMR. The configurations with phonons also nicely explain a part of the low-lying spectrum in the neighboring odd mass nuclei. configurations with phonons are used in many theoretical approaches [88, 65, 66]).

There is, however, an additional fact that greatly simplifies the problem, and that is the existence of a small parameter for closed shell nuclei [88]:

$$\alpha = \frac{\langle 1 \| \mathrm{g} \| 2 \rangle^2}{(2j_1 + 1)\omega_s^2} < 1, \tag{1.44}$$

where $\langle 1 \| \mathrm{g} \| 2 \rangle$ is a reduced matrix element of the amplitude for low-lying phonon creation with the energy ω_s and 1 represents the set of single-particle quantum numbers n_1, l_1 and j_1 for spherical nuclei. Henceforth, when we refer to the g^2 approximation, it will be understood that the dimensionless α is small. Using this small parameter affords following advantages:

1. We obtain a general principle for selecting terms: as α is small, we may restrict ourselves to 1p1h$\otimes$phonon configurations, which correspond to second order in g (two-phonon configurations correspond to terms of order g^4). Because we use the g^2 approximation in the propagators of our integral equations, our approach is not the usual perturbative theory in g^2.

2. For the widths of GMR the most important contribution comes from low-lying phonons, which give rise to a strong energy dependence in the energy range of the high lying collective (1p1h) RPA solutions. Therefore we may confine ourselves to the most collective low-lying phonons, which are restricted in number. The effects of the other phonons are effectively already included through the phenomenological parameters of our approach.

3. The restriction to a small number of collective phonons noticeably reduces the numerical difficulties. This is especially important for the present approach, in which non-separable forces and the GSC induced by complex configurations are considered.

4. As some of the 1p1h$\otimes$phonon configurations are treated explicitly in the ETFFS, one should expect that the Landau-Migdal parameters that are determined within the 1p1h approximation may change. As we restrict

ourselves to the g^2 approximation and the collective low-lying phonons, this effect in the actual calculations is found to be small.

The ETFFS approach is, like the original TFFS, a semi-microscopic theory. As our approach is based on the TFFS, we actually do not need additional experimental input beyond that used already in the TFFS. We must, however, "correct" some of the parameters in order to avoid double counting. Such corrections can be performed fully consistently within our approach. The most important corrections refer to the single–particle energies which are taken—as far as possible—from experiment, or else from a shell model potential that is carefully adjusted to the corresponding closed shell nucleus. The single-particle wave functions are also taken from that model. These quantities contain contributions from the same phonons that enter the complex configurations under consideration. In order to avoid double counting due to these phonons, the single–particle model has to be "refined" from this mixing. The procedure for this will be described in section 3.1.2. The complex configurations that we treat explicitly in our extended approach are also included implicitly in the force parameters of the original TFFS approach. Therefore, in principle, one has also to correct the Landau-Migdal parameters.

There exists so far no self-consistent theoretical approach that includes all the effects discussed above. In such an approach one would start with an effective two-body interaction that would allow to determine the single-particle energies and wave functions and the ph interaction. As within such a procedure the phonon effects are not included, our extended theory would be the natural formalism in which to do it. In our extended version of the TFFS we did not include the so-called tadpole graphs with the low-lying phonons under consideration that have been used in the self-consistent version of the TFFS [6]. Their contribution is contained effectively in our "refined" mean field.

2 Framework of the extended theory

2.1 General description of collective excitations, including the particle-hole and quasiparticle-phonon interaction

As mentioned before, the original TFFS allows to calculate only the centroid energies and total transition strength of giant resonances because the approach is restricted to 1p1h configurations. In order to describe more detailed nuclear structure properties one has to include higher configurations. Here we describe the derivation of the main ETFFS equations. These equations contain both the quasiparticle-phonon interaction and the effective ph interaction in a general form.

2.1.1 General relations

Equation (1.19) can be considered a definition of the response function R only if the quantities Σ and U are known. In order to obtain realistic numerical results, however, we have at our disposal only model approximations of these quantities because it is not possible to solve the whole system of nonlinear equations for R, Σ and U for a realistic case [2, 19]. For that reason Landau introduced phenomenological elements into his microscopic theory. In Landau‘s approach Σ is irreducible in the one-particle (one-hole) channel and U is irreducible in the ph channel. As the strongest energy dependence in Eqs. (1.19) and (1.21) is considered explicitly through the one-particle (one-hole) and the ph propagators, respectively, the irreducible parts are weakly energy dependent and are parameterized in an energy–independent way. In our extended theory the complex configurations give rise to a strong energy dependence in the previously weakly energy–dependent irreducible quantities Σ and U, which we now have to consider explicitly. In order to do so we represent Σ and U as a sum of two terms, in which the first terms are again assumed to depend only weakly on the energy. As we shall see in the next section, these terms are irreducible with respect to the complex configurations that are considered explicitly and can, for the same reasons as in Landaus original theory, be parameterized.

$$\Sigma_{12}(\epsilon) = \tilde{\Sigma}_{12} + \Sigma^{e}_{12}(\epsilon), \tag{2.1}$$

$$U_{12,34}(\omega, \epsilon, \epsilon') = \tilde{U}_{12,34} + U^{e}_{12,34}(\omega, \epsilon, \epsilon'), \tag{2.2}$$

Using Eq. (2.2) one can transform our main equation, Eq. (1.19), into the symbolic form

$$R = R^e - R^e \tilde{U} R, \tag{2.3}$$

where the quantity R^e satisfies the equation

$$R^e = -GG + GGU^e R^e. \tag{2.4}$$

Equation (2.3) is more convenient in the sense that it contains only the energy-independent amplitude $\tilde{U}$, which allows us to apply the known renormalization procedure [2] to it.

Furthermore, we rewrite the Dyson Eq. (1.21) as

$$G_{12}(\epsilon) = \tilde{G}_{12}(\epsilon) + \sum_{34} \tilde{G}_{13}(\epsilon)\Sigma^e_{34}(\epsilon)G_{42}(\epsilon), \tag{2.5}$$

where $\tilde{G}$ is the solution of the Dyson equation with the mass operator $\tilde{\Sigma}$,

$$\tilde{G}_{12}(\epsilon) = G^0_{12}(\epsilon) + \sum_{34} G^0_{13}(\epsilon)\tilde{\Sigma}_{34}\tilde{G}_{42}(\epsilon). \tag{2.6}$$

The quantities denoted with tildes, $\tilde{\Sigma}$ and $\tilde{U}$, as mentioned before, are the analogs of the quantities Σ and U in the standard TFFS. The energy-dependent quantities Σ^e and U^e introduce the effects of the complex configurations into the formalism. The corresponding single-particle basis $\{\tilde{\varphi}_\lambda, \tilde{\epsilon}_\lambda\}$, which is defined by $\tilde{\Sigma}$, is a new or "refined" basis and should be obtained from the basis $\{\varphi_\lambda, \epsilon_\lambda\}$ used in the TFFS.

2.1.2 Renormalization of the general equation for the response function and the equation for the density matrix.

Equation (2.3), as well as Eq. (1.19), contains summation and integration over all states, including those that are far away from the Fermi surface and which, in actual calculations can be taken into account only effectively. For that reason it is necessary to perform the Landau-Migdal renormalization procedure in order to obtain an equation with the summation restricted to the vicinity of the Fermi energy. This is performed in analogy with the renormalization of the equation for the response function within the standard TFFS described in section 1.2.3. [18].

In analogy with Eqs. (1.25) and (1.26), we introduce the generalized propagator A, which will explicitly be given by the complex configurations under consideration and the quantity B, which contains all the rest:

$$\int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi\imath} R^e_{12,34}(\omega,\epsilon) = A_{12,34}(\omega) + B_{12,34}. \tag{2.7}$$

It is assumed that the quantity B depends only weakly on ω compared to A. One obtains from Eq. (2.3) the renormalized response function $\tilde{R}(\omega)$, which is connected with our initial response function by the relation

$$R_{12,34} = \sum_{5678} (\tilde{e}^+_q)_{12,56} \tilde{R}_{56,78}(\omega)(\tilde{e}_q)_{78,34} + \sum_{56} B_{12,56}(\tilde{e}_q)_{56,34}, \tag{2.8}$$

where $\tilde{R}(\omega)$ satisfies the equation

$$\tilde{R}_{1234}(\omega) = A_{12,34}(\omega) - \sum_{56,34} A_{12,56}(\omega)\tilde{F}_{5678}\tilde{R}_{7834}(\omega). \tag{2.9}$$

Here the new effective charge operator $\tilde{e}_q$ and the effective ph interaction amplitude $\tilde{F}$ are given by

$$(\tilde{e}_q)_{12,34} = \delta_{13}\delta_{24} - \sum_{56} \tilde{F}_{12,56} B_{56,34} \tag{2.10}$$

$$\tilde{F}_{12,34} = \sum_{56} (\tilde{e}_q)_{12,56} \tilde{U}_{56,34} \tag{2.11}$$

Equation (2.4) still contains the full GF and, in fact, according to Eq. (2.7), determines the desired propagator $A(\omega)$ in Eq. (2.9). It is useful to transform it to a more convenient form. Acting on both sides of Eq. (2.4) with the operator $(G^{-1}\tilde{G})(\tilde{G}G^{-1})$ and using Eq. (2.5), we obtain, in the time-representation,

$$R^e(12,34) = \tilde{R}^0(12,34) + \sum_{5678;t} \tilde{R}^0(12,56) W^e(56,78) R^e(78,34), \tag{2.12}$$

where

$$\tilde{R}^0(12,34) = -\tilde{G}(3,1)\tilde{G}(2,4) \tag{2.13}$$

$$W^e(12,34) = W^e_0(12,34) - \imath\Sigma^e(31)\Sigma^e(24) \tag{2.14}$$

$$W^e_0(12,34) = U^e(12,34) + \imath\Sigma^e(3,1)\tilde{G}^{-1}(2,4) + \imath\tilde{G}^{-1}(3,1)\Sigma^e(2,4). \tag{2.15}$$

Because the quantity B only weakly depends on the energy as compared with A, it follows from Eq. (2.8) and Eq. (1.18) that $S(E) = \lim_{\Delta \to +0} S(E, \Delta)$, with

$$S(E, \Delta) = \frac{1}{\pi} \text{ Im } \sum_{1234} (\tilde{e}_q V^0)^*_{21} \tilde{R}_{12,34}(E + \imath\Delta)(\tilde{e}_q V^0)_{43} \tag{2.16}$$

$$(\tilde{e}_q V^0)_{12} = \sum_{34} (\tilde{e}_q)_{21,43} V^0_{34}. \tag{2.17}$$

We see that the strength function is completely determined by the renormalized response function $\tilde{R}(\omega)$.

The advantage of these transformations is that, as in the original TFFS, the experimental quantity $S(E)$ is now connected with the renormalized response function, which can be calculated from Eq. (2.9). In contrast to Eq. (2.4) for the full response function R, which cannot be used for numerical calculations, the equation for the renormalized response function, Eqs. (2.9), (2.16) contain only quantities which will be defined in the next section: the generalized propagator A, which can be explicitly calculated from the complex configurations, and the effective ph interaction $\tilde{F}$, and the effective charge operator $\tilde{e}_q$ that are treated in the same way as in the original TFFS approach.

As in the case of the TFFS (section 1.2.3), we also introduce the change of the density matrix ρ_{12} due to an external field V^0 for the generalized case under consideration:

$$\rho_{12}(\omega) = -\sum_{34} \tilde{R}_{1234}(\omega)(\tilde{e}_q V^0)_{43}. \tag{2.18}$$

The equation for ρ_{12} follows directly from Eq. (2.9) and has the form

$$\rho_{12}(\omega) = -\sum_{34} A_{12,34}(\omega)(\tilde{e}_q V^0)_{43} - \sum_{3456} A_{12,34}(\omega)\tilde{F}_{3456}\rho_{56}(\omega), \tag{2.19}$$

and the expression for the strength function can be written as:

$$S(\omega, \Delta) = -\frac{1}{\pi} \text{ Im } \sum_{12} (\tilde{e}_q V^0)^*_{21} \rho_{12}(\omega + \imath\Delta). \tag{2.20}$$

Equations(2.20) and (2.19) are our main general results. We stress that in Eqs. (2.1), (2.2), (2.4) and (2.14), (2.15) we did not yet specify the form of our energy-dependent terms Σ^e and U^e and the transformation from R to $\tilde{R}$ is formally exact. So far we have only assumed that the quantities B and

$\tilde{U}$, as in the standard TFFS, depend only weakly on the energy. The same holds for $\tilde{e}_q$ and $\tilde{F}$. After we specify the mass operator and the irreducible amplitude in section 2.2, we shall obtain the generalized propagator A in section 2.3. With this information we are able to apply the ETFFS to GMR, the results of which we shall discuss in sections 3 and 4.

Equations (2.8) and (2.9) for our renormalized response function $\tilde{R}(\omega)$ and the final equations, Eqs. (2.20) and (2.19), have a structure similar to the relations in the TFFS given in section 1.2.3. Let us point out the main differences as compared with the TFFS.

First of all, the generalized propagator A, will differ the most from the TFFS form, Eqs. (1.23) and (1.26), because we must now solve an equation, either (2.4) or (2.12) to obtain the propagator. In the standard theory one considers the propagation of a particle–hole pair, whereas in the extended theory the propagator has to include also the configurations that go beyond the RPA. In our specific case these will be 1p1h$\otimes$phonon configurations. In the next sections we shall specify these contribution and derive explicit expression for A. Here we already point out, that the final Eqs. (2.9) or (2.19) will explicitly include both the ph interaction $\tilde{F}$ and (in the propagator) the quasiparticle–phonon interaction. In this respect our formulation differs from the approaches containing only the quasiparticle–phonon interaction [65, 66], and we do not use a phonon representation.

Secondly, the formulas of the extended theory contain also the quantities $\tilde{F}$ and $\tilde{e}_q$ that describe the effective interaction and local charge of our "refined" quasiparticles. They play the same role as F and e_q in the TFFS and will be parameterized in a similar way. In general they should differ numerically from the quantities F and e_q because the "dangerous" pole terms corresponding to the complex configurations, which will be considered explicitly in the propagator A, have been removed from $\tilde{F}$ and $\tilde{e}_q$. As discussed in section 1.4, however, these differences turn out to be small, as we shall see in sections 3 and 4.

Finally, we have written Eqs. (2.9), (2.20) and (2.19) in a new shell model representation $\tilde{\lambda}$, which differs from the representation used in the TFFS. These new single-particle wave functions $\tilde{\varphi}_\lambda$ and energies $\tilde{\epsilon}_\lambda$ that represent the new basis $\{\tilde{\varphi}_\lambda, \tilde{\epsilon}_\lambda\}$ instead of the phenomenological one $\{\varphi_\lambda, \epsilon_\lambda\}$, do not contain the phonon mixing, which is included in the mass operator Σ^e. In a sense, the basis corresponds to a Hartree-Fock basis. The mass operator $\tilde{\Sigma}$ defines the basis functions of the configuration space. The reason for this is that in the GF formalism the basis $\{\tilde{\varphi}_\lambda, \tilde{\epsilon}_\lambda\}$ is chosen in such a way that it

diagonalizes the Greens function $\tilde{G}$, see Eq. (2.6):

$$\tilde{G}_{12}(\epsilon) = \delta_{12}\tilde{G}_1(\epsilon), \quad \tilde{G}_1(\epsilon) = \frac{1}{\epsilon - \tilde{\epsilon}_1 + \imath\sigma_1\delta}, \ \ \delta \to +0. \tag{2.21}$$

In this connection the mass operator is nothing else than a generalized single–particle potential. In Eq. (2.21) we introduced the quantity σ_1, which is equal to +1 for particles and –1 for holes, and is related to the level occupation number by $\sigma_1 = 1 - 2n_1$. The "refined" basis $\{\tilde{\varphi}_\lambda, \tilde{\epsilon}_\lambda\}$ can be calculated if the old phenomenological basis $\{\varphi_\lambda, \epsilon_\lambda\}$ and the quantity Σ^e are known. In section 3.1.2 we shall describe our method of calculating $\{\tilde{\varphi}_\lambda, \tilde{\epsilon}_\lambda\}$.

2.2 Approximation for the mass operator and the irreducible amplitude in the case of 1p1h⊗phonon configurations

Let us return to Eq. (1.19), which is the starting point of our approach. We have shown in section 1.2.3. that by an appropriate choice of the ph propagator A and the effective interaction F, that is, in fact, using the approximations $\Sigma = \tilde{\Sigma}$ and $U = \tilde{U}$, one obtains from this equation Eq. (1.31) for the renormalized response functions $\tilde{R}$, which is the basis of Migdal's theory. Therefore our problem of including more complex configurations in the generalized renormalized response function defined by Eq. (2.9) requires the appropriate choice of the corresponding quantities Σ^e and U^e, which describe the coupling of ph configurations with the more complex ones and represent, therefore, the basic input to our extended theory. Depending on the approximations made for these quantities, we obtain the various versions of extended RPA theories mentioned in the introduction to this chapter:

1. *The Extended Second RPA.* If we include in Σ^e and U^e complex configurations in the lowest order of perturbation theory in the effective nucleon-nucleon interaction, we obtain the Extended Second RPA of Ref. [88]. Here one may or may not consider GSC. Such a variant of inclusion of "pure" 2p2h configurations within the GF method is briefly discussed in ref. [90]. As it is known that such a perturbation series in the nucleon-nucleon interaction converges very slowly, one has to consider partial summations that include the interaction to all orders.

2. *The Phonon Coupling Model.* Such a possibility is given by the phonon coupling model. Here the interaction between the ph pairs that build up the phonons is included to all orders. Moreover, in the case of closed shell nuclei, one can show that the coupling between the particle (or hole) with the phonons, the dimensionless parameter g (Eq. (1.44)), is a small quantity. For that reason one can restrict oneself to the coupling of one phonon to that particle (or hole), which is the g^2 approximation discussed earlier. In the g^2 approximation the expressions for Σ^e and U^e are represented graphically in Fig. 2.1 and have the following analytical form:

$$\Sigma^e_{12}(\epsilon) = \sum_{3,m} \frac{g_{13}^{m(\sigma_3)^*} g_{23}^{m(\sigma_3)}}{\epsilon - \tilde{\epsilon}_3 - \sigma_3(\omega_m - \imath\delta)} \tag{2.22}$$

$$U^e_{12,34}(\omega, \epsilon, \epsilon') = \sum_{\sigma,m} \frac{\sigma g_{31}^{m(\sigma)^*} g_{42}^{m(\sigma)}}{\epsilon - \epsilon' + \sigma(\omega_m - \imath\delta)}, \tag{2.23}$$

Here we have introduced the abbreviation:

$$g_{12}^{m(\sigma)} = \delta_{\sigma,+1} g_{12}^m + \delta_{\sigma,-1} g_{21}^{m*}.$$

where g^m_{12} is a matrix element of the creation phonon amplitude and ω_m is the phonon energy. Upon substituting Eqs. (2.22) and (2.23) into Eqs. (2.1), (2.2) and (2.4), (2.5), (2.7) one obtains the propagator A and Eq. (2.9) for the renormalized response function in the ETFFS.

The simplest propagator A, obtained from Eq. (2.12) in the g^2 approximation, is given in graphical representation in Fig. 2.2. The corresponding model, which have used this propagator, was suggested in refs. [91, 92] and realized in refs. [93, 94, 95] for the M1 resonances in magic nuclei ^{16}O, ^{40,48}Ca and ^{208}Pb in the λ representation. A reasonably good description of the experimental data, including solution of the long-standing problem of the isovector M1 resonance in ^{208}Pb was obtained. In addition, as mentioned in section 1.3.1, the model also reproduced the experimentally known 1^+ levels in ^{16}O and ^{40}Ca as a result of including the complex 1p1h$\otimes$phonon configurations in the ground state. Such an approach, however, leads to the following problem: the propagator A contains in this approximation poles of second order in the variable ω at the points of the simple poles of the RPA–like propagator. The problem is that the second order poles do not have the same physical meaning as the simple poles of the exact response

function. This can result in distortion of the energy dependence of the strength function near these poles. It was shown, however, that for the M1 resonances this distortion was negligible [94, 95].

The way out of this difficulty is to perform the summation of the g^2 terms in the propagator. In ref. [18] the method of summation was developed both for the linearized mass operator (Eq. (2.22)) and for a nonlinear case, which uses the full GF G in the mass operator instead of $\tilde{G}$. Even so, the numerical difficulties are still rather serious in both cases. There exist, however, a simpler method, which treats the summation approximately. The main idea in this method is to perform a summation of an infinite series of *some* of the g^2 terms in the propagator.

2.3 Constructing the generalized propagator: the method of chronological decoupling of diagrams

In this section we consider the method of constructing our generalized ph propagator $A(\omega)$ [77], where the problem of second-order poles of the model [92, 95] has been solved by a partial summation of the diagrammatic series in the propagator.

2.3.1 The basic idea.

Here we present the technical aspects of the method we use to construct the generalized ph propagator $A(\omega)$. As in section 1.2.2, we denote the set of one-particle quantum numbers and the time variable by $1 = \{_1, t_1\}$ and so forth. In particular, in the time representation the Green function $\tilde{G}$ given by Eq. (2.21) becomes

$$\tilde{G}(1,2) = -\imath\sigma_1\delta_{12}\theta(\sigma_1 t_{12})e^{-\imath\tilde{\epsilon}_1 t_{12}}, \tag{2.24}$$

where $t_{12} = t_1 - t_2$ and θ is the Heaviside (step) function.

Let us consider an operator in the time representation whose matrix elements in the basis $\{\tilde{\varphi}_\lambda, \tilde{\epsilon}_\lambda\}$ are given by

$$\tilde{D}(12,34) = \delta_{\sigma_1,-\sigma_2}\theta(\sigma_1 t_{41})\theta(\sigma_1 t_{32})\tilde{G}(3,1)\tilde{G}(2,4). \tag{2.25}$$

Multiplying the right-hand side by the sum $\theta(\sigma_1 t_{12}) + \theta(\sigma_1 t_{21}) = 1$ and using Eq. (2.24) we find

$$\begin{aligned}\tilde{D}(12,34) &= \delta_{\sigma_1,-\sigma_2}\delta_{13}\delta_{24}e^{-\imath(\tilde{\epsilon}_1 t_{31}+\tilde{\epsilon}_2 t_{24})} \\ &\times [\theta(\sigma_1 t_{12})\theta(\sigma_1 t_{41})\theta(\sigma_1 t_{31}) + \theta(\sigma_1 t_{21})\theta(\sigma_1 t_{32})\theta(\sigma_1 t_{42})]\end{aligned} \tag{2.26}$$

Here we have used the easily verified identity ($\sigma = \pm 1$),

$$\theta(\sigma t_{13})\theta(\sigma t_{12})\theta(\sigma t_{23}) = \theta(\sigma t_{12})\theta(\sigma t_{23}). \tag{2.27}$$

We further introduce, in accordance with Eq. (1.19) and ref. [18], the time difference variables $\tau_1 = t_3 - t_1, \tau_2 = t_2 - t_1, \tau_3 = t_3 - t_4$, so that

$$\tilde{D}(12,34) = \tilde{D}_{12,34}(\tau_1,\tau_2,\tau_3), \tag{2.28}$$

and transform to the energy representation:

$$\tilde{D}_{12,34}(\omega,\epsilon,\epsilon') = \int_{-\infty}^{+\infty} d\tau_1 d\tau_2 d\tau_3 e^{\imath(\omega\tau_1+\epsilon\tau_2+\epsilon'\tau_3)}\tilde{D}_{12,34}(\tau_1,\tau_2,\tau_3). \tag{2.29}$$

Substituting Eq. (2.26) into Eq. (2.29) and using the well-known formulas

$$\int_{-\infty}^{+\infty} d\tau e^{\imath\omega\tau}\theta(\sigma\tau) = \frac{\imath\sigma}{\omega + \imath\sigma\delta}, \quad \sigma = \pm 1, \quad \delta \to +0, \tag{2.30}$$

we find

$$\begin{aligned}\tilde{D}_{12,34}(\omega,\epsilon,\epsilon') = \delta_{\sigma_1,-\sigma_2}\delta_{13}\delta_{24}\frac{\imath\sigma_1}{(\epsilon'+\omega-\tilde{\epsilon}_1+\imath\sigma_1\delta)(\epsilon'-\tilde{\epsilon}_2+\imath\sigma_2\delta)}\times \\ \left(\frac{1}{\epsilon+\omega-\tilde{\epsilon}_1+\imath\sigma_1\delta} - \frac{1}{\epsilon-\tilde{\epsilon}_2+\imath\sigma_2\delta}\right).\end{aligned} \tag{2.31}$$

¿From this, using the definition Eq. (2.21), we obtain the final result

$$\begin{aligned}\tilde{D}_{12,34}(\omega,\epsilon,\epsilon') = -\imath\delta_{\sigma_1,-\sigma_2}\delta_{13}\delta_{24}\sigma_1(\omega-\tilde{\epsilon}_{12}+\imath\sigma_1\delta) \\ \times \tilde{G}_1(\epsilon+\omega)\tilde{G}_2(\epsilon)\tilde{G}_3(\epsilon'+\omega)\tilde{G}_4(\epsilon').\end{aligned} \tag{2.32}$$

We see from this relation that the dependence of the function $\tilde{D}_{12,34}(\omega,\epsilon,\epsilon')$ on the variables ϵ and ϵ' is separable. This is essential for the subsequent model transformations. We note that the simple product of two Green functions $\tilde{G}(3,1)\tilde{G}(2,4)$ in Eq. (2.13) does not possess this property in the energy representation:

$$\tilde{R}^0_{12,34}(\omega,\epsilon,\epsilon') = -\delta_{13}\delta_{24}2\pi\delta(\epsilon-\epsilon')\tilde{G}_1(\epsilon+\omega)\tilde{G}_2(\epsilon). \tag{2.33}$$

In order to illustrate the effect of this separability, let us consider a function $F(12,34)$ which, in the time representation, is given by

$$F(12,34) = \sum_{56,78;t} V^L(12,56)\tilde{D}(56,78)V^R(78,34), \tag{2.34}$$

where V^L and V^R are different functions whose dependence on the time arguments reduces to a dependence on three time difference variables, e.g.,

$$V^L(12,34) = V^L_{12,34}(t_{31}, t_{21}, t_{34}) \tag{2.35}$$

and $\sum_{12\ldots;t}$ denotes summation over the one-particle indices of the basis wave functions $\tilde{\varphi}_1$ and integration over the time variables. Then, in the energy representation, Eq. (2.34) is given by

$$F_{12,34}(\omega,\epsilon,\epsilon') = \sum_{5678}\int_{-\infty}^{+\infty}\frac{d\epsilon_1 d\epsilon_2}{(2\pi)^2}V^L_{12,56}(\omega,\epsilon,\epsilon_1)\tilde{D}_{56,78}(\omega,\epsilon_1,\epsilon_2)V^R_{78,34}(\omega,\epsilon_2,\epsilon'). \tag{2.36}$$

Substituting Eq. (2.32) into this, we see that, owing to the separability of the energy dependence of $\tilde{D}_{56,78}(\omega,\epsilon_1,\epsilon_2)$, the integrations over the variables ϵ_1 and ϵ_2 in Eq. (2.36) decouple. From the technical point of view this is due to the presence of the additional (compared to $\tilde{R}^0$) factor of two θ functions on the right-hand side of Eq. (2.25) written in the time representation. The replacement of $\tilde{R}^0$ by -$\tilde{D}$ in the expressions determining the generalized ph propagator is the main element of the method of constructing the propagator in our approach. The physical meaning of it will become clear with the specification of the quantities entering into equations such as Eqs. (2.34) and (2.36) and in terms of Feynmann diagrams. This also provides the name of the method of constructing of the generalized propagator, which we refer to as the method of chronological decoupling of diagrams (MCDD) [77].

2.3.2 Formulation of the method

Using symbolic notation one can write the solution of Eq. (2.12) in the form

$$R^e = \tilde{R}^0 + \imath\tilde{R}^0\Gamma^e\tilde{R}^0, \tag{2.37}$$

where the amplitude Γ^e satisfies the equation

$$\Gamma^e = W^e + \imath W^e\tilde{R}^0\Gamma^e. \tag{2.38}$$

We introduce the new amplitude Γ^e defined by the equation

$$\tilde{\Gamma}^e = \tilde{W}^e + \imath\tilde{W}^e(-\tilde{D})\tilde{\Gamma}^e, \tag{2.39}$$

which is obtained from Eq. (2.38) by the replacement

$$\tilde{R}^0 \to -\tilde{D}, W^e \to \tilde{W}^e. \tag{2.40}$$

Here the function $\tilde{D}$ is given by Eq. (2.25) and the amplitude $\tilde{W}^e$ is given by

$$\tilde{W}^e(12,34) = W_0^e(12,34) + W^{comp}(12,34), \tag{2.41}$$

which differs from Eq. (2.14) by the replacement of $-\imath\Sigma^e\Sigma^e$ by W^{comp}. The amplitude W^{comp} in Eq. (2.41) plays the same role as the term $-\imath\Sigma^e\Sigma^e$ in Eq. (2.14) and will be defined below. We now replace in Eq. (2.37) the amplitude Γ^e by $\tilde{\Gamma}^e$:

$$\tilde{R}^e = \tilde{R}^0 + \imath\tilde{R}^0\tilde{\Gamma}^e\tilde{R}^0. \tag{2.42}$$

In the energy representation this function defines the desired propagator,

$$A_{12,34}(\omega) = \int_{-\infty}^{+\infty} \frac{d\epsilon}{2\pi\imath}\tilde{R}^e_{12,34}(\omega,\epsilon). \tag{2.43}$$

Let us discuss the physical meaning of these model transformations. It is clear that W^{comp} contains g^4 terms (see also Eq. (2.44), below) and that the solution of the integral equation (2.39) (after the substitution of Eqs. (2.22) and (2.23)) gives the function $\tilde{R}^e$, and therefore the propagator A, which contains an infinite sum of the terms of higher order in the g^2 terms. An additional—and physical—condition, whose fulfilment must be verified, is that this sum must correspond to a subset of the series of Feynmann graphs that correspond to the initial quantity R^e. In this sense the replacement of $\tilde{R}^0$ (Eq. (2.13)) by $-\tilde{D}$ (Eq. (2.25)) is justified by the fact that the quantity $-\tilde{D}$ contains θ functions, which are projection operators in the space of time variables; that is, the graphs corresponding to different combinations of arguments of the function $-\tilde{D}(12,34)$ belong to the set of graphs corresponding to the function $\tilde{R}^0(12,34)$. Physically, however, it is very important that the replacement of $\tilde{R}^0$ by $-\tilde{D}$ *eliminates* from the diagrammatic expansion of R^e the terms corresponding to processes in which configurations more complex than 1p1h⊗phonon are excited while leaving most of the 1p1h⊗phonon configurations. In our model with 1p1h⊗phonon configurations the discarded

graphs include those in which a time cut through two or more phonon lines is possible in the time representation. As an example, in Figs. 2.3a, 2.3b and 2.3c we show three graphs of order g^4 that are excluded from the expansion, and in Figs. 2.3d, 2.3e and 2.3f, we show three similar graphs that remain in the diagrammatic expansion of $\tilde{R}^e$.

The sum of the contributions of higher order in g^2, which is contained in the function $\tilde{R}^e$, corresponds to the sum of chains of graphs similar to those shown in Figs. 2.3d, 2.3e and 2.3f. The contributions of the graphs shown in Figs. 2.3a, 2.3b and 2.3c are excluded due simply to the θ functions in the definition of $\tilde{D}$, Eq. (2.25).

The statements that in the method under consideration all the contributions of configurations more complex than 1p1h⊗phonon are excluded, while all 1p1h⊗phonon contributions are included, are completely valid only when GSC are ignored. The role played by their effect will be discussed below. These effects correspond to the so-called backward–going diagrams which, on the one hand, take us beyond the 1p1h⊗phonon (or 2p2h) approximation (see Fig. 2.4) but, on the other hand, not all the 1p1h⊗phonon contributions related to them (those of order g^4 and above) are included in the MCDD. Since g^2 is a small parameter, however, these neglected contributions of higher order are small. Our task in constructing the generalized propagator is to avoid the second order poles, that is, to perform approximately an infinite summation of the graphs of the type shown in Figs. 2.3d, 2.3e and 2.3f and, simultaneously, to take into account the remaining graphs—at least within the accuracy of g^2.

The last formula, which is necessary to obtain our generalized propagator, is an expression for W^{comp} in Eq. (2.41). The role of this quantity is to remove the multiple counting of graphs with self-energy insertions Σ^e. This problem therefore arises only for the terms of order g^4 and above and for the inclusion of the GSC induced by 1p1h⊗phonon configurations because, if there are no backward-going graphs like those shown in Fig. 2.4, the multiple counting is eliminated by the θ functions in the definition of the function $\tilde{D}$ (Eq. (2.25)), and in that case $W^{comp} = 0$. The expression for W^{comp} was found from the condition of cancellation of the double-counting of self-energy contributions

obtained after the first iteration of Eq. (2.39) [77]:

$$W^{comp}(12,34) = -\imath\delta_{\sigma_1,-\sigma_2}\delta_{\sigma_3,-\sigma_4}\delta_{\sigma_1,\sigma_3}\times$$
$$\sum_{1'2'3'4';t}\sum_{5'6'7'8';t}\tilde{G}^{-1}(1',1)\tilde{G}(5',1')\tilde{G}^{-1}(2,2')\tilde{G}(2',6')\times$$
$$\Sigma^e(7',5')\Sigma^e(6',8')\tilde{G}(3',7')\tilde{G}^{-1}(3,3')\tilde{G}(8',4')\tilde{G}^{-1}(4',4)\Theta(1'2',3'4'), \tag{2.44}$$

where $\Theta(1'2',3'4') = \theta(\sigma_{1'}t_{4'1'})\theta(\sigma_{1'}t_{5'8'})\theta(\sigma_{1'}t_{3'2'})\theta(\sigma_{1'}t_{6'7'})$. This expression was used in our calculations (the explicit form of it is given in the Appendix). An improved form of this term was discussed in ref. [90]. All these effects, however, are of order of g^4, so that the difference is small in our case.

2.3.3 The generalized propagator in the energy representation and its properties

Equations (2.25), (2.39), (2.41), (2.42), (2.43) and (2.44) completely determine the generalized ph propagator $A(\omega)$ in the MCCD. Since most of them are written in the time representation it is necessary to transform to the energy representation. After some algebraic effort we obtain

$$A_{12,34}(\omega) = \sum_{56,78}[\delta_{15}\delta_{26} + Q^{(+-)}_{12,56}(\omega)]A^{(--)}_{56,78}(\omega)[\delta_{73}\delta_{84} + Q^{(-+)}_{78,34}(\omega)] + P^{(++)}_{12,34}(\omega), \tag{2.45}$$

where

$$Q^{(+-)}_{12,34}(\omega) = -\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,-\sigma_4}A^{[1]}_{12,34}(\omega)\sigma_3(\omega - \tilde{\epsilon}_{34}) \tag{2.46}$$

$$Q^{(-+)}_{12,34}(\omega) = -\delta_{\sigma_1,-\sigma_2}\delta_{\sigma_3,\sigma_4}A^{[1]}_{12,34}(\omega)\sigma_1(\omega - \tilde{\epsilon}_{12}) \tag{2.47}$$

$$P^{(++)}_{12,34}(\omega) = \delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}A^{[1]}_{12,34}(\omega) \tag{2.48}$$

$$A^{[1]}_{12,34}(\omega) = -\int_{-\infty}^{+\infty}\frac{d\epsilon d\epsilon'}{(2\pi\imath)^2}\tilde{G}_1(\epsilon+\omega)\tilde{G}_2(\epsilon)U^e_{12,34}(\omega,\epsilon,\epsilon')\tilde{G}_3(\epsilon'+\omega)\tilde{G}_4(\epsilon') -$$
$$\int_{-\infty}^{+\infty}\frac{d\epsilon}{(2\pi\imath)}[\delta_{31}\tilde{G}_1(\epsilon+\omega)\tilde{G}_2(\epsilon)\Sigma^e_{24}(\epsilon)\tilde{G}_4(\epsilon) +$$
$$\delta_{24}\tilde{G}_3(\epsilon+\omega)\Sigma^e_{31}(\epsilon+\omega)\tilde{G}_1(\epsilon+\omega)\tilde{G}_2(\epsilon)], \tag{2.49}$$

with $\tilde{\epsilon}_{12} = \tilde{\epsilon}_1 - \tilde{\epsilon}_2$ and the GF $\tilde{G}_1(\epsilon)$ given by Eq. (2.21).

The only equation that must be solved to find the propagator $A(\omega)$ is the equation for the ph-ph part $A^{(--)}_{56,78}$ of the propagator:

$$A^{(--)}_{12,34}(\omega) = \tilde{A}_{12,34}(\omega) - \sum_{56,78} \tilde{A}_{12,56}(\omega)\Phi_{56,78}(\omega)A^{(--)}_{78,34}(\omega), \tag{2.50}$$

where the propagator

$$\tilde{A}_{12,34}(\omega) = -\delta_{13}\delta_{24}\sigma_1\delta_{\sigma_1,-\sigma_2}/(\omega - \tilde{\epsilon}_{12}) \tag{2.51}$$

is the "refined" RPA propagator and

$$\Phi_{12,34}(\omega) = \tilde{\Phi}_{12,34}(\omega) + \Phi^{comp}_{12,34}(\omega) \tag{2.52}$$

$$\tilde{\Phi}_{12,34}(\omega) = -\delta_{\sigma_1,-\sigma_2}\delta_{\sigma_3,-\sigma_4}\sigma_1(\omega - \tilde{\epsilon}_{12})A^{[1]}_{12,34}(\omega)\sigma_3(\omega - \tilde{\epsilon}_{34}). \tag{2.53}$$

The expression for $\Phi^{comp}(\omega)$ is given in the Appendix.

Properties of the generalized propagator $A(\omega)$ have already been discussed in detail in ref. [90]. Here we enumerate only the most important ones. Let us first introduce for convenience a new terminology. Let us represent our propagator in the form

$$\begin{gathered} A(\omega) = A^{(--)}(\omega) + A^{(-+)}(\omega) + A^{(+-)}(\omega) + A^{(++)}(\omega) \\ A^{(-+)}(\omega) = A^{(--)}(\omega)Q^{(-+)}(\omega), \;\; A^{(+-)}(\omega) = Q^{(+-)}(\omega)A^{(--)}(\omega), \\ A^{(++)}(\omega) = Q^{(+-)}(\omega)A^{(--)}Q^{(-+)}(\omega) + P^{(++)}(\omega) \end{gathered} \tag{2.54}$$

and refer to the term $A^{(--)}$ as the *unlike* component (e.g., the ph-ph component), and to the remaining three terms as the *associated* components.

First we consider the question of the second order poles, which is the main technical problem that the MCDD is designed to solve. It is obvious, in fact, that $A(\omega)$ does not contain second-order poles at the points $\omega = \pm(\tilde{\epsilon}_p - \tilde{\epsilon}_h)$, which are simple poles of the RPA propagator $\tilde{A}(\omega)$. Moreover it follows from Eq. (2.50) that the function $A^{(--)}$, and consequently $A(\omega)$ too, is regular at these points, as would be expected if the matrix $[\tilde{A}^{-1}(\omega) + \Phi(\omega)]$ is invertible at these values of ω, as is true except for accidental cases. One can also prove [90] that: (i) in solving the problem of constructing the MCCD propagator in the complete one-particle basis, all its components are regular at singular

points of the function $A^{[1]}(\omega)$ (Eq. (2.49)), and (ii) when the full one-particle basis $\{\tilde{\phi}_1, \tilde{\epsilon}_1\}$ is used, all the poles of the propagator $A(\omega)$ coincide with the poles of its unlike component $A^{(--)}(\omega)$.

As for the poles of the function $A^{(--)}(\omega)$ itself, the number of poles of $A^{(--)}(\omega)$ can be greater than the total number of poles of the functions $\tilde{A}(\omega)$ and $\Phi(\omega)$ due to splitting of the poles of $\Phi(\omega)$ in the complete or partial lifting of their degeneracy in the solution of Eq. (2.50). These effects are of order g^2.

Let us also point out a role of the term $Q^{(+-)}A^{(--)}Q^{(-+)}$ in Eq.(2.54) for the MCDD propagator, although formally this term is of order of g^4. The analysis of its poles shows that they coincide with the poles of the unlike component $A^{(--)}$ and can ensure the removal of the singularities of the function $A^{(++)}$, which coincide with the poles of $P^{(++)}$ of Eq. (2.54). This absence of "extra" poles provides an indirect confirmation of our model approximation.

Using Eqs. (2.45)–(2.49) or formulas from the Appendix, one obtains for the MCDD propagator

$$\sum_1 A_{11,34}(\omega) = \sum_3 A_{12,33}(\omega) = 0. \tag{2.55}$$

It follows from Eq. (2.9) that analogous identities are valid for the renormalized response function $\tilde{R}(\omega)$. We then find that the relation

$$\sum_1 \rho_{11} = 0 \tag{2.56}$$

holds in our approach. This means that an external field does not change the total number of particles; that is, we have the particle number conservation law. Were the problem to be solved exactly, this would be obvious from very general conditions. As in many other models, however, the validity of the law is not obvious *a priori*. This implies that the graphs with self-energy insertions Σ^e and phonons in the cross ph channel U^e must be taken into account simultaneously.

2.3.4 Ground state correlations induced by 1p1h⊗phonon configurations

As discussed earlier, in the GF graphical language any GSC correspond to the so-called backward-going diagrams. In Fig. 2.5 we show some examples of the graphs of order g^2 that correspond to the GSC caused by 1p1h⊗phonon

configurations (GSC_{phon}). In the approach under discussion there are two types of GSC_{phon} effects: those whose inclusion, as in the RPA GSC case, affect only the location and the magnitude of the residues of our ph propagator $A(\omega)$ and those of new type that can give additional poles of the function $A(\omega)$. In the latter case our approach is qualitatively different from the RPA. Some of the corresponding graphs are shown in Figs. 2.5e–h. These new poles are caused by the associated components of the MCDD propagator, which are directly connected with GSC_{phon} effects. The same components can also give GSC_{phon} effects that are not connected with the new poles and, in this sense, are analogous to the RPA GSC.

Another important feature of the associated components of the MCDD propagator is that the solution of Eqs. (2.9) or (2.19) using this propagator also contains associated components. Therefore—and contrary to the RPA case, according to the spectral expansion of the response function, non-zero components of the transition densities $\rho^{n0}_{pp'}$ and $\rho^{n0}_{hh'}$ appear, where $\rho^{n0}_{12} =< n|a^+_1 a_2|0 >$. These effects are caused by the GSC_{phon} corresponding to the above-mentioned new poles. Physically, the corresponding graphs describe processes of creation from the vacuum or annihilation of two ph pairs simultaneously, i.e, 2p2h configurations. In other words, the external field can directly excite 2p2h configuration from the correlated ground state in such a way that a particle of the 2p2h configuration makes a transition from one state to another (pp' transition) or a hole of this configuration makes a transition to another hole state (hh' transition). This means that new components, $\rho^{n0}_{pp'}$ or $\rho^{n0}_{hh'}$, appear in the spectral expansion of the response function in addition to the usual ρ^{n0}_{ph} and ρ^{n0}_{hp} components. The inclusion of these effects leads to a change of the sum rules for the moments of the strength function. Within the ESRPA, that is, for the case of "pure" 2p2h configurations, this question has been treated analytically in ref. [96] and in ref. [89], where calculations for the M1 and Gamov-Teller resonances in ^{48}Ca have been reported. Our calculations (see section 3), where perturbation theory on the ph interaction was not used, show the quantitative importance of the inclusion of the associated components of the propagator.

We can already see from this brief discussion of the GSC_{phon} effects that they manifest themselves in a considerably more complicated way than the RPA GSC effects. This question therefore merits further study.

2.4 Inclusion of the single-particle continuum

In the coordinate representation the equation for the density matrix is obtained from Eq. (2.19):

$$\rho(\mathbf{r},\omega) = -\int A(\mathbf{r},\mathbf{r}',\omega)\tilde{e}_q V^0(\mathbf{r}',\omega)d^3r' - \int A(\mathbf{r},\mathbf{r_1},\omega)\tilde{F}(\mathbf{r_1},\mathbf{r_2})\rho(\mathbf{r_2},\omega)d^3r_1 d^3r_2. \tag{2.57}$$

The present equation for the change of the density matrix is formally identical with the corresponding equation (1.35) for the case of the standard TFFS. However, the quantities which enter $\{\tilde{e}_q, \tilde{F}\}$ and the propagator A of the extended theory are defined differently compared with the quantities $\{e_q, F\}$ and A of the standard theory. The propagator of the ETFFS differs the most from the standard RPA propagator. It consists of two parts: the RPA-like part and the part that contains much more complicated physics caused by 1p1h$\otimes$phonon configurations, including GSC$_{phon}$.

As discussed for the case of the standard TFFS (section 1.2.4), use of the coordinate representation allows accounting for the single-particle continuum exactly. The effects associated with resonance decay into the continuum (the width $\Gamma \uparrow$), that is, with nucleon emission from the nucleus, are thereby automatically included in the width.

The systematic use of this technique for the case of complex configurations leads to great computational difficulties. (See, for example, the general expression for $A(\mathbf{r},\mathbf{r}',\omega)$ in the g^2 approximation obtained in ref. [94].) At this stage we therefore included the continuum only in the RPA part of the propagator, using the idea of the so-called combined $(\mathbf{r},\lambda)$ representation that was developed within the TFFS for nuclei with pairing [97]. In our ETFFS the expression for the propagator A is given by

$$A(\mathbf{r},\mathbf{r}',\omega) = \tilde{A}^{RPA}_{cont}(\mathbf{r},\mathbf{r}',\omega) + \sum_{1234}(A_{1234}(\omega) - \tilde{A}^{RPA}_{1234}(\omega))\tilde{\varphi}^*_1(\mathbf{r})\tilde{\varphi}_2(\mathbf{r})\tilde{\varphi}_3(\mathbf{r}')\tilde{\varphi}^*_4(\mathbf{r}')\,, \tag{2.58}$$

where $\tilde{A}^{RPA}_{cont}$ is the refined RPA propagator in which the single-particle continuum is taken into account exactly. In the following applications the summation in Eq. (2.58) is performed over two shells above and all shells below the Fermi level. If one would sum over all configurations, the terms with $\tilde{A}^{RPA}$ in Eq. (2.58) would cancel each other and we would have the exact expression for $A(\mathbf{r},\mathbf{r}',\omega)$. It is necessary to subtract the $\tilde{A}^{RPA}_{1234}$ term in order to avoid double counting as the full propagator A_{1234} contains the RPA part

already. Thus, in this method the single-particle continuum is taken into account completely only in the RPA part of the generalized propagator. Our calculations show that this approximation is satisfactory. The expression for the propagator A_{1234} in the "refined" single-particle basis $\{\tilde{\varphi}_\lambda, \tilde{\epsilon}_\lambda\}$, is given explicitly in the Appendix.

The distribution of the transition strength is then given by

$$S(\omega, \Delta) = -\frac{1}{\pi} \text{ Im } \int d^3r [\tilde{e}_q V^0(\mathbf{r}, \omega)]^* \rho(\mathbf{r}, \omega + \imath\Delta) \,, \tag{2.59}$$

from which one can easily obtain the transition probabilities and the EWSR summed in an energy interval.

Equations (2.57), (2.58) and (2.59) are the basic relations used in the calculations reported below.

3 Application to giant resonances

3.1 Scheme of the calculations

In the previous section the basic equation (2.57) of the ETFFS for the case of closed shell nuclei has been formulated in coordinate space for the change of the density matrix in an external field $V^0(\mathbf{r}, \omega)$ with the energy ω. The strength function (Eq. (2.59)) gives the energy distribution of the excitation strength under consideration:

$$S(\omega, \Delta) = \frac{dB(EL)}{d\omega} = -\frac{1}{\pi} \text{ Im } \Pi(\omega + \imath\Delta), \tag{3.1}$$

where

$$\Pi(\omega + \imath\Delta) = \int d^3r [\tilde{e}_q V^0(\mathbf{r})]^* \rho(\mathbf{r}, \omega + \imath\Delta) \tag{3.2}$$

is the polarizability propagator and Δ is a smearing parameter. By using this we take into account phenomenologically those complex configurations that are not explicitly treated and we simulate the finite experimental resolution. The use of a sufficiently large Δ also greatly reduces numerical difficulties in the calculations. From the solutions of this equation one obtains the transition probabilities and the EWSR, summed over the energy interval $[E_1, E_2]$, —e.g., the linear EWSR:

$$S_L = \sum_{[E_1, E_2]} E_i B_i(EL) \uparrow = \frac{1}{2\pi\imath} \oint d\omega \, \omega \Pi(\omega). \tag{3.3}$$

Here the integration contour in the complex plain intersects the real energy axis at the points E_1 and E_2.

3.1.1 Electric sum rules

In the following we discuss electromagnetically induced E0, E1 and E2 excitations. In order to avoid possible confusion we will define all relevant quantities explicitly. The electric operators are given as:

$$Q_{LM} = e\sum_{i=1}^{Z} r_i^L Y_{LM}(\Omega_i), \qquad L \geq 2 \tag{3.4}$$

$$Q_{00} = e\sum_{i=1}^{Z} r_i^2 \tag{3.5}$$

$$Q_{1M} = \frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\Omega_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\Omega_i), \tag{3.6}$$

where the electric isovector dipole operator contains the well-known kinematic charges $e_p = Ne/A$, $e_n = -Ze/A$. The linear (full) energy–weighted sum rules for these operators

$$S_L \equiv EWSR = \sum_k (E_k - E_0)B_k(EL) \uparrow$$

are then given by [117]

$$S_L = \frac{\hbar^2 e^2}{8\pi m_p} L(2L+1)^2 Z < r^{2L-2} >_p , \qquad L \geq 2, \tag{3.7}$$

$$S_0 = \frac{2\hbar^2 e^2}{m_p} Z < r^2 >_p, \tag{3.8}$$

$$S_1 = \frac{9\hbar^2 e^2}{8m_p}\frac{NZ}{A}, \tag{3.9}$$

where the radial average is usually taken over the proton distribution in the ground state. The electric dipole EWSR, Eq. (3.9), corresponds to the well-known Thomas-Reiche-Kuhn sum rules for photoabsorption cross section,

$$\sigma_0^{cl} = \int \sigma(E)dE = \frac{2\pi^2 e^2 \hbar}{mc}\frac{NZ}{A}.$$

The sums S_2 and S_0 include both the isoscalar ($\Delta T = 0$) and isovector ($\Delta T = 1$) contributions, where T is the isospin of the nucleus. These contributions may be separated for the E0 and EL (L$\geq$ 2) transitions by means of the additional physical approximation $< \tau_3 > = (N - Z)/A$ [117] so that

$$Q_{LM} = Q^0_{LM} + Q^1_{LM},$$

where for $L \geq 2$

$$Q^0_{LM} = \frac{eZ}{A} \sum_{i=1}^{A} r_i^L Y_{LM}(\Omega_i) \tag{3.10}$$

and

$$Q^1_{LM} = \frac{eN}{A} \sum_{i=1}^{Z} r_i^L Y_{LM}(\Omega_i) - \frac{eZ}{A} \sum_{i=1}^{N} r_i^L Y_{LM}(\Omega_i) \tag{3.11}$$

and for $L = 0$

$$Q^0_{00} = \frac{eZ}{A} \sum_{i=1}^{A} r_i^2 \tag{3.12}$$

$$Q^1_{00} = \frac{eN}{A} \sum_{i=1}^{Z} r_i^2 - \frac{eZ}{A} \sum_{i=1}^{N} r_i^2. \tag{3.13}$$

The effective charges in Eqs. (3.11) and (3.13) coincide with the kinematic charges for isovector E1 excitations, but they have another physical origin.

These operators give the following isoscalar and isovector electric EWSR

$$S^0_L = \frac{\hbar^2 e^2}{8\pi m_p} L(2L+1)^2 \frac{Z^2}{A} < r^{(2L-2)} >_p \tag{3.14}$$

$$S^1_L = \frac{\hbar^2 e^2}{8\pi m_p} L(2L+1)^2 \frac{ZN}{A} < r^{(2L-2)} >_p \tag{3.15}$$

for $L \geq 2$ and

$$S^0_0 = \frac{2\hbar^2 e^2}{m_p} \frac{Z^2}{A} < r^2 >_p \tag{3.16}$$

$$S^1_0 = \frac{2\hbar^2 e^2}{m_p} \frac{NZ}{A} < r^2 >_p \tag{3.17}$$

for $L = 0$.

We have calculated our strength distributions $S(E)$ with the electric operators (3.4), (3.5) and (3.6) for the full $S(E)$, (3.10) and (3.12) for the isoscalar $S(E)$ and (3.11) and (3.13) for the isovector $S(E)$.

3.1.2 Input quantities for the ETFFS

As discussed earlier, as in the standard TFFS, the initial numerical input to the ETFFS are two sets of phenomenological parameters that describe (i) the Woods-Saxon single-particle potential and (ii) the effective interaction of the Landau-Migdal type.

In all of the approaches that include complex configurations in this way, two problems concerning double counting of the complex configurations (in our case, 1p1h⊗phonon) arise. They concern the necessity to extract the contribution of the complex configurations that are treated explicitly from the "old" phenomenological quantities, that is,(i) from the mean field and (ii) from the effective interaction and local charges of quasiparticles.

As far as the procedure for the mean field is concerned, this "refining" from our phonons goes as follows: To obtain the new ("refined") single-particle basis $\{\tilde{\epsilon}_\lambda, \tilde{\varphi}_\lambda\}$ we must subtract the contribution of the phonon mixing from the "old" phenomenological Woods-Saxon energies ϵ_λ [91, 92],

$$\tilde{\epsilon}_\lambda = \epsilon_\lambda - \Sigma^e_{\lambda\lambda}(\epsilon_\lambda), \tag{3.18}$$

where Σ^e is the energy–dependent part of the mass operator in Eq. (2.1). In our g^2 approximation it is defined in Eq. (2.22). More detailed expressions are given in the Appendix.

As for the effective interaction and local charges of quasiparticles, in all our equations the new effective interaction $\tilde{F}$ and local charges $\tilde{e}_q$ enter and play the same role as the corresponding quantities F and e_q of the TFFS. By definition they should not contain the complex configurations that are considered explicitly and, therefore, the parameters describing in this approximation $\tilde{F}$ and $\tilde{e}_q$ have to be determined, in principle, from experiment. However, as discussed in section 1.4, we use only a small number of low-lying phonons, which give the main contribution to the giant resonances characteristics under consideration. The corrections to the quantities F and e_q due to these phonons must be irregular and of a long-range character. Therefore one can hope that these non-local corrections of the local quantities F and e_q are not very important and that we can use the old parameters of F and e_q instead of the new ones. A detailed investigation of the Landau-Migdal parameters shows indeed that such changes, except in the case of f_{ex}, are negligibly small.

Thus, although additional configurations, are treated in the ETFFS explicitly, it is possible to use the the parameters of the TFFS approach. This

means that all but one, of the input quantities of the ETFFS are, at least to a good approximation, already known.

3.1.3 Numerical details

For the reasons mentioned above, we use in the following application of the ETFFS the same Landau-Migdal interaction as we had used previously in the standart TFFS, where we also investigated low- and high-lying collective states. The parameters are given in Eq. (1.42). The only exception is the parameter f_{ex}. Because of the non-self-consistency of the Landau-Migdal approach, f_{ex} should be chosen in such a way that the spurious 1^- state lays at zero energy. In our calculations we used for the density interpolation in Eq. (1.11) the theoretical ground state density $\rho_0(r)$ given by Eq. (1.43). In our opinion, such a choice of $\rho_0(r)$ instead of the usual Woods-Saxon form—which we have used starting with ref. [83]—makes our calculations more consistent. (See also the discussion of previous values of f_{ex} in ref. [90].)

For the medium mass nuclei ^{40}Ca, ^{48}Ca and ^{56}Ni, using Eq. (1.43) gives noticeably different results (mainly for the IS E0 resonances) from those obtained using the Woods-Saxon form of $\rho_0(r)$ as compared with the difference for ^{208}Pb. In particular, this change in the numerical scheme has shown that the effects of GSC_{phon} there were numerically overestimated by us [137] although their qualitative features were left unchanged [85].

The phonon characteristics, that is, their energies and transition probabilities, which have been calculated within the standard TFFS [2] with the same parameters (Eq. (1.42)), are given in Table 3.1 and in ref. [137]. The criteria for the selection of these phonons were that they should have the strongest coupling parameters—and therefore the largest transition probabilities, and that the energies of the corresponding configurations should be in the energy region of the giant resonance.

The influence of the different number of phonons under consideration was investigated for ^{208}Pb. In ref. [99] it was shown that the role played by the "unnatural parity" states (e.g., $2^-_1, 3^+_1, 4^-_1$ and 5^+_1) is very small. A similar result for a much larger number of unnatural parity phonons has been obtained within a simpler model [98]. In refs. [99, 78] the contribution of the four 2^+ phonons corresponding to the isoscalar E2 resonance at an energy of about 10 MeV in ^{208}Pb has also been checked. It turned out that their contribution was only noticeable in the high-energy tail of the E1 resonance.

The single-particle levels and wave functions were calculated with the

standard Woods-Saxon potential [100]. In order to get good agreement with the experimental single-particle energies, the well depth of the central part of the potential was adjusted by changing the depth parameter by less than 5 percent. The single-particle energies thus obtained are given in refs. [80, 85].

Numerical values of all the EWSR used in the calculations are presented in Table 3.2. The quantities $< r^2 >_p$ were obtained using the proton density distribution in the ground state, Eq. (1.43), calculated from the single-particle scheme described above.

The mean energies and dispersion D are determined by

$$E_{1,0} = \frac{m_1}{m_0}, \quad E_{3,1} = (\frac{m_3}{m_1})^{1/2}, \quad E_{2,0} = (\frac{m_2}{m_0})^{1/2}, \quad D = \sqrt{\frac{m_2}{m_0} - (\frac{m_1}{m_0})^2}, \tag{3.19}$$

where the energy moments m_k corresponding to the energy interval $\Delta E = E_{max} - E_{min}$ have been calculated using

$$m_k = \int_{E_{min}}^{E_{max}} dE\ E^k S(E). \tag{3.20}$$

3.2 Electric and magnetic resonances in magic nuclei

3.2.1 Photoabsorption cross sections; E1 excitations.

The E1 absorption cross section has been calculated using the formula

$$\sigma_{E1}(\omega) = 4.022\,\omega S_{E1}(\omega), \tag{3.21}$$

where ω is taken in MeV, S in fm^2-MeV^{-1} and σ in mb.

The results of these calculations in ^{40}Ca, ^{48}Ca and ^{208}Pb and the comparisons with experiment are presented in Figs. 3.1–3.3 and Table 3.3 [78]. The label 1p1h+continuum in Table 3.3 denotes the calculations done within the CRPA (or CTFFS) and the label 1p1h+2p2h denotes those without the continuum but with the smearing parameter $\Delta \neq 0$. In order to obtain the integral characteristics of the resonances we used a Lorentz function to approximate the resonance curves, as is usually done in analyzing experiments. The parameters of this function—the mean energy $\overline{E}$, the maximum value of the cross section σ_{max} and the resonance width Γ—were found from the condition that the three lowest energy moments in Eq. (3.19) ($k = 0, 1, 2$) coincide for the exact and approximate resonance curves. The characteristics of the fitted lorentzians—the mean energy $\overline{E}$, the width Γ and the integral

cross sections σ_0 —were obtained for the same experimental energy interval and are compared with the corresponding experimental values in Table 3.3. For details see ref. [78].

In the case of CRPA we see that the mean energy values $\overline{E}$ of the E1 resonances and the widths do not agree with experiment. The energies are too low by 1 to 2 MeV, where as the widths are off by more than a factor of two. The inclusion of the 1p1h$\otimes$phonon configurations noticeably improves the agreement with experiment as compared with the CRPA. As can be seen from Figs. 3.1-3.3, the complex correlations lead to a large reduction of the maxima of the photoabsorption cross sections and to a better description of the E1 resonance tails.

The final results (denoted in Table 3.3 as 1p1h+2p2h+continuum) show that inclusion of the continuum changes the values of $\overline{E}$ by 0.6–0.8 MeV in the required direction as compared with the 1p1h+2p2h approximation, except for the case of ^{40}Ca. The theoretical mean energies for ^{48}Ca and ^{208}Pb are now in good agreement with the data.

The most important result of these calculations, however, is the now satisfactory explanation of E1 widths. To be precise, they are explained to an accuracy of the value of the smearing parameter used, which is smaller than the widths. The two new (as compared with the standard RPA) elements, namely the 1p1h$\otimes$phonon configurations and the single-particle continuum, increase the values of Γ by about a factor of two. As should be expected, the contribution of the continuum to Γ is small for the heavy nucleus ^{208}Pb and somewhat larger for the lighter ones. Specifically, its contribution to the calculated total width is 14 percent for ^{40}Ca and 28 percent for ^{48}Ca, but only 7 percent for ^{208}Pb. Now we underestimate Γ_{exp} by only 14 percent for ^{40}Ca, 11 percent for ^{48}Ca and 4 percent for ^{208}Pb. These small differences may be connected with the approximate treatment of the single-particle continuum.

In Figs. 3.1–3.3 the theoretical curves were approximated by lorentzians in the energy interval in which there are experimental points and, correspondingly, the integral characteristics of Table 3.3 were calculated for the same interval. In order to study the EWSR questions and the role of the GSC$_{phon}$ it is necessary, at least at first, to increase the energy interval under consideration. Therefore, in our later calculations [85] for ^{40}Ca, ^{48}Ca and ^{56}Ni, considerably larger energy intervals were studied and the corresponding theoretical curves are presented there. Comparing these results with experimental curves for ^{40}Ca [103] and ^{48}Ca [104], one can conclude that we have also obtained agreement with experiment for mean energies, total widths and

maximum values of photoabsorption cross sections, although the agreement is better for the previous analysis, as should be expected. This agreement is mainly obtained by inclusion of 1p1h⊗phonon configurations, but the role of our GSC_{phon} is noticeable, for example, in the integral quantity E1 EWSR (see Table 3.4. and section 3.2.4.). We have also calculated the depletion of EWSR σ_0 in the experimentally studied intervals 10.0–32.0 MeV (σ_0^{exp} = 637.7 mb-MeV) for ^{40}Ca and 11.0–27.5 MeV (σ_0^{exp} = 836.6 mb-MeV) for ^{48}Ca and obtained reasonable agreement with experiment: 599.2 mb-MeV (99.9 percent of σ_0^{cl}) and 635.3 mb-MeV (90.8 percent of σ_0^{cl}), respectively.

The integral characteristics calculated for the large intervals are given in Table 3.4. The depletion of the corresponding EWSR (in percentages of S_1, Eq. (3.9)) in Table 3.4 were obtained using the values of the reference EWSR from Table 3.2. We have obtained 97–101 percent of the E1 EWSR for the RPA case as well as for the RPA+1p1h⊗phonon configurations (without GSC_{phon}). The case with GSC_{phon} will be discussed in section 3.2.4.

3.2.2 E0 and E2 resonances

The isoscalar electric monopole (IS E0) giant resonance in nuclei is a unique source of information on the compressibility and equation of state of nuclei. The extrapolation to nuclear matter and neutron stars requires that the energy of the resonance be known over a wide range of the mass number A and for very different numbers of protons and neutrons. Here we discuss ETFFS results simultaneously for the E0 and E2 resonances because these resonances have rather similar excitation energies and overlap to a large extent. The results of isoscalar (IS) and also isovector (IV) E0 and E2 resonances are given in Figs. 3.4, 3.5, Tables 3.5, 3.6 for ^{40}Ca, ^{48}Ca and ^{56}Ni and in Figs. 3.6, 3.7 and Table 3.7 for ^{208}Pb.

E0 and E2 resonances in ^{40}Ca *and* ^{48}Ca

The main part of the IS E0-resonance in ^{40}Ca —65 percent of S_0^0 as taken from Table 3.2, is in the 11.0–23.0 MeV interval (see Fig. 3.5). In the 6.0–30.0 MeV interval we have 85.5 percent.. The comparison with experiment for isoscalar E0 and E2 excitations[1] in ^{40}Ca has been made in ref. [83]. It

[1]In our comparisons with experiments there is an uncertainty because in the experimental works, as a rule, there is no exact information about numerical values of the reference EWSR used. Therefore there may be a discrepancy due to the different values of $< r^2 >_p$ used here and in the experimental articles.

was shown there that in order to explain the electron scattering experiments [111] where both of the resonances (E0 and E2) are excited, it was necessary to take into account GSC_{phon} for both of them. This can be seen in Fig. 3.4 for the full electromagnetic E0 and E2 strengths in ^{40}Ca and also in Fig. 3.5. See also the discussion in section 4.

For the mean energies of the isoscalar E0 resonance in ^{40}Ca we obtained $E^{th}_{1,0} = 18.4$ MeV (Table 3.5), compared with the experimental value of $E^{exp}_{1,0}$ $= 18.89 \pm 0.11$ MeV [128]. Our theoretical results agree with experiment reasonably well not only for the integral characteristics, but we also for gross structure of the isoscalar E0 strength function in Fig. 3.5 and for of the cross section in refs. [128, 129], as we shall see in section 4.

It was first observed in ref. [109] that the full isoscalar E2 strength in ^{40}Ca is located in the 0–22 Mev region and is divided into approximately equal parts around 13.5 Mev and 18 Mev. This splitting was confirmed later [110, 111]. Our calculations give 73.8 percent of S^0_2 for the 0–22 MeV region, 18.3 percent for the 12–15 MeV region and 27.9 percent in the broader 10.5-16.5 MeV interval. For the 16.5–19.5 MeV and 15–21 MeV regions, we obtain 25 percent and 44.6 percent, respectively. It is therefore difficult to speak about the *equal* distribution between the regions around 13.5 Mev and 18 MeV, but the trend is the same and corresponds roughly to experiment. The splitting can be seen in Fig. 3.5. In agreement with the experiment in ref. [111], we obtained also the third maximum at 12 MeV.

As can be seen from Fig. 3.5, the isovector E0 as well as isovector E2 resonances are spread out over larger regions than the corresponding isoscalar E0 and E2 resonances, for one obvious reason: the attractive isoscalar interaction shifts the isoscalar strength down and that reduces the (escape) width; the repulsive isovector interaction has the opposite effect. There are also noticeable low-lying tails of isovector strength in the regions of the isoscalar E0 and E2 resonances. (Table 6 of ref. [85] contains numerical results for all the nuclei under consideration.) The total isovector E0+E2 contributions to the main region of the isoscalar E0 resonance are about one-half for ^{40}Ca and ^{56}Ni and about one-fourth for ^{48}Ca. These results may be important for electron scattering on these nuclei; at least a similar effect has been obtained in $^{28}\mathrm{Si}(e, e'\alpha)$ coincidence scattering [118].

Recently, the comparison of our calculations for the low-energy isoscalar E2 strength in ^{40}Ca (in the 10–18 MeV interval) and ^{48}Ca (in the 11–14 MeV interval) with the strength that was extracted from $(p, p'x)$ reactions ($x = \alpha_0$, p_0 and n_0) has been performed in ref. [138]. In order to do this the authors

multiplied the calculated E2 strength by the ground state branching ratios deduced from statistical model calculations. They obtained good overall agreement with our calculations, which probably also supports the need to take the GSC_{phon} into account. On the other hand, a very large percentage of the E2 EWSR observed in both ^{40}Ca and ^{48}Ca [149] between 6 and 12 MeV (about 40 percent and 23 percent, respectively), which means in fact a large shift of these E2 resonances—especially in ^{40}Ca—to the low-energy region, is in strong conflict with the results of our calculations shown in Fig. 3.5. In these experiments a heavy ion ^{86}Kr beam of 60 MeV/nucleon was used to excite the Ca isotopes.

E0 and E2 resonances in ^{208}Pb

The E0 and E2 hadronic response functions in ^{208}Pb have been calculated here for both the IS and IV resonances within the ETFFS and for large energy intervals (see Figs. 3.6 and 3.7 and Table 3.7). The parameter f_{ex} = –2.2 was obtained by fitting to the energy of the 2^+_1 level. The difference from the value of –1.9 obtained in the CTFFS calculations (section 1.2.5) is caused by the complex configurations that have been included here. This small difference of the parameters shows the numerical effect of inclusion of our complex configurations for this low-lying level in ^{208}Pb.

It is natural (and in accord with the experiment) that, as one can see from Table 3.7, the mean energy values and depletion of the EWSR depend rather sensitively on the energy interval used in the calculation. In most cases we obtained reasonable agreement with the experimental data available for these integral characteristics if the energy intervals under consideration are comparable. (A recent summary of the experimental data on the E0 and E2 resonances in ^{208}Pb is presented in ref. [30].) As for the cases of ^{40}Ca and ^{48}Ca, there is a noticeable amount of the IS strength in the main region of the IV strength and vice-versa for both E0 and E2 resonances.

Comparison of the E2 results shown in Fig. 3.7 and Table 3.7 with those of the CTFFS in Figs. 1.3, 1.4 and Table 1.1, calculated with the same smearing parameter, Δ = 250 keV, shows the role of 1p1h$\otimes$phonon configurations: (i) its inclusion gives the resonance widths that are numerically similar to the observed values of 3.1 $\pm$ 0.3 MeV [35] and 5 $\pm$ 0.5 MeV [37] for the IS E2 and IV E2, respectively and (ii) as for the isovector E1, the mean energy of the IV E2 resonance is changed in the desired direction (see Table 3.7).

In the calculations with the smearing parameter Δ = 250 keV shown in Fig. 3.7, a gross structure of the IS E2 hadronic response was obtained

that is absent, of course, in the calculations with $\Delta = 800$ keV, shown also in Fig. 3.7. Thus one can see a natural and noticeable difference between these two calculations that could be checked in hadron experiments. The calculated fine structure of the *electromagnetic* response function is given in Fig. 3.9 and discussed below.

For the E0 IS and IV resonances in ^{208}Pb calculated with $\Delta = 250$ keV, shown in Fig. 3.6 and Table 3.6, we obtained approximately the same results as for the E2 case: agreement of the integral characteristics with experiment, as well as a gross structure that probably has not been observed.

On the fine structure of E1 and E2 resonances

Generally speaking, the description of the giant resonance fine structure [2] is a natural step once the single-particle continuum and some complex configurations have been taken into account. A principal motivation for such a step is the rapidly improving experimental resolution. The phenomenological smearing parameter used in our and many other calculations in practice unifies (or simulates, to be more precise) two quite different effects: realistic experimental resolution and complex configurations that are not treated explicitly in the approach under consideration.

To exclude the influence of smearing and to imitate, in a sense, the results of future E1 experiments with very good resolution, we have repeated the calculations for the E1 resonance in ^{208}Pb with $\Delta = 10$ keV. The results are shown in Fig. 3.8. Of course, it is unrealistic to expect to observe the fine structure obtained on the high-lying slope of the E1 resonance, but the one on the low-lying slope could probably be observed if the "strongest" complex configurations were chosen correctly. A more detailed discussion of the low-energy strength is presented in section 3.2.5.

The results of the calculations of the E2 resonance fine structure are presented in Fig. 3.9 for the electromagnetic IS E2 strength function in ^{208}Pb. They improve our earlier calculations [86] in the following way: (i) as in the previous calculations under discussion, our GSC$_{phon}$ have been taken into account; (ii) as in Fig. 3.7, the value of the $f_{ex} = -2.2$ has been adjusted to obtain the energy of the 2^+_1 level and (iii) as mentioned above, Eq. (1.43) for the nuclear density in the ground state has been used. The smearing parameter was taken as $\Delta = 40$ keV so as to be comparable with the experimental

[2] For definiteness we will refer to an observed structure as *fine* if the experimental resolution is less than 100 keV and *gross* if the resolution is more than 100 keV.

resolution in the (e, e') and (p, p') experiments, results of which are presented in Fig. 3.9.

As can be seen from Fig. 3.9 and corresponding Fig. 1 in ref. [86], we obtained here better agreement with experiment than in ref. [86]. In that work it was shown that a large part of the fine structure under consideration was due to complex configurations. The same result can be clearly seen in Fig. 3.9. All of this confirms the decisive role of our complex 1p1h⊗phonon configurations in the formation of the fine structure—at least for this resonance. On the whole, however, the situation is very involved, due in particular to the fact that the escape width depends strongly on the excitation energy and that there are very many non-collective configurations. If we do not yet include additional complex configurations (which is, of course, quite possible to do within the GF formalism using the general prescription described in section 2.1), then one of the first steps towards clarifying the situation is probably to take into account the energy dependence of the smearing parameter and/or to use a "refined" optical potential that effectively takes into account the complex configurations that we do not consider explicitly [67, 84].

In any case, the explanation of the giant resonance fine structure is a challenge for any microscopic theory. It is clear that after improving the experimental resolution it will be possible to observe more complex configurations in the fine structure. In this case it will be necessary to improve the ETFFS by the addition of new configurations, which should be done following the methods of construction of the ETFFS described in section 2. One can also expect successes in this direction for the low-lying structures in the nucleon separation energy region (see section 3.2.5).

3.2.3 M1 resonances

There is a rich history of study of M1 excitations in nuclei [120]. The calculations within the standard TFFS with the Landau-Migdal interaction were able to reproduce the excitation energies of the isoscalar and isovector M1 excitations with the universal values of spin interaction parameters g and g' defined in Eq. (1.42) [121, 122, 123, 102].

The observed total transition strengths were also reproduced in these calculations because the standard TFFS contains the universal spin local charges determined from [2]:

$$e_q^p V^{0p} = (1 - \xi_l)\mathbf{j}^p + [(1 - \xi_s)\gamma^p + \xi_s\gamma^n + \frac{1}{2}\xi_l - \frac{1}{2}]\boldsymbol{\sigma}^p \qquad (3.22)$$

$$e_q^n V^{0n} = \xi_l \mathbf{j}^n + [(1-\xi_s)\gamma^n + \xi_s \gamma^p + \frac{1}{2}\xi_l]\boldsymbol{\sigma}^n,$$

where $\gamma^p = 2.79\mu_0$, $\gamma^n = -1.91\mu_0$, $\mu_0 = \frac{e\hbar}{2m_p}$ and

$$\xi_s^p = \xi_s^n = 0.1, \quad \xi_l^p = \xi_l^n = -0.03, \tag{3.23}$$

as obtained earlier [102, 15]. These values yield for the spin local charges $e_q^p = 0.64\gamma^p$ and $e_q^n = 0.74\gamma^n$, which explains the observed quenching of M1 strength. This quenching was one of the reasons for the long-standing problem of the " missing" M1 strength in heavy nuclei like ^{208}Pb, which was greatly clarified by the polarized photon scattering experiments of Laszewski et al. [124]. (For details see the reviews [41, 120] and also ref. [95].)

Such an approach, however, fails to explain the resonance widths that, especially in heavy nuclei, are caused by a coupling to more complex configurations than those accounted for in the RPA. In other words, the RPA or TFFS calulations did not answer the important question of why there are so many 1^+ levels with B(M1)$\uparrow \leq (2-3)\mu_0^2$ observed in ^{208}Pb [120, 124]. That was another part of the problem of the "missing" M1 strength.

The existence of deviations from the RPA predictions was confirmed by the discovery of the low-lying M1 resonance in the magic nuclei ^{16}O (at about 15 MeV) [126] and ^{40}Ca (at about 10 MeV) [125] because, according to the RPA, no M1 resonance with a similar energy can exist if the p shell in ^{16}O or the sd shell in ^{40}Ca is fully occupied. In order to illustrate this, we show in Fig. 3.10 the results of calculations of the M1 resonance in ^{40}Ca [93, 95] performed within a simpler model of taking 1p1h$\otimes$phonon configurations into account and without inclusion of the single-particle continuum, as was briefly discussed in section 2.2. As was shown in refs. [93, 95], only the terms that correspond to the "backward-going" graphs—that is, the GSC$_{phon}$ graphs—are responsible for the effect. We found that inclusion of the GSC$_{phon}$ eliminated the prohibition of the existence of the low-lying M1 excitations in ^{16}O and ^{40}Ca and, as one can see in Fig. 3.10, reasonably explained at least the strong M1 excitations observed in ref. [125]. One should note, however, that the total M1 strength calculated is substantially larger than that observed [125] and that the calculations were performed using the Woods-Saxon form of the nuclear density in the ground state and with the phonons treated within the Bohr-Mottelson model.

In Fig. 3.11 and Table 3.8 the results of the ETFFS calculations using Eqs. (3.22) and (3.23) for both isoscalar and isovector M1 excitations in

^{208}Pb are presented. In spite of the above-mentioned quenching due to the spin local charges, it was necessary to include the complex 1p1h⊗phonon configurations as well as our GSC$_{phon}$ in order to obtain agreement with experiment. The width, which can be deduced from the curve, agrees with the experimental value of 1 MeV [124] (smearing parameter $\Delta = 100$ keV). The $\Sigma_i B_i(M1) \uparrow$ value of 11.57 μ_0^2 in the 6.3–8.7 Mev interval and the mean energy $\overline{E} = 7.7$ MeV agree reasonably well with $\Sigma_i B_i(M1) \uparrow_{exp} \approx 15.6\ \mu_0^2$ and $\overline{E}_{exp} = 7.3$ MeV [124] . A reasonable agreement for the isoscalar 1^+ level at $E_{exp} = 5.85$ MeV was obtained. A more detailed discussion of the M1 calculations, including those for ^{48}Ca and ^{54}Fe, can be found in refs. [80, 95, 75].

3.2.4 Effects of GSC$_{phon}$

We define the case of absence of GSC$_{phon}$ as that when the associated components of the full propagator $A(\omega)$, Eq. (2.54) or—in other words—when the quantities $Q^{(-+)}, Q^{(+-)}, P^{(++)}$, Eq. (2.45) are equal to zero. In principle also the unlike components A$^{(--)}$ give rise to GSC$_{phon}$ but it turns out that this effect is very small, as indicated in Tables 3.4, 3.5 and 3.6, where the third row includes the effects of the A$^{(--)}$ only. These correlations are similar to the RPA GSC in the sense that they affect only the locations and values of the residues of the poles but do not change the EWSR. We will discuss in the following only GSC$_{phon}$, which are created by the associated components of the full propagator and which gives some new effects, as were discussed in section 2.3.4. and demonstrated for the case of a "pure" GSC^{phon} effect in section 3.2.3.

In all the calculations for ^{40,48}Ca and ^{56}Ni in the large energy intervals, which are given in Tables 3.4, 3.5 and 3.6, we obtained 97–102 percent of the corresponding EWSR for the RPA case as well as for that of the RPA + 1p1h⊗configurations (without the above-mentioned GSC$_{phon}$). Taking into account our GSC$_{phon}$, however, increases the EWSR by 4–7 percent as a rule. This result is in accord with the result obtained analytically in ref. [96], although our model and the ESRPA used in that work differ greatly. The main differences are that we use complex configurations with collective phonons while in the ESRPA "pure" 2p2h configurations are used and, in contrast to the ESRPA, we do not use perturbation theory in the effective interaction. The complex configurations in the ground state give rise to an increase of $< r^2 >_p$ and, therefore, of the EWSR, and perhaps to changes in other

ground state characteristics.

The role of GSC$_{phon}$ is especially noticeable in the low-lying energy region of the E0 and E2 resonances in the nuclei under consideration. We have from Table 3.2 for the 5–12 MeV interval in ^{40}Ca 7 percent of our S_0^0 and 7.4 percent of S_2^0. The contribution of GSC$_{phon}$ to these figures is 3.6 percent and 4.8 percent, respectively—i.e, more than half. (For comparison, the contributions to this interval obtained within the RPA are 1.0 percent and 0.4 percent). There is also additional low-lying strength due to GSC$_{phon}$ in ^{56}Ni (Fig. 3.4). There it is 4.8 percent of S_0^0 in the 5.0–14.0 MeV interval as compared with 2.5 percent without GSC$_{phon}$—i.e., about half. For higher energies the role of GSC$_{phon}$ is diminished; for example, for the 10–20.5 MeV interval in ^{40}Ca their contribution to the full E2 EWSR decreases it by about one-fourth.

For M1 resonances, as can be seen from Table 3.8 and Fig. 3.12 for ^{56}Ni and ^{78}Ni and Fig. 3.11 for ^{208}Pb, the role of GSC$_{phon}$ is also noticeable, although probably not to the extent that it is for the electric resonances under discussion. This is in agreement with the results obtained for M1 resonances within the ESRPA [89]. Their role in the fine structure of the E1 strength in ^{208}Pb has been calculated [84] and will be discussed briefly in the next subsection.

3.2.5 The pygmy resonance and low-lying structures

One of the impressive examples of the gross or fine structure in a broad region near the nucleon separation energy is the so-called pygmy resonance. At present this subject is the subject of active discussion in connection with the general interest in nuclei with a large neutron excess because, phenomenologically, the pygmy resonance is described as the vibration of the neutron excess against the core with $N = Z$. Thus, properties of the pygmy resonance should strongly depend on the N/Z ratio [141].

Resonances of the same name were observed in (n, γ) spectra at neutron energies from 10 keV up to several MeV in many nuclei [55, 142]. They were described by means of an additional lorentzian in the radiative strength functions with the fitted parameters $\overline{E} \approx$ 2–6 MeV, $\Gamma \approx$ 1.0–1.7 MeV and an integral strength of 0.1–1.0 percent of the classical sum rule [55]. In the (γ, n) cross sections in ^{208}Pb there are well-known structures in the 7.6- 12.0 MeV interval [56] and between 9.9 and 11.2 MeV [143], which were partly manifest earlier in the photoabsorbtion cross section [105] too.

In different experiments for many nuclei with $A = 58$–208, resonance-like structures were observed as "the low-lying $52\mathrm{A}^{-1/3}$ MeV dipole resonance" [54]. Its characteristics are $\Gamma = 1.2$–2.0 MeV and an integral strength of 1.65–2.5 percent of the experimental integral strength in the region of the isovector E1 resonance. There are also other low-lying structures in the broad energy region near the nucleon separation energy; see, for example, ref. [67].

In the broad energy region under consideration, one should observe at least E2, M1 and—mainly—E1 transitions. As yet, even for ^{208}Pb, there are no consistent microscopic calculations with non-separable forces performed within the *same* calculational scheme for all three multipoles. It is clear that the usual RPA or QRPA calculations are not able to explain these structures. The calculations should take into account more complex configurations and the single-particle continuum for the energies above the threshold. Moreover, as calculations in ^{208}Pb have shown [84], the role of GSC_{phon} is also essential. The E1 photoabsorption cross section in the 4.5–7.5 MeV interval has been calculated there within the ETFFS, both with and without GSC_{phon} (with smearing parameter $\Delta = 80$ keV), and a very noticeable difference between these two cases was found. In particular, the known structure at 5.5 MeV was explained just by the GSC_{phon} in these calculations. The experimental value of $\Sigma B(E1) = 1.338\, e^2$-fm^2 [53] for six 1^- levels is in good agreement with the corresponding theoretical value of 1.304 e^2-fm^2. See also a discussion of this question in ref. [78].

The results of nuclear resonance fluorescence experiments should significantly clarify the situation because they are able in principle to solve the long-standing problem of the identification of separate $1^-, 2^+$ and 1^+ states [144]. In connection with the numerous and successful results of these experiments, and with the interest in neutron-rich nuclei, the term "pygmy dipole resonance" (PDR) is used now for the E1 part of the "old" pygmy resonance.

The new experiments in the neutron-rich oxygen isotopes ^{18}O, ^{20}O and ^{22}O, which used electromagnetic excitation in heavy ion collisions at beam energies of about 600 MeV/nucleon, gave up to 10 percent of the classical E1 sum rule in the region up to 15 MeV [145], which is in sharp contrast to the dipole response of stable nuclei where the experiments usually give about 1 percent. It is clear that the role of the single-particle continuum is essential here but, as was discussed earlier, the role of complex configurations and, for non-magic nuclei, of a new (phonon) mechanism of pairing [146] should be important for such delicate properties of nuclei, especially of unstable ones. Because there are already many experimental data [144, 148] for stable

nuclei, the comparison with the theory, which uses a non-separable nucleon-nucleon interaction and takes into account simultaneously the single-particle continuum and complex configurations, will enable the choice of a reasonable variant of the microscopic theory to apply to calculations in unstable nuclei.

3.3 Giant resonances in unstable magic nuclei

Because the ETFFS takes into account the single-particle continuum and uses the universal parameters of the effective interaction and local charges, which are the minimum necessary for calculating properties of unstable nuclei (see sections 1.3.2 and 1.3.3), this approach is applicable to such calculations. On the other hand, the corresponding measurements could be a convincing verification of the universality of the ETFFS parameters.

The results of the calculations of E0 and E2 resonances in the unstable nucleus ^{56}Ni are presented in Figs. 3.4 and 3.5 and in Tables 3.5 and 3.6. The E1 calculations for this nucleus are given in Table 3.4 and in ref. [85]. These results are largely similar to those for ^{40}Ca and ^{48}Ca discussed above.

As discussed in section 1, the general interest in unstable nuclei is connected first of all with the neutron-rich nuclei. We have therefore calculated the isovector E1 [81] and the isoscalar and isovector M1 resonance [81, 80] (see Fig. 3.12) in the very exotic nucleus ^{78}Ni.

The calculations of the E1 resonance in refs. [81, 80] were performed using our Woods-Saxon potential to calculate the density in the nuclear ground state in the interpolation formula (1.11), and thus with another value of the parameter f_{ex}, but that is not critical—not, at least, for integral characteristics. It was found that the isovector E1 resonance in ^{78}Ni had a width of 5.2 MeV. Its mean energy $\overline{E} = 15.2$ MeV is much less than that obtained from the empirical formula $\overline{E} = 78A^{-1/3}$ (= 18.3 MeV). In this case the influence of the 1p1h$\otimes$phonon configurations is less than in other, "less exotic" nuclei: within the CTFFS we obtained $\overline{E} = 14.7$ MeV and $\Gamma = 4.8$ MeV.

As can be seen from Fig. 3.12, the M1 resonance in ^{78}Ni has an asymmetric shape with a width of about 1 MeV, whereas this resonance in ^{56}Ni and ^{132}Sn has no fragmentation width.

The M1 resonance in the neutron-deficient ^{100}Sn nucleus was split into two major peaks, with $\overline{E}_1 = 9.8$ MeV and $\overline{E}_2 = 10.5$ MeV (note the smearing parameter $\Delta = 100$ keV). This result was obtained without GSC$_{phon}$, however in ref. [140] they were taken into account in a simpler model and the result did not change. We see from Fig. 3.12 that these features for ^{78}Ni and ^{100}Sn

were caused by the inclusion of the complex configurations and for ^{100}Sn also by the small value of the proton separation energy $B_p = 2.91$ MeV (for ^{78}Ni $B_n = 5.98$ MeV was used).

4 Microscopic transition densities and the calculations of cross sections

4.1 Comparison of microscopic and phenomenological transition densities

The transition densities ρ^L_{tr}, which are necessary to describe the nuclear structure in calculations of cross sections, are simply connected with our density matrix $\rho_L(r, E + \imath\eta)$ determined in Eq. (2.57):

$$\rho^L_{tr}(r, \Delta E) = \frac{1}{\pi\sqrt{\Sigma B(EL)}} \ \mathrm{Im} \int_{E_{min}}^{E_{max}} dE \rho_L(r, E + \imath\eta), \tag{4.1}$$

where $\Sigma B(EL)$ is the $B(EL)$ value summed over the interval ΔE.

The isovector E1 transition densities calculated in our approach for the large energy interval were obtained in ref. [85]. There is no significant difference between the continuum RPA and our full calculation. In all cases there are maxima of proton and neutron transition densities on the nuclear surface with approximately equal amplitudes and opposite signs. This corresponds to the isovector nature of these giant vibrations. The existence of the maxima on the nuclear surface corresponds to the usual phenomenological models used, especially the Goldhaber-Teller model for ^{40}Ca and ^{48}Ca. We pointed out also that the transition densities defined in Eq. (4.1) are also large inside the nucleus, especially for the protons in ^{48}Ca and for the neutrons in ^{56}Ni, which may be important in the analysis of (e, e') experiments done at large momentum transfer.

In the experiments, however, a smaller energy interval is usually investigated. In order to understand better the dependence of $\rho(r)$ on the energy region, we have calculated it for the small energy intervals of 5 MeV [85] and 2 MeV (see Fig. 4.1). There is a considerable difference in the radial form of the transition densities as compared with the large interval, as well as with the different intervals under consideration. In this respect the microscopic ρ_{tr} are very different from the phenomenological ones used in conventional

analyses, where they are taken to be independent of energy over the whole energy interval [157]. Such a strong dependence on the energy region needs to be checked experimentally. It is clear that this dependence may be important for the description of the gross structure of cross sections.

In Fig. 4.1 we compare the phenomenological transition densities for the IS E0 resonance in ^{40}Ca with the theoretical transition densities obtained for two 2 MeV intervals in the main part of the resonance. We see noticeable differences for the two intervals, both between the theoretical and phenomenological densities and between the theoretical densities themselves. The behavior of the microscopic and phenomenological densities near the nuclear surface, to which the (α, α') cross sections are most sensitive, is very different in the two intervals considered. Inside the nucleus the behavior of the transition densities also differs strongly for the two intervals; for example, for the 17–19 MeV interval the neutron microscopic density has three nodes, whereas the IS E0 phenomenological transition density has always just one node.

Differential (α, α') cross sections calculated with the transition densities of Fig. 4.1 are shown in Fig. 4.2 (details of the calculations are given in section 4.2.2, below). The values of the ratio M_n/M_p of the nuclear transition momenta, which are taken to be 1 for ^{40}Ca in the phenomenological approach, are also given in Figs. 4.1 and 4.2. In the microscopic approach there is a noticeable difference in the M_n/M_p ratios, both with respect to themselves for the two intervals and between the phenomenological and microscopic densities. The cross sections calculated with phenomenological densities taken at microscopic values of M_n/M_p are also shown. The various cross sections differ most around zero degrees, where our theoretical cross sections are smaller by 5–8 percent compared to the phenomenological analysis.

In order to demonstrate the role of the size of the energy interval, we compare two theoretical ^{40}Ca(α, α') differential cross sections for the IS E0 giant resonance in Fig. 4.3. The dashed line is the result of using a transition density averaged over the whole energy range, 12–30 MeV; the full line corresponds to the differential cross sections that were obtained by the procedure discussed, performed for the 2 MeV bin and summed over the same energy range. As one can see, around zero degrees the two cross sections calculated with microscopic densities differ by nearly 25 percent. The difference between the results with the phenomenological [157] and our 2 MeV microscopic transition densities is even larger.

In addition to the differences in energy dependence, there are further im-

portant differences between the microscopic and phenomenological transition densities: (i) In the microscopic approach, the simple relations for the proton and neutron components of the nuclear transition momenta (as discussed above) and transition densities, such as the ones with the ratio N/Z, do not exist. This results in a specific energy dependence of the Coulomb-nuclear interference in total transition potentials, which may even change a destructive interference into a constructive one. (ii) The multipole decompositions of the cross sections are also very different in the phenomenological approach as compared to the microscopic one. In the first case one tries to extract the corresponding multipole composition by fitting to various total and differential experimental cross sections, whereas in the microscopic approach these multipole decompositions are determined within the theoretical model with known parameters. Indeed, our microscopic results deviate appreciably from the ones derived from phenomenological approaches.

4.2 Microscopic analyses of inelastic electron and alpha scattering experiments

4.2.1 Electron scattering

In order to compare with electron scattering experiments, in which neither of the isoscalar E0 and E2 resonances are disentangled, it is necessary to sum the E2 and E0 strengths in the same proportion that they enter into the longitudinal electron form factor [36]. Thus the quantity $[dB(E2)/d\omega + (25/16\pi)dB(E0)/d\omega]$ should be calculated and compared with experiment.

In our calculations of electron scattering in ^{40}Ca [83] we obtained a good description of the experiments [111]. As can be seen in Fig. 4.4, the description of the splitting of the isoscalar (E2+E0) strength into three peaks observed in ref. [111] at about 12 MeV, 14 MeV and 17 MeV was obtained. The value of the E2+E0 theoretical isoscalar EWSR in the observed interval 10.0–20.5 MeV, which is equal to $[6581+(25/16\pi)3729] = 8436$ e^2-fm^{4}MeV, agrees with the experimental value (7899 ± 1580) e^2-fm^4 MeV.

Our calculations of the electromagnetic E2 strength in ^{40}Ca [85] gave a nearly uniform distribution in the 12.0–22.0 MeV interval. If we take the isoscalar part of it, however, the E2 strength shows more structure, which is washed out by the isovector contribution. In earlier experiments [108] the contribution of $(23.5 \pm 4.7)\%$ of the isoscalar E0 EWSR in the 10.5–15.7 MeV was determined by two independent methods: $(\alpha, \alpha\prime)$ and $(\alpha, \alpha\prime\alpha_0)$ reactions.

A contribution of $(30 \pm 6)\%$ in the 10.5–20.0 MeV interval just from $(\alpha, \alpha\prime)$ reactions was also found. We have obtained good agreement with experiment for the first interval (20.7 percent of our S_0^0 in the 10.5–15.5 MeV region) and somewhat more (50.2 percent of S_0^0) in the second interval. See also a discussion of the IS E2 and E0 resonances in section 3.2.2.

Thus the ETFFS describes reasonably well not only the summed strengths but also the gross structure of the E2 and E0 strength extracted from electron and alpha scattering experiments with ^{40}Ca. A noticeable influence of our GSC_{phon} was also found [83]. We shall discuss below some of the ETFFS results for cross sections.

4.2.2 α particle scattering

The calculations of the cross sections were carried out with the modified code DWUCK4. The modification was done in such a way that the multipole transition potentials were constructed by single-folding the complex density–dependent Gaussian effective α-nucleon interaction [130] with our microscopic ρ_{tr}^L (Eq. (4.1)) following the prescription given in ref. [134]. The four parameters of the strength and range of the real and imaginary parts of the effective interaction were adjusted to the experimental cross sections of the low-lying 2^+ and 3^- collective states [32]. In Fig. 4.5 we show the results of these calculations for the 3_1^- and 2_1^+ levels in ^{58}Ni. The agreement between the theory and experiment allows us to conclude that the choice of the above-mentioned four parameters is good, so that they have also been used in the cross section calculations in the 10–35 MeV interval. The parameters of the optical model potential were taken from ref. [135]. As in the experimental analyses of refs. [32, 33], contributions of the IS and IV E1 and IS E0, E2, E3 and E4 resonances, which were calculated within our approach in the region under consideration, have been taken into account.

In our analysis of the $(\alpha, \alpha\prime)$ cross sections the following procedure was used: The theoretical transition densities for each of the six EL resonances were analyzed over 5 MeV energy intervals for ^{58}Ni and these densities were used as input into the DWUCK4 code. For every energy interval the DWBA cross sections $d\sigma_L(\overline{E}_L, \theta)/d\Omega$ were calculated. The inelastic α spectrum was then obtained from the expression

$$\frac{d^2\sigma}{d\Omega dE}(E, \theta) = \sum_L \frac{2}{\pi\Gamma_L} \frac{(\Gamma_L/2)^2}{(E - \overline{E}_L)^2 + (\Gamma_L/2)^2} \frac{d\sigma_L}{d\Omega}(\overline{E}_L, \theta). \qquad (4.2)$$

Here the summation runs over the six multipolarities considered and the parameters $\overline{E}_L$ and Γ_L were obtained from a lorentzian fit to the calculated strength function $S_L(E)$ defined in Eq. (3.1).

For the cross section calculations in ^{40}Ca the same method was used, but it was necessary to use 2 MeV bins because the IS E0 strength and the total spectra have much more structure. As in the case of ^{58}Ni, the contributions of the same six resonances were taken into account.

In the calculations of the giant resonances we used a standard Woods-Saxon single-particle basis. The residual Landau-Migdal interaction given in Eqs. (1.10), (1.11) and (1.43), with the parameters of Eq. (1.42), was used. Additional information about numerical details can be found in section 3.1. In all these calculations the smearing parameter $\Delta = 500$ keV was used.

4.3 Calculation of the ^{58}Ni and ^{40}Ca $(\alpha, \alpha\prime)$ cross sections; comparison with experiment.

A general problem in nuclei with $A < 90$ is that the isoscalar monopole resonance is very broad and no longer concentrated in one single peak. In moving to lighter nuclei the role of the surface becomes more important than in heavy nuclei and the $(\alpha, \alpha\prime)$ reaction is very sensitive to the nuclear surface. Moreover, for light and medium mass nuclei the single-particle continuum becomes very important. All of this has to be considered if theoretical models for giant resonances in lighter nuclei are to be developed. Therefore the conventional RPA approach to collective states has to be extended by the inclusion of surface modes and the single-particle continuum.

Until recently there were several open questions concerning the experimental information about the IS E0 resonance in nuclei with $A < 90$ [30, 106]. In some cases the magnitude of the detected strength was much smaller than the EWSR limit. An important example in this connection is ^{58}Ni, where originally only 32 percent of the EWSR was observed with inelastically scattered α-particles [32]. These authors used standard data analysis techniques with the same phenomenological transition densities ρ^L_{tr} for different excitation energies. For comparison, the same type of experiment in ^{40}Ca and a similar analysis by the same authors showed $(92 \pm 15)\%$ of the EWSR [128]. Such uncertainties might have serious consequences for nuclear matter compressibility and its applications to astrophysics.

Our analysis within the ETFFS [131, 132] of the same experimental data,

where microscopic transition densities were used instead, gave 71.4 percent of the IS E0 EWSR in the observed 12.0–25.0 MeV interval, compared with the 32 percent of ref. [32]. In these calculations the energy interval considered was divided into 5 MeV bins, for which the calculations were performed separately. The results of the calculations allowed the assumption [131, 132] that a part of the IS E0 strength in ^{58}Ni might be hidden in the experimental background.

In the meantime the 12–25 MeV energy range of the original experiment in ^{58}Ni has been extended [33]. Here the authors present results from the 12.0–31.1 MeV excitation region. Compared with the original analysis [32], two new ingredients were included by the same authors in their new work [33]: (i) a nuclear reaction description following the methods of ref. [130] was used and (ii) the giant resonance peak obtained after subtraction of the continuum was divided into 15 energy intervals from 1 to 3 MeV, each of which was analyzed separately.

In what follows we discuss the ETFFS analysis of the (α, α') experiments in ^{58}Ni and ^{40}Ca, including the newest ones [33, 129] for both nuclei, where the isoscalar giant resonance region was investigated. This analysis includes both of the necessary effects mentioned at the beginning of this subsection. Within this model we calculated the distribution of isoscalar strength of the isoscalar E0–E4 resonances and the corresponding transition densities. From these transition densities we obtained, in the standard way, (α, α') cross sections that we compare with the experimental data in ^{40}Ca in the observed 8.0–29.0 MeV interval [128] and for the isotope ^{58}Ni in several energy regions: 12–25 MeV [32], 12.0–31.1 MeV [33] and 12–35 MeV [133].

4.3.1 ^{58}Ni results

In Fig. 4.6 we compare our microscopic calculations with the newest experimental results, given in ref. [33], for the IS E0 and E2 strength distributions. For the observed 12.0–31.1 MeV interval we obtained a value of the mean energy (defined as m_1/m_0) of the IS E0 resonance of 19.9 MeV and find 81.5 percent of the EWSR. The experimental data are $(20.30^{+1.69}_{-0.14})$ MeV and $(74^{+20}_{-6})\%$, respectively. For the "old" 12.0–25.0 MeV interval the new phenomenological analysis finds (58±6)% of the EWSR [33], which is now much closer to the microscopic value of 71.4 percent [131] obtained for the same interval. Thus both analyses now give very similar results.

As one can see from the lower part of Fig. 4.6, however, for the IS E2

strength the two approaches still arrive at quite different results. Our E2 resonance mean energy value and the depletion of the IS E2 EWSR defined in the 10.0–20.0 MeV energy interval are 19.1 MeV and 47 percent, while the phenomenological analysis gives 16.1 MeV and (115±15)%. There is also a disagreement over the IS E1 and E3 strengths. Our results are presented in Fig. 4.7. (See also the discussion at the end of this subsection.)

In Figs. 4.8 and 4.9 we compare the same experimental data with our theoretical results in a slightly different way. There we obtain in both cases good agreement between our theory and the data for the total cross sections.

In Fig. 4.8 we show the role of the background and the contribution of the various giant resonances to the total cross section. We compare our theoretical results (full line with dots) with the experimental cross section at $\theta = 1.08^\circ$ (histogram) of ref. [33]. We obtained these data by subtracting an instrumental background (denoted by "Backgr. (2000)") from the original experimental spectrum. The theoretical cross section is the sum of six different multipoles, of which we only show E0 and E2. The straight line in the lower part of the figure denoted by "Backgr. (1996)" corresponds to the analysis of ref. [32], where the considered energy interval, 12–25 MeV, was smaller. In the original analysis with phenomenological transition densities, only 32 percent of the IS E0 EWSR limit was found. With the improved—but still conventional—analysis, the authors of ref. [130] obtained about 50 percent.

In Fig. 4.8, we also compare the 12–25 Mev interval and the previous background subtraction with the new, extended interval and the experimental data, where a different background has been subtracted. We find here that the total IS E0 resonance cross section in the 12–25 MeV energy range is 138.3 mb/sr and corresponds to 71.4 percent of EWSR. The area under the old background line in this region, which is included in our 71.4 percent of the EWSR, corresponds to 22 percent of the IS E0 EWSR limit, or 42.6 mb/sr. This strength had been subtracted as a part of the background in ref. [32] and was therefore not included in the analysis of ref. [130]. If we extend the analysis to the larger 12–35 MeV interval our theoretical model predicts 89.6 percent of the IS E0 EWSR limit. For the comparison with experiment see ref. [133].

In Fig. 4.10 we compare our theoretical cross sections with the data at $\theta = 4.08^\circ$. We reconstructed the experimental cross section from Fig. 4 and Fig. 1 of ref. [32]. Good agreement between the theory and experiment [32] is obtained. We also see that at this angle the IS E2 resonance and

higher multipoles contribute most to the cross section, whereas the monopole contribution is small.

Thus one can conclude that the new experimental data for the IS E0 resonance in ^{58}Ni are in good agreement with the microscopic calculations, which do not contain any fitting parameters for the nuclear structure part. The values of integral characteristics correspond now to the known experimental systematics. The disagreement between our microscopic results and the phenomenological analysis [33] for the IS E2 resonance only confirms our earlier conclusion [131] regarding the necessity of using microscopic transition densities in the experimental analysis. The same is true for the IS E1 resonance, where the two approaches also lead to very different conclusions. The authors of ref. [33] obtained only 41 percent of the IS E1 EWSR and this strength was spread more or less uniformly from 12 MeV to 35 MeV. Our distribution of this strength is shown in Fig. 4.7. One can see that the distribution is not uniform; rather, it has a resonance structure. We obtained 89 percent of the IS E1 EWSR and a value of 25.0 MeV for the mean energy in the interval under consideration. These figures are consistent with the results of ref. [136] for other nuclei. Our E3 IS strength is more uniformly distributed.

4.3.2 ^{40}Ca results

Our improved calculations of the IS E0 resonance in ^{40}Ca, presented in ref. [85], show that it has a more compact form than in our earlier calculations [137], but it remains strongly structured and spread out over a large energy interval: 65 percent of the EWSR is in the 11–23 MeV interval and 106.7 percent is in the 5.0–45.0 MeV interval. Thus it is important that the large energy interval, 8.0–29.0 MeV, was studied in ref. [128] and that, in fact, most of the IS E0 strength was found in their analysis.

Our theoretical distributions of the IS E0 and E2 strengths in ^{40}Ca are shown in Fig. 4.11. Compared with the corresponding results of the analysis of ref. [128], which were obtained from the difference between spectra taken at $\theta = 1.1^\circ$ and $\theta = 2.4^\circ$ (see Fig. 6 of ref. [128]), a reasonable agreement for the central part (10–23 MeV) of the IS E0 resonance was obtained. The percentages of the IS E0 EWSR in the four observed intervals 7.5–12.5 MeV, 12.5–22.5 MeV, 22.5–28.8 MeV and 7.5–28.8 MeV are 6.0(7.6±0.2)%, 60.0(50.0±1.4)%, 16.0(34.7±1.7)% and 81.6(92±2)%, respectively. (The quantities in parentheses are the results of ref.[128] with

statistical errors only.) The final result of the analysis in ref. [128] of the IS E0 EWSR for E_x between 8 and 29 MeV is (92±15)%, which agrees with our value of 81.6 percent. There is, however, noticeable disagreement in the low-lying and high-lying regions of the excitation spectrum.

In ref. [128] the authors find (33±4)% of the IS E0 EWSR and (57±6)% of the IS E2 EWSR at a peak energy of 17.5±0.4 MeV. Our results for the IS E0 and E2 resonance mean energies, obtained (as m_1/m_0) from averaging over the observed 8.0–29.0 MeV interval, are 17.2 MeV and 17.1 MeV, respectively. It is impossible to compare the experimental IS E0 EWSR depletion value with our value of 81.6 percent, one of the reasons being that the experimental value of (33±4)% does not contain the continuum [128]. For the IS E2 resonance, however, the difference between our value of 88.0 percent, obtained for the 8.0–29.0 MeV interval, and the 57 percent in ref. [128] is smaller than in the IS E0 case, which may indicate that there is less E2 strength in the experimental continuum.

A clearer comparison with experiment is shown in Fig. 4.12 for the double-differential cross section for ^{40}Ca. Our full calculations reproduce the experimental gross structure reasonably well but, again, we see differences at low and high energies. The general difference between the values of the theoretical and experimental cross sections may be hinting that multipoles other than the IS E0 contribute to that cross section.

In order to understand better the role of complex configurations, we also show in Fig. 4.12 (α, α') cross sections obtained with the microscopic transition density calculated within the continuum RPA only; that is, without inclusion of our complex 1p1h⊗phonon configurations. It can easily be seen that there is a big difference between the two theoretical curves. We conclude that the gross structure of the IS E0 resonance in ^{40}Ca is caused by the complex 1p1h⊗phonon configurations. A similar conclusion for the fine structure of the IS E2 resonance in ^{208}Pb was reached in our earlier calculations [86] and in section 3.2.2.

As seen in section 4.3.1 and Figs. 4.8, 4.9 and 4.10, the calculations for ^{58}Ni explained reasonably well not only the IS E0 resonance in this nucleus but also the total spectra. We therefore calculated the total spectrum of the ^{40}Ca(α, α') reaction observed in ref. [128]. It has a detailed structure but, unfortunately, is given there only as counts. We obtained the following results, shown in Fig. 4.13:

1. The ETFFS describes reasonably well the gross structure of total spec-

tra, except for the region below 9 MeV, which may be connected with the excitations of the α+^{36}Ar system.

2. The difference between the two horizontal axes in Fig. 4.13 gives an instrumental background of the experiment under discussion. This difference is about 7 mb/sr-MeV, so that the integrated cross section is about 7×25 = 175 mb/sr. (These numbers are, of course, very approximate because they were obtained by imposing the experimental curve on the theoretical one). Integration of the theoretical curve gives 422 mb/sr and of the experimental one 676 mb/sr. Thus we should compare (676-175) = 501 mb/sr (experiment) and 422 mb/sr (theory).

3. As can be seen in Fig. 4.13, the experimental background shown by the dot-dashed line contains a noticeable contribution from giant resonances, including that from the IS E0.

Thus, in the calculations presented in Fig. 4.12 and 4.13, a reasonably good description of the gross structure, both in the 0° cross section for the IS E0 strength and in the total spectra in ^{40}Ca, has been obtained. One can, however, see some disagreement with the experiments around 19 MeV. Fortunately, the results of the new and improved experiments [129] have been published recently, and these should be analyzed within the ETFFS.

Description of the improved E0 cross section data in $^{40}\mathrm{Ca}(\alpha,\alpha')$ *at* E_α *= 240 MeV.*

According to the authors of ref. [129], the new experimental results were obtained using an analysis technique that unambiguously identifies multipole strength, whereas the old spectrum subtraction technique [128] is very sensitive, particularly to experimental background. In ref. [128] a definite assignment could be made only for (33±4)% of the E0 strength in the peak. What is also important is that the new method allows considerable extension of the observed energy interval to obtain thereby some additional E0 strength in the high energy tail region, so that a new experimental background must be taken. In other words, the authors have extracted an additional E0 strength from the old background, as was assumed in our work on ^{58}Ni beginning in 1996 [131, 132]. In the new ^{40}Ca measurements [129] (97±11)% of the EWSR was observed in the 10–55 MeV energy interval, whereas the original interval was 4–27 MeV. This result agrees with our calculations: 81.6 percent in the (7.5-28.8) Mev interval [87] and 106.7 percent in the (5-45) MeV interval [85].

A possible reason for the above-mentioned disagreement with the experiment around 19 MeV could be that the number of low-lying phonons used—the 3^- and 5^-)—is insufficient. We have therefore added a third one. Because this additional 2^+_1-phonon at $E = 3.90$ MeV has another (probably 2p2h) nature as compared with those used, we had to make further approximations for this specific phonon. We have therefore calculated, first of all, the distribution of the EWSR for the IS E0 resonance with three phonons and compared it with experiment in Fig. 4.14. In Fig. 4.15 we compare the new [129] and old [128] experimental results for the E0 cross section. One can see a noticeable difference between the experimental results of 1997 and 2001. In Fig. 4.15 we show also our previous calculations within the ETFFS [87] with two phonons, which were used earlier in the calculations for ^{40}Ca.

With the inclusion of the third phonon, very good agreement with the new experimental results was obtained, both for the EWSR distribution (Fig. 4.14) and for the IS E0 cross section (Fig. 4.16).[3]

We conclude that experiments of this type can shed some light on low-lying excitation spectra and, for a microscopic theory, can demonstrate the necessity of accounting for—and identification of—additional 1p1h⊗phonon configurations, or even of inclusion of more complex configurations than the 1p1h⊗phonon.

5 Conclusion

In this review we presented a new microscopic many-body theory for the structure of closed shell nuclei. The extended theory of finite Fermi systems (ETFFS) is based on the Landau-Migdal theory of finite Fermi systems (TFFS) and includes in a consistent way configurations beyond the 1p1h level. A large part of this review is concerned with the application of this new approach to giant resonances in closed shell nuclei. As in the standard TFFS [2], we formulate the theory within the framework of many-body Green functions. As in the original approach, one makes an ansatz for the propagators in the equation for the response function that takes into account these higher configurations and one renormalizes the resulting equation in the standard way. After a long mathematical procedure one ends up with an equation that includes effective charges and an effective interaction which are

[3]Our estimates of the contribution of the fourth, i.e. 0^+_1, low-lying phonon at 3.35 MeV, have shown that its contribution is small.

parameterized in the same way as in the standard theory. The corresponding parameters are universal for all the nuclei, investigated so far. The ETFFS extends the standard TFFS in the following directions:

1. In addition to the 1p1h configurations, it considers in a consistent way complex configurations of the 1p1h⊗phonon type. The corresponding formulation is a natural extension of the conventional RPA.

2. It includes explicitly the single-particle continuum. For that reason one is able to obtain a microscopically determined envelope of the resonance without using a purely phenomenological smearing parameter. (For numerical convenience we still use a small smearing parameter of a few hundred keV.) This feature of the ETFSS is especially important for the nuclei with the nucleon separation energy near zero.

3. The approach takes into account in a consistent way ground state correlations caused by the conventional RPA and by the more complex configurations. It is demonstrated that the latter ones (GSC_{phon}) are at least as important as the conventional RPA ground state correlations.

Another important feature of the ETFFS is that the final equation explicitly contains both the effective ph interaction and the quasiparticle-phonon interaction. This new interaction, however, does not introduce additional parameters. It is completely determined by the original ph interaction that enters into the conventional RPA equation. From this equation one obtains the structure of the phonons and the corresponding quasiparticle-phonon interaction.

We have reviewed the main results obtained within the extended theory and discussed in detail the following physical results:

1. Quantitative explanation of the widths of GMR. It is shown that the 1p1h⊗phonon configurations give the most important contributions to the widths of the GMR. In addition, a large fraction of the observed gross and fine structure can be directly traced back to specific 1p1h⊗phonon configurations. Detailed results have been presented for ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{58}Ni and ^{208}Pb. These calculations give not only the solution to a long-standing problem, but they are also crucial for the analyses of the experimental data in medium mass nuclei.

2. Necessity of using *microscopic* transition densities: In order to extract the parameters of GMR from the experimental cross sections one needs the corresponding form factors. So far, in nearly all the analyses of hadron and electron scattering experiments, phenomenological transition densities derived from some macroscopic models have been used. In heavy mass nuclei, where the GMR are relatively narrow and in general well separated from each other, such a procedure seems to work. For nuclei with A $<$ 90, where the giant resonances show pronounced structure and are spread out over a large energy interval and where various multipoles overlap, such a procedure is no longer appropriate. First of all, the microscopic transition densities are energy dependent and vary over the ranges of the GMR in such nuclei. Secondly, as the resonances overlap, it seems difficult with the conventional method to extract in an unique way the width and the absolute strength of the individual GMR. Those effects were discussed in some detail for ^{58}Ni and ^{40}Ca. It was shown in particular that overlapping multipole resonances give rise to a very smooth angular distribution that looks like an experimental background. Such a misinterpretation gives rise to "missing strength," as occurred some years ago. The analysis of the corresponding experimental cross sections with microscopic transition densities derived from the ETFFS has clarified that point.

3. It has been shown that the ground state correlations induced by complex 1p1h$\otimes$phonon configurations give rise to a 5-7 percent contribution to the EWSR. This additional strength violates the sum rule. One should note that this important question still lacks a completely satisfactory answer.

In the next stage of the development of this new microscopic nuclear structure theory one has to include pairing. Such a generalization will be crucial for the application of the ETFFS to open shell nuclei. One may expect some new effects for the theory of nuclear pairing if one takes the quasiparticle-phonon interaction into account [146]. Such new effects could probably be observed in experiments with high experimental resolution. The first steps towards extending the ETFFS in this direction have already been taken [147] and the corresponding calculations are in progress.

As mentioned earlier, the ETFFS and the original TFFS start with phenomenological single-particle potentials and parameterize the residual ph interaction; i.e., the theories are not self-consistent. For the applications of

these theories to nuclei far from stability the self-consistency, however, is crucial. Here one starts with, for example, a density functional or some relativistic Lagrangian, which provides a single-particle scheme and, simultaneously, the residual interaction. Actually, the ETFFS is most appropriate for a self-consistent approach because it takes into account explicitly the effects of the phonons on the single-particle energies and single-particle wave functions as well as in the residual interaction. In this respect the self-consistent ETFFS will go far beyond all existing theories for the structure of unstable nuclei. For closed shell nuclei, initial steps in this direction have already been taken [150]. We are convinced that for a quantitative understanding of *unstable* nuclei one has to consider simultaneously non-separable universal forces, complex configurations, the single-particle continuum and—perhaps—the GSC_{phon}, as discussed in section 1.3.2. We are also convinced that the new effects, such as the GSC_{phon} (section 3.2.4) and the second (phonon) mechanism of pairing [146], can manifest themselves in unstable nuclei in a clearer form than in the stable ones. The advent of modern radioactive beam accelerators and large gamma detector arrays will open possibilities to study these exciting new effects.

The general outline for the construction of a theory with complex configurations used in the formulation of the nuclear ETFFS under discussion may be used, of course, for other Fermi systems—with and without superconductivity, such as metallic clusters and quantum dots, where phenomena similar to zero sound have already been observed.

Last but not least, all the formulas of the ETFFS can be straightforwardly generalized to the finite temperature case because the GF formalism used here has a natural generalization in the Matsubara temperature technique.

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References

[1] L.D. Landau, JETP 3 (1957) 920; 5 (1957) 101; 8 (1959) 70

[2] A.B. Migdal, Theory of Finite Fermi Systems and Application to Atomic Nuclei (John Wiley and Sons, New York, 1967)

[3] A.I. Larkin, A.B. Migdal, Zh. Eksperim. Teor. Fiz. 44 (1963) 1703.

[4] M. Baranger, Phys. Rev. 120 (1960) 957.

[5] E.E. Saperstein, S.A. Fayans, V.A. Khodel, Fiz. Elem. Chastits. At. Yadra 9 (1978) 221

[6] V.A. Khodel and E.E. Saperstein, Phys. Rep. 92 (1982) 183

[7] A.B. Migdal, Theory of Finite Fermi System and Properties of Atomic Nuclei (Nauka, Moscow,1983) [in Russian]

[8] D. Zawischa, J. Speth, and D. Pal, Nucl. Phys. A311 (1978) 445

[9] J. Speth, Z. Phys. 239 (1970) 249.

[10] P. Ring and J. Speth, Nucl. Phys. A235 (1974) 315.

[11] J. Meyer and J. Speth, Nucl. Phys. A203 (1973) 17.

[12] F. Osterfeld, Rev. Mod. Phys. 64 (1992) 491.

[13] J. Speth, V. Klemt, J.Wambach, and G.E. Brown, Nucl. Phys. A343 (1980) 382.

[14] I.N. Borzov, E.E. Saperstein, S.V. Tolokonnikov and S.A. Fayans, Fiz. Elem. Chastits. At. Yadra 12 (1981) 848

[15] J.Speth, E. Werner, W. Wild, Phys. Rep. 33 (1977) 127-208

[16] I.N. Borzov, S.P. Kamerdzhiev, Bull. Acad. Sci. USSR, Phys.Ser., 41 (1977) No.1

[17] S. Krewald, K. Nakayama, J. Speth, Phys. Rep. 161 (1988) 103.

[18] S.P. Kamerdzhiev and V.I. Tselyaev, Sov. J. Nucl. Phys. 44 (1986) 214

[19] A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics (Prentice-Hall, N.J., 1963).

[20] S. Shlomo and G. Bertsch, Nucl. Phys. A243 (1975) 507.

[21] E.E. Saperstein, S.A. Fayans, V.A. Khodel, Kurchatov Atomic Energy Institute preprint IAE-2580 (1976).

[22] R.Pitthan and Th. Walcher, Phys. Lett. 37 B (1971) 563.

[23] S. Fukuda and Y. Torizuka, Phys. Rev. Lett. 29 (1972) 1109.

[24] M.B. Lewis and F. Bertrand, Nucl. Phys. A 196 (1972) 337.

[25] J. Speth (Ed.), Electric and Magnetic Giant resonances in Nuclei (World Scientific, Singapore, 1991)

[26] M. Thoennessen (Ed.), Giant Resonances. Proc. of the Gull Lake Nucl. Phys. Conf.on Giant Resonances, Gull Lake, Michigan, Aug.17-21, 1993. Nucl. Phys. A 569 (1994)

[27] J.C. Bacelar, M.N. Harakeh and O. Scholten (Eds.), Giant Resonances. Proc. of the Groningen Conf. on Giant Resonances, Groningen, The Nitherlands, 28 June-1July, 1995. Nucl. Phys. A 599 (1996).

[28] Angela Bracco and Pier Francesco Bortignon (Eds.), Giant Resonances GR98. Proc. of the Sixth Intern. Topical Conf.on Giant Resonances, Varenna, Italy, 11-16 May 1998. Nucl. Phys. A 649 (1999).

[29] M.Fujiwara, H.Ejiri, M.Momachi and T.Wakasa (Eds.), Giant Resonances GR2000. Proc.of the Seventh Intern. Topical Conf.on Giant Resonances ,Osaka, Japan, 12-15 June 2000. Nucl. Phys. A 687 (2001)

[30] M.Harakeh and A. van der Woude, Giant Resonances (Clarendon Press, Oxford, 2001)

[31] S. Drożdż, S. Nishizaki, J. Speth, J. Wambach, Phys. Rev. C49 (1994) 867

[32] D.H. Youngblood, H.L. Clark, and Y.-W- Lui, Phys. Rev. Lett. 76 (1996) 1426.

[33] Y.-W. Lui, H.L. Clark, and D.H. Youngblood, Phys. Rev. C 61 (2000) 067307.

[34] K.van der Borg, M.H.Harakeh and A. van der Woude, Nucl. Phys. A365 (1981) 243.

[35] S. Brandenburg ,W.T.A. Borghols, A.G. Drentje, L.P. Ekström, M.N. Harakeh and A. van der Woude, Nucl. Phys. A466 (1987) 29.

[36] G.O. Bolme, L.S. Cardman, R. Doerfler, L.J. Koester, Jr.B.L. Miller, C.N. Papanicolas, H. Rothhaas, and S.E. Williamson, Phys. Rev. Lett. 61 (1988) 1081.

[37] D.S. Dale, R.M. Laszewski, R. Alarson, Phys. Rev. Lett. 68 (1992) 3507.

[38] F.E. Bertrand and J.R. Beene, in: M.S. Hussein (Ed.), Proc. 1989 Int. Nucl. Phys. Conf. (World Scientific, Singapore, 1990) Vol. 2., p.397.

[39] A. Erell, J. Alster, J. Lichtenstadt et al., Phys. Rev. C 35 (1986) 1822.

[40] M.N. Harakeh, B. Van Heyst, K. van der Borg and A. van der Woude, Nucl. Phys. A327 (1979) 373.

[41] J. Speth and J. Wambach, in Ref. [25], p.2

[42] I. Tanihata (Ed.), Special Issue on Research Opportunties with Acceleratrd Beam of Radioactive Ions. Nucl. Phys. A693 (2001) Nos.1,2.

[43] E. Norman, L. Schröder, G. Wozniak (Eds.), Nuclear Physics in the 21.st Century (International Nuclear Physics Conference, INPC 2001, Berkeley, 30 July - 3 August, 2001), AIP Conf. Proceedngs, v.610, 2002.

[44] J.J. Cowan, F.K. Thielemann, J.W. Truran, Phys. Rep. 208 (1991) 267

[45] K. Langanke, Nucl. Phys. A687 (2001) 303c.

[46] I. Hamamoto, H.Sagawa, and X.Z. Zhang, J. Phys. G: Nucl. Part. Phys. 24 (1998) 1417.

[47] A. Vitturi, J. Phys. G: Nucl. Part. Phys. 24 (1998) 1439.

[48] F. Ghielmetti, G. Colo, E. Vigezzi, P.F. Bortignon and R.A. Broglia Phys. Rev. C54 (1996) R2143.

[49] T.M. Brack, Rev. Mod. Phys. 65 (1993) 677

[50] I.V. Hertel, H. Steger, J.De Vries, B. Weisser, C. Mentzl, B. Kamke, W. Kamke Phys. Rev. Lett. 68 (1992) 784.

[51] A.Richter, Nucl. Phys. A 553 (1993) 417c.

[52] K.N. Knöpfle and G.J. Wagner, in [25], p.234.

[53] R.M.Laszewski and P. Axel, Phys. Rev. 19 (1979) 342.

[54] B.S. Dolbilkin, Proc. of the 5-th Course of Int. School on Intermediate Energy Nucl. Phys., Verona, Italy, 1985, p.208.

[55] M. Igashira, H. Kitazawa, M. Schimiru et al., Nucl. Phys. A457 (1986) 301.

[56] S.N. Belyaev, A.B. Kozin, A.A. Nechkin et al., Phys. At. Nucl. 55 (1992) 157.

[57] M.A. Hofstee, S.Y. van der Werf, A.M. van den Berg et al., Nucl. Phys. A588 (1995) 729.

[58] A. Fässler (Ed.), 4π High Resolution Gamma Ray Spectroscopy and Nuclear Structure, Proc. of the Intern. School of Nucl. Phys., Erice, 16-24 September 1996 (Pergamon, Oxford, 1997).

[59] S. Lunardi, Plenary talk at the Intern. Nucl. Phys. Conf., Berkeley, July 30-August 3, 2001.

[60] A.Covello (Ed.), Proceedings of the 6th Intern. Spring Seminar on Nuclear Physics, Highlighhts of Modern nuclear Structure, Italy, May 18-22, 1998 (World Scientific, Singapore, 1999)

[61] J.Reif, P. von Brentano, J. Eberth et al., Nucl. Phys. A620 (1997) 1.

[62] J. Bryssinck, L. Govor, V.Yu. Ponomarev et al., Phys. Rev. C62 (2000) 014309.

[63] P. Ring and P. Schuck, The Nuclear Many Body Problem (Springer, New York, 1980)

[64] V.G. Soloviev, Theory of Complex Nuclei (Pergamon Press, Oxford, 1976) [Russ. original, Nauka, 1971]

[65] V.G. Soloviev, Theory of Atomic Nuclei: Quasiparticlws and Phonons (Institute of Physics , Bristol and Phyladelphia, USA, 1992)

[66] G.F. Bertsch, P.F. Bortignon and R.A. Broglia , Rev. Mod. Phys. 55 (1983) 287

[67] S.P. Kamerdzhiev, Bull. of Russ. Acad. of Sci., Physics, 61 (1997) 122.

[68] D. Karadjov, V.V. Voronov and F. Catara, Phys. Lett. B 306 (1993) 197.

[69] S.P. Kamerdzhiev, V.N. Tkachev, Sov. J. Nucl. Phys. 36 (1982) 43.

[70] M. Buballa, A. Cattone, R. de Haro et al., Nucl. Phys. A517 (1990) 61.

[71] Nguyen van Giai, Ch. Stoyanov, Phys. Lett. B252 (1990) 9.

[72] S. Adachi, S. Yoshida, Nucl. Phys. A462 (1987) 61

[73] E. Migli, S. Drożdż, J. Speth, and J. Wambach, Z. Phys. 340 (1991) 111.

[74] G. Colo, P.F. Bortignon, Nguyen Van Giai, A.Bracco, R. Broglia, Phys. Lett. B276 (1992) 279

[75] S.P. Kamerdzhiev and G.Ya. Tertychny , JETP Lett. 53 (1991) 412

[76] G. Colo, Nguyen Van Giai, P.F. Bortignon, R.A Broglia, Phys. Rev. C 50 (1994) 1496

[77] V.I. Tselyaev, Sov. Nucl. Phys. 50 (1989) 780

[78] S.Kamerdzhiev, J. Speth, G. Tertychny, and V. Tselyaev, Nucl. Phys. A555 (1993) 90

[79] S.Kamerdzhiev, G. Tertychny, W. Unkelbach, Phys. Lett. B278 (1992) 293.

[80] S.Kamerdzhiev, J. Speth, G. Tertychny, and J. Wambach, Z. Phys. A 336 (1993) 253.

[81] S.Kamerdzhiev, J. Speth, G. Tertychny, in: C.M. Class, and L.Cohen (Eds.) Variations on Nuclear Themes, (World Scientific, Singapore, 1994) p.103

[82] S.Kamerdzhiev, G. Tertychny,and J. Speth , Nucl. Phys. A 569 (1994) 313c.

[83] S.Kamerdzhiev, J. Speth, G. Tertychny, Phys. Rev. Lett. 74 (1995) 3943.

[84] S.Kamerdzhiev and J. Speth, Nucl. Phys. A599 (1996) 373c.

[85] S.Kamerdzhiev, J.Speth, G. Tertychny, Nucl. Phys. A 624 (1997) 328.

[86] S.Kamerdzhiev, J. Lisantti, P. von Neumann-Cosel, A. Richter, G. Tertychny, and J.Wambach, Phys. Rev. C 55 (1997) 2101.

[87] S. Kamerdzhiev, J. Speth, G. Tertychny, Eur. Phys. J. A 7 (2000) 483.

[88] A. Bohr and B. Mottelson, Nuclear Structure, Vol.2 (Benjamin , New York, 1975)

[89] S. Drożdż, J. Nishizaki, J. Speth, and J. Wambach, Phys. Rep. 197 (1990) 1.

[90] S.P. Kamerdzhiev, G.Ya. Tertychny and V.I. Tselyaev, Phys. Part. Nucl. 28 (1997) 134.

[91] S.P. Kamerdzhiev , JETP Lett. 30 (1979) 500.

[92] S.P. Kamerdzhiev, Sov. J. Nucl. Phys. 38 (1983) 188.

[93] S. Kamerdzhiev and V. Tkachev, Phys. Lett. B142 (1984) 225.

[94] S.P. Kamerdzhiev and V.N. Tkachev, Sov. J. Nucl. Phys. 46 (1986) 918.

[95] S.P. Kamerdzhiev and V.N. Tkachev, Z. Phys. A 344 (1989) 19.

[96] S. Adachi and E. Lipparini, Nucl. Phys. A 489 (1988) 445.

[97] A.P. Platonov and E.E. Saperstein , Sov. J. Nucl. Phys. 48 (1987) 231.

[98] J.Wambach and V.K.Mishra and LI CHU-HSIA, Nucl. Phys. A380 (1982) 285.

[99] S.P. Kamerdzhiev, G.Ya. Tertychny and V.I. Tselyaev, Phys. Lett. B 267 (1991) 12.

[100] V.A. Chepurnov, Sov. J. Nucl. Phys. 6 (1967) 696.

[101] A.V. Smirnov, S.V. Tolokonnikov, S.A. Fayans, Sov. J. Nucl. Phys. 48 (1988) 995.

[102] I.N. Borzov, S.V. Tolokonnikov, S.A. Fayans, Sov. J. Nucl. Phys. 40 (1984) 732.

[103] J. Ahrens, H. Borchert, K.H. Czock, H.B. Eppler, H. Gimm, H.Gundrum, M. Krönig, P. Riehn, G. Sita Ram, A. Ziegler, and B. Ziegler, Nucl. Phys. A251 (1975) 479.

[104] G.J. O'Keefe, M.N. Thompson, Y.I. Assafiri, R.E. Pywell, and K. Shoda, Nucl.Phys. A469 (1987) 239.

[105] A. Veyssiere, H. Bell, R. Bergere, P. Carlos and A. Lepretre, Nucl. Phys. A159 (1970) 561.

[106] A. van der Woude, in Ref. [25] p. 101.

[107] A. van der Woude, Nucl. Phys. A 599 (1996) 393c.

[108] S. Brandenburg, R. De Leo, A.G. Drentje, M.N. Harakeh, H. Sakai, and A.van der Woude, Phys. Lett. 130B (1983) 9.

[109] F. Zwartz, A.G. Drentje, M.N. Harakeh, and A. van der Woude, Nucl. Phys. A 439 (1985) 117.

[110] J. Lisantti, D.J. Horen, F.E. Bertrand, R.L. Auble, B.L. Burke, E.E. Gross, R.O. Sayer, D.K. McDaniels, K.W. Jones, J.B. McClelland, S.J. Seestrom-Morris, and L.W. Swenson, Phys. Rev. C 40 (1989) 211.

[111] H. Diesener, U. Helm, G. Herbert, V. Huck, P. von Neumann-Cosel, C. Rangacharyulu, A. Richter, G. Schrieder, A. Stascheck. A Stiller, J. Ryckebusch, and J. Carter, Phys. Rev. Lett. 72 (1994) 1994.

[112] H. Diesener, U. Helm, P. von Neumann-Cosel, A Richter, G. Schrieder, and S. Strauch, Phys. Lett. B 352 (1995) 201.

[113] P.F. Bortignon and R.A. Broglia, Nucl. Phys. A 371 (1981) 405.

[114] H. Horiuchi, K. Ikeda, K. Sato, Y. Suzuki, and I. Tanihata, eds., Nucl. Phys. A 588 (1995) No.1;

[115] R. Wyss (Ed.), Physica Scripta 56 (1995).

[116] S. Adachi and E.L. Lipparini, Nucl. Phys. A489 (1988) 445.

[117] E.K. Warburton and J. Weneser, in: D.H. Wilkinson (Ed.),Isospin in nuclear physics (North-Holland, Amsterdam, 1969) p. 173.

[118] Th. Khim, K.T. Knöpfle, H. Riedesel, P. Voruganti, H.J. Emrich, G. Fricke, R. Neuhausen, and R.K.M. Schneider, Phys. Rev. Lett. 56 (1986) 2789.

[119] D.H. Youngblood,H.L. Clark, and Y.-W. Lui, Phys. Rev.Lett. 82 (1999) 691.

[120] S.Raman, L.W. Fagg and R.S. Hicks, in Ref. [25], p.356.

[121] P. Ring, J. Speth, Phys. Lett. B44 (1973)477.

[122] V.N. Tkachev, I.N. Borzov, S.P. Kamerdzhiev, Sov. J. Nucl. Phys. 24 (1976) 373.

[123] I.N. Borzov, V.N. Tkachev, Izv. Akad. Nauk SSSR, ser.fiz. 41 (1977) 1263.

[124] R.M. Laszewski, R. Alarcon, D.S. Dale and S.D. Hoblit, Phys. Rev. Lett. 61 (1988) 1710.

[125] D.M. Pringle, E.F. Gaman, S.H. Chew et al., Phys. Lett. B 115 (1982) 291.

[126] G. K*ü*chler, A. Richter, E. Spamer, W. Steffen, W. Kn*ü*pfer, Nucl. Phys. A406 (1983) 473.

[127] G. Kilgus, G. K*ü*hner, S. M*ü*ller, A. Richter and W. Kn*ü*pfer, Z. Phys. A-Atomic nuclei 326 (1987) 41.

[128] D.H. Youngblood, Y.-W. Lui, and H.L. Clark, Phys. Rev. C **55**, 2811 (1997).

[129] D.H. Youngblood, Y.-W. Lui, and H.L. Clark, Phys. Rev. C 63 (2001) 067301.

[130] G.R. Satchler and Dao T. Khoa, Phys. Rev. C 55 (1997) 285.

[131] S. Kamerdzhiev, J. Speth, and G. Tertychny, in: S.N. Ershov, R.V. Jolos and V.V. Voronov (Eds.), Proceedings of the Intern. Conf.on Nuclear Structure and Related Topics, Dubna, September 1997, p. 347;

[132] S. Kamerdzhiev, J. Speth, G. Tertychny, Acta Phys. Pol. B 29 (1998) 2231.

[133] A.van der Woude, Nucl. Phys. A649 (1999) 97c.

[134] G.R. Satchler and W.G. Love, Phys. Rep. 55 (1979) 183.

[135] H.L. Clark, Y.-W. Lui, and D.H. Youngblood, Nucl. Phys. A589 (1995) 416.

[136] H.L. Clark, Y.-W.Lui, D.H. Youngblood, K. Bachtr, U.Garg, M.H. Harakeh, and N. Kalantar-Nayestanaki, Nucl.Phys. A649 (1999) 57c.

[137] S. Kamerdzhiev, J. Speth, G. Tertychny, and J. Wambach, Nucl. Phys. A577 (1994) 641.

[138] K.Schweda, J.Carter, A.A. Cowley, H. Diesener, R.W. Fearick, S.V. Foertsch, J.J. Lawrie, P.von Neumann-Cosel, J.V. Pilcher, A. Richter, F.D. Smit, G.E. Steyn, S. Strauch, Phys. Lett. B 506 (2001) 247.

[139] S.P. Kamerdzhiev and V.I. Tselyaev, Izvestia Akademii Nauk SSSR. Ser.Fiz. 55 (1991) 49.

[140] A.V. Avdeenkov, S.P.Kamerdzhiev, Bull.of Russ. Acad. of Sci., Physics, 60 (1996) 1716.

[141] J.P. Adams, B. Castel, and H. Sagawa, Phys. Rev. C 53 (1996) 1016.

[142] J. Kopecky and M.Uhl, Phys. Rev. C 41 (1990) 1941.

[143] Z.W. Bell, L.S. Cardmann, and P. Axel, Phys. Rev. C 25 (1982) 791.

[144] U. Kneissl, H.H. Pitz, and A. Zilges, Progr. Part. Nucl. Phys. 37 (1996) 349.

[145] T. Aumann, A. Leistenschneider, U. Datta Pramanik et al., Nucl. Phys. A687 (2001) 103c.

[146] A.V. Avdeenkov and S.P. Kamerdzhiev, JETP Lett. 69 (1999) 715.

[147] S. Kamerdzhiev, E. Litvinova, and D. Zawischa, Eur. Phys. J. A 12 (2001) 285.

[148] T. Hartmann, J. Enders, P. Mohr, K. Vogt, S. Volz, and A. Zilges, Phys. Rev. C 65, 0343301.

[149] S. Ottini-Hustache, N. Alomanos, F. Auger et al., Phys. Rev. C 59 (1999) 3429.

[150] V.I. Tselyaev, Phys. At. Nucl. 61 (1998) 739.

[151] A. Hakansson, J. Blomgern, A. Lokar et al., Nucl. Phys. A512 (1989) 399.

[152] T. Murakami, I. Halpern, D.W. Storm et al., Phys. Rev. 35 (1987) 479.

[153] W. Steffen, H.-D. Gräf, A. Richter et al., Nucl. Phys. A404 (1983) 413.

[154] K. Wienhard, K. Ackermann, K. Bangert et al., Phys. Rev. Lett. 49 (1982) 18.

[155] S. Müller, G. Küchler, A. Richter et al., Phys. Rev. Lett. 54 (1985) 293.

[156] R. Köhler, J.A. Wartena, H. Weigmann et al., Phys. Rev. C35 (1987) 1646.

[157] G.R. Satchler, Direct Nuclear Reactions (Clarendon Press, Oxford, 1983).

[158] D.H. Youngblood, H.L. Clark and Y.-W. Lui, Nucl. Phys. A649 (1999) 49c.

Table 1.1. Integral characteristics of the E2 IS and IV resonances in ^{40}Ca,^{208}Pb calculated within the Continuum TFFS

Nucleus, resonance	Interval, MeV	$\frac{EWSR}{EWSR_{th}}$,%	$E_{1,-1}$, MeV	$E_{1,0}$, MeV	$E_{2,0}$, MeV	E_{exp}, MeV
^{40}Ca, IS	5-45	100	20.7	21.6	22.5	$\sim$ 18 [34]
^{208}Pb, IS	0-45	95	6.1	8.1	10.2	10.9 $\pm$ 0.3 [35] 10.4 [36]
^{208}Pb, IV	0-45	87	16.6	18.2	19.3	20.2 $\pm$ 0.5 [37] 22.6 $\pm$ 0.4 [38]

Figure captions for section 1

Fig. 1.1 Graphical representation of the Bethe-Salpeter equation for the response function in the ph channel.

Fig. 1.2 The hadron strength function of the isoscalar E2 resonance in ^{40}Ca calculated within the continuum TFFS (solid line). The dashed line gives the same quantity obtained without the ph interaction ("free response").

Fig. 1.3 The same as in Fig. 1.2 but for ^{208}Pb.

Fig. 1.4 The same as in Fig. 1.2 but for the isovector E2 resonance in ^{208}Pb.

Figure captions for section 2

Fig. 2.1 The energy–dependent mass operator Σ^e and ph amplitude U^e in the g^2 approximation.

Fig. 2.2 Graphs corresponding to the propagator of the model [91, 95] including RPA (a) and 1p1h$\otimes$phonon configurations (b,c,d). The wavy line and the circle denote the phonon Green function and the amplitude g, and the thin lines denote the Green function $\tilde{G}$.

Fig. 2.3 Typical graphs of fourth order in the quasiparticle-phonon interaction amplitude g entering into the diagrammatic expansion in the time representation that correspond to the MCDD propagator used in the ETFFS. The notation is the same as in Fig. 2.2, but here the direction of the arrow on a fermion line denotes the particle or hole GF $\tilde{G}$. The dotted line denotes the time cut at fixed time. Only graphs d, e and f are explicitly included in the ETFFS.

Fig. 2.4 Graphs of order g^4 with self-energy insertions in the time representation that are included in the ETFFS and give a part of the GSG_{phon}.

Fig. 2.5 Typical "backward-going" graphs of order g^2 due to the GSC_{phon}: graphs a, b, c, and d are similar to the RPA GSC case, whereas graphs e, f, g and h are the GSC_{phon} of new type that correspond to direct excitation of a complex configuration by an external field.

Figure captions for section 3

Fig. 3.1 The E1 photoabsorption cross section for ^{40}Ca obtained within CTFFS (dash-dotted), in the 1p1h+1p1h⊗phonon (dashed) and 1p1h+1p1h⊗phonon + continuum (solid) approximation. The experimental data are taken from ref. [103]. From [78].

Fig. 3.2 Same as Fig. 3.1 but for ^{48}Ca. The experimental data are taken from ref. [104]. From [78].

Fig. 3.3 Same as Fig. 3.1, but for ^{208}Pb. The experimental data are taken from ref. [105]. From [78].

Fig. 3.4. The full E0 and E2 electromagnetic strength functions calculated in the 5.0–45.0 MeV energy interval. The final results are shown by the solid curve. The RPA + 1p1h⊗phonon (without GSC$_{phon}$) and the continuum RPA results are shown by the dashed and dotted curves, respectively. ¿From [85].

Fig. 3.5. The full isoscalar and isovector E0 and E2 strength functions calculated with taking into account the RPA and 1p1h⊗phonon configurations, the single-particle continuum and GSC$_{phon}$. From [85].

Fig. 3.6 The E0 isoscalar (full line) and isovector (dashed line) hadronic strength functions in ^{208}Pb calculated in the ETFFS (smearing parameter $\Delta = 250$ keV).

Fig. 3.7 The E2 hadronic strength function in ^{208}Pb calculated within the ETFFS with smearing parameter $\Delta = 250$ keV (upper panel) and 800 keV (lower panel)

Fig. 3.8 The E1 photoabsorption cross section for ^{208}Pb calculated in the RPA + 1p1h⊗phonon + continuum approximation with the smearing parameter $\Delta = 250$ keV (heavy line) and $\Delta = 10$ keV (thin line) compared with experiment. From [78].

Fig. 3.9. Upper: high resolution inelastic proton and electron scattering spectra in the region of the isoscalar E2 resonance in ^{208}Pb [51, 86]. Lower: the isoscalar E2 strength function calculated in the RPA + 1p1h⊗phonon + continuum approximation. See text for details.

Fig. 3.10 M1 in ^{40}Ca calculated in the RPA + 1p1h⊗phonon approximation [93, 95]. The dashed lines represent the experiment(s) [125]. From [93].

Fig. 3.11. Upper: the strength function for the M1 excitation in ^{208}Pb in the CRPA (dotted line) and in the RPA + 1p1h⊗phonon + continuum without (dashed line) and with GSC$_{phon}$ [80] (full line). Lower: experimental data [124].

Fig. 3.12. The M1 strength function for unstable nuclei calculated within the CTFFS (dotted curve) and the ETFFS without (gs-) and with (gs+) GSP$_{phon}$. From [80]

Figure captions for section 4.

Fig. 4.1. The microscopic (dotted line) and phenomenological [157] (solid line) IS E0 transition densities in ^{40}Ca, calculated for two different energy intervals of 2 MeV each. Results for proton and neutron microscopic transition densities are also given (denoted by triangles). The values of the corresponding ratio M_n/M_p of the nuclear transition moments are given in the legend. From [87].

Fig. 4.2 Angular distributions for the IS E0 resonance in the same intervals as in Fig. 1, calculated with phenomenological [157] and microscopic transition densities. For comparison, the results with the phenomenological transition density, obtained with the microscopic value of the ratio M_n/M_p of the nuclear transition moments (dashed line) are also shown. From [87].

Fig. 4.3. Angular distributions for the IS E0 resonance in ^{40}Ca in the 12–30 MeV interval, calculated using two different methods: the division of the interval into 9 bins of 2 MeV each (solid line) and the interpretation of the 12–30 MeV interval as one bin (dashed line), respectively. For comparison, the results obtained with the phenomenological transition density [157] are also given (solid curve with dots). From [87].

Fig. 4.4 (a) The experimental data [111] for the strength function vs. the theoretical one for ^{40}Ca (see text). (b) The theoretical electromagnetic E2 (dashed line) and E0 (dotted line) strength functions. From [83].

Fig. 4.5 The experimental [32] and theoretical α particle scattering differential cross sections for the 3^-_1 (below) and 2^+_1 (above) levels in ^{58}Ni.

Fig. 4.6. Distribution of the IS E0 and E2 strengths in ^{58}Ni. The experimental data are taken from [33]. From [87].

Fig. 4.7. Distribution of the IS E1 and E3 strengths in ^{58}Ni as calculated within our theoretical model. From [87].

Fig. 4.8. Cross sections of ^{58}Ni(α,α') at $E_\alpha = 240$ MeV and $\theta = 1.08°$. The experimental data (histogram) were taken from Ref. [33], where an instrumental background has been subtracted. The solid curve with dots shows the calculated total (summed) cross sections, the dashed line ("Rest(EL)") corresponds to the sum of the IS and IV E1, and IS E3 and E4 multipoles. The shaded area shows an additional IS E0 strength which has been subtracted in the previous experiments as background [32]. This area corresponds to 22 percent of the IS E0 EWSR, see text. From [87].

Fig. 4.9. Cross sections of ^{58}Ni(α,α') at $E_\alpha = 240$ MeV and $\theta = 1.08°$. The experimental data (histogram), including the instrumental background, are taken from ref. [33]. The solid curve with dots gives the calculated total (summed) cross sections. In the lower part of the picture the components of the total cross sections are shown without the background. In particular, the dashed line gives the IS E0 contribution. From [87].

Fig. 4.10. Same as in Fig. 4.9 but for $\theta = 4.08°$. The experimental data were obtained by using the results of ref. [32] (see text). From [87].

Fig. 4.11 Distribution of the IS E0 and E2 strengths in ^{40}Ca (theory). From [87].

Fig. 4.12. The 0° cross section for the IS E0 strength in ^{40}Ca(α, α') at $E_\alpha = 240$ MeV, calculated with (solid line) and without (dotted line) taking into account complex 1p1h⊗phonon configurations. The experimental data (histogram) are taken from ref. [128]. One can see that the gross structure of the IS E0 resonance is caused by complex 1p1h⊗phonon configurations. From [87].

Fig. 4.13 Cross sections of ^{40}Ca(α, α') at $E_\alpha = 240$ MeV and $\theta = 1.1°$. The experimental data dotted line) and the background (dot-dashed line) are taken from ref. [128]. Because in Fig. 2 of ref. [128] there are only counts, the experimental histogram was imposed on the theoretical curve in such a way that the maxima of the two curves coincided. In this way it was possible to estimate roughly the contribution of the instrumental background as the difference between two horizontal axes (see text).

Fig. 4.14 Upper: distribution of the IS E0 EWSR calculated with 2 and 3 low-lying phonons. Lower: comparison of the 3-phonon calculation with experiment [129].

Fig. 4.15 E0 cross sections of ^{40}Ca(α, α') at $E_\alpha = 240$ MeV and $\theta = 1.1°$. The experimental data 2001, 1997 and the theoretical results with 2 phonons are taken from refs. [129, 128] and [87], respectively.

Fig. 4.16 Comparison of the improved experimental data [129] for cross sections and the theoretical calculations with 3 phonons.

Appendix.
1.Generalized propagator in the method of chronological decoupling of diagrams

The equation for the density matrix $\rho_{12}(\omega)$, Eq. (2.19), in the $\tilde{\lambda}$ representation is given by (the tilde on $\tilde{\epsilon_\lambda}$ and $\tilde{n}_\lambda$ has been omitted in the equations that follow)

$$\delta\rho_{12}(\omega) = \delta\rho^0_{12}(\omega) - \sum_{56,34} A_{12,34}\tilde{F}_{34,56}\delta\rho_{56}(\omega)$$
$$\delta\rho^0_{12}(\omega) = -\sum_{34} A_{12,34}(\omega)(\tilde{e}_q V^0)_{43} \tag{A.1}$$

The generalized propagator A has the form

$$A_{12,34}(\omega) = \sum_{56,78}[\delta_{15}\delta_{26} + Q^{+-}_{12,56}(\omega)]A^{--}_{56,78}(\omega)[\delta_{73}\delta_{84} + Q^{-+}_{78,34}(\omega)] + P^{++}_{12,34}(\omega). \tag{A.2}$$

For the ph-ph part $A^{--}_{56,78}$ of the propagator the following equation must be solved

$$A^{--}_{12,34}(\omega) = \tilde{A}_{12,34}(\omega) - \sum_{56,78}\tilde{A}_{12,56}(\omega)\Phi_{56,78}(\omega)A^{--}_{78,34}(\omega) \tag{A.3}$$

where the propagator

$$\tilde{A}_{12,34}(\omega) = \delta_{13}\delta_{24}\frac{n_1 - n_2}{\omega - \epsilon_{12}}$$
$$\epsilon_{12} = \epsilon_1 - \epsilon_2, n_i = 1(\epsilon_i < \epsilon_F), n_i = 0(\epsilon_i > \epsilon_F), \tag{A.4}$$

is the RPA propagator and

$$\Phi_{12,34} = \tilde{\Phi}_{12,34} + \Phi^{comp}_{12,34} \tag{A.5}$$

$$\tilde{\Phi}_{12,34} = (n_2 - n_1)(n_4 - n_3) \sum_q \Big\{ \delta_{13} \sum_6 \frac{(n_6 - n_1)\beta^q_{24,6}}{\omega - \epsilon_{16} - (1-2n_1)\omega_q} +$$

$$\delta_{24} \sum_5 \frac{(n_2 - n_5)\beta^{q*}_{13,5}}{\omega - \epsilon_{57} - (1-2n_1)\omega_q} +$$

$$(-1)^{J+J_q} \left\{ \begin{array}{ccc} J & j_1 & j_2 \\ J_q & j_4 & j_3 \end{array} \right\} \Big[(-1)^{j_3 - j_2} g^q_{13} g^{q*}_{24} \left(\frac{(1-n_1)(1-n_3)}{\omega - \epsilon_{32} - \omega_q} - \frac{n_1 n_3}{\omega + \epsilon_{41} + \omega_q} \right) +$$

$$(-1)^{j_1 - j_4} g^{q*}_{31} g_{42} \left(\frac{(1-n_1)(1-n_3)}{\omega - \epsilon_{14} - \omega_q} - \frac{n_1 n_3}{\omega + \epsilon_{23} + \omega_q} \right) \Big] +$$

$$\delta_{13} \sum_6 \frac{(n_6 - n_2)[\omega - \epsilon_{12} - \epsilon_{64} - (1-2n_1)\omega_q]}{[\epsilon_{62} + (1-2n_1)\omega_q][\epsilon_{64} + (1-2n_1)\omega_q]} \beta^q_{24,6} +$$

$$\delta_{24} \sum_5 \frac{(n_1 - n_5)[\omega - \epsilon_{12} - \epsilon_{35} - (1-2n_1)\omega_q]}{[\epsilon_{15} + (1-2n_1)\omega_q][\epsilon_{35} + (1-2n_1)\omega_q]} \beta^{q*}_{13,5} +$$

$$(-1)^{J+J_q} \left\{ \begin{array}{ccc} J & j_1 & j_2 \\ J_q & j_4 & j_3 \end{array} \right\} \Big[(-1)^{j_3 - j_2} g^q_{13} g^{q*}_{24} \left(\frac{(1-n_1)n_3}{\epsilon_{42} + \omega_q} + \frac{(1-n_3)n_1}{\epsilon_{31} + \omega_q} \right) +$$

$$(-1)^{j_1 - j_4} g^{q*}_{31} g^q_{42} \left(\frac{(1-n_1)n_3}{\epsilon_{31} + \omega_q} + \frac{(1-n_3)n_1}{\epsilon_{24} + \omega_q} \right) \Big] \Big\}, \tag{A.6}$$

$$\Phi^{comp}_{12,34} = (n_2 - n_1)[(1-n_1)(1-n_3) + n_1 n_3][(1-n_2)(1-n_4) + n_2 n_4] \times$$

$$\sum_{q_1, q_2, 56} \beta^{q_1}_{31,5} \beta^{q_2}_{24,6} \times$$

$$\Big\{ \frac{(1-n_1)(1-n_5)(1-n_6) + n_1 n_5 n_6}{[\epsilon_{62} + (1-2n_1)\omega_{q_2}][\epsilon_{64} + (1-2n_1)\omega_{q_2}][\omega - \epsilon_{52} - \epsilon_{64} - (1-2n_1)(\omega_{q_1} + \omega_{q_2})]} +$$

$$\frac{(1-n_2)(1-n_5)(1-n_6) + n_2 n_5 n_6}{[\epsilon_{15} + (1-2n_1)\omega_{q_1}][\epsilon_{35} + (1-2n_1)\omega_{q_1}][\omega - \epsilon_{15} - \epsilon_{36} - (1-2n_1)(\omega_{q_1} + \omega_{q_2})]} -$$

$$\frac{[(1-n_2)(1-n_5)n_6 + n_2 n_5 (1-n_6)]}{[\epsilon_{15} + (1-2n_1)\omega_{q_1}][\epsilon_{35} + (1-2n_1)\omega_{q_1}][\epsilon_{62} + (1-2n_1)\omega_{q_1}]} \times$$

$$\frac{[\omega - \epsilon_{12} - \epsilon_{34} + \epsilon_{56} - (1-2n_1)(\omega_{q_1} + \omega_{q_2})]}{[\epsilon_{64} + (1-2n_1)\omega_{q_2}]} \Big\}. \tag{A.7}$$

The quantities Q^{+-} and P^{++} are given by

$$Q^{+-}_{12,34} = [(1-n_1)(1-n_2)+n_1n_2][(1-n_3)n_4+n_3(1-n_4)]\times$$
$$\sum_q \Big\{\delta_{13}\sum_6 \Big[\Big(\frac{1-n_1}{\epsilon_{24}(\epsilon_{64}+\omega_q)}+\frac{n_1}{\epsilon_{62}+\omega_q}(\frac{1}{\epsilon_{24}}-\frac{1}{\omega+\epsilon_{61}+\omega_q})\Big)\ (1-n_6)\beta^{q(+)}_{24}+$$
$$+\Big(\frac{n_1}{\epsilon_{42}(\epsilon_{46}+\omega_q)}+\frac{1-n_1}{\epsilon_{26}+\omega_q}(\frac{1}{\epsilon_{42}}+\frac{1}{\omega-\epsilon_{16}-\omega_q})\Big)\ n_6\beta^{q(-)}_{24,6}\Big]-$$
$$-\delta_{24}\sum_5\Big[\Big(\frac{n_1}{\epsilon_{13}(\epsilon_{35}+\omega_q)}+\frac{1-n_1}{\epsilon_{15}+\omega_q}(\frac{1}{\epsilon_{13}}-\frac{1}{\omega+\epsilon_{25}+\omega_q})\Big)\ n_5\beta^{q(-)*}_{13,5}+$$
$$\Big(\frac{1-n_1}{\epsilon_{31}(\epsilon_{53}+\omega_q)}+\frac{n_1}{\epsilon_{51}+\omega_q}(\frac{1}{\epsilon_{31}}+\frac{1}{\omega-\epsilon_{52}-\omega_q})\Big)\ (1-n_5)\beta^{q(+)*}_{13,5}\Big]+$$
$$(-1)^{J+J_q}\begin{Bmatrix} J & j_1 & j_2\\ J_q & j_4 & j_3\end{Bmatrix}[(-1)^{j_3-j_2}g^q_{13}g^{q*}_{24}n_1\times$$
$$\Big(\frac{(1-n_3)}{(\epsilon_{31}+\omega_q)(\omega-\epsilon_{32}-\omega_q)}-\frac{n_3}{(\epsilon_{42}+\omega_q)(\omega+\epsilon_{41}+\omega_q)}\Big)+$$
$$(-1)^{j_1-j_4}g^{q*}_{31}g^q_{42}(1-n_1)\ \Big(\frac{(1-n_3)}{(\epsilon_{24}+\omega_q)(\omega-\epsilon_{14}-\omega_q)}-\frac{n_3}{(\epsilon_{13}+\omega_q)(\omega+\epsilon_{23}+\omega_q)}\Big)\Big]\Big\},$$
$$Q^{-+}_{12,34} = Q^{+-*}_{34,12} \tag{A.8}$$

$$P^{++}_{12,34}(\omega) = [(1-n_1)(1-n_2)+n_1n_2]\,[(1-n_3)(1-n_4)+n_3n_4]\times$$
$$\sum_q\Big\{\delta_{13}\sum_6\frac{(n_1-n_6)\beta^q_{24,6}}{[\epsilon_{26}+(1-2n_1)\omega_q]\,[\epsilon_{46}+(1-2n_1)\omega_q]\,[\omega-\epsilon_{16}+(1-2n_1)\omega_q]}-$$
$$\delta_{24}\sum_5\frac{(n_1-n_5)\beta^{q*}_{13,5}}{[\epsilon_{15}+(1-2n_1)\omega_q]\,[\epsilon_{35}+(1-2n_1)\omega_q]\,[\omega+\epsilon_{25}+(1-2n_1)\omega_q]}+$$
$$(-1)^{J+J_q}\begin{Bmatrix} J & j_1 & j_2\\ J_q & j_4 & j_3\end{Bmatrix}\Big[(-1)^{j_3-j_2}g^q_{13}g^{q*}_{24}\times$$
$$\frac{n_1(1-n_3)}{(\epsilon_{31}+\omega_q)(\epsilon_{42}+\omega_q)}\Big(\frac{1}{\omega+\epsilon_{41}+\omega_q}-\frac{1}{\omega-\epsilon_{32}-\omega_q}\Big)+$$
$$(-1)^{j_1-j_4}g^{q*}_{31}g^q_{42}\frac{n_3(1-n_1)}{(\epsilon_{13}+\omega_q)(\epsilon_{24}+\omega_q)}\Big(\frac{1}{\omega+\epsilon_{23}+\omega_q}-\frac{1}{\omega-\epsilon_{14}-\omega_q}\Big)\Big]\Big\}. \tag{A.9}$$

Here the following notations have been used:

$$\beta^{q(+)}_{12,3} = \frac{\delta_{j_1 j_2}\delta_{l_1 l_2}}{2j_1+1} g^{q*}_{13} g^{q}_{23}, \beta^{q(-)}_{12,3} = \frac{\delta_{j_1 j_2}\delta_{l_1 l_2}}{2j_1+1} g^{q}_{31} g^{q*}_{32},$$
$$\beta^{q}_{12,3} = (1-n_3)\beta^{q(+)}_{12,3} + n_3 \beta^{q(-)}_{12,3}. \tag{A.10}$$

Into all these formulae the reduced matrix elements $g^q_{12} \equiv < 1||g^q||2 >$ ($q \equiv JLS$) of the phonon creation amplitude at J = L.

$$g(\mathbf{r}, \boldsymbol{\sigma}) = \sum_{JLSM} g_{JLS}(r) T^M_{JLS}(\mathbf{n}, \boldsymbol{\sigma}) \tag{A.11}$$

enter, where $\mathbf{n} = \mathbf{r}/r$ and

$$T^M_{JLS}(\mathbf{n}, \boldsymbol{\sigma}) = \sum_{\mu,\nu} (L\mu S\nu|JM) Y_{L\mu}(\mathbf{n}) \sigma_{S\nu} \tag{A.12}$$

with $\nu = \pm 1, 0, \sigma_{0\nu} = 1$ and $\sigma_{1\nu} = \sigma_\nu, \sigma_\nu$ being cyclic components of spin matrices. In the spherical single-particle basis they are determined by

$$< m_1 l_1 j_1 n_1 | g_{JLS} T^M_{JLS} | m_2 l_2 j_2 n_2 > = (-1)^{-j_2+J+m_1} \times$$
$$\times \begin{pmatrix} j_1 & J & j_2 \\ -m_1 & M & m_2 \end{pmatrix} < 1||g^{JLS}||2 >, \tag{A.13}$$

where

$$< 1||g^{JLS}||2 > = < l_1 j_1||T_{JLS}||l_2 j_2 > \int R_{l_1 j_1 n_1} g_{JLS}(r) R_{l_2 j_2 n_2} r^2 dr. \tag{A.14}$$

The reduced matrix element of the spherical tensor operator T_{JLS} has the form

$$< l_1 j_1||T_{JLS}||l_2 j_2 > =$$
$$= \tfrac{1}{2}[1 + (-1)^{L+l_1+l_2}](-1)^{S+j_2-\frac{1}{2}} \sqrt{\tfrac{(2J+1)(2L+1)(2j_1+1)(2j_2+1)}{4\pi}} \times$$
$$\Big[\begin{pmatrix} J & L & S \\ 0 & 0 & 0 \end{pmatrix} + (x_1 + (-1)^{J+L+S} x_2) \sqrt{\tfrac{S(S+1)}{J(J+1)}} \begin{pmatrix} J & L & S \\ 1 & 0 & -1 \end{pmatrix}\Big] \begin{pmatrix} j_1 & j_2 & J \\ 1/2 & -1/2 & 0 \end{pmatrix}, \tag{A.15}$$

where $x_k = (l_k - j_k)(2j_k + 1)$.

2.The refinement procedure

Eq. (3.18) for the refined single-particle energies $\tilde{\epsilon}_\lambda$ has the form

$$\tilde{\epsilon}_1 = \epsilon_1 - \sum_{3,q} \frac{\beta^q_{12,3}}{\epsilon_1 - \tilde{\epsilon}_3 - (1-2n_3)\omega_q} \tag{A.16}$$

where index 1 stands for the set of the single-particle quantum numbers and ϵ_1 are the phenonomenological single-particle energies.

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