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On the strong energy dependence of the $e^+e^- \leftrightarrow p\bar{p}$ amplitude near threshold

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Abstract

We study the energy dependence of the $e^+e^- \to p\bar{p}$ cross section close to the two-nucleon threshold, recently reported by the BaBar collaboration. Our analysis also includes the $\bar{p}p \to e^+e^-$ data collected by PS170 collaboration and the $e^+e^- \to N\bar{N}$ data from the FENICE collaboration. We show that the near-threshold enhancement in the $e^+e^- \to p\bar{p}$ cross section can be explained by the final-state interaction between proton and antiproton in the ${}^3S_1$ partial wave, utilizing the Jülich nucleon-antinucleon model. As a consequence, the strong dependence of the proton electromagnetic form factors on the momentum transfer close to the two-nucleon threshold is presumably also driven by this final-state interaction effect. This result is in line with our previous studies of the near-threshold enhancement of the $p\bar{p}$ invariant mass spectrum seen in the $J/\Psi \to \gamma p\bar{p}$ decay by the BES collaboration and in the $B^+ \to p\bar{p}K^+$ decay by the BaBar collaboration.

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The observation of a steep energy dependence of the proton electromagnetic form factors (EMFF) in the timelike region at momentum transfers $q^2 \approx (2m_p)^2$, where m_p is the proton mass, was first reported by the PS170 collaboration [1], based on a measurement of the $\bar{p}p \to e^+e^-$ reaction cross section close to the $p\bar{p}$ threshold at LEAR. Later the FENICE collaboration at Frascati measured the cross section for the time-reversed process $e^+e^- \to p\bar{p}$ [2, 3]. However, their data were taken at energies not close enough to the threshold in order to confirm this strong energy dependence and, furthermore, had very large uncertainties. The FENICE collaboration also made the first and only measurement of the $e^+e^- \to n\bar{n}$ cross section [3] which turned out, within the large experimental errors, to be close to the $e^+e^- \to p\bar{p}$ one. Only recently the BaBar collaboration reported very precise data on the $e^+e^- \to p\bar{p}$ cross section down to energies very close to the $p\bar{p}$ threshold [4]. The form factor deduced from those data substantiates the finding of the PS170 collaboration.

A steep dependence of the proton EMFF on the momentum transfer simply reflects the fact that the underlying (measured) $e^+e^- \to p\bar{p}$ cross section shows a significant enhancement near the $p\bar{p}$ threshold. It is interesting that a near-threshold enhancement was also reported recently in an entirely different reaction involving the $p\bar{p}$ system, namely the radiative decay $J/\Psi \to \gamma p\bar{p}$ [5]. For the latter case several explanations have been put forth, including scenarios that invoke $N\bar{N}$ bound states or so far unobserved meson resonances. However, it was also shown that a rather conventional but plausible interpretation of the data can be given in terms of the final-state interaction (FSI) between the produced proton and antiproton [6, 7, 8, 9, 10]. Specifically, in the calculation of our group [6] utilizing the Jülich $N\bar{N}$ model [11, 12], the mass dependence of the $p\bar{p}$ spectrum close to the threshold could be nicely reproduced by the S-wave $p\bar{p}$ FSI in the isospin $I = 1$ state within the Watson-Migdal [13] approach.

The success of those investigations suggests that the same effects, namely the FSI between proton and antiproton, could be also responsible for the near-threshold enhancement in the $e^+e^- \to p\bar{p}$ cross section and, accordingly, for the strong momentum-transfer dependence of the proton EMFF in the timelike region near $q^2 \approx (2m_p)^2$. In the present paper we report results of a corresponding calculation, utilizing again the scattering amplitudes of the Jülich $N\bar{N}$ model and applying the Watson-Migdal approach.

Fig. 1 shows the $e^+e^- \to p\bar{p}$ and $e^+e^- \to n\bar{n}$ cross sections measured by the FENICE [3] and BaBar [4] collaborations as a function of the excess energy, $M(p\bar{p}) - 2m_p$, with $M(p\bar{p}) = \sqrt{s}$ the invariant energy of the $p\bar{p}$ system. In order to compare the $\bar{p}p \to e^+e^-$ data (also shown in the figure) with the $e^+e^- \to p\bar{p}$ results, we apply detailed balance assuming time-reversal invariance, i.e.

$$\sigma(e^+e^- \to p\bar{p}) \simeq \left[1 - \frac{4m_p^2}{M^2(p\bar{p})}\right] \sigma(\bar{p}p \to e^+e^-) \ , \tag{1}$$

where we neglect the electron mass. Although there seems to be a systematical difference between the $e^+e^- \to p\bar{p}$ and $\bar{p}p \to e^+e^-$ cross section data, the latter are by a factor of about 1.3 smaller, their energy dependence is very similar. The dashed line in Fig. 1 shows the energy dependence due to the two-body phase space given by

$$\sigma(e^+e^- \to p\bar{p}) = \frac{|A|^2}{16\pi\, M^2(p\bar{p})} \left[1 - \frac{4m_p^2}{M^2(p\bar{p})}\right]^{1/2} , \tag{2}$$

where the squared Lorenz invariant amplitude, $|A|^2 = 46\,\mathrm{MeV}^2\cdot\mathrm{fm}^2$, was normalized to the data at the excess energy of 136 MeV. The experimental results clearly exhibit an energy

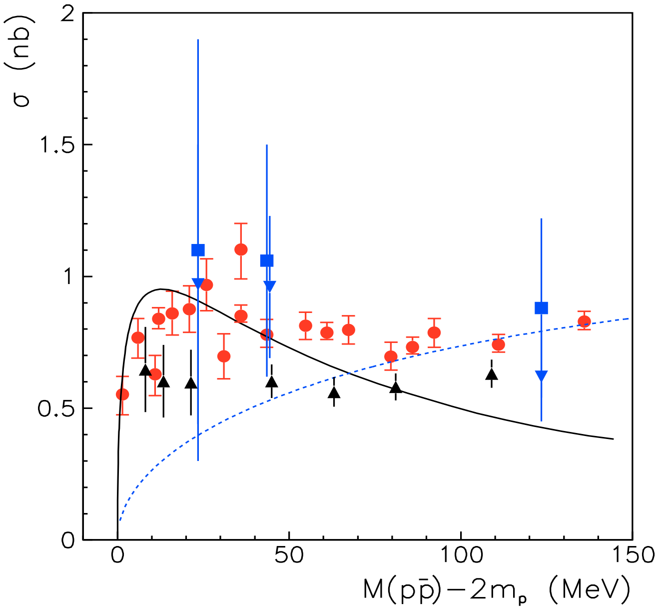

FIG. 1: Cross section of the $e^+e^- \to p\bar{p}$ and $e^+e^- \to n\bar{n}$ reactions as a function of the excess energy. The data are from the FENICE [3] (inverse triangles and squares) and BaBar [4] (circles) collaborations. Triangles represent results obtained by applying detailed balance to the $\bar{p}p \to e^+e^-$ cross section measured by the PS170 collaboration [1]. The dashed line indicates the energy dependence of the two-body phase space. The solid line is the scattering amplitude squared predicted by the Jülich $N\bar{N}$ model A(OBE) [11] for the $^3S_1$ partial wave, multiplied by appropriate phase-space factors.

dependence that differs from the phase space especially at excess energies below 50 MeV. This implies that the transition amplitude A varies substantially for energies close to the $p\bar{p}$ threshold.

To illustrate this conjecture more transparently we extract the squared invariant amplitude $|A|^2$ from the near-threshold data [1, 4] by dividing out the phase space factor according to Eq. (2). The corresponding results are shown in Fig. 2. They clearly indicate that the squared transition amplitude depends rather strongly on the energy within the range $M(p\bar{p}) - 2m_p \leq 50\,\mathrm{MeV}$, say.

Since the $e^+e^- \to p\bar{p}$ and $\bar{p}p \to e^+e^-$ data are used for the extraction [14] of the proton EMFF, the strong energy dependence of the transition amplitude is reflected in the behaviour of the EMFF in the time-like region close to threshold. Phenomenological models such as vector dominance model (VDM), which assumes that the photon couples to hadrons through intermediate vector mesons [15, 16], fail to describe that steep energy dependence. To resolve this discrepancy the VDM was extended to include also heavier vector mesons [16, 17] besides the light ρ, ω and ϕ mesons. Taking the couplings of the heavy vector mesons to the proton as free parameters it was possible to reproduce the steep dependence of the $\bar{p}p \to e^+e^-$ cross section close to $p\bar{p}$ threshold. For a discussion of this issue in the context of dispersion relations, see [18, 19].

On the other hand, the success of $p\bar{p}$ FSI effects in explaining the near-threshold enhancement in the $p\bar{p}$ mass spectrum of $J/\Psi \to \gamma p\bar{p}$ suggests that the same mechanisms could be also responsible for the behaviour of the EMFF. Indeed FSI effects have been already considered before [20, 21] to describe the near-threshold energy dependence of the $\bar{p}p \to e^+e^-$

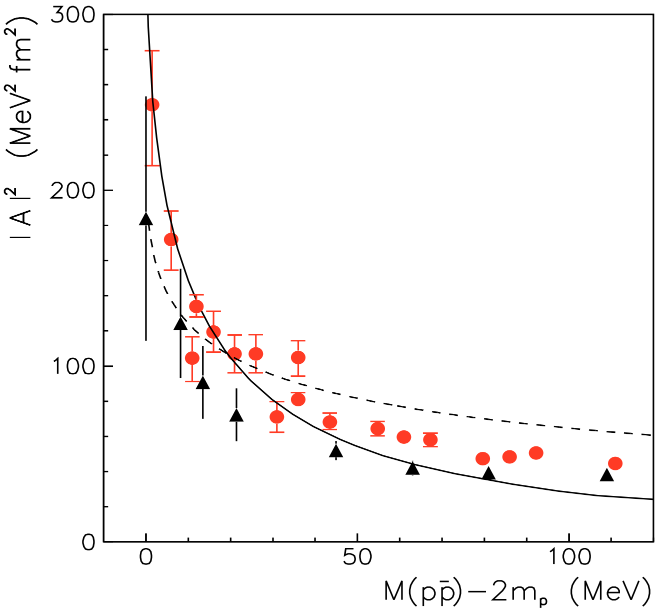

FIG. 2: The Lorentz invariant amplitude squared for the $e^+e^-\to p\bar{p}$ (circles) and $\bar{p}p\to e^+e^-$ (triangles) reactions extracted from the data [1, 4] by Eq. (2) shown as a function of the excess energy. The dashed line is the result based on Eq. (3) with N fixed to the threshold data, while the solid line is the scattering amplitude squared predicted by the Jülich $N\bar{N}$ model A(OBE) [11] for the ${}^3S_1$ partial wave.

reaction by the $p\bar{p}$ initial-state-interaction, though at a time when only the less accurate LEAR data were available. Based on the usual assumption that one-photon exchange constitutes the main reaction mechanism the reaction can only proceed from the $J^{PC}{=}1^{--}$ state.[1] Then $\bar{p}p\to e^+e^-$ as well as the time-reversed reaction $e^+e^- \to p\bar{p}$ can only involve a single partial wave, namely the coupled ${}^3S_1 - {}^3D_1$ $p\bar{p}$ state. Obviously, close to the $p\bar{p}$ threshold the reaction amplitude will be dominated by the ${}^3S_1$ component. Invoking the Watson-Migdal prescription for the treatment of final-state effects [13] and using the scattering length approximation with keeping only the term linear in the antiproton momentum in the center-of-mass system, the squared transition amplitude should behave like

$$|A|^2 \approx N \,/\, \left(1 - \;\mathrm{Im}\, a\sqrt{M^2(p\bar{p}) - 4m_p^2}\right), \tag{3}$$

where $\mathrm{Im}\, a$ is the imaginary part of the ${}^3S_1$ scattering length and N a normalization constant.

Eq. (3) has the advantage that one can obtain a rough but model-independent estimate of the FSI effects by utilizing available experimental values for the $p\bar{p}$ scattering lengths extracted from $1s$ level shifts and widths of antiprotonic hydrogen atoms [28, 29]. The most recently published value for the imaginary part of the pure strong-interaction spin–averaged scattering length is $\mathrm{Im}\, a = (-0.73 \pm 0.03)$ fm [29]. The corresponding result, the dashed line

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[1] There are indications that two-photon exchange contributions are important in the space-like region and can account for the discrepancy between the form factor values extracted from polarization data and Rosenbluth separation of cross section data [22, 23, 24, 25]. Their importance in the time-like region is less clear. For a recent analysis, see Refs. [26, 27].

in Fig. 2, is in line with the trend shown by the data and, therefore, definitely an indication that FSI effects might be responsible for the near-threshold enhancement in the $\bar{p}p \to e^+e^-$ amplitude. One should say, however, that there are uncertainties in using the experimental $\mathrm{Im}\, a$ since the value extracted from $p\bar{p}$ atoms is, in fact, an average of the ${}^3S_1$ and ${}^1S_0$ states and not the one corresponding to the ${}^3S_1$ alone. Moreover, only data extremely close to the threshold are expected to be in line with Eq. (3), i.e. to exhibit a linear dependence on the antiproton momentum k in the center-of-mass system. To include higher orders $\sim k^2$ would require the real part of the $p\bar{p}$ scattering length, but also the (complex) effective range which is not known experimentally.

Therefore, in our analysis we use explicitely the full ${}^3S_1$ $p\bar{p}$ amplitude of the Jülich $N\bar{N}$ model A(OBE) [11]. This model is constrained by the available data on $N\bar{N}$ interactions and it will be interesting to see whether it can reproduce the strong energy dependence of $|A|^2$. The purely nuclear $p\bar{p}$ scattering length predicted by this model for the ${}^3S_1$ partial wave is $a = (0.96 - i0.83)$ fm. The value for the imaginary part is in reasonable agreement with the experimental information, cited above, considering the fact that the latter is actually a spin-averaged result. As already mentioned above, in a previous study [6] we have demonstrated that the near-threshold enhancement in the $p\bar{p}$ invariant mass spectrum from the $J/\Psi \to \gamma p\bar{p}$ decay observed by the BES collaboration [5] is presumably due to the FSI between the outgoing proton and antiproton, utilizing this $N\bar{N}$ model. Similar conclusions on the origin of the near-threshold enhancement in the $p\bar{p}$ mass spectrum were drawn by other groups, employing the Paris $N\bar{N}$ model [7] but also within the effective range approximation [8, 9, 10].

The solid line in Fig. 2 is the $p\bar{p}$ isospin-averaged scattering amplitude squared predicted by the $N\bar{N}$ model A(OBE) [11] for the ${}^3S_1$ partial wave. It is normalized to the low-energy data in order to facilitate the comparision with the $e^+e^- \to p\bar{p}$ amplitude. The same result is also shown in Fig. 1, multiplied by appropriate phase-space factors, cf. Eq. (2), in order to enable a comparision with the $e^+e^- \to \bar{p}p$ cross section. It is obvious that the energy dependence of the $e^+e^- \to \bar{p}p$ transition amplitude squared for energies $M(p\bar{p}) - 2m_p <$ 50 MeV is indeed rather similar to that of the $N\bar{N}$ scattering amplitude. This results strongly suggests that, like for $J/\Psi \to \gamma p\bar{p}$, the FSI in the $p\bar{p}$ system is predominantly responsible for the near-threshold enhancement observed in the $e^+e^- \to \bar{p}p$ cross section, and consequently for the strong dependence of the proton EMFF on the momentum transfer near $q^2 \approx (2m_p)^2$, extracted from those data.

We want to mention that we also performed analogous calculations utilizing other $N\bar{N}$ models of the Jülich group, specifically the potentials A(BOX) and D, which are described in Refs. [11] and [12]. In all these cases the obtained results were rather similar to the ones for the model A(OBE) and, therefore, we refrain from showing them here. Note that the disagreement with the experiment at higher excess energies is not a reason of concern and, in particular, does not discredit the interpretation of the data in terms of FSI effects. We have omitted the contribution from the ${}^3D_1$ state in our calculation, which is negligible in the near-threshold region. However, at energies around $M(p\bar{p}) - 2m_p \approx$ 100-150 MeV its contribution is presumably no longer small and, therefore, most likely responsible for the underestimation of the experimental cross section by our model analysis in this energy range.

In summary, we have analyzed the energy dependence of the squared transition amplitudes for the $\bar{p}p \to e^+e^-$ [1] and $e^+e^- \to p\bar{p}$ [4] reactions utilizing the Jülich $N\bar{N}$ model [11, 12]. Our investigation demonstrates that the strong energy dependence of the $e^+e^- \to \bar{p}p$ cross section is driven by the initial or final-state-interaction in the ${}^3S_1$ partial wave of the $p\bar{p}$

system. This explanation is in line with our previous studies [6, 30] of the near-threshold enhancement in the $p\bar{p}$ invariant mass spectrum from the $J/\Psi \rightarrow \gamma p\bar{p}$ decay observed by the BES collaboration [5] and the $B^+ \rightarrow p\bar{p}K^+$ decay reported by the BaBar collaboration [31]. As a consequence, the steep dependence of the proton electromagnetic form factor on the momentum transfer in the time-like region near $q^2 \approx (2m_p)^2$ is presumably a reflection of this initial or final-state-interaction effect. This leaves not much room for other non-standard dynamics in the time-like EMFF close to threshold, such as the narrow resonance scenario put forth in Refs. [3, 32]. Nevertheless, the dynamics of the EMFF in the time-like region is far from well understood and many important problems, such as the asymptotic ratio of the space-like and time-like form factors or the reliable separation of electric and magnetic form factors, remain.

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