

Correction to “Prediction of fingering in porous media”

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1. Introduction

[1] In the paper “Prediction of fingering in porous media” by Zhi Wang, Jan Feyen, and David E. Elrick (*Water Resources Research* 34(9), 2183–2190, 1998), the authors developed 24 specific criteria for predicting unstable flow in porous media, based on the Chuoke equation [Chuoke *et al.*, 1959]. However, some of the results were inconsistent with predictions by others. For instance, *de Rooij* [2000] noted that the Wang criteria showed unstable flow for overly rapid upward flow in contrary to *Philip’s* [1975] theory that predicted an unconditionally stable flow. These discrepancies resulted from three mistakes in the theoretical development, namely, (1) an inconsistency in the transcription of the Chuoke equation (equation (1) in the original paper), (2) a discrepancy between the definition of σ^* compared to the Chuoke definition (leading to a wrong equation (3a)) and, (3) sign errors in the development of the criteria (Table 1). In the following, we give the correct development and revise the 24 stability criteria of Wang *et al.* [1998].

2. Initial Chuoke Formulation

[2] The initial formulation by Chuoke *et al.* [1959] of unstable wetting front criterion (equation (13) in their paper) is

$$V(\mu_2 - \mu_1) + (\rho_2 - \rho_1)gk \cos(\alpha) - k\sigma^*\alpha^2 > 0 \quad (1)$$

where V is the Darcy’s velocity (m s^{-1}), ρ is the density (kg m^{-3}) and μ the viscosity of the fluids ($\text{kg m}^{-1} \text{s}^{-1}$), g is the acceleration due to gravity (m s^{-2}), α is the angle between the vertical axis z' (positive upward) and z another axis with a direction normal to the initial planar interface between fluid 1 to fluid 2 (positive from fluid 2 to fluid 1 and negative from fluid 2 to fluid 1), σ^* is the effective macroscopic interfacial tension (N m^{-1}) defined as $\sigma^* = C\sigma$ with C a scale constant (greater than 1) and σ the

microscopic interfacial tension, k is the permeability of the porous medium (m^2) and α is the wave number of the sinusoidal perturbation (m^{-1}). Chuoke *et al.* [1959] also defined the sign of V as positive when fluid 1 displaces fluid 2 and negative otherwise. Equation (1) in the original paper should therefore be replaced by this equation (1).

3. Reformulation of Equation (3a)

[3] Chuoke’s formulation seemed ambiguous because the wetting characteristics of the fluids were not clearly defined. Therefore Wang *et al.* [1998] stipulated that fluid 1 be the nonwetting fluid and fluid 2 be the wetting fluid. This was based on the experimental setup by Chuoke *et al.* [1959]. Wang *et al.* [1998] also conceptualized that the macroscopic interfacial tension σ^* is inherently negative for the nonwetting fluid (fluid 1) displacing the wetting fluid (fluid 2). The wettability of the displacing fluid was defined by introducing a wettability variable e , valued $e = 1$ for the wetting fluid displacing the nonwetting fluid, and $e = -1$ for the reversed displacement. Hence the interfacial tension, $\sigma^* = e|\sigma^*|$, is positive when $e = 1$ and negative when $e = -1$. Accordingly, equation (1) as a case for $e = -1$ in the analysis of Chuoke *et al.* [1959] was generalized into equations (2) and (3) in the paper of Wang *et al.* [1998].

[4] However, reexamining the definition of Chuoke *et al.* [1959], we found that σ^* should always be positive rather than signed, because the center of the interfacial curvature always falls in the domain of the nonwetting fluid (fluid 1). According to Chuoke *et al.* [1959], fluid wettability (e) was rather associated with the interfacial velocity V , because they specified that V “is positive for (nonwetting) fluid 1 displacing (wetting) fluid 2, negative for the reverse displacement”. Therefore equations (1) as a case for $e = -1$ in the analysis of Chuoke *et al.* [1959] can be rewritten as

$$-e|V|(\mu_w - \mu_{nw}) + (\rho_w - \rho_{nw})gk \cos(\alpha) - k|\sigma^*|\alpha^2 > 0 \quad (2)$$

Furthermore, we introduce a directional variable, $n_z = \cos(\alpha)/|\cos(\alpha)|$, to characterize the direction of flow and the relative position of the two fluids at the beginning of displacement. On the basis of Chuoke *et al.*’s [1959] definitions of z and z' , if the wetting fluid 2 is located above the nonwetting fluid 1, $n_z = 1$, otherwise, $n_z = -1$. It is therefore logical to see that the direction of flow is indicated by two variables, e and n_z . For instance, if $e = 1$ and $n_z = 1$ the flow is driven by the wetting fluid which is above the nonwetting fluid (a typical condition of water infiltration into water wettable soils partially or completely filled with air).

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Table 1. Criteria for Unstable Flow in Porous Media

Wettability of the Driving Fluid to Porous Medium	Density and Viscosity Contrast	Displacement Direction (Capillary Effect)		
		Downward	Upward	Horizontal/Capillary
Wettable ($e = 1$)	$\rho_w > \rho_{nw}, \mu_w > \mu_{nw}$	$ V < V_{crit} - V_{cap}$	$ V < -V_{crit} - V_{cap}$ stable	$ V < -V_{cap}$ stable
Wettable ($e = 1$)	$\rho_w > \rho_{nw}, \mu_w < \mu_{nw}$	$ V > -V_{crit} + V_{cap}$	$ V > V_{crit} + V_{cap}$	$ V > V_{cap}$
Wettable ($e = 1$)	$\rho_w < \rho_{nw}, \mu_w > \mu_{nw}$	$ V < -V_{crit} - V_{cap}$ stable	$ V < V_{crit} - V_{cap}$	$ V < -V_{cap}$ stable
Wettable ($e = 1$)	$\rho_w < \rho_{nw}, \mu_w < \mu_{nw}$	$ V > V_{crit} + V_{cap}$	$ V > -V_{crit} + V_{cap}$	$ V > V_{cap}$
Nonwettable ($e = -1$)	$\rho_{nw} > \rho_w, \mu_{nw} > \mu_w$	$ V < V_{crit} - V_{cap}$	$ V < -V_{crit} - V_{cap}$ stable	$ V < -V_{cap}$ stable
Nonwettable ($e = -1$)	$\rho_{nw} > \rho_w, \mu_{nw} < \mu_w$	$ V > -V_{crit} + V_{cap}$	$ V > V_{crit} + V_{cap}$	$ V > V_{cap}$
Nonwettable ($e = -1$)	$\rho_{nw} < \rho_w, \mu_{nw} > \mu_w$	$ V < -V_{crit} - V_{cap}$ stable	$ V < V_{crit} - V_{cap}$	$ V < -V_{cap}$ stable
Nonwettable ($e = -1$)	$\rho_{nw} < \rho_w, \mu_{nw} < \mu_w$	$ V > V_{crit} + V_{cap}$	$ V > -V_{crit} + V_{cap}$	$ V > V_{cap}$

[5] Following *Wang et al.* [1998], a density directional variable, $n_\rho = (\rho_w - \rho_{nw})/|\rho_w - \rho_{nw}|$, and a viscosity directional variable, $n_\mu = (\mu_w - \mu_{nw})/|\mu_w - \mu_{nw}|$, are reused here. Substituting the variables n_z , n_ρ and n_μ into (2) and rearranging after dividing by $|\mu_w - \mu_{nw}|$ yields:

$$-e|V|n_\mu + n_\rho n_z V_{crit} - V_{cap} > 0 \quad (3)$$

where, V_{crit} is the critical velocity and V_{cap} is the capillary and viscosity driven velocity defined by *Wang et al.* [1998]. This equation must replace equation (3a) in the original paper.

4. New Criteria

[6] On the basis of (3), we delineated *Chuoque et al.*'s [1959] criterion into 24 subcriteria (as shown in Table 1) according to specific combinations of fluid properties and relative positions in porous media. Table 1 provides a direct comparison of the individual criteria with those in Table 1 of *Wang et al.* [1998].

[7] Note that, with these corrections, the wettable and nonwettable halves of Table 1 are now identical, indicating that parameter e does not affect the onset of instability. Moreover, rows 3 and 4 coalesce into rows 1 and 2,

respectively. Therefore these identical pairs of criteria can be regrouped in terms of relative position of the heavier and lighter fluid and in terms of the viscosity contrast.

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