On threshold of radial detachment in tokamaks

M. Z. Tokar
Institut für Plasmaphysik, Forschungszentrum Jülich GmbH, Association KFA-Euratom, 52425, Jülich, Germany

(Received 28 January 2000; accepted 3 February 2000)

The threshold for radial plasma detachment in limiter tokamaks is theoretically investigated. It is shown by taking into account of realistic boundary conditions at the last closed magnetic surface that detachment can start at a radiation level significantly lower than the input power as it occurs in experiments. The origin of contradictions between previous approaches to the problem is clarified. © *2000 American Institute of Physics.* [S1070-664X(00)00306-2]

I. INTRODUCTION

Detachment of the plasma column from the last closed magnetic surface (LCMS) in limiter tokamaks is probably one of the first manifestations of the exceptional role of impurities in fusion devices.[1] A ramp of the mean plasma density above a certain critical value $\overline{n}_{cr}$ leads to a sudden increase of the radiation level to 100% of the input power and contraction of the plasma column. Depending on the safety factor at the LCMS, this process results in a plasma disruption or in the formation of a stable ''detached plasma'' with a significantly reduced effective radius.[2–4]

Two different scenarios of detachment have been identified in the experiments on numerous tokamaks. It was found that under the conditions of ohmic discharges in the devices of moderate dimensions, like the Divertor Injection Tokamak Experiment (DITE)[2] or Tokamak Experiment for Technology Oriented Research (TEXTOR),[3] a detachment starts simultaneously on the whole LCMS so that the poloidal symmetry of the plasma column is preserved. Conversely, in large tokamaks, e.g., in the Tokamak Fusion Test Reactor (TFTR)[4] or in the small devices with a high density of input power,[5] a region with a cold dense plasma, named Multi-Facetted Radiation From the Edge (MARFE) because of its brightness, arises at first at the inner tokamak board. Later, the MARFE smears out in a detached radiative shell. In the present paper, we confine ourselves to consideration only of the radial type of the detachment. Although this phenomenon is of interest by itself, an interpretation of the mechanisms at work is also of importance for understanding of the MARFE.

For a long time, the experimental fact that detachment can start at a radiation level significantly lower than 100% of the input power[1,3] was ignored. In models $\overline{n}_{cr}$ was identified with the density limit and estimated under the assumption that the launched power is completely radiated from the plasma (see, e.g., Refs. 6, 7). An important step has been done by Ohyabu[8] by demonstrating that the radiative layer at the plasma edge becomes unstable if $\overline{n}$ exceeds some critical level. It was also shown that the transport in the scrape-off layer plays a crucial role for the existence of this threshold. The importance of convective heat losses in the radiative layer in estimating of $\overline{n}_{cr}$ has been stressed in Ref. 9.

The results obtained in Ref. 8 on the basis of an approximate variational technique have been reexamined by Drake[10] by the exact solution of the eigenvalue problem for small perturbations of the temperature profile. Conversely to Ref. 8, it has been demonstrated that the edge radiative layer is stable against radial detachment for any γ_{rad} equal to the ratio of the radiation losses to the input power. Simultaneously, it has been stated that poloidal inhomogeneities, which probably lead to the formation of MARFE, become unstable when $\gamma_{rad}>\gamma_{rad}^{cr}=0.75$. These results contradict, however, the observations of radial poloidally symmetric detachments in experiments and numerical simulations of this process.[11,12]

The above said demonstrates the necessity for further reexamining of the detachment mechanism. This is indispensable not only from the academic point of view, in order to resolve the contradictions outlined, but also can bring a new insight into the physics of phenomena caused by impurity radiation. A deeper understanding of detachment in limiter devices can be useful for a further progress in understanding of the nature of this phenomenon in divertor machines,[13] which is still a matter of discussion. Moreover, this is an unavoidable step in the analysis of factors limiting the plasma performance in the radiatively improved modes emerging as an interesting scenario for reactor operation.[14–16]

In the present paper, it will be shown that the model developed in Ref. 10 also describes the radial detachment at a γ_{rad} significantly less that 1 if proper boundary conditions are imposed at the LCMS. This partly rehabilitates the conclusions drawn by Ohyabu[8] on the basis of a crude variational approach, which failed to predict the stability for conditions considered by Drake.[10] The structure of the paper is as follows. The model used for the heat transport in the radiative layer at the plasma edge and the results concerning stationary states are presented in Sec. II. Stability of these states is considered in Sec. III by using both the perturbation analysis and an ''energetic'' approach. Here, we also take into account the influence of convective heat transport and density profile on the detachment conditions. The results obtained are discussed in Sec. IV and the details of mathematical treatment are given in the Appendix.

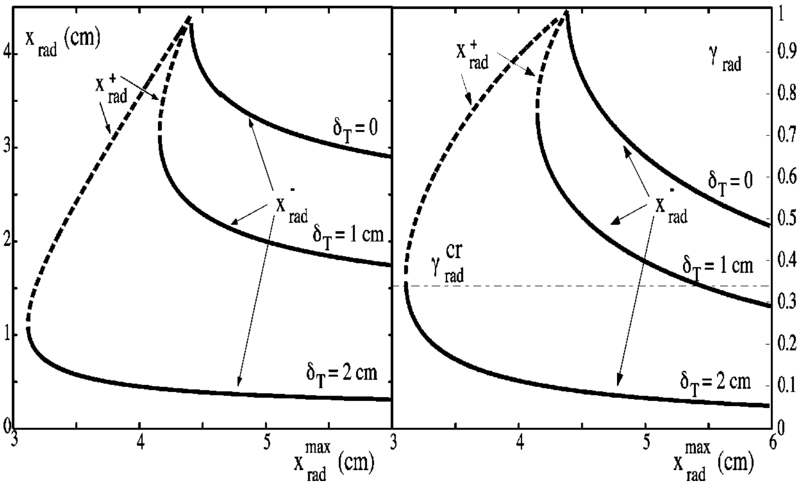

FIG. 1. Width of the radiative layer and radiation level versus $x_{\rm rad}^{\rm max}=\phi_{\rm core}/Q_0$.

II. HEAT BALANCE IN THE RADIATIVE LAYER AT THE PLASMA EDGE

It is well founded that the line radiation from light impurities like carbon and oxygen is generated mostly in the plasma periphery in a layer much narrower that the minor radius. This is justified already for devices of very modest dimensions and allows one to perform an analysis of the edge heat balance in a planer geometry and neglecting heat sources. We start our consideration with a very crude model[10,17] which neglects convective energy losses, assumes the same temperature T for electrons and ions and adopts the plasma heat conductivity $\kappa_\perp$, electron and impurity densities, n and n_I, respectively, independent of the distance x from the LCMS toward the plasma axis. In such a case, the stationary heat balance in the radiative layer is governed by the equation:[10]

$$-\kappa_\perp \frac{d^2T}{dx^2}=-nn_I L_I . \tag{1}$$

The ''cooling rate'' of the impurity L_I, is assumed in this section in the simplest ''box'' approximation:[10,17] $L_I \equiv L_I^{\rm max}$ for $T \leqslant T_{\rm max}$ and $L_I \equiv 0$ for higher temperatures. Hence, Q_0 will be used instead of $n\, n_I L_I^{\rm max}$.

One of the boundary conditions needed to solve Eq. (1) is posed at the interface between the radiative layer and the plasma core. The position of this interface, $x_{\rm rad}$, is not known *a priori* but has to be determined from the requirement $T(x_{\rm rad})=T_{\rm max}$. Here, we prescribe the density of the heat flux from the plasma interim:

$$-\kappa_\perp \frac{dT}{dx}=\phi_{\rm core} . \tag{2}$$

This implies that the total heat source does not vary with $x_{\rm rad}$. This is a good approximation for ''attached plasmas'' because in this case, $x_{\rm rad}$ is much less than the plasma minor radius $r_{\rm LCMS}$ and $\phi_{\rm core}$ changes insignificantly with $x_{\rm rad}$. After the detachment in ohmic discharges $\phi_{\rm core}$ increases noticeably, which is of principle importance for the stability of ''detached plasmas.''[12] Therefore, the results of our analysis are relevant only for conditions before the detachment.

The main distinction of our consideration from those performed by other authors is in the boundary condition for the plasma temperature at the LCMS $(x=0)$, $T_{\rm LCMS}$. In previous models,[10,17–20] this was kept constant, normally equal to zero. But in reality, $T_{\rm LCMS}$ cannot be fixed arbitrarily, since it is controlled both by the heat flux from the plasma core and by the transport in the scrape-off layer (SOL) out of the LCMS. This problem has been discussed in Ref. 8 with respect to perturbations of stationary temperature profiles. But as it will be shown below, also considering the stationary heat balance, this is of importance to impose an adequate boundary condition at the LCMS. We adopt this in the following form:

$$\frac{dT}{dx}=\frac{T}{\delta_T}. \tag{3}$$

The e-folding length δ_T is determined by the competition between the heat flows in the SOL along and perpendicular to the magnetic field (see Appendix C and Ref. 21). In experiments, it is normally in the range of 1–5 cm. The condition $T_{\rm LCMS}=0$ used in previous considerations[10,17,18] corresponds to $\delta_T=0$. This would mean an extremely effective transport in the SOL, completely controlling $T_{\rm LCMS}$ and counteracting a development of perturbations. With a nonzero δ_T, the stability of the radiative layer can be reduced significantly.[8]

Under the assumptions made, Eq. (1) is easy to integrate analytically. For the width of the radiative layer, one gets a quadratic algebraic equation with the roots:

$$x_{\rm rad}^{\pm}=x_{\rm rad}^{\rm max}-\delta_T \pm \sqrt{(x_{\rm rad}^{\rm max})^2+\delta_T^2-2\,\delta_0 x_{\rm rad}^{\rm max}}, \tag{4}$$

where $\delta_0=(\kappa_\perp T_{\rm max})/\phi_{\rm core}$ and $x_{\rm rad}^{\rm max}=\phi_{\rm core}/Q_0$ is maximally conceivable width for which the total power is radiated by impurities. For given δ_0 and $x_{\rm rad}^{\rm max}$, the temperature at the LCMS has also two possible values:

$$T_{\rm LCMS}^{\pm}=\frac{\delta_T}{\kappa_\perp}(\phi_{\rm core}-Q_0 x_{\rm rad}^{\pm}). \tag{5}$$

If one assumes $\delta_T=0$, only the root $x_{\rm rad}^-$ satisfies the obvious constraints $x_{\rm rad} \leqslant x_{\rm rad}^{\rm max}$ and $T_{\rm LCMS} \geqslant 0$. In this case, an increase of Q_0, e.g., with a ramp of the plasma density, leads to a reduction of $x_{\rm rad}^{\rm max}$ and results in a monotonous increase of $x_{\rm rad}^-$. This is shown in Fig. 1(a) for typical conditions of ohmic discharges in TEXTOR: $\phi_{\rm core}\approx 1\,{\rm W/cm^2}$, $\kappa_\perp=2.5\times 10^{17}\,{\rm cm^{-1}\,s^{-1}}$ and carbon as the main impurity ($T_{\rm max}\approx 50\,{\rm eV}$). When the density achieves the maximum level corresponding to $x_{\rm rad}^{\rm max}=2\delta_0$, the expression in the square root in Eq. (4) reduces to zero, $x_{\rm rad}=x_{\rm rad}^{\rm max}$ and the radiation level $\gamma_{\rm rad}\equiv x_{\rm rad}/x_{\rm rad}^{\rm max}$ becomes equal to 1. This variation of $\gamma_{\rm rad}$ with $x_{\rm rad}^{\rm max}$ is shown in Fig. 1(b). The stability analysis performed in Ref. 10 has shown that for $T_{\rm LCMS}\equiv 0$, all states with $\gamma_{\rm rad}\leqslant 1$ are stable with respect to small perturbations homogeneous on the magnetic surfaces, i.e., potentially leading to a radial detachment. Therefore, the conclusion was drawn there that the instability of the radiative layer demonstrated in Ref. 8 by means of an approximate variational technique actually does not exist.

This conclusion fails, however, if one considers a realistic situation with $\delta_T>0$ also demonstrated in Fig. 1. Now, the root $x_{\rm rad}^-$ exists when

$$\delta_0+\sqrt{\delta_0^2-\delta_T^2}\equiv x_{\rm rad}^*\leqslant x_{\rm rad}^{\rm max}$$

and describes stationary states with $0\leqslant\gamma_{\rm rad}\leqslant\gamma_{\rm rad}^{\rm cr}$, where

$$\gamma_{\rm rad}^{\rm cr}=1-\frac{\delta_T}{x_{\rm rad}^*}. \tag{6}$$

For $x_{\rm rad}^*\leqslant x_{\rm rad}^{\rm max}\leqslant 2\delta_0$, the roots $x_{\rm rad}^+$ also do not exceed $x_{\rm rad}^{\rm max}$ and have physical meaning. This branch corresponds to the states with $\gamma_{\rm rad}^{\rm cr}<\gamma_{\rm rad}<1$. From a general point of view, one of two stationary states is unstable and it will be shown in the following section that this is the case for $x_{\rm rad}=x_{\rm rad}^+$

III. STABILITY OF STATIONARY STATES

To analyze the stability of stationary states, two approaches will be exploited. The first one is a rigorous analysis of a response of the system to a small perturbation. This approach when applied, e.g., in Refs. 10, 18, 19, allows one to determine the growth rate of perturbations, but leads to manageable analytical expressions only for a restricted class of equilibria, like that considered in the previous section. For more realistic cases, e.g., when a spatial variation of the plasma density or convective heat losses are taken into account or a more realistic temperature dependence of the impurity cooling rate is assumed, arising equations can be solved only numerically. Therefore, we additionally consider an alternative ''energetic'' approach which operates with the properties of steady states. This method is less rigorous, but allows one to find the stability domains for more complex equilibria.

A. Perturbation analysis

As in Ref. 10, we consider small perturbations of the stationary temperature profile, $\delta T(t,x)=\tilde{T}(x)\exp(\gamma t)$. For $x\leqslant x_{\rm rad}$, the function $\tilde{T}(x)$ is governed by the linearized Eq. (1) with a nonstationary term:

$$3n\gamma\overline{T}-\kappa_\perp\frac{d^2\tilde{T}}{dx^2}=Q_0\,\delta(T_{\rm max}-T)\tilde{T}. \tag{7}$$

Here, the Dirac's δ function results from the linearization of the radiation loss in the ''box'' approximation for the impurity cooling rate. Beyond the radiation layer ($r<r_{\rm rad}\equiv r_{\rm LCMS}-x_{\rm rad}$), we neglect perturbations in the heat source and take into account the cylindrical geometry of the plasma column:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\tilde{T}}{dr}\right)=k^2\tilde{T}. \tag{8}$$

By matching of both solutions presented in Appendix A at the interface of these regions, one gets a dispersion equation for $k=\sqrt{3n\gamma/\kappa_\perp}$:

$$kx_{\rm rad}\left[\frac{k\delta_T+\coth(kx_{\rm rad})}{k\delta_T\coth(kx_{\rm rad})+1}+\frac{I_1(kr_{\rm rad})}{I_0(kr_{\rm rad})}\right]=\frac{Q_0x_{\rm rad}}{\phi_{\rm core}}\equiv\gamma_{\rm rad}. \tag{9}$$

With $\delta_T=0$ and asymptotics for the Bessel functions of the imaginary argument, $I_{0,1}$, for $kr_{\rm rad}\gg 1$, one gets from Eq. (9), the dispersion equation obtained by Drake:[10]

$$\hat{k}+\hat{k}\coth\hat{k}=\gamma_{\rm rad}$$

with $\hat{k}=kx_{\rm rad}$ Since the left-hand side of this relation is not less than 1 for any positive $\hat{k}$, the radiative layer is stable against radial detachment for any $\gamma_{\rm rad}\leqslant 1$.[10]

This is not the case for $\delta_T\neq 0$. To show this, we expand in series the left-hand side of Eq. (9) near the instability threshold when $k\to 0$. Here, the approximation $kr_{\rm rad}\gg 1$ fails and the asymptotics for the Bessel functions for small arguments are more relevant. As a result, one gets by taking into account $\gamma_{\rm rad}\equiv x_{\rm rad}/x_{\rm rad}^{\rm max}$:

$$\gamma\approx\frac{2\kappa_\perp}{3nx_{\rm rad}^{\rm max}}\frac{\delta_T+x_{\rm rad}-x_{\rm rad}^{\rm max}}{(\delta_T+x_{\rm rad})(r_{\rm rad}+2\delta_T)}$$
$$\sim\pm\sqrt{(x_{\rm rad}^{\rm max})^2+\delta_T^2-2\delta_0x_{\rm rad}^{\rm max}}, \tag{10}$$

where the signs $\pm$ correspond to the roots $x_{\rm rad}^{\pm}$, respectively [see Eq. (4)].

According to Eq. (10), $\gamma<0$ for the roots $x_{\rm rad}^-$ and these roots describe stable states of the radiative layer. Conversely, $\gamma>0$ for $x_{\rm rad}^+$, i.e., the corresponding states with a radiation level higher than $\gamma_{\rm rad}^{\rm cr}$, are unstable. Qualitatively, this instability can be explained as follows. A spontaneous increase of the width of the radiative layer results on the one hand in an increase of the total radiated power proportional to $x_{\rm rad}$, but on the other hand, in a reduction of conductive losses, since $T_{\rm LCMS}$ decreases. For $x_{\rm rad}^-$, the latter effect dominates and the total energy losses decrease, which suppresses the perturbation. For $x_{\rm rad}^+$, the total losses increase, $T_{\rm LCMS}$ drops further and the perturbation develops.

Thus, the conductive losses to the LCMS dominate the energy balance of the radiative layer for stable roots, but the radiation is more important for unstable ones. This dominance has nothing to do with the absolute contributions of these channels to the total loss. It is easy to see in the limiting case of $\delta_T=\delta_0$, when $\gamma_{\rm rad}^{\rm cr}=0$ and the stable equilibrium corresponds to $x_{\rm rad}^-\equiv 0$ and $T_{\rm LCMS}\equiv T_{\rm max}$ for any $Q_0\leqslant\phi_{\rm core}/\delta_T$. When, however, this value is exceeded, e.g., by a ramp of the density, detachment starts immediately.

The decrease of $\gamma_{\rm rad}^{\rm cr}$ with increasing δ_T is in a qualitative agreement with the conclusion drawn in Ref. 8 that a less effective SOL with a larger δ_T should reduce the stability of the radiative layer. This is caused by a worse control of the temperature at the LCMS, which allows spreading of perturbations from $x=x_{\rm rad}$ where they are generated due to $dL_I/dT=-\infty$.

The present analysis predicts that the critical radiation level is achieved and detachment starts at the critical density given by the relation:

$$n_{\rm cr}=\frac{\phi_{\rm core}}{\sqrt{\kappa_\perp T_{\rm max} c_I L_I^{\rm max}}}*\left(1+\sqrt{1-\frac{\delta_T^2}{\delta_0^2}}\right)^{-1/2},$$

where c_I is the impurity concentration.

One can see that $n_{\rm cr}$ increases with δ_T. This is explained by the fact that increasing δ_T, one reduces conductive energy losses which should lead for a given $\phi_{\rm core}$ for an increase in $T_{\rm LCMS}$. Therefore, the radiative layer becomes narrower, which allows a higher level of Q_0 or n. This tendency is opposite to the decrease of $n_{\rm cr}$ with increasing δ_T predicted in Ref. 8. There the stationary temperature profile whose properties are of importance for the analysis of perturbations was settled arbitrary without taking the boundary condition at the LCMS into account.

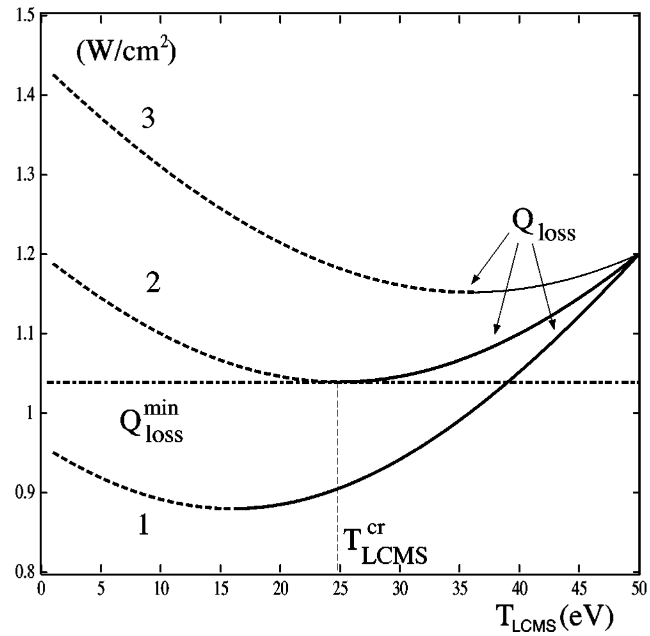

FIG. 2. Energy loss from the plasma edge with conduction and impurity radiation vs the temperature at the LCMS for different plasma densities: $1-2\times10^{13}$, $2-2.5\times10^{13}$, and $3-3\times10^{13}$ cm^{-3}.

B. ''Energetic'' approach

Consider the integral balance of the energy in the plasma column. This can be written as follows:

$$\frac{dE_{\rm tot}}{dt}=W_{\rm heat}-S_{\rm LCMS}Q_{\rm loss}, \tag{11}$$

where $E_{\rm tot}$ and $W_{\rm heat}$ are the total thermal energy and heat source in the plasma, correspondingly, $S_{\rm LCMS}$ the area of the LCMS and $Q_{\rm loss}$ the surface density of the energy losses with the plasma conduction and impurity radiation. The latter can be straightforwardly computed as a function of $T_{\rm LCMS}$. Indeed, if one multiplies Eq. (1) by $2\kappa_\perp dT$ and integrates from $T_{\rm LCMS}$ to $T_{\rm max}$, one gets

$$Q_{\rm loss}=\sqrt{\left(\kappa_\perp\frac{T_{\rm LCMS}}{\delta_T}\right)^2+2\int_{T_{\rm LCMS}}^{T_{\rm max}}\kappa_\perp n n_I L_I\,dT}.$$

With the ''box'' model for the impurity cooling rate exploited above, $Q_{\rm loss}(T_{\rm LCMS})$ is shown in Fig. 2 for several magnitudes of the plasma density and 1% of carbon ($L_I^{\rm max}\approx3\times10^{-7}$ eV cm^3 s^{-1}).

It is reasonable to assume that $E_{\rm tot}$ increases with increasing $T_{\rm LCMS}$, which is obvious if $\kappa_\perp$ and $W_{\rm heat}$ are not influenced by $T_{\rm LCMS}$. In this case, a perturbation in $T_{\rm LCMS}$, $\delta T_{\rm LCMS}$, causes a perturbation in $E_{\rm tot}$. Adopting $\delta T_{\rm LCMS}\sim\exp(\gamma t)$ and linearizing Eq. (9) with respect to $\delta T_{\rm LCMS}$, we get

$$\gamma\sim-\frac{\partial Q_{\rm loss}}{\partial T_{\rm LCMS}}\sim n n_I L_I(T_{\rm LCMS})-\kappa_\perp\frac{T_{\rm LCMS}}{\delta_T^2}.$$

Thus, the stability of stationary states is determined by the sign of the derivative of the total energy losses with respect to the temperature at the LCMS. Qualitatively, this seems to be a transparent criterium: if $\partial Q_{\rm loss}/\partial T_{\rm LCMS}<0$, a spontaneous reduction of $T_{\rm LCMS}$ would lead to an increase in energy losses and a further reduction of the temperature. However, it is necessary to stress that $Q_{\rm loss}$ is not a local but the integral energy loss. In numerous investigations of radiative instabilities, the local losses are analyzed to estimate the growth rate. Such an analysis ignores the role of boundary conditions which can forbid an instability which seems to be existing for an *a priori* assumed form of perturbations. This was clearly demonstrated by Drake[10] for $T_{\rm LCMS}\equiv0$.

The stationary states are given by the equality of $Q_{\rm loss}$ to the heat flux from the plasma interim, $\phi_{\rm core}$. It is easy to prove that the stable states with $\partial Q_{\rm loss}/\partial T_{\rm LCMS}>0$ correspond to $x_{\rm rad}^-$ and unstable ones with $\partial Q_{\rm loss}/\partial T_{\rm LCMS}<0$–to $x_{\rm rad}^+$. Thus, the ''perturbation'' and ''energetic'' approaches lead to the same results concerning the stability border.

The ''energetic'' approach is easy to apply to analyze the stability properties of a wide class of thermal equilibria. Consider, for instance, the following more realistic temperature dependence of the cooling rate: $L_I=L_I^{\rm max}\psi(T)$ with $\psi(T)=\exp[-(\sqrt{T_1/T}-\sqrt{T/T_2})^2]$. This form takes into account that impurities are excited to rare when electrons are too cold, $T<T_1$, and are overwhelmingly in the form of ''dim'' helium-like ions if the electron temperature is too high, $T>T_2$.[22,23] Consider the limiting stationary stable state given by the condition $\partial Q_{\rm loss}/\partial T_{\rm LCMS}=0$. The latter results in the equation for the critical edge temperature, $\alpha T_{\rm LCMS}^{\rm cr}=\psi(T_{\rm LCMS}^{\rm cr})$, with $\alpha=\kappa_\perp/(\delta_T^2 n n_I L_I^{\rm max})$. Analyzing the shape of the function ψ, we see that, depending on α, this equation has three or one, $T_{\rm LCMS}^{\rm cr}=0$, root. Simple analysis shows that the latter case with $\gamma_{\rm rad}^{\rm cr}=1$ is realized if $\alpha>\alpha_*=\psi(T_*)/T_*$ with $T_*=\sqrt{(T_2/2)^2+T_1T_2}-T_2/2$. So, for the considered $L_I(T)$, the radiative layer is stable to a radial detachment only if the e-folding length of the temperature at the LCMS is small enough:

$$\delta_T < \sqrt{\frac{\kappa_\perp}{\alpha_* n n_I L_I^{\max}}}.$$

Analogous conclusions can be drawn for other forms of $L_I(T)$, which can be found in the literature,[20,24] e.g., $L_I \sim T^\alpha \exp[-(T/T_*)^\beta]$ or $L_I \sim [(T_1/T)^\alpha + (T/T_2)^\beta]^{-\gamma}$.

Thus, the destabilizing role of the e-folding length for the temperature at the LCMS is independent to a certain degree on the model for the impurity cooling rate.

C. Analysis with inhomogeneous density profile and convective heat transfer

In reality, the plasma density varies significantly across the radiative layer, which makes the density of radiation losses inhomogeneous even with the ''box'' model for the impurity cooling rate. In Ref. 18, the effect of this was considered in the approximation that the width of the radiative layer is much less than the penetration depth of recycling neutrals, which determines the characteristic dimension for the density change. Even in this case, the perturbation analysis of stability was performed by means of further approximations. The ''energetic'' approach described above allows one to avoid such restrictions in the analysis of the density profile effect on the stability of the edge radiative layer. Moreover, another important effect directly related to the density inhomogeneity, namely, convective losses of energy can be taking into account. As before, we confine ourselves to Alcator-like transport scaling both for the plasma heat conductivity and particle diffusivity $D_\perp : \kappa_\perp = 3nD_\perp = \text{const}$. By taking into account the convective heat flux in the form $-3D_\perp(dn/dr)T$, we rewrite Eq. (1) as follows:

$$\frac{d}{dx}\left(-\frac{\kappa_\perp}{n}\frac{dnT}{dx}\right) = -nn_I L_I. \qquad (12)$$

The density profile is found for $D_\perp \sim n^{-1}$ in Appendix B. Retaining the ''box'' model for $L_I(T)$ and $n_I(x) = \text{const}$, one obtains from Eq. (12) the temperature profile in the radiative layer:

$$\frac{T(u)}{T_{\rm LCMS}} = \frac{n_{\rm LCMS}}{n(u)}\left[\frac{T_*}{T_{\rm LCMS}}u^2 + \left(\frac{1}{n_0\sigma\delta_T} + 1 - \frac{n_{\rm LCMS}}{n_0}\right)u + 1\right],$$

where $T_* = L_I^{\max}/(2\kappa_\perp\sigma^2)(n_I/n_0)$ with n_0 and σ being the central plasma density and the cross section for attenuation of recycling neutrals in the plasma, correspondingly; $u = \int_0^x n\sigma\, dx$ is the dimensionless distance from the LCMS. The plasma density $n_{\rm LCMS}$ and the e-folding length of the temperature at the LCMS are computed in Appendix C on the basis of a simple model for the particle and heat transport in the SOL under assumption that the transport coefficients, $\kappa_\perp$ and $D_\perp$, are here the same as in the radiative layer.

The dimensionless width of the radiative layer $u_{\rm rad}$ is calculated by solving the transcendental equation $T(u_{\rm rad}) = T_{\max}$. The total energy losses, $Q_{\rm loss} = \kappa_\perp(T_{\rm LCMS}/\delta_T) + [(L_I^{\max} n_I)/\sigma]u_{\rm rad}$, are presented in Fig. 3 versus $T_{\rm LCMS}$ for $n_0 = 4.7\times10^{13}\,\text{cm}^{-3}$. The latter corresponds roughly to the critical central density for $\phi_{\rm core} \approx 1\,\text{W/cm}^2$ typical in TEXTOR ohmic discharges.[3] The variation of the radiation level is also shown in Fig. 3. A critical level of $\gamma_{\rm rad}$ of 0.4 is in good agreement with the experimental data[3] in spite of the simplicity of our model. This magnitude of $\gamma_{\rm rad}^{\rm cr}$ is significantly less than a critical level of 0.75 predicted in Ref. 10 for the development of MARFE. This could explain why under conditions of ohmic plasmas detachment occur without MARFE stage.

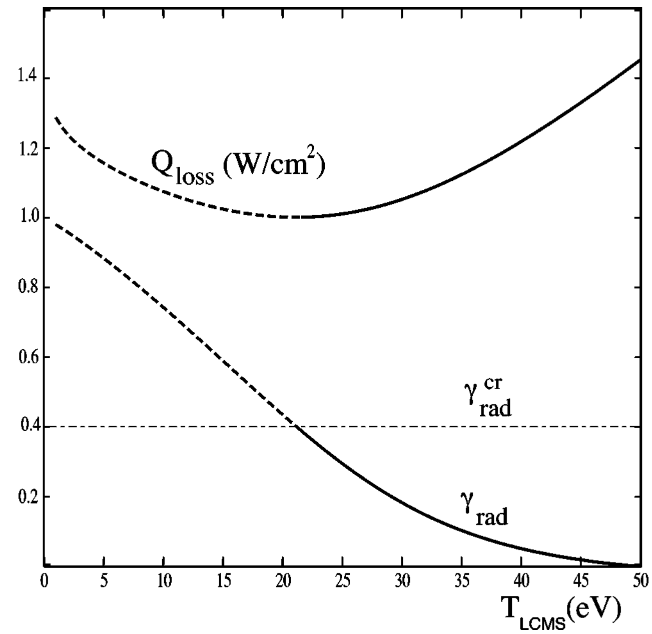

FIG. 3. Energy loss from the plasma edge and the radiation level computed taking into account the density profile and convective energy losses.

In concluding, we can describe a radial detachment caused by a ramp of the density as follows. The increase of the density results in an increase of $Q_{\rm loss}^{\min}$. For a low enough density when $Q_{\rm loss}^{\min} < \phi_{\rm core}$, the radiative layer is in a stable state on the solid branch of the curve $Q_{\rm loss}(T_{\rm LCMS})$. When the density corresponding to $Q_{\rm loss}^{\min} = \phi_{\rm core}$ is reached, there are no more stable attached states and a dynamic detachment occurs.

IV. DISCUSSION

Starting from the model proposed previously,[10] the energy balance at the plasma edge in tokamaks influenced by the radiation from light impurities has been analyzed by taking into account a more realistic boundary condition at the LCMS, namely, a nonzero e-folding length of the temperature, δ_T. The latter is determined by the heat transport in the scrape-off layer and the role of δ_T for the stability of the radiative layer is governed by the controlling function of this process with respect to the temperature at the LCMS. If $\delta_T = 0$ and $T_{\rm LCMS} \equiv 0$, a spreading of temperature perturbations from the border of the radiative layer with the plasma core, where they arise due to a sharp slop in the impurity cooling rate, is forbidden and there is not any radial detachment.[10] With a nonzero δ_T, the edge radiative layer becomes un stable and undergoes a radial detachment if the radiation level exceeds the critical value. This value decreases with increasing δ_T. For experimentally measured δ_T of 1–3 cm, $\gamma_{\rm rad}^{\rm cr}$ is significantly less than 0.75 predicted in Ref. 10 as a threshold for MARFE. This provides a plausible explanation for the experimental observations of radial detachments in tokamaks of moderate dimensions without a MARFE stage. Qualitatively, the found effect of nonzero δ_T agrees with the results obtained earlier[8] by an approximate variational technique. However, our consideration shows that the main effect is the reduction of the critical radiation level but not of the critical density.

Besides the conventional perturbation analysis, an ''energetic'' approach has been applied in the present paper to investigate the stability of stationary states. This method considers the variation of the total energy losses from the plasma with the temperature at the LCMS. For a simple equilibrium found with the ''box'' model for the impurity cooling rate, this method reproduces the threshold of the radial detachment predicted by the perturbation approach. An important advantage of the ''energetic'' method is its applicability to more complex thermal equilibria. It is shown, in particular, that also for more realistic temperature dependences of the cooling rate, the destabilizing role of a nonzero δ_T remains in force. Applying the ''energetic'' method, the convective heat losses in the radiative layer and spatial variation of the plasma density have been taken into account. The critical radiation level for detachment, $\gamma_{\rm rad}^{\rm cr}\approx 0.4$, is in agreement with the experimental observations.

APPENDIX A

In the region $x<x_{\rm rad}$, Eq. (7) has the solution:

$$\tilde{T}(x<x_{\rm rad})=A_+\exp(kx)+A_-\exp(-kx), \tag{A1}$$

where the integration constants $A_\pm$ related each other through the linearized boundary condition (3):

$$k\delta_T(A_+-A_-)=A_++A_-. \tag{A2}$$

At the plasma axis $d\tilde{T}/dr(r=0)=0$ and the solution of Eq. (8) is given by

$$\tilde{T}(r<r_{\rm rad}=r_{\rm LCMS}-x_{\rm rad})=BI_0(kr) \tag{A3}$$

with I_0 being the modified Bessel function of the zero order.

The solutions for $x<x_{\rm rad}$ and $r<r_{\rm rad}$ should be matched at the border between the radiative layer and core. The δ function in Eq. (7) results in a discontinuity in $d\tilde{T}/dx$ which can be found by integrating Eq. (7) from $x_{\rm rad}-0$ to $x_{\rm rad}+0$ with the usage of the relation $dx=dT(\kappa_\perp/\phi_{\rm core})$:

$$\frac{d\tilde{T}}{dx}(x_{\rm rad}-0)-\frac{d\tilde{T}}{dx}(x_{\rm rad}+0)=\frac{\tilde{T}(x_{\rm rad})}{x_{\rm rad}^{\rm max}}.$$

This condition together with the continuity of $\tilde{T}$ at $x_{\rm rad}$ and Eq. (A2) results in Eq. (9).

APPENDIX B

In a stationary state, the influx of recycling neutrals and outflow of charged particles are in balance:[21]

$$D_\perp\frac{dn}{dx}=\Gamma_0\exp\left(-\int_0^x\sigma n\,dx\right). \tag{B1}$$

The exponent in Eq. (B1) takes into account that neutrals are attenuated in the plasma due to ionization by electrons. The rate of attenuation is proportional to n and the effective cross section $\sigma\approx 5\times 10^{-15}\,{\rm cm}^2$ which takes into account both ionization and charge exchange with ions (see, e.g., Ref. 21). With Alcator scaling for $D_\perp$ and σ=const, Eq. (B1) is integrated analytically:[21]

$$n(x)=n_0\times\frac{\exp(n_0\sigma x)}{(n_0/n_{\rm LCMS})-1+\exp(n_0\sigma x)}.$$

Here n_0 is the density far from the LCMS and related to the mean density in the tokamak; the flux density at the LCMS, Γ_0, was eliminated in favor of $n_{\rm LCMS}$. With the dimensionless distance from the LCMS, u, we have

$$n(u)=n_0-(n_0-n_{\rm LCMS})\times\exp(-u).$$

APPENDIX C

To determine $n_{\rm LCMS}$ and δ_T in consistent way, we consider the particle and heat balances in the SOL under the assumption that there are only sources and sinks of charged particles and energy through the plasma neutralization at the limiter:[21]

$$\frac{d}{dx}\left(-\frac{\kappa_\perp}{3n}\frac{dn}{dx}\right)=-\frac{nc_s}{2L_\parallel}, \tag{C1}$$

$$\frac{d}{dx}\left(-\frac{\kappa_\perp}{n}\frac{dnT}{dx}\right)=-\frac{\gamma_L nc_sT}{2L_\parallel}, \tag{C2}$$

where $c_s=\sqrt{2T/m_i}$ is the sound speed, $L_\parallel$ the parallel connection length, and γ_L the heat transmission factor to the limiter. Assume that both the density and temperature decay exponentially with the distance from the LCMS toward the wall: $n(|x|)=n_{\rm LCMS}\exp(-|x|/\delta_n)$ and $T(|x|)=T_{\rm LCMS}\cdot\exp(-|x|/\delta_T)$. Then, from Eqs. (C1) and (C2) and assuming continuity of the density and flux of particles at the LCMS, one gets algebraic equations for n_0, δ_n, and δ_T:

$$n_{\rm LCMS}=n_0-\frac{1}{\sigma\delta_n},$$

$$\delta_n=\frac{1}{2n_0\sigma}\times\left[1+\sqrt{1+\frac{4\kappa_\perp\sigma^2n_0L_\parallel}{3c_s}(2+\zeta)}\right],$$

$$\delta_T=\frac{\delta_n}{\zeta}$$

with $\zeta=\frac{1}{18}(\gamma_L-15+\sqrt{9+42\gamma_L+\gamma_L^2})\approx 0.73$ for a typical $\gamma_L\approx 8$.

[1]V. A. Vershkov and S. V. Mirnov, Nucl. Fusion **14**, 383 (1974).
[2]G. M. McCracken, J. Allen, K. Axon *et al.*, J. Nucl. Mater. **145–147**, 181 (1987).
[3]U. Samm, P. Bogen, H. A. Claassen *et al.*, J. Nucl. Mater. **176–177**, 273 (1990).
[4]F. P. Boody, C. E. Bush, S. S. Medley, H. K. Park, and J. F. Schivell, J. Nucl. Mater. **145–147**, 196 (1987).
[5]B. Lipschultz, B. LaBombard, E. S. Marmar *et al.*, Nucl. Fusion **24**, 977 (1984).
[6]P. H. Rebut and B. J. Green, in Proceedings of 6th Conference on Plasma Physics and Controlled Nuclear Fusin Res., Berchtesgaden, 1976, (International Atomic Energy Agency, Vienna, 1977) Vol. 2, p. IAEA-CN-35/B6.
[7]A. Gibson, Nucl. Fusion **16**, 546 (1976).
[8]N. Ohyabu, Nucl. Fusion **19**, 1491 (1979).
[9]M. Z. Tokar, Nucl. Fusion **23**, 1395 (1983).
[10]J. F. Drake, Phys. Fluids **30**, 2429 (1987).
[11]D. P. Stotler, Phys. Fluids **31**, 3713 (1988).
[12]M. Z. Tokar, Plasma Phys. Controlled Fusion **36**, 1819 (1994).
[13]A. Loarte, R. D. Monk, J. R. Martin-Solis *et al.*, Nucl. Fusion **38**, 331 (1998).
[14]R. R. Weynants, A. M. Messiaen, J. Ongena *et al.*, Nucl. Fusion **39**, 1637 (1999).
[15]G. M. Staebler, G. L. Jackson, W. P. West *et al.*, Phys. Rev. Lett. **82**, 1692 (1999).
[16]K. W. Hill, S. D. Scott, M. Bell *et al.*, Phys. Plasmas **30**, 877 (1999).
[17]K. Itoh and S.-I. Itoh, J. Phys. Soc. Jpn. **57**, 1269 (1988).
[18]S. Roy Choudhury and P. K. Kaw, Phys. Fluids B **1**, 1646 (1989).
[19]J. Mandrekas, W. M. Stacey, and F. A. Kelly, Contrib. Plasma Phys. **36**, 245 (1996).
[20]P. C. Georgiou, M. G. Haines, and R. J. Hastie, Contrib. Plasma Phys. **38**, 136 (1998).
[21]A. V. Nedospasov and M. Z. Tokar, in *Reviews of Plasma Physics*, edited by B. B. Kadomtsev (Consultants Bureau, New York, 1993), Vol. 18, p. 77.
[22]M. Z. Tokar, Nucl. Fusion **34**, 835 (1994).
[23]H. Lasaar and M. Z. Tokar, in *Fusion Theory*, Proceedings of 7th European Fusion Theory Conference, Jülich 1997 (Forschungszentrum Jülich GmbH, Julich, 1998), ISSN 1433-5522, ISBN 3-89336-219-3, Vol. 1, p. 189.
[24]P. K. Kaw, S. Deshpande, K. Avinash, and S. Rath, Phys. Rev. Lett. **65**, 2873 (1990).