Challenges of Josephson Junction Arrays for ac Voltage Generation by Microwave Pulse Power Modulation

Alexander S. Katkov, Alexander M. Klushin, Gennady P. Telitchenko, Ralf Behr, and Juergen Niemeyer

***Abstract*—Voltage pulses with a determined amplitude and width might be the basis for high-precision ac voltage measurements in the future. The pulse width modulation technique makes possible the generation of arbitrary waveforms. In this paper, arrays of SINIS Josephson junctions were investigated which produce a 1 V voltage step under irradiation at a frequency of the order of 70 GHz. This step has an amplitude of 1 mA whereof a region of 0.3 mA overlaps with the critical current. This allows the generation of a 1 V voltage pulse by switching the microwave power at constant dc bias current. The results of this investigation will be presented and also the array parameters which are required for obtaining a relative uncertainty of 0.1 ppm in ac voltage measurements.**

***Index Terms*—Ac voltage, Josephson array, Josephson effect, pulse modulation.**

I. INTRODUCTION

THE reproduction of ac voltage by pulse modulation of a constant voltage is well known and used in industrial calibrators [1]. The idea to increase the accuracy of ac measurements by implementing the Josephson effect in superconductors is developing. In accordance with the Josephson equation the output voltage of an array is

$$U_0 = F \cdot \frac{N}{K_J}, \tag{1}$$

where F is a microwave frequency applied to the Josephson junction, $\boldsymbol{N}$ is the number of Shapiro steps and K_{J} is the Josephson constant.

Several methods exist which are used for ac waveform synthesis investigations. In 1995, Hamilton *et al.* proposed a Josephson D/A converter based on binary sequence of series arrays [2]. Investigations of this method—which produces an ac voltage by switching bias currents of selected sub-arrays—shows that the frequency of a synthesized ac voltage will be limited to frequencies clearly below 10 kHz because the undefined voltage transients between steps of quantized voltage enhance the uncertainty [2]–[4]. Benz and Hamilton have developed another approach that biases the array with short pulses where the repetition rate of pulses is changed [5]. But it is difficult to reach an output voltage of 1 V, as the path to every junction should be reasonably independent of frequency from dc to about 30 GHz.

Another method is based on pulse modulation of the array output voltage at constant bias frequency [6], [7]. It is possible to change the voltage of the array from the nominal value to zero by switching an array bias current, or by reducing the applied microwave power to less than the determined critical value, or by applying both methods, current and power changing. Compared with current control, the method of power control [7] has some advantages. The pulse power modulation method uses a microwave guide which reduces the problem with load matching existing in cables for current modulation. The present work is devoted to the investigation of the Josephson junction array (JJA) parameters that are important for reproducing the ac voltage by pulse power modulation.

II. THEORY

Any ac voltage can be written as

$$U(t) = \frac{U_0}{2} + \sum_{1}^{\infty} U_n \sin(n2\pi f t + \varphi_n), \tag{2}$$

where n is the number of the harmonic, U_{n} is the amplitude of the harmonic, f is the fundamental frequency, t is time, and φ_{n} is a phase angle.

For ac voltage synthesis using pulses it is obvious that some of the higher harmonics will be lost due to the time constant in the equipment. This means that it is important to have a time constant as low as possible in the system. To reach, for example, an uncertainty near 10^{-7} by reproducing the effective ac voltage value at 1 kHz by bipolar meander pulses we should have a time constant of less than 0.1 ns.

To produce stable pulses at 1 V, an array with nonhysteretic current-voltage characteristic must be used. There are superconductor-normal-superconductor (SNS), super-conductor-insulator-normal-insulator-superconductor (SINIS), and externally shunted superconductor-insulator-super-conductor (SIS) types of Josephson junctions that can be used in such an array. The most important parameter for microwave power switching is an overlap between the end of the current step $I_{\min}$ and the critical current I_{c} of an array (Fig. 1). This constraint requires an optimized microwave design of the programmable Josephson array to ensure homogenous microwave power distribution and, furthermore, sets very small margins for the

Manuscript received October 5, 2004.

A. S. Katkov and G. P. Telitchenko are with the Mendeleyev Institute for Metrology (VNIIM), 198000 St. Petersburg, Russia (e-mail: A.S.Katkov@vniim.ru).

A. M. Klushin is with the Institut fuer Schichten und Grenzflaechen, 52428 Juelich, Germany (e-mail: a.klushin@fz-juelich.de).

R. Behr and J. Niemeyer are with the Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany (e-mail: Ralf.Behr@ptb.de).

Digital Object Identifier 10.1109/TASC.2005.849833

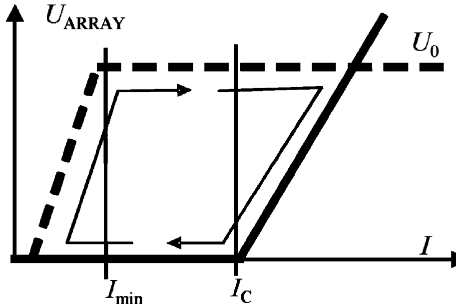

Fig. 1. Drawing of the $I-V$ characteristic of an array with (dashed line) and without millimeter wave power applied (solid line).

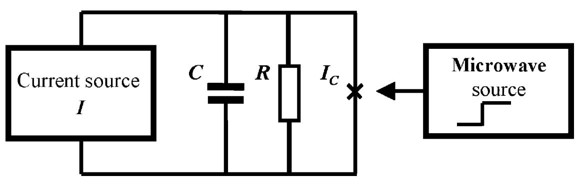

Fig. 2. Model of the array bias circuit.

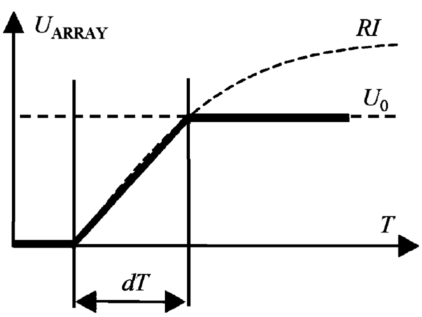

Fig. 3. Diagram of the voltage rise at the array.

fabrication process as the junction parameters, e.g. the I_c R_n product of the junctions, have to meet the range of F/Φ_0. R_n is the normal resistance of a junction and Φ_0 the flux quanta. If we apply a bias current larger than I_{min} and smaller than I_c, the array output voltage is changed from zero to U_0 by a microwave power switching at constant bias current.

The circuit, which is used for a dynamic response calculation, is presented in Fig. 2. Here C is the capacitance of the circuit; R is determined by the dynamic resistance of the array (dU/dI) and includes the shunting effect by the output resistance of the current source.

The voltage output behavior (Fig. 3) when switching the power on is similar to the model of charging a capacitance. The equation for the time needed to switch the voltage from zero to U_0 (for $I_{min} < I < I_C$ and $RI > U_0$) can be written as

$$dT_{up} = R_{up} C \ln \left(\frac{1}{\left(1 - \frac{U_0}{R_{up} I}\right)} \right). \tag{3}$$

When the voltage at the capacitance reaches the step voltage U_0, the charging process is finished and the output voltage is

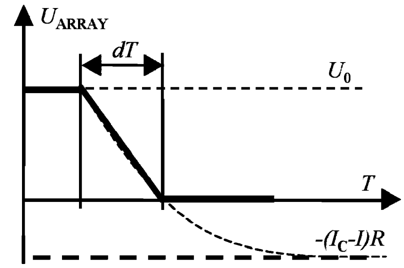

Fig. 4. Diagram of the voltage descent at the array.

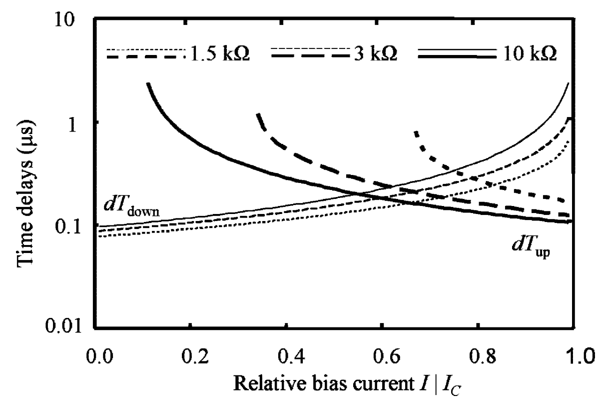

Fig. 5. Diagram of time delay dependence at $\boldsymbol{I}_C = 1$ mA.

stable. If the power is switched off (Fig. 4), the equation of the descent time (for $I < I_C$) can be written as

$$dT_{down} = R_{down} C \ln \left(\frac{1}{\left(1 - \frac{U_0}{((I_C - I)R_{down} + U_0)}\right)} \right). \tag{4}$$

Equations (3) and (4) show that for minimizing the value of dT_{up} the current I must be close to I_c, but to minimize the value of dT_{down} the current I must be close to I_{min}. To minimize both values of dT at $U_0 = \text{constant}$, we must increase I_c and decrease C.

The diagram in Fig. 5 shows the dependencies $dT(I/I_c)$ for $C = 100$ pF at different I_c and R. The result of these calculations shows that we can estimate the time delays dT_{up} and dT_{down} at the level of 0.1 μs for the parameters adopted for calculation.

III. Experiment

A 1 V SINIS array [8] with a critical current of 1 mA was used in our experiments. Its $I-V$ characteristic is presented in Fig. 6. Pulses were modulated by means of an HP 8346 A synthesizer that has a pulse rise time of less than 4 ns. The output frequency of the HP 8346 A of 8.75 GHz was multiplied to 70 GHz. The microwave pulses were applied to the SINIS array with a dc bias current in the range from 0.75 mA to 0.95 mA. A first measurement shows large rise and descent times of about 10 μs to 30 μs. This high value of transient times is explained by the high value of 0.6 nF capacitance of measuring cable and the output resistance of the bias current source of 10 kΩ. To reduce the influence of the cable capacitance we changed the measuring circuit by inserting a resistive divider $(10\ \text{k}\Omega/20\ \Omega)$

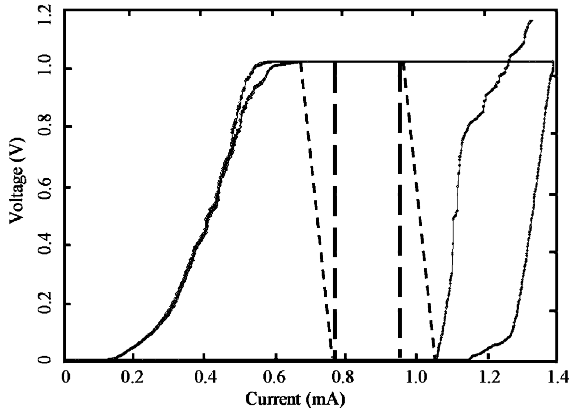

Fig. 6. $I - V$ characteristic of the 1 V SINIS array. Dashed lines limit the region of bias currents and the dotted lines show the load line given by the $10\,\mathrm{k\Omega}$ output resistance of the bias source.

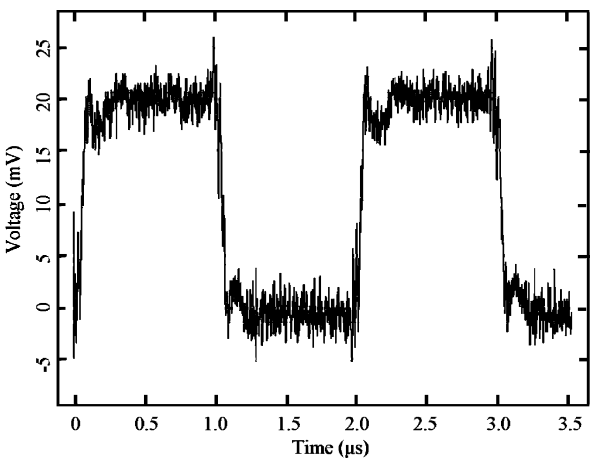

Fig. 7. Output voltage pulses of the 1 V SINIS array using a resistive divider closely placed to the array.

between array and cable. According to this scheme we obtain transients of about 0.3 μs (Fig. 7).

To measure the dependence of the time delay on the dc bias current more precisely we have carried out additional dc measurements. First we measured the array output voltage U_1 without pulse power modulation. After that we modulate this power by a meander-like profile and measured the output voltage at $U_{0.5}(I) \approx U_1/2$. To reduce the capacitance of the measuring cable, a resistor of 100 $\mathrm{k\Omega}$ was inserted between cable and array. To calculate the difference between the rise and descent times of the pulse, we use the equation

$$\Delta T(I) = dT_{down}(I) - dT_{up}(I) = \frac{U_{0.5}(I) - \frac{U_1}{2}}{F_{\mathrm{mod}} \cdot \frac{U_1}{2}}, \quad (5)$$

where F_{mod} is the modulation frequency.

The results of the ΔT measurement and calculation are presented in Fig. 8 as circles and, correspondingly, as a solid line. The dashed and dotted lines in the Fig. 8 present dT_{up} and dT_{down} that were calculated using (3) and (4).

Calculations are performed with the following array parameters: $U_0 = 1.15$ V, $I_{\mathrm{c}} = 1.08$ mA, $R_{\mathrm{up}} = 1.4\,\mathrm{k\Omega}$, $R_{\mathrm{down}} =$

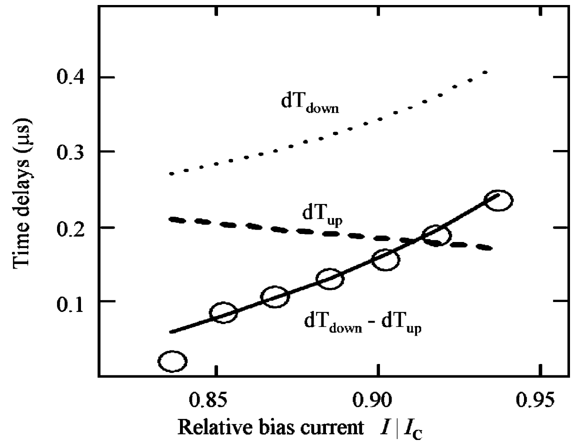

Fig. 8. Theoretical calculation of the rise time (dot line) and the descent time (dashed line) and its difference (solid line) as dependence of the dc bias current. The measured difference of these time constants are presented as circles.

$3\,\mathrm{k\Omega}$. The resistance values R_{up} and R_{down} include the influence of the dc bias current output resistance of $10\,\mathrm{k\Omega}$. The calculations and measurements coincide at $C = 90$ pF a capacitance that can be assigned the array capacitance. In the range from 100 Hz to 500 kHz we do not see any dependence of ΔT on the frequency modulation. The measuring points at currents lower than 0.85 I/I_{C} are not taken into account because the step of the array is not flat in that region.

The measurements show that by using this SINIS Josephson array, it is possible to reach transients near 0.3 μs at bias currents of 0.85 I/I_{c}.

IV. Discussion

The result of this work shows that the equations presented describe the experimental data measured by an oscilloscope and dc techniques very well. Furthermore, it demonstrates that the voltage transient of the investigated 1 V SINIS Josephson array is limited to 0.1 μs when switching the microwave power. By means of this approach it is possible to calculate the parameters for the array and the connected circuit used to obtain smaller time delays for microwave power switching. There are some applications that would make it possible to reach an uncertainty of close to 0.1 ppm by using an advanced array technology:

1. The dc bias current should be adjustable in such a way that dT_{down} and dT_{up} can be set to equal values. That means that the ideal signal of the array (without any time delay in the measuring system) has the same area under the transients.
2. It is easily possible to reproduce the effective value of a signal with an uncertainty near 10^{-7} at a frequency lower than 1 Hz. To increase the frequency of such a synthesized waveform signal at the same level of uncertainty we should change the parameters of the array and of the connected circuit. According to (3) and (4) we should decrease the capacitance and increase the critical current. The value of $dT = 0.1$ ns at an output voltage 1 V can be reached with an I_{c} of more than 10 mA and a C of less than 1 pF. In this case we can estimate an uncertainty at the level of $dT \cdot f = 10^{-7}$ at $f = 1$ kHz.

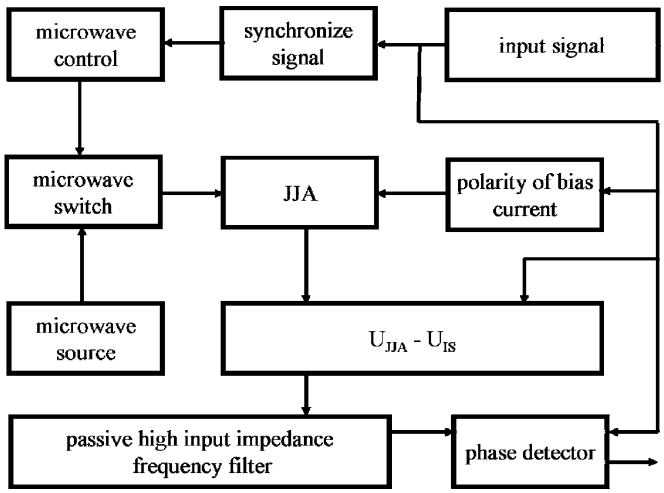

Fig. 9. Block diagram for the comparison of fundamental frequency amplitudes.

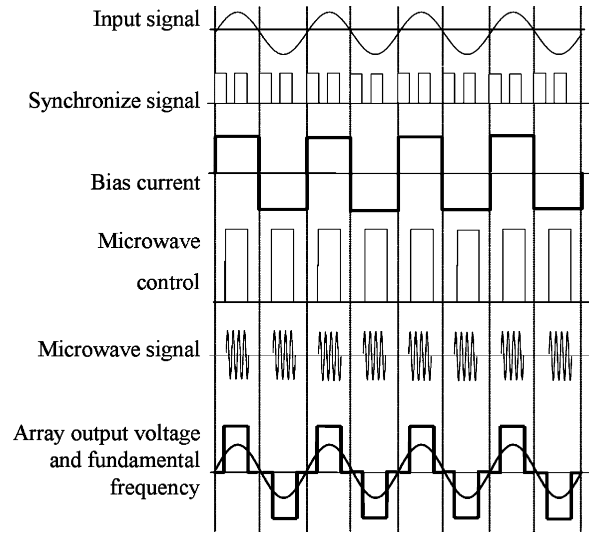

Fig. 10. Schematic time diagram of the control signals for a fundamental frequency amplitude comparison.

3. Another method of sinusoidal signal measurements uses the fundamental frequency to compare amplitudes [6], [9]. The uncertainty of this method can be estimated as $K(dT \cdot f)^2$. Here K is the coefficient in the range from $2\pi^2$ to $\pi^2/6$ depending on the shape of the corresponding time delay dT. For $dT = 0.1\ \mu$s and $f = 1$ kHz a relative uncertainty of comparison near 10^{-7} can be reached.

A scheme for the method carried out by means of the power modulation technique is presented in Fig. 9. The sinusoidal input signal to be calibrated controls the synchronized signal, the polarity of the bias current and the phase detector. The input signal is subtracted from the Josephson junction array signal, which is formed by modulation of the microwave power. The difference of the compared signals is first delivered to the passive frequency filter with a high input impedance and then to the phase detector. A null at the phase detector output means that the amplitudes of the fundamental waves of both signals are equal. The amplitude of the fundamental frequency f from the JJA is calculated according to the amplitude of the pulse defined by (1). The time diagram of the control signals of this scheme is presented in Fig. 10.

V. Conclusion

We have evaluated the method of microwave pulse power modulation on Josephson arrays for ac waveform synthesis. On the one hand this method is easy to apply to already available 1 V SINIS Josephson arrays. On the other hand it is difficult to achieve uncertainties at the level of 10^{-7} at frequencies in the kHz range using this method. This demand requires very fast transient times of better than 100 ns and thus to very stringent conditions for the parameters of programmable Josephson arrays.

References

[1] *Calibration: Philosophy in Practice*, 2nd ed. Everett, WA: Fluke Corp., 1994.

[2] C. A. Hamilton, C. J. Burroughs, and R. L. Kautz, "Josephson D/A converter with fundamental accuracy," *IEEE Trans. Instrum. Meas.*, vol. 44, pp. 223–225, Apr. 1995.

[3] C. A. Hamilton, C. J. Burroughs, S. P. Benz, and J. R. Kinard, "AC Josephson voltage standard. Progress report," *IEEE Trans. Instrum. Meas.*, vol. 46, pp. 224–228, Apr. 1997.

[4] R. Behr, J. M. Williams, P. Patel, T. J. B. M. Janssen, T. Funk, and M. Klonz, "Synthesis of precision ac waveforms using a SINIS Josephson junction array," in *CPEM Conf. Dig.*, London, U.K., 2004, pp. 4–5.

[5] S. P. Benz and C. A. Hamilton, "A pulse-driven programmable Josephson voltage standard," *Appl. Phys. Lett.*, vol. 68, pp. 3171–3173, May 1996.

[6] J. Nissilä, K. Ojasalo, A. Kemppinen, A. Manninen, J. Hassel, P. Helistö, and H. Seppä, "Development of a quantum ac voltage standard based on externally shunted Josephson arrays," in *CPEM Conf. Dig.*, London, U.K., 2004, pp. 158–159.

[7] O. V. Karpov, V. D. Koutovoi, S. V. Sherstobitov, V. N. Krutikov, and J. Niemeyer, "Pulse width modulation type Josephson ac waveform synthesizer," in *CPEM Conf. Dig.*, Ottawa, ON, Canada, 2002, p. 497.

[8] R. Behr, H. Schulze, F. Müller, J. Kohlmann, and J. Niemeyer, "Josephson arrays at 70 GHz for conventional and programmable voltage standard," *IEEE Trans. Instrum. Meas.*, vol. 48, pp. 270–273, Apr. 1999.

[9] A. S. Katkov, E. D. Koltik, V. I. Krzhimovsky, and G. P. Telitchenko, "DC-AC quantum voltage standard. Basic design," *Izmeritel'naja technicka*, vol. 40, no. 3, pp. 39–42, 1997.