Isospin dependence of the three-nucleon force

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We classify A-nucleon forces according to their isospin dependence and discuss the most general isospin structure of the three-nucleon force. We derive the leading and subleading isospin-breaking corrections to the three-nucleon force using the framework of chiral effective field theory.

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I. INTRODUCTION

Three-nucleon forces (3NFs) are well established in nuclear physics. Although small compared to the dominant two-nucleon force (2NF), they are nevertheless needed to gain a quantitative understanding of nuclei and nuclear physics. A recent example in this context is the discussion of the 3NF effects in proton-deuteron scattering (see, e.g., [1,2]). Other examples are the binding energy difference between ^{3}H and ^{3}He or the saturation properties of nuclear matter. Only in the past decade has a theoretical tool become available to systematically analyze few-nucleon forces and consider such fine but important aspects as isospin violation in such forces and in systems made of a few nucleons. This tool is the extension and application of chiral perturbation theory to systems with more than one nucleon that require an additional nonperturbative resummation to deal with the shallow nuclear bound states and large S-wave scattering lengths. Although 3NFs in the isospin limit have been analyzed in some detail (see, e.g., [3–6]), the question of isospin violation in the 3NF has not yet been addressed in this framework. The work reported here is intended to fill this gap.

Without further ado, let us address the issues considered here. First, we generalize the classification of the isospin dependence of two-nucleon forces due to Henley and Miller [7] to the case of A nucleons ($A \geqslant 3$), with particular emphasis on the three-nucleon system (see Sec. II). This is essentially a quantum-mechanical exercise and reveals no underlying dynamics. The keywords here are isospin mixing, charge independence (breaking), and charge symmetry (breaking). We stress that although such language, which precedes quantum chromodynamics (QCD) and originates from Heisenberg's definition of isospin to account for the almost degeneracy of the proton and the neutron combined with their almost equally strong forces, is useful to categorize few-nucleon forces, in QCD the underlying broken symmetry is isospin of the light up and down quarks. This symmetry is broken in pure QCD by the light quark mass difference and further by electromagnetism when external electroweak interactions are considered. Thus, in the second part of this work, Sec. III, we derive the leading and next-to-leading order isospin-violating contributions of the 3NF based on chiral effective field theory (EFT).[1] We briefly recall the counting rules for the inclusion of strong and electromagnetic isospin violation presented in [8] and discuss the pertinent terms of the effective chiral Lagrangian in Sec. III A. We present the leading and subleading isospin-breaking contributions to the 3NF in momentum space in Sec. III B, followed by a brief estimate of the relative strength of these forces in Sec. III C. We end with a short summary. The Appendix contains the coordinate space representation of the isospin-violating 3NF.

II. GENERAL CONSIDERATIONS

This section deals with a novel classification scheme for the isospin dependence of the A-nucleon forces. To derive this scheme, one makes no assumption about the dynamics underlying such forces but only utilizes their transformation properties under isospin-symmetry and charge-symmetry operations on the level of nucleons.

A. Definitions and notation

The nonrelativistic A-nucleon system is described by the Hamilton operator H as follows:

$$H = H_0 + V^{2N} + V^{3N} + \cdots + V^{AN}, \tag{1}$$

where H_0 is the nucleon kinetic energy and V^{nN} represents the potential corresponding to the n-nucleon force. The total isospin operator $\boldsymbol{T}$ is given by the sum of the isospin operators $\boldsymbol{t}$ of the individual nucleons as follows:

$$\boldsymbol{T} = \sum_{a=1}^{A} \boldsymbol{t}(a). \tag{2}$$

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[1]We eschew here pionless nuclear EFT as it is not the appropriate tool to analyze this particular problem.

The total isospin operator $\boldsymbol{T}$ as well as the operators $\boldsymbol{t}(i)$ satisfy the Lie algebra of the SU(2) isospin group:

$$[T_i,\ T_j] = i\epsilon_{ijk}T_k,$$
$$[t_i(a),\ t_j(b)] = i\delta_{ab}\epsilon_{ijk}\ t_k(a), \tag{3}$$

with $i, j, k = 1, 2, 3$. The single-nucleon isospin operators $t_i(a)$ can be conveniently represented in terms of Pauli matrices τ_i as follows:

$$t_i(a) = \tfrac{1}{2}\tau_i(a). \tag{4}$$

The charge operator Q is defined for the A-nucleon system as follows:

$$Q = e\left(\frac{A}{2} + T_3\right). \tag{5}$$

Because the baryon number and the charge are conserved in nuclear reactions, the operator T_3 commutes with H even if isospin symmetry is broken.

Charge symmetry represents invariance under reflection about the 1–2 plane in charge space. The charge symmetry operator P_{cs} transforms proton and neutron states into each other and is given by the following [7]:

$$P_{cs} = e^{i\pi T_2} = \prod_{a=1}^{A} e^{i\pi t_2(a)} = \prod_{a=1}^{A} (i\tau_2(a)). \tag{6}$$

Thus charge symmetry conservation means the equivalence of nn and pp, nnn and $ppp, \ldots,$ forces. Obviously, charge symmetry is valid if isospin is conserved, that is, if

$$[H,\ \boldsymbol{T}^2] = [H,\ T_i] = 0. \tag{7}$$

B. Two nucleons

The classification of the two-nucleon forces according to their isospin dependence has been worked out by Henley and Miller [7]. For the sake of completeness, we briefly remind the reader of this classification scheme in what follows.

The two-nucleon forces fall into four classes:

- Class (I) forces, V_I^{2N}, are isospin invariant and can be expressed as follows:

$$V_I^{2N} = \alpha_1 + \alpha_2\, \boldsymbol{t}(1)\cdot \boldsymbol{t}(2), \tag{8}$$

  where α_i are space and spin operators.
- Class (II) forces, V_{II}^{2N}, maintain charge symmetry but break charge independence (i.e., are not isospin invariant[2]):

$$\left[V_{II}^{2N},\ \boldsymbol{T}\right] \neq 0,$$
$$\left[V_{II}^{2N},\ P_{cs}\right] = 0. \tag{9}$$

  The class (II) forces are proportional to the isotensor:

$$V_{II}^{2N} = \alpha\, \tau_3(1)\, \tau_3(2). \tag{10}$$

  It is easy to verify that these forces do not mix isospin in the two-nucleon system and thus satisfy, in addition, the following relation:

$$\left[V_{II}^{2N},\ \boldsymbol{T}^2\right] = 0. \tag{11}$$

- Class (III) forces break charge symmetry but do not lead to isospin mixing in the two-nucleon system:

$$\left[V_{III}^{2N},\ \boldsymbol{T}\right] \neq 0,$$
$$\left[V_{III}^{2N},\ P_{cs}\right] \neq 0, \tag{12}$$
$$\left[V_{III}^{2N},\ \boldsymbol{T}^2\right] = 0.$$

  Such forces have the following general structure:

$$V_{III}^{2N} = \alpha(\tau_3(1) + \tau_3(2)). \tag{13}$$

  and are symmetric under the interchange of the nucleons 1 and 2.
- Finally, class (IV) forces break charge symmetry and cause isospin mixing, that is,

$$\left[V_{IV}^{2N},\ \boldsymbol{T}\right] \neq 0,$$
$$\left[V_{IV}^{2N},\ P_{cs}\right] \neq 0, \tag{14}$$
$$\left[V_{IV}^{2N},\ \boldsymbol{T}^2\right] \neq 0.$$

  They can be expressed as follows:

$$V_{IV}^{2N} = \alpha_1(\tau_3(1) - \tau_3(2)) + \alpha_2[\boldsymbol{\tau}(1)\times\boldsymbol{\tau}(2)]_3. \tag{15}$$

  The operator α_2 has to be odd under a time reversal transformation.

C. Three and more nucleons

Let us now generalize the above treatment to systems with more than two nucleons. Considering the commutation relations of the Hamilton operator H with the operators $\boldsymbol{T}^2$ and P_{cs}, one can distinguish between four different cases for isospin-violating forces: the Hamilton operator may commute with both $\boldsymbol{T}^2$ and P_{cs}, with one of those operators or with none.[3] The problem with such a classification scheme is that conservation of $\boldsymbol{T}^2$ depends on the number of particles. In general, an A-nucleon force that commutes with the squared total isospin operator in the A-nucleon system, $\boldsymbol{T}_A^2 \equiv (\boldsymbol{t}(1) + \boldsymbol{t}(2) + \cdots + \boldsymbol{t}(A))^2$, will not commute with the operator $\boldsymbol{T}_{>A}^2$. For example, all isospin-breaking two-nucleon forces, which cause no isospin mixing in the two-nucleon system, lead to isospin mixing in the three-nucleon system. Conversely, the property of charge symmetry is independent on the number of nucleons and suitable for generalization. Thus in systems with more than two nucleons it is convenient to distinguish between the following three classes of forces: class (I) isospin symmetric forces; class (II) forces, which break isospin but maintain charge symmetry; and class (III) forces, which break both isospin and charge symmetry. For two nucleons, our class

[2]Clearly, V_{II}^{2N} as well as all other considered isospin-violating interactions still commute with the third components of the total isospin for the reason explained before.

[3]In case of two nucleons, only three of these four cases appear, because there are no forces which commute with P_{cs} and do not with the operator $\boldsymbol{T}^2$.

(III) interactions obviously include the class (III) and (IV) forces in the classification by Henley and Miller.

Let us now concentrate on the 3NF and list all possible isospin structures.

- Class (I) forces are isospin scalars and have the following structure:

$$V_I^{3N} = \sum_{i\neq j\neq k} \big(\alpha_I^{ijk} + \beta_I^{ijk}\, \boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j) + \gamma_I^{ijk}[\boldsymbol{\tau}(i)\times\boldsymbol{\tau}(j)]\cdot\boldsymbol{\tau}(k)\big), \qquad (16)$$

where α_I^{ijk}, β_I^{ijk}, and γ_I^{ijk} are space and spin operators with the superscripts being the nucleon labels.

- Class (II) forces satisfy the following:

$$[V_{II},\ \boldsymbol{T}] \neq 0, \quad [V_{II},\ P_{cs}] = 0, \qquad (17)$$

and can be expressed as follows:

$$V_{II}^{3N} = \sum_{i\neq j\neq k} \big(\alpha_{II}^{ijk}\, t_3(i) t_3(j) + \beta_{II}^{ijk}\, [\boldsymbol{\tau}(i)\times\boldsymbol{\tau}(j)]_3 \tau_3(k)\big). \qquad (18)$$

The forces in Eq. (18) give rise to isospin mixing in the 3N system except in the following two cases:

$$\begin{aligned}\alpha_{II}^{123}+\alpha_{II}^{213} &= \alpha_{II}^{132}+\alpha_{II}^{312} = \alpha_{II}^{231}+\alpha_{II}^{321},\\ \beta_{II}^{123}-\beta_{II}^{213} &= \beta_{II}^{312}-\beta_{II}^{132} = \beta_{II}^{231}-\beta_{II}^{321}.\end{aligned} \qquad (19)$$

- Class (III) forces satisfy the following:

$$[V_{II},\ \boldsymbol{T}] \neq 0, \quad [V_{II},\ P_{cs}] \neq 0. \qquad (20)$$

There are four types of such isospin-breaking forces:

$$\begin{aligned}V_{III}^{3N} = \sum_{i\neq j\neq k} \big(&\alpha_{III}^{ijk}\, \tau_3(i) + \beta_{III}^{ijk}\, [\boldsymbol{\tau}(i)\times\boldsymbol{\tau}(j)]_3\\ &+ \gamma_{III}^{ijk}\, \tau_3(i)\, \boldsymbol{\tau}(j)\cdot\boldsymbol{\tau}(k)\\ &+ \kappa_{III}^{ijk}\, \tau_3(i)\,\tau_3(j)\,\tau_3(k)\big).\end{aligned} \qquad (21)$$

The first three terms in Eq. (21) cause isospin mixing in the 3N system except in the following special cases:

$$\begin{aligned}\alpha_{III}^{123}+\alpha_{III}^{132} &= \alpha_{III}^{213}+\alpha_{III}^{231} = \alpha_{III}^{312}+\alpha_{III}^{321},\\ \beta_{III}^{123}-\beta_{III}^{213} &= \beta_{III}^{312}-\beta_{III}^{132} = \beta_{III}^{231}-\beta_{III}^{321},\\ \gamma_{III}^{123}+\gamma_{III}^{132} &= \gamma_{III}^{213}+\gamma_{III}^{231} = \gamma_{III}^{312}+\gamma_{III}^{321}.\end{aligned} \qquad (22)$$

The last term in Eq. (21) does not lead to isospin mixing in the 3N system. Notice further that the quantities β_{ijk} are time reversal odd.

In what follows, we perform explicit calculation of the dominant isospin-violating three-nucleon forces based on chiral effective field theory.

III. ISOSPIN-BREAKING THREE-NUCLEON FORCE IN CHIRAL EFFECTIVE FIELD THEORY

A. Power counting and effective Lagrangian

Isospin-breaking two-nucleon forces have been extensively studied within effective field theory approaches (see, e.g., [9–15]), as well as using more phenomenological methods (see, e.g., [16,17] for some recent references). In the standard model, isospin-violating effects have their origin in both strong (i.e., due to the different masses of the up and down quarks) and electromagnetic interactions (due to different charges of the up and down quarks). The electromagnetic effects can be separated into the ones due to soft and hard photons. Although effects of hard photons are incorporated in effective field theory by inclusion of electromagnetic short distance operators in the effective Lagrangian, soft photons have to be taken into account explicitly.

Consider first isospin breaking in the strong interaction. The QCD quark mass term can be expressed as follows:

$$\mathcal{L}_{\rm mass}^{\rm QCD} = -\bar{q}\,\mathcal{M}\,q = -\frac{1}{2}\bar{q}\,(m_u+m_d)(1-\epsilon\tau_3)\,q, \qquad (23)$$

where

$$\epsilon \equiv \frac{m_d-m_u}{m_d+m_u} \sim \frac{1}{3}. \qquad (24)$$

The above numerical estimation is based on the light quark mass values utilizing a modified $\overline{\rm MS}$ subtraction scheme at a renormalization scale of 1 GeV [18]. The isoscalar term in Eq. (23) breaks chiral but preserves isospin symmetry. It leads to the nonvanishing pion mass, $M_\pi^2 = (m_u+m_d)B \neq 0$, where B is a low-energy constant (LEC) that describes the strength of the bilinear light quark condensates. All chiral-symmetry-breaking interactions in the effective Lagrangian are proportional to positive powers of M_π^2. The isovector term ($\propto\tau_3$) in Eq. (23) breaks isospin symmetry and generates a series of isospin-breaking effective interactions $\propto(\epsilon M_\pi^2)^n$ with $n \geqslant 1$. It therefore appears to be natural to count strong isospin violation in terms of ϵM_π^2. However, we note already here that isospin-breaking effects are in general much smaller than indicated by the numerical value of ε, because the relevant scale for the isospin-conserving contributions is the chiral-symmetry-breaking scale Λ_χ rather than m_u+m_d.

Electromagnetic terms in the effective Lagrangian can be generated using the method of external sources (see, e.g., [19–21] for more details). All such terms are proportional to the nucleon charge matrix $Q = e\,(1+\tau_3)/2$, where e denotes the electric charge.[4] More precisely, the vertices that contain (do not contain) the photon fields are proportional to Q^n (Q^{2n}), where $n = 1, 2, \ldots$. Because we are interested here in nucleon-nucleon scattering in the absence of external fields, so that no photon can leave a Feynman diagram, it is convenient to introduce the small parameter $e^2 \sim 1/10$ for isospin-violating effects caused by the electromagnetic interactions. As discussed below, three-nucleon forces, because of virtual photon exchange, do not contribute at the leading and subleading

[4]Or equivalently, one can use the quark charge matrix $e\,(1/3+\tau_3)/2$.

orders. We therefore do not consider virtual photons in the present work. Notice, however, that electromagnetic effects might be enhanced at low energy due to the long range of the corresponding interaction (see [8] for more details). A systematic study of such effects should therefore be performed in the future. For the first step in this direction see Ref. [22].

In the present study we adopt the same power counting rules for isospin-breaking contributions as introduced in Ref. [8]. Specifically, we count the following:

$$\epsilon \sim e \sim \frac{q}{\Lambda}, \tag{25}$$

where $q \sim M_\pi$ refers to a generic low-momentum scale and Λ to the hard scale that enters the values of the corresponding low-energy constants. In addition, we keep track of the additional factors $1/(4\pi)^2$ arising from the photon loops by counting the following:

$$\frac{e^2}{(4\pi)^2} \sim \frac{q^4}{\Lambda^4}. \tag{26}$$

Notice further that contrary to the standard practice in the single-nucleon sector, the nucleon mass is considered as a much larger scale compared to the chiral-symmetry-breaking scale for reasons explained in Ref. [3]. In this work we adopt the counting rule $q/m \sim (q/\Lambda)^2$, which has also been used in Ref. [23]. Counting the nucleon mass in this way ensures that all iterations of the leading-order NN potential contribute to the scattering amplitude at leading order $(q/\Lambda)^0$ and thus have to be resummed. The N-nucleon force receives contributions of the order $\sim(q/\Lambda)^\nu$, where

$$\nu = -4 + 2n_\gamma + 2N + 2L + \sum_i V_i \Delta_i. \tag{27}$$

Here, L and V_i refer to the number of loops and vertices of type i and n_γ is the number of virtual photons. Further, the vertex dimension Δ_i is given by the following:

$$\Delta_i = d_i + \tfrac{1}{2}n_i - 2, \tag{28}$$

where n_i is the number of nucleon field operators and d_i is the q power of the vertex, which accounts for the number of derivatives and insertions of pion mass, ε and $e/(4\pi)$ according to Eqs. (25) and (26).

Let us now specify the relevant terms in the effective Lagrangian. In the purely pionic sector, we have to take into account the following structures:

$$\mathcal{L}_{\pi\pi} = \frac{F_\pi^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + C\left\langle Q_+^2 - Q_-^2\right\rangle, \tag{29}$$

where F_π refers to the pion decay constant and the brackets $\langle\ \rangle$ denote traces in the flavor space. We remark that various LECs appearing in the effective Lagrangian correspond to bare quantities in the chiral SU(2) limit. Throughout this manuscript we do not specify this explicitly and use physical values for the LECs to express our results for the 3NF. Mass and coupling constant renormalization is detailed, for example, in Refs. [24, 25]. Further,

$$\begin{aligned} u_\mu &= i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger), \quad u = \sqrt{U}, \quad \chi = 2B\mathcal{M}, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u, \qquad Q_\pm = \tfrac{1}{2}(u^\dagger Q u \pm uQu^\dagger), \end{aligned} \tag{30}$$

The unitary 2×2 matrix U in the flavor space collects the pion fields. In the σ-model gauge, it takes the following form:

$$U = \frac{1}{F_\pi}\left[\sqrt{F_\pi^2 - \boldsymbol{\pi}^2} + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right]. \tag{31}$$

The pion mass resulting from Eq. (29) is given by the following:

$$\begin{aligned} M_{\pi^0}^2 &= B(m_u + m_d), \\ M_{\pi^\pm}^2 &= B(m_u + m_d) + \frac{2}{F_\pi^2}e^2 C. \end{aligned} \tag{32}$$

The experimentally known pion mass difference $M_{\pi^\pm} - M_{\pi^0} = 4.6$ MeV allows us to fix the value of the LEC C, $C = 5.9\cdot 10^{-5}$ GeV^4. Notice that the natural scale for this LEC is $F_\pi^2\Lambda^2/(4\pi)^2 \sim 3\cdot 10^{-5}$ GeV^4 if one adopts $\Lambda \sim M_\rho$.

Utilizing the heavy baryon framework, the relevant structures in the single-nucleon Lagrangian are [26] (for a more detailed discussion, see, e.g., the review [27]) as follows:

$$\begin{aligned} \mathcal{L}_{\pi N} = \bar{N}_v\bigg[& iv\cdot D + g_A S\cdot u + c_1\langle\chi_+\rangle + \frac{c_3}{2}\langle u\cdot u\rangle \\ & + \frac{c_4}{2}[S^\mu,\ S^\nu][u_\mu,\ u_\nu] + c_5\hat{\chi}_+ + f_1 F_\pi^2\left\langle Q_+^2 - Q_-^2\right\rangle \\ & + f_2 F_\pi^2\langle Q_+\rangle Q_+ + f_3 F_\pi^2\langle Q_+\rangle^2\bigg]N_v, \end{aligned} \tag{33}$$

where N_v refers to the field operator of a nucleon moving with the velocity v_μ; $c_{1,3,4,5}$ and $f_{1,2,3}$ are the strong and the electromagnetic LECs, respectively; and

$$\begin{aligned} D_\mu &= \partial_\mu + \Gamma_\mu, & \Gamma_\mu &= \tfrac{1}{2}[u^\dagger,\ \partial_\mu u], \\ \hat{\chi}_+ &= \chi_+ - \tfrac{1}{2}\langle\chi_+\rangle, & S_\mu &= \tfrac{1}{2}i\gamma_5\sigma_{\mu\nu}v^\nu. \end{aligned} \tag{34}$$

Keeping the terms with at most two pion fields and switching to the nucleon rest-frame system, the Lagrangian density in Eq. (33) can be expressed in a more convenient form:[5] as follows:

$$\begin{aligned} \mathcal{L}_{\pi N} = N^\dagger\bigg[& i\partial_0 - \Delta m + \frac{g_A}{2F_\pi}\boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi} - \frac{1}{4F_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}) \\ & - \frac{2c_1}{F_\pi^2}M_\pi^2\boldsymbol{\pi}^2 + \frac{c_3}{F_\pi^2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) \\ & - \frac{c_4}{2F_\pi^2}\epsilon_{ijk}\,\epsilon_{abc}\,\sigma_i\tau_a(\nabla_j\pi_b)(\nabla_k\pi_c) \\ & - \frac{c_5}{F_\pi^2}\epsilon M_\pi^2(\boldsymbol{\pi}\cdot\boldsymbol{\tau})\pi_3 + f_1 e^2\left(\pi_3^2 - \boldsymbol{\pi}^2\right) \\ & + \frac{1}{4}f_2 e^2((\boldsymbol{\pi}\cdot\boldsymbol{\tau})\pi_3 - \boldsymbol{\pi}^2\tau_3)\bigg]N. \end{aligned} \tag{35}$$

Notice that at the order we are working, there is no need to distinguish between M_{π^0} and $M_{\pi^\pm}$ in Eq. (35). We have

[5]Notice that only terms with three and more pion fields depend on the specific parametrization of the matrix U.

therefore used the same symbol M_π for both charged and neutral pion masses. The nucleon mass shift Δm in the above equation is given by the following:

$$\Delta m = -4c_1 M_\pi^2 - \tfrac{1}{2}F_\pi^2 e^2(2f_1 + f_2 + 2f_3) - \tfrac{1}{2}\tau_3\left(4c_5\epsilon M_\pi^2 + f_2\, e^2 F_\pi^2\right). \quad (36)$$

The isospin-invariant shift given by the first two terms in Eq. (36) is of no importance and can be absorbed by a redefinition of the bare nucleon mass. The proton-to-neutron mass difference $\delta m \equiv m_p - m_n$ fixes the values of the LECs c_5 and f_2 through the following:

$$\begin{aligned}(\delta m)^{\text{str.}} \equiv (m_p - m_n)^{\text{str.}} &= -4c_5\epsilon M_\pi^2 \\ &= (-2.05 \pm 0.3)\text{ MeV}, \\ (\delta m)^{\text{em.}} \equiv (m_p - m_n)^{\text{em.}} &= -f_2\, e^2 F_\pi^2 \\ &= (0.7 \pm 0.3)\text{ MeV}. \end{aligned} \quad (37)$$

These values are taken from Ref. [28]. The electromagnetic shift is based on an evaluation of the Cottingham sum rule. In principle, this contribution could also be evaluated in chiral perturbation theory, including virtual photons. Although the formalism exists (see, e.g., [19–21]), there are still some subtleties to be addressed [29]. Therefore, we consider the electromagnetic mass shifts for the ground state baryon octet collected in Ref. [28] the best values available. Notice that according to the counting rules of Eqs. (25) and (26), the strong and electromagnetic shifts in Eq. (37) are effects of order q^3 and q^4, respectively. Although the constants c_5 and f_2 can be fixed from Eq. (37), the value of the LEC f_1, which contributes to the isospin-violating $\pi\pi NN$ vertex [see Eq. (35)], is unknown. This term plays an important role in the analysis of isospin violation in pion-nucleon scattering and the evaluation of the ground state characteristics of pionic hydrogen (see [26] and [31], respectively). In the two-nucleon sector, it leads only to an isospin-invariant contribution to the TPEP at NNLO, which has so far not been considered (it can be absorbed in the normalization of the term $\sim c_1$). On the contrary, the resulting contribution to the 3NF is isospin breaking. It, however, does not violate charge symmetry and, therefore, does not contribute to the binding-energy difference of ^{3}H and ^{3}He. We further stress that the f_1 term has not been included in the Lagrangian used in Refs. [17,30,32] because another power counting for the electromagnetic effects was employed (see the discussion in Ref. [9]).

In the few-nucleon sector we need only the following isospin invariant structures:

$$\begin{aligned}\mathcal{L}_{NN} = &-\tfrac{1}{2}C_S(\bar N_v N_v)(\bar N_v N_v) + 2C_T(\bar N_v S_\mu N_v)(\bar N_v S^\mu N_v) \\ &- \tfrac{1}{2}D(\bar N_v N_v)(\bar N_v S\cdot u N_v) \\ &- \tfrac{1}{2}E(\bar N_v N_v)(\bar N_v \boldsymbol{\tau} N_v)\cdot(\bar N_v \boldsymbol{\tau} N_v). \end{aligned} \quad (38)$$

where $C_{S,T}$, D, and E are the corresponding low-energy constants. The Lagrangian density defined in Eq. (38) gives rise to the following relevant terms in the nucleon rest-frame system:

$$\begin{aligned}\mathcal{L}_{NN} = &-\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger\vec\sigma N)(N^\dagger\vec\sigma N) \\ &- \frac{D}{4F_\pi}(N^\dagger N)(N^\dagger\vec\sigma\boldsymbol{\tau} N)\cdot\vec\nabla\boldsymbol{\pi} \\ &- \frac{1}{2}E\,(N^\dagger N)(N^\dagger\boldsymbol{\tau} N)\cdot(N^\dagger\boldsymbol{\tau} N). \end{aligned} \quad (39)$$

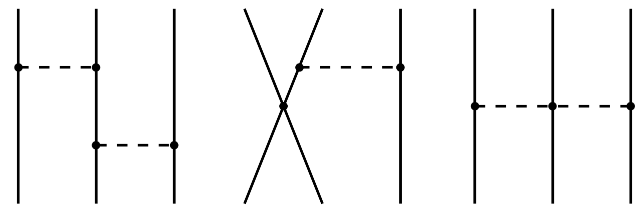

FIG. 1. Leading contribution to the 3NF at the order $(q/\Lambda)^2$ that vanish, as discussed in the text. Solid and dashed lines are nucleons and pions, respectively. Heavy dots denote the leading-order vertices with $\Delta_i = 0$.

B. Three-nucleon force in momentum space

We are now in the position to discuss the leading and subleading isospin-breaking contributions to the 3NF.[6] For the sake of completeness, we briefly remind the reader of the structure of the isospin-conserving 3NF. The leading 3NF contribution of the order $(q/\Lambda)^2$ represented by the graphs in Fig. 1 is well known to vanish. More precisely, the first two graphs [(a) and (b)] in this figure vanish in the static limit if one adopts an energy-independent formalism such as the method of unitary transformation [34]. Alternatively, one can use old-fashioned perturbation theory to derive a corresponding energy-dependent 3NF potential. The latter is known to cancel out the recoil corrections to the 2N potential iterated in the scattering equation [4,35]. It should be understood that the first two diagrams shown in Fig. 1 only specify the topology and do not correspond to Feynman graphs. Clearly, the corresponding contributions to the 3NF do not include the pieces generated by the iteration of the 2NF. We remind the reader that the operators associated with these diagrams depend on the scheme and on the definition of the potential. In the method of unitary transformation, these graphs subsume both irreducible and reducible time-ordered topologies. However, the reducible diagrams do not contain anomalously small energy denominators, which correspond to the purely two-nucleon intermediate states in old-fashioned perturbation theory. The last diagram in Fig. 1 is suppressed by a factor of q/m because the time derivative entering the Weinberg-Tomozawa vertex in Eq. (35).

[6]After submission of our manuscript, related work on charge symmetry breaking in the 3N system by Friar, Payne, and van Kolck [33] appeared.

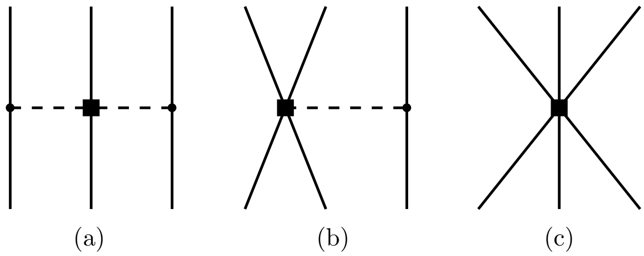

FIG. 2. Subleading contribution to the 3NF at the order $(q/\Lambda)^3$. Solid rectangles refer to vertices with $\Delta_i = 1$. For remaining notation, see the legend to Fig. 1.

The first nonvanishing 3NFs arise at order $(q/\Lambda)^3$ from the diagrams shown Fig. 2 with one subleading vertex of dimension $\Delta_i = 1$. The contribution from the first graph in Fig. 2 is also incorporated in various phenomenological models such as, for example, the TM99 3NF [36] and given by Ref. [4] (see also Ref. [5] for a related discussion):

$$V_{2\pi}^{3N} = \sum_{i\neq j\neq k} \frac{1}{2}\left(\frac{g_A}{2F_\pi}\right)^2 \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(\vec{q}_i^{\,2}+M_\pi^2)(\vec{q}_j^{\,2}+M_\pi^2)} F_{ijk}^{\alpha\beta}\tau_i^\alpha\tau_j^\beta, \tag{40}$$

where $\vec{q}_i \equiv \vec{p}_i^{\,\prime} - \vec{p}_i$; $\vec{p}_i$ ($\vec{p}_i^{\,\prime}$) are initial (final) momenta of the nucleon i and

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-\frac{4\tilde{c}_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2}\vec{q}_i\cdot\vec{q}_j\right] + \sum_\gamma \frac{c_4}{F_\pi^2}\epsilon^{\alpha\beta\gamma}\tau_k^\gamma\vec{\sigma}_k\cdot[\vec{q}_i\times\vec{q}_j].$$

Here and in what follows, we use the usual notation to express the nuclear force: the quantity $V_{2\pi}^{3N}$ is an operator with respect to spin and isospin quantum numbers and a matrix element with respect to momentum quantum numbers. Notice also that we have changed the notation of Sec. II and write the nucleon labels as subscripts of the spin and isospin matrices [i.e., use $\boldsymbol{\tau}_i$ and $\vec{\sigma}_i$ instead of $\boldsymbol{\tau}(i)$ and $\vec{\sigma}(i)$], whereas the superscripts denote corresponding vector indices. Further,

$$\tilde{c}_1 = c_1 + \frac{e^2F_\pi^2 f_1}{2M_\pi^2}. \tag{41}$$

Note that this renormalization of the sigma-term related LEC c_1 by the electromagnetic LEC f_1 was already discussed in the analysis of pion-nucleon scattering [26]. Clearly, this electromagnetic shift of the LEC c_1 represents a higher-order effect and only needs to be taken into account at order $(q/\Lambda)^5$ and higher. The remaining contributions from graphs (b) and (c) in Fig. 2 are given by [6] the following:

$$V_{1\pi}^{3N} = -\sum_{i\neq j\neq k}\frac{g_A}{8F_\pi^2}D\frac{\vec{\sigma}_i\cdot\vec{q}_i}{\vec{q}_i^{\,2}+M_\pi^2}(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i)(\vec{\sigma}_j\cdot\vec{q}_i),$$
$$V_{\rm cont}^{3N} = \frac{1}{2}\sum_{j\neq k}E\,(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k), \tag{42}$$

First isospin-conserving corrections to the 3NF arise at order $(q/\Lambda)^4$, where one has to consider tree diagrams

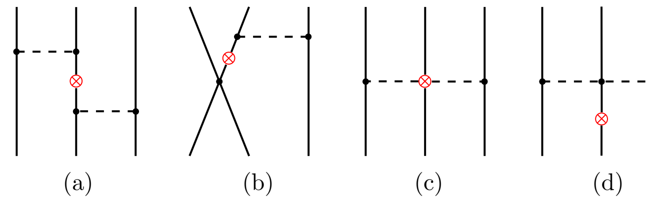

FIG. 3. (Color online) Leading isospin-violating contribution to the 3NF at the order $(q/\Lambda)^4$. Crossed circles refer to isospin-breaking vertices with $\Delta_i = 2$. For remaining notation, see the legend to Fig. 1.

with one vertex of the dimension $\Delta_i = 2$ as well various one-loop diagrams with the leading vertices. Derivation of these corrections to the 3NF will be published elsewhere. The main focus of the present work is related to isospin-breaking corrections that first appear at the same order $(q/\Lambda)^4$ and are given by the graphs in Fig. 3. The first two diagrams (a) and (b) and the last one (d) are due to strong nucleon mass shift and of the order $\epsilon(q/\Lambda)^3 \sim (q/\Lambda)^4$. It should be understood that the proton-to-neutron mass difference has to be taken into account not only for intermediate but also for incoming and outgoing nucleon states. The corresponding corrections to the two-nucleon force have been recently studied in [14,15]. In what follows, we do not separate the electromagnetic and strong shifts in the nucleon mass and express the result in terms of the proton-to-neutron mass difference $\delta m = m_p - m_n$. We use the method of unitary transformation as detailed in Ref. [37] to calculate the relevant 3NF contributions. Utilizing the notation of this reference, the corresponding two-pion exchange potential can be written as follows:

$$\begin{aligned} V_{2\pi} = \eta'\Bigg[&\frac{1}{2}H_1\frac{\lambda^1}{(H_0-E_{\eta'})}H_1\,\tilde{\eta}\,H_1\frac{\lambda^1}{(H_0-E_{\tilde{\eta}})(H_0-E_{\eta'})}H_1 \\ &-\frac{1}{8}H_1\frac{\lambda^1}{(H_0-E_{\eta'})}H_1\,\tilde{\eta}\,H_1\frac{\lambda^1}{(H_0-E_{\tilde{\eta}})(H_0-E_\eta)}H_1 \\ &+\frac{1}{8}H_1\frac{\lambda^1}{(H_0-E_{\eta'})(H_0-E_{\tilde{\eta}})}H_1\,\tilde{\eta}\,H_1\frac{\lambda^1}{(H_0-E_{\tilde{\eta}})}H_1 \\ &-\frac{1}{2}H_1\frac{\lambda^1}{(H_0-E_\eta)}H_1\,\frac{\lambda^2}{(H_0-E_\eta)} \\ &\times H_1\frac{\lambda^1}{(H_0-E_\eta)}H_1\Bigg]\eta + \text{h.c.} \end{aligned} \tag{43}$$

Here η, η', and $\tilde{\eta}$ denote the projectors on the purely nucleonic subspace of the Fock space, whereas λ^i refers to the projector on the states with i pions. Further, H_1 is the leading πNN vertex corresponding to the third term in the first line of Eq. (35), H_0 denotes the free Hamilton operator for pions and nucleons corresponding to the density

$$\mathcal{H}_0 = \tfrac{1}{2}\dot{\boldsymbol{\pi}}^2 + \tfrac{1}{2}(\vec{\nabla}\boldsymbol{\pi})^2 + \tfrac{1}{2}M_\pi^2\boldsymbol{\pi}^2 + \tfrac{1}{2}N^\dagger\delta m\tau_3 N, \tag{44}$$

and E_η, E'_η, and $E_{\tilde{\eta}}$ refer to the energy of the nucleons in the states η, η', and $\tilde{\eta}$, respectively. Notice that the first three terms in Eq. (43) subsume the contributions of the reducible graphs, whereas the last term refers to the irreducible topology. Neglecting the proton-to-neutron mass difference in Eq. (44) one recovers the isospin symmetric result of [37]:

$$V_{2\pi} = \eta' \left[\frac{1}{2} H_1 \frac{\lambda^1}{(\omega)^2} H_1 \, \tilde{\eta} \, H_1 \frac{\lambda^1}{\omega} H_1 + \frac{1}{2} H_1 \frac{\lambda^1}{\omega} H_1 \, \tilde{\eta} \, H_1 \frac{\lambda^1}{(\omega)^2} H_1 - H_1 \frac{\lambda^1}{\omega} H_1 \frac{\lambda^2}{\omega_1 + \omega_2} H_1 \frac{\lambda^1}{(\omega)^2} H_1 \right] \eta, \tag{45}$$

where ω denotes the pionic free energy. We remark that Eq. (43) can also be used to calculate relativistic $1/m$ corrections to the two-pion exchange potential if one keeps the nucleon kinetic energy term in Eq. (44). An additional unitary transformation should, however, be performed to end up with the potential used in Ref. [8].

Explicit evaluation of the 3NF using Eq. (43) leads to the following result:

$$V_{2\pi}^{3\mathrm{N}} = \sum_{i \neq j \neq k} 2\delta m \left(\frac{g_A}{2F_\pi} \right)^4 \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{\left(\vec{q}_i^{\,2} + M_\pi^2\right)^2 \left(\vec{q}_j^{\,2} + M_\pi^2\right)} \times \left\{ [\vec{q}_i \times \vec{q}_j] \cdot \vec{\sigma}_k \, [\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j]^3 + \vec{q}_i \cdot \vec{q}_j \times \left[(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) \tau_j^3 - (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \tau_k^3 \right] \right\}. \tag{46}$$

Notice that we have expanded the energy denominators in powers of δm in Eq. (43) and kept only the linear terms. Similarly to the case of the two-nucleon potential [14], the resulting 3NF is entirely due to irreducible diagrams. As a cross-check of our approach, we have also calculated the two-pion exchange 2NF corresponding to Eq. (43) and recovered the results of Ref. [14]. The contribution of the one-pion exchange diagram (b) in Fig. 3 is given by the following operators:

$$V_{1\pi} = \eta' \left[-\frac{1}{2} H_1 \frac{\lambda^1}{(H_0 - E_\eta)(H_0 - E_{\tilde{\eta}})} H_1 \, \tilde{\eta} \, H_2 + \frac{1}{2} H_1 \frac{\lambda^1}{(H_0 - E_\eta)} H_2 \frac{\lambda^1}{(H_0 - E_\eta)} H_1 \right] \eta + \mathrm{h.c.}, \tag{47}$$

where H_2 corresponds to the first two terms in Eq. (39). Similarly to the previously considered case, we recover the result of Ref. [37] in the limit $\delta m \to 0$:

$$V_{1\pi} = \eta' \left[-\frac{1}{2} H_1 \frac{\lambda^1}{(\omega)^2} H_1 \, \tilde{\eta} \, H_2 - \frac{1}{2} H_2 \, \tilde{\eta} \, H_1 \frac{\lambda^1}{(\omega)^2} H_1 + H_1 \frac{\lambda^1}{\omega} H_2 \frac{\lambda^1}{\omega} H_1 \right] \eta. \tag{48}$$

We find the following expression for the isospin-breaking one-pion exchange 3NF:

$$V_{1\pi}^{3\mathrm{N}} = \sum_{i \neq j \neq k} 2\, \delta m \, C_T \left(\frac{g_A}{2F_\pi} \right)^2 \times \frac{\vec{\sigma}_i \cdot \vec{q}_i}{\left(\vec{q}_i^{\,2} + M_\pi^2\right)^2} \, [\boldsymbol{\tau}_k \times \boldsymbol{\tau}_i]^3 \, [\vec{\sigma}_j \times \vec{\sigma}_k] \cdot \vec{q}_i. \tag{49}$$

Notice that $V_{1\pi}^{3\mathrm{N}}$ can be rewritten in an equivalent form making use of the following relation:

$$[\boldsymbol{\tau}_k \times \boldsymbol{\tau}_i]^3 \, [\vec{\sigma}_j \times \vec{\sigma}_k] \cdot \vec{q}_i = \left((\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \tau_k^3 - (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) \tau_j^3 \right) (\vec{\sigma}_j \cdot \vec{q}_i), \tag{50}$$

which holds true when the corresponding operators act on antisymmetrized states with respect to j and k.

The diagram in Fig. 3(c) is due to the c_5 term in Eq. (35) and of the order $\epsilon (q/\Lambda)^3 \sim (q/\Lambda)^4$ as well. Denoting the interaction $\propto c_5$ by H_3, the contribution of this graph is given by the following:

$$V_{2\pi} = \eta' \left[H_1 \frac{\lambda^1}{\omega} H_3 \frac{\lambda^1}{\omega} H_1 + H_1 \frac{\lambda^1}{\omega} H_1 \frac{\lambda^2}{(\omega_1 + \omega_2)} H_3 + H_3 \frac{\lambda^2}{(\omega_1 + \omega_2)} H_1 \frac{\lambda^1}{\omega} H_1 \right] \eta. \tag{51}$$

Alternatively, one can use the Feynman graph technique to evaluate the corresponding 3NF. We find the following:

$$V_{2\pi}^{3\mathrm{N}} = \sum_{i \neq j \neq k} \frac{(\delta m)^{\mathrm{str.}}}{4F_\pi^2} \left(\frac{g_A}{2F_\pi} \right)^2 \times \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{\left(\vec{q}_i^{\,2} + M_\pi^2\right) \left(\vec{q}_j^{\,2} + M_\pi^2\right)} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) \tau_j^3. \tag{52}$$

Notice that all leading (i.e., $\sim (q/\Lambda)^4$) isospin-violating 3NFs given by Eqs. (46), (49), and (52) are charge-symmetry-breaking, that is, of class (III) in the notation of Sec. II C. We further point out that although the $M_{\pi^\pm} \neq M_{\pi^0}$ corrections to the graphs in Fig. 1, given by the first three graphs, (a), (b), and (c), in Fig. 6, are formally also of the order $(q/\Lambda)^4$, they lead to $1/m$-suppressed contributions to the 3NF for the same reason as do the corresponding isospin-conserving terms.

The contribution of the last diagram (d) in Fig. 3 is given by the following:

$$V_{2\pi} = \frac{1}{2} \eta' \left[H_1 \frac{\lambda^1}{(H_0 - E_\eta)} H_1 \frac{\lambda^2}{(H_0 - E_\eta)} H_1^{\mathrm{WT}} + H_1^{\mathrm{WT}} \frac{\lambda^2}{(H_0 - E_\eta)} H_1 \frac{\lambda^1}{(H_0 - E_\eta)} H_1 + H_1 \frac{\lambda^1}{(H_0 - E_\eta)} H_1^{\mathrm{WT}} \frac{\lambda^1}{(H_0 - E_\eta)} H_1 \right] \eta + \mathrm{h.c.}, \tag{53}$$

where H_1^{WT} refers to the Weinberg-Tomozawa vertex. Explicit evaluation of this graph can be performed expanding the above expression in powers of δm and keeping the terms $\propto \delta m$. Alternatively, one can use the Feynman graph technique. In that case one should use for the energy transfer of the nucleon i: $q_i^0 = (p_i')^0 - p_i^0 = \Delta m + \mathcal{O}(m^{-1})$, where Δm denotes the nucleon mass difference in the final and initial state. We find the following:

$$V_{2\pi}^{3\mathrm{N}} = \sum_{i \neq j \neq k} \frac{\delta m}{4F_\pi^2} \left(\frac{g_A}{2F_\pi} \right)^2 \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{\left(\vec{q}_i^{\,2} + M_\pi^2\right) \left(\vec{q}_j^{\,2} + M_\pi^2\right)} \times \left[(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) \tau_j^3 - (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \tau_k^3 \right]. \tag{54}$$

The first corrections to the leading isospin-breaking 3NFs arise from diagrams (a), (b), and (e) in Fig. 4 and are of the order $(e/4\pi)^2 q/\Lambda \sim (q/\Lambda)^5$. Notice that the contributions of the graphs (c), (d), and (f) in this figure are already included in Eqs. (46), (49), and (54). The first two graphs in Fig. 4 represent isospin-violating corrections to graphs (a) and (b) in Fig. 2 because of the pion mass difference and lead to the

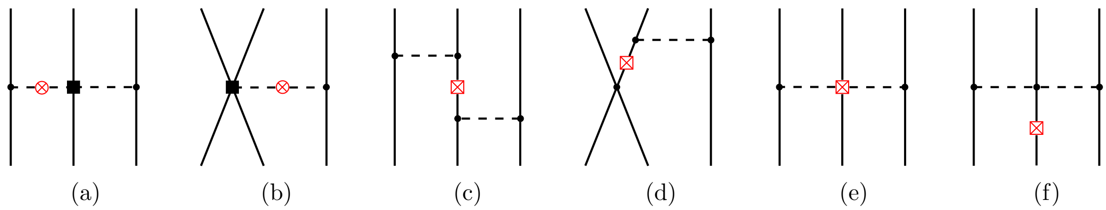

FIG. 4. (Color online) Subleading isospin-violating contribution to the 3NF at the order $(q/\Lambda)^5$. Crossed rectangles refer to isospin-breaking vertices with $\Delta_i = 3$. For remaining notation, see the legend to Fig. 1.

following:

$$V_{2\pi}^{3N} = \sum_{i\neq j\neq k} \delta M_\pi^2 \left(\frac{g_A}{2F_\pi}\right)^2 \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(\vec{q}_i^{\,2}+M_\pi^2)^2(\vec{q}_j^{\,2}+M_\pi^2)} \times \left\{\tau_i^3\tau_j^3\left[-\frac{4c_1M_\pi^2}{F_\pi^2}+\frac{2c_3}{F_\pi^2}(\vec{q}_i\cdot\vec{q}_j)\right] + \frac{c_4}{F_\pi^2}\tau_i^3[\boldsymbol{\tau}_j\times\boldsymbol{\tau}_k]^3[\vec{q}_i\times\vec{q}_j]\cdot\vec{\sigma}_k\right\}$$

$$V_{1\pi}^{3N} = -\sum_{i\neq j\neq k} \delta M_\pi^2 \frac{g_A}{8F_\pi^2} D \frac{\vec{\sigma}_i\cdot\vec{q}_i}{(\vec{q}_i^{\,2}+M_\pi^2)^2}\tau_i^3\tau_j^3(\vec{\sigma}_j\cdot\vec{q}_i), \tag{55}$$

where we have defined

$$\delta M_\pi^2 = M_{\pi^\pm}^2 - M_{\pi^0}^2. \tag{56}$$

Notice that at this order [i.e., at $(q/\Lambda)^5$] one has to distinguish between the charged and neutral pion masses in the pion propagators in Eqs. (40) and (42). Isospin-violating corrections in Eq. (55) are consistent with taking $M_{\pi^\pm}$ in the pion propagators in Eqs. (40) and (42). The contribution of diagram (e) can be obtained from Eq. (51) as follows:

$$V_{2\pi}^{3N} = \sum_{i\neq j\neq k}\left(\frac{g_A}{2F_\pi}\right)^2\frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(\vec{q}_i^{\,2}+M_\pi^2)(\vec{q}_j^{\,2}+M_\pi^2)}\left\{\frac{(\delta m)^{\rm em.}}{4F_\pi^2} \times \left((\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\tau_k^3-(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)\tau_j^3\right)+f_1e^2\tau_i^3\tau_j^3\right\}. \tag{57}$$

The 3NFs resulting from $M_{\pi^\pm}\neq M_{\pi^0}$ in graphs (a) and (b) of Fig. 4 are charge symmetry conserving (i.e., class (II)), whereas diagram (e) in this figure gives rise to both charge-symmetry-conserving ($\propto f_1$) and charge-symmetry-breaking [$\propto(\delta m)^{\rm em.}$] 3NFs. We stress again that the contribution $\sim f_1$ is considered here for the first time. Notice further that the charge-symmetry-breaking 3NFs in Eqs. (52), (54), and (57) can be combined into the following:

$$V_{2\pi}^{3N} = \sum_{i\neq j\neq k}\frac{(\delta m)^{\rm str.}}{4F_\pi^2}\left(\frac{g_A}{2F_\pi}\right)^2\frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(\vec{q}_i^{\,2}+M_\pi^2)(\vec{q}_j^{\,2}+M_\pi^2)} \times\left[2(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)\tau_j^3-(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\tau_k^3\right]. \tag{58}$$

The coordinate space representation of the obtained 3NFs is given in the Appendix. Notice that there exist further diagrams at this order which, however, lead to vanishing contributions and are not considered in the present work.

Let us now comment on the obtained results. First, we notice a (formally) larger relative size of the isospin-breaking corrections compared to the two-nucleon sector. Indeed, isospin-breaking 3NFs are suppressed by q/Λ compared to the isospin-conserving 3NFs, whereas the suppression factor in the case of the 2NF is $(q/\Lambda)^2$. Second, the leading isospin-breaking corrections to the 2N and 3N forces arise from different sources. In particular, the dominant contribution to the 3NF is governed by the proton-to-neutron mass difference, which only gives a sub-subleading isospin-breaking correction to the 2N force. Further, charge dependence of the pion-nucleon coupling constant does not show up in the 3NF at the considered order. Similarly, the leading isospin-breaking 3N contact interaction is of the order $\epsilon M_\pi^2(q/\Lambda)^3 \sim (q/\Lambda)^6$ and therefore does not need to be included. Last but not least, we notice that the hierarchy of isospin-violating forces observed in the two-nucleon system (i.e., charge-independence-breaking forces are stronger than charge-symmetry-breaking forces [9]) is not valid for three-nucleon forces.

C. Estimation of the size of the isospin-breaking 3NFs

Having derived the dominant isospin-breaking 3NF corrections it would be very interesting to see how large the effects actually are. This, however, requires explicit calculations of few-nucleon observables, which goes beyond the scope of the present study. Here we restrict ourselves to the following very rough estimation. Consider the two-pion-exchange correction given in Eq. (46). Approximating $1/(\vec{q}_i^{\,2}+M_\pi^2)\sim 1/M_\pi^2$ we obtain the same spin-space structures as the ones that enter the leading isospin conserving 3NF in Eq. (40). Neglecting the isospin structure one observes that the strength of the isospin-breaking terms in Eqs. (46), (52), and (54) reaches a few percentages of the strength of the corresponding isospin-conserving pieces in Eq. (40). Based on the above estimates and on the fact that two-pion exchange 3NFs typically contribute several hundred kiloelectron volts to the binding energy of ^{3}H and ^{3}He,[7] one might expect the contribution of the isospin-breaking 3NF in Eq. (46) to the ^{3}He-^{3}H binding-energy difference to reach 10 . . . 20 keV. Conversely, the relative

[7]In [38] contributions of various pieces of the Tucson–Melbourne 3NF to the ^{3}H are considered. While the so-called a term (it corresponds to the c_1 term in the chiral 3NF) was found to provide only a tiny contribution, the $b(\propto c_3)$ and d terms ($\propto c_4$) give about 250 . . . 300 keV each. In the analysis [6] based on chiral EFT, the expectation value of the two-pion exchange 3NF for ^{3}H (with the reduced values of the LECs $c_{3,4}$) was found to be 390 . . . 730 keV depending on the cutoff chosen.

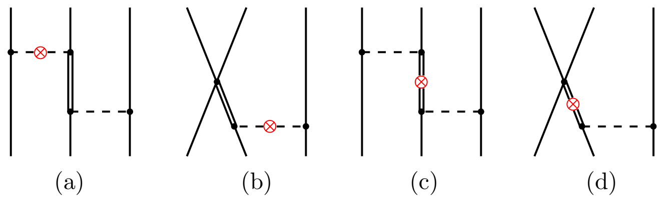

FIG. 5. (Color online) Isospin-violating contribution to the 3NF due to intermediate Δ-excitation (double lines), which are not considered explicitly in the present work. The effect of such diagrams is hidden in certain contact operators that originates from integrating out the delta in the approach considered here. For remaining notation, see the legend to Fig. 1.

strength of the formally subleading two-pion exchange terms in Eqs. (55) and (57) reaches even $2\delta M_\pi^2/M_\pi^2 \sim 15\%$. This surprisingly large size of the subleading isospin-breaking corrections compared to the leading ones is due to the LECs $c_{1,3,4}$, which enter Eq. (55) and are numerically large (the physics behind this enhancement of the LECs is well understood [39]).[8] Notice that a similar situation occurs for the isospin-conserving two-pion exchange 2N force, where the numerically dominant contributions are provided by subleading terms. One should, however, keep in mind that the isospin-breaking 3NFs $\propto c_{1,3,4}$ do not lead to charge symmetry breaking and thus do not contribute to the ^{3}He-^{3}H binding-energy difference. The leading charge-symmetry-breaking one-pion-exchange 3NF in Eq. (49) is numerically smaller in size than the corresponding subleading charge-symmetry-conserving contribution in Eq. (55) as well, although the reason is now completely different. The 3NF in Eq. (49) is proportional to the LEC C_T, which is numerically small [40].[9] It should be noted in this context that the size of the isospin-breaking 3NFs would be more natural if one would treat the Δ isobar as an explicit degree of freedom. In that case a large portion of the subleading 3NFs $\propto c_{3,4}$ and D because of graphs (a) and (b) in Fig. 5 would be promoted to the leading order. Note also that such an approach with explicit deltas is much more complicated because one has to deal with more structures and also needs to reanalyze pion-nucleon scattering (for an attempt see, e.g., Ref. [42]). Further, one should keep in mind that the above numerical estimates are very rough. In particular, taking into account the neglected isospin structure will change the numbers by several times depending on the process considered. Thus, only explicit calculation of various few-nucleon observables will provide quantitative insights on the size of the derived 3NFs.

Finally, we point out that there are many $1/m$ corrections to the obtained results, some of which are depicted in Fig. 6. Because we consider the nucleon mass as a larger scale compared to Λ, such relativistic corrections are irrelevant at the order considered in this work. Notice, however, that if one would adopt the counting rule $m \sim \Lambda$, various $1/m$ corrections (including the ones due to virtual photons) would have to be included at the subleading order $(q/\Lambda)^5$. Some 3NF diagrams due to virtual photon exchange have been considered by Yang and found to provide relatively small contributions of the order of $\sim$7 keV to the ^{3}He-^{3}H binding-energy difference [43,44]. Furthermore, we remind the reader that the long-range electromagnetic 3NFs might, in principle, give rise to large contributions to scattering observables under certain kinematic conditions [8].

IV. SUMMARY

Here we summarize the pertinent results of this investigation:

i. We have given a classification scheme for A-nucleon forces according to their isospin dependence. In the 3N system, one finds three different classes of forces, according to their transformation properties under isospin and charge-symmetry transformations.

ii. We have worked out the leading and subleading isospin-violating 3NFs. The leading contributions are generated by one- and two-pion exchange diagrams with their strength given by the strong neutron-proton mass difference. The subleading corrections are again given by one- and two-pion exchange diagrams, driven largely by the charged-to-neutral pion mass difference and also by the electromagnetic neutron-proton mass difference and the dimension two electromagnetic LEC f_1, that plays an important role in the pion-nucleon system.

iii. We have estimated the relative strength of the leading and subleading corrections compared to the isospin-conserving 3NF at the same order. Isospin-violating 3NFs are expected

[8]In that article it was shown that the smallness of the $N\Delta$ mass splitting enhances certain pion-nucleon LECs when one integrates out the δ. Furthermore, scalar and vector mesons make large contributions to c_1 and c_4, respectively.

[9]In EFT without or with perturbative pions, one has $C_T = 0$ in the limit when both NN S-wave scattering lengths go to infinity [41].

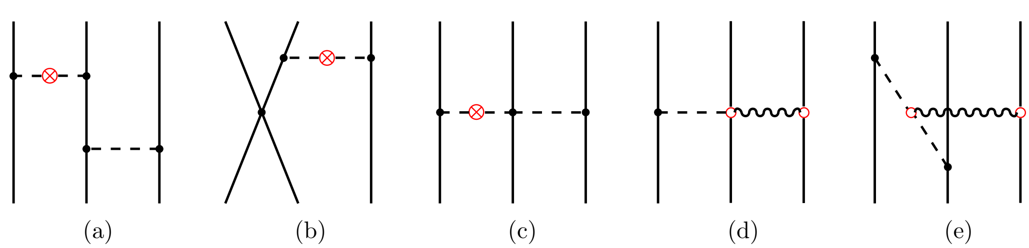

FIG. 6. (Color online) Selected $1/m$ corrections to the isospin-violating 3NF, which are not considered in the present work. Wavy lines refer to photons, and open circles denote vertices with photons. For remaining notation, see the legend to Fig. 1.

to provide a small but nonnegligible contribution to the ^{3}He-^{3}H binding-energy difference.

In the future, these isospin-breaking forces should be used to analyze three- and four-nucleon systems based on chiral EFT, extending, for example, the work presented in Ref. [6].

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APPENDIX: COORDINATE SPACE REPRESENTATION

The leading and subleading 3NFs are local and can easily be transformed into coordinate space. We first define the following operators:

$$
\begin{aligned}
O^1_{ijk} &= \int \frac{d^3q_i}{(2\pi)^3}\frac{d^3q_j}{(2\pi)^3}\, e^{i\vec q_i\cdot\vec r_{ik}}\, e^{i\vec q_j\cdot\vec r_{jk}} \\
&\quad\times \frac{(\vec\sigma_i\cdot\vec q_i)(\vec\sigma_j\cdot\vec q_j)}{\left(\vec q_i^{\,2}+M_\pi^2\right)^2\left(\vec q_j^{\,2}+M_\pi^2\right)}\,[\vec q_i\times\vec q_j]\cdot\vec\sigma_k \\
&= (\vec\sigma_i\cdot\vec\nabla_{ik})(\vec\sigma_j\cdot\vec\nabla_{jk})[\vec\nabla_{ik}\times\vec\nabla_{jk}]\cdot\vec\sigma_k \\
&\quad\times h_2(r_{ik})\,h_1(r_{jk}) \\
&= \frac{1}{32\pi^2}\frac{e^{-x_{ik}}}{r_{ik}}\frac{e^{-x_{jk}}}{r_{jk}^3}\Big((\vec\sigma_i\cdot\hat{\vec r}_{ik})(\vec\sigma_j\cdot\hat{\vec r}_{jk})\,[\hat{\vec r}_{ik}\times\hat{\vec r}_{jk}]\cdot\vec\sigma_k \\
&\quad\times(1+x_{ik})\left(3+3x_{jk}+x_{jk}^2\right)+[\vec\sigma_i\times\vec\sigma_j]\cdot\vec\sigma_k\,(1+x_{jk}) \\
&\quad-(\vec\sigma_i\cdot\hat{\vec r}_{ik})\,[\hat{\vec r}_{ik}\times\vec\sigma_j]\cdot\vec\sigma_k\,(1+x_{ik})(1+x_{jk}) \\
&\quad-[\vec\sigma_i\times\hat{\vec r}_{jk}]\cdot\vec\sigma_k\,(\vec\sigma_j\cdot\hat{\vec r}_{jk})\left(3+3x_{jk}+x_{jk}^2\right)\Big) \\
&\quad+\frac{1}{24\pi}\frac{e^{-x_{ik}}}{r_{ik}}\,\delta^3(r_{jk})\,\Big((\vec\sigma_i\cdot\hat{\vec r}_{ik})\,[\hat{\vec r}_{ik}\times\vec\sigma_j]\cdot\vec\sigma_k \\
&\quad\times(1+x_{ik})-[\vec\sigma_i\times\vec\sigma_j]\cdot\vec\sigma_k\Big)
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
O^2_{ijk} &= \int \frac{d^3q_i}{(2\pi)^3}\frac{d^3q_j}{(2\pi)^3}\, e^{i\vec q_i\cdot\vec r_{ik}}\, e^{i\vec q_j\cdot\vec r_{jk}} \\
&\quad\times \frac{(\vec\sigma_i\cdot\vec q_i)(\vec\sigma_j\cdot\vec q_j)}{\left(\vec q_i^{\,2}+M_\pi^2\right)^2\left(\vec q_j^{\,2}+M_\pi^2\right)}\,(\vec q_i\cdot\vec q_j) \\
&= (\vec\sigma_i\cdot\vec\nabla_{ik})(\vec\sigma_j\cdot\vec\nabla_{jk})(\vec\nabla_{ik}\cdot\vec\nabla_{jk})\,h_2(r_{ik})\,h_1(r_{jk}) \\
&= \frac{1}{32\pi^2}\frac{e^{-x_{ik}}}{r_{ik}}\frac{e^{-x_{jk}}}{r_{jk}^3}\Big((\vec\sigma_i\cdot\hat{\vec r}_{ik})(\vec\sigma_j\cdot\hat{\vec r}_{jk})\,(\hat{\vec r}_{ik}\cdot\hat{\vec r}_{jk}) \\
&\quad\times(1+x_{ik})\left(3+3x_{jk}+x_{jk}^2\right)+(\vec\sigma_i\cdot\vec\sigma_j)\,(1+x_{jk}) \\
&\quad-(\vec\sigma_i\cdot\hat{\vec r}_{ik})\,(\vec\sigma_j\cdot\hat{\vec r}_{ik})\,(1+x_{ik})(1+x_{jk}) \\
&\quad-(\vec\sigma_i\cdot\hat{\vec r}_{jk})\,(\vec\sigma_j\cdot\hat{\vec r}_{jk})\left(3+3x_{jk}+x_{jk}^2\right)\Big) \\
&\quad+\frac{1}{24\pi}\frac{e^{-x_{ik}}}{r_{ik}}\,\delta^3(r_{jk})\,\Big((\vec\sigma_i\cdot\hat{\vec r}_{ik})\,(\vec\sigma_j\cdot\hat{\vec r}_{ik}) \\
&\quad\times(1+x_{ik})-(\vec\sigma_i\cdot\vec\sigma_j)\Big)
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
O^3_{ijk} &= \int \frac{d^3q_i}{(2\pi)^3}\frac{d^3q_j}{(2\pi)^3}\, e^{i\vec q_i\cdot\vec r_{ik}}\, e^{i\vec q_j\cdot\vec r_{jk}} \\
&\quad\times \frac{(\vec\sigma_i\cdot\vec q_i)(\vec\sigma_j\cdot\vec q_j)}{\left(\vec q_i^{\,2}+M_\pi^2\right)\left(\vec q_j^{\,2}+M_\pi^2\right)} \\
&= -(\vec\sigma_i\cdot\vec\nabla_{ik})(\vec\sigma_j\cdot\vec\nabla_{jk})\,h_1(r_{ik})\,h_1(r_{jk}) \\
&= \frac{1}{16\pi^2}\frac{e^{-x_{ik}}}{r_{ik}^2}\frac{e^{-x_{jk}}}{r_{jk}^2}(\vec\sigma_i\cdot\hat{\vec r}_{ik})(\vec\sigma_j\cdot\hat{\vec r}_{jk} \\
&\quad\times(1+x_{ik})(1+x_{jk})
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
O^4_{ijk} &= \int \frac{d^3q_i}{(2\pi)^3}\frac{d^3q_j}{(2\pi)^3}\, e^{i\vec q_i\cdot\vec r_{ik}}\, e^{i\vec q_j\cdot\vec r_{jk}} \\
&\quad\times \frac{(\vec\sigma_i\cdot\vec q_i)(\vec\sigma_j\cdot\vec q_j)}{\left(\vec q_i^{\,2}+M_\pi^2\right)^2\left(\vec q_j^{\,2}+M_\pi^2\right)} \\
&= -(\vec\sigma_i\cdot\vec\nabla_{ik})\left(\vec\sigma_j\cdot\vec\nabla_{jk}\right)h_2(r_{ik})\,h_1(r_{jk}) \\
&= \frac{1}{32\pi^2}\,e^{-x_{ik}}\,\frac{e^{-x_{jk}}}{r_{jk}^2}(\vec\sigma_i\cdot\hat{\vec r}_{ik}) \\
&\quad\times(\vec\sigma_j\cdot\hat{\vec r}_{jk})\,(1+x_{jk})
\end{aligned}
\tag{A4}
$$

$$
\begin{aligned}
O^5_{ijk} &= \int \frac{d^3q_i}{(2\pi)^3}\frac{d^3q_j}{(2\pi)^3}\, e^{i\vec q_i\cdot\vec r_{ik}}\, e^{i\vec q_j\cdot\vec r_{jk}} \\
&\quad\times \frac{(\vec\sigma_i\cdot\vec q_i)}{\left(\vec q_i^{\,2}+M_\pi^2\right)^2}\,(\vec\sigma_j\cdot\vec q_i) \\
&= -(\vec\sigma_i\cdot\vec\nabla_{ik})(\vec\sigma_j\cdot\vec\nabla_{ik})\,h_2(r_{ik})\,g(r_{jk}) \\
&= -\frac{1}{8\pi}\frac{e^{-x_{ik}}}{r_{ik}}\,\delta^3(r_{jk})\,\Big((\vec\sigma_i\cdot\hat{\vec r}_{ik})(\vec\sigma_j\cdot\hat{\vec r}_{ik}) \\
&\quad\times(1+x_{jk})-(\vec\sigma_i\cdot\vec\sigma_j)\Big).
\end{aligned}
\tag{A5}
$$

Here $\vec r_{ij}$ is the relative distance between the nucleons i and j, $r_{ij}=|\vec r_{ij}|$, $\hat{\vec r}_{ij}=\vec r_{ij}/r_{ij}$, and $x_{ij}=M_\pi r_{ij}$. Further,

$$
\begin{aligned}
h_1(r) &= \int\frac{d^3q}{(2\pi)^3}\,\frac{e^{i\vec q\cdot\vec r}}{\left(\vec q^{\,2}+M_\pi^2\right)}=\frac{1}{4\pi r}e^{-M_\pi r}, \\
h_2(r) &= \int\frac{d^3q}{(2\pi)^3}\,\frac{e^{i\vec q\cdot\vec r}}{\left(\vec q^{\,2}+M_\pi^2\right)^2}=\frac{1}{8\pi M_\pi}e^{-M_\pi r}, \\
g(r) &= \int\frac{d^3q}{(2\pi)^3}\,e^{i\vec q\cdot\vec r}=\delta^3(r).
\end{aligned}
\tag{A6}
$$

The isospin-violating 3NF in Eqs. (46), (49), (52), (55), and (57) can now be expressed in terms of the operators $O^{1\ldots5}_{ijk}$ defined above:

$$
V^{3\mathrm{N}}=\sum_{i\neq j\neq k}\left(\frac{g_A}{2F_\pi}\right)^2\Bigg\{(\boldsymbol\tau_i\cdot\boldsymbol\tau_j)\tau_k^3\Bigg[-2\left(\frac{g_A}{2F_\pi}\right)^2\delta m\; O^2_{ijk}
$$

$$
\begin{aligned}
& - \frac{1}{4F_\pi^2}(\delta m)^{\text{str.}} O^3_{ijk} + 2\delta m\, C_T\, O^5_{ijk} \Bigg] + (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)\tau_j^3 \\
& \times \left[2\left(\frac{g_A}{2F_\pi}\right)^2 \delta m\ O^2_{ijk} + \frac{1}{2F_\pi^2}(\delta m)^{\text{str.}}\ O^3_{ijk} \right. \\
& \left. - 2\delta m\, C_T\, O^5_{ijk} \right] + [\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j]^3 \left(\frac{g_A}{2F_\pi}\right)^2 2\delta m\ O^1_{ijk} \\
& + \tau_i^3\, [\boldsymbol{\tau}_j \times \boldsymbol{\tau}_k]^3\, \frac{1}{F_\pi^2}\, \delta M_\pi^2\, c_4\ O^1_{ijk} + \tau_i^3 \tau_j^3 \\
& \times \left[\frac{2}{F_\pi^2} \delta M_\pi^2 c_3 O^2_{ijk} + f_1 e^2 O^3_{ijk} - \frac{4}{F_\pi^2} c_1 M_\pi^2 \delta M_\pi^2 O^4_{ijk} \right. \\
& \left. - \frac{1}{2g_A} D \delta M_\pi^2 O^5_{ijk} \right] \Bigg\} \qquad \text{(A7)}
\end{aligned}
$$

Notice that the expressions for the operators $O^{1\ldots5}_{ijk}$ in Eqs. (A1)–(A5) are singular at short distance and need to be regularized. If one chooses to work with the local regulating functions, the regularized expressions can easily be obtained by an appropriate modification of the functions $h_1(r)$, $h_2(r)$, and $g(r)$.

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