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Can one extract the π-neutron scattering length from π-deuteron scattering?

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Abstract

We give a prove of evidence that the original power counting by Weinberg can be applied to estimate the contributions of the operators contributing to the π-deuteron scattering length. As a consequence, π-deuteron observables can be used to extract neutron amplitudes—in case of π-deuteron scattering this means that the π-neutron scattering length can be extracted with high accuracy. This result is at variance with recent claims. We discuss the origin of this difference.

1. In absence of neutron targets, it became common practice to use few–body nuclei as effective neutron targets. To extract π-neutron (π-n) amplitudes, π-deuteron (π-d) scattering has been studied in the past. This program can be successful only when both the proton observables and the few–body corrections are known to high accuracy. As the former can be measured directly, they do not cause any problem. For the latter the development of chiral perturbation theory for few–nucleon systems promised a controlled, model independent, high precision evaluation of the corresponding amplitudes. This program was put forward in a series of publications, e.g., for π-d scattering (see [1] and references therein).

All those analyses are based on the conjecture of Ref. [2] that the transition operators for reactions on nuclei with external sources can be constructed perturbatively within chiral perturbation theory. The resulting operators are then to be convoluted with the appropriate nuclear wave functions. For this to work it needs to be assumed that the contribution of few–nucleon counter terms to the transition operators can be estimated on the basis of naïve dimensional analysis. If we apply this recipe to πd scattering, the leading counter term (Fig. 1 (d)) appears at 5th order—two orders down compared to the leading few body correction (Fig. 1 (c)). This was recently confirmed by an explicit calculation of the counter term contribution assuming natural strength for the transition operator [4].

In contrast to this it was found recently that a logarithmic scale dependence shows up in the leading few–body correction to π-d scattering (Fig. 1 (c)) that calls for a counter term already at this very order [3,4] (see also [5]), which would preclude any high accuracy extraction of π-n scattering parameters from π-d data. This finding is based on a perturbative treatment of one–pion exchange.

In contrast, in this letter, we demonstrate by an explicit numerical calculation that the logarithmic divergence disappears, if we treat the one–pion exchange non–perturbatively to obtain the wave function. This explains, why previous studies basically lead to identical numbers for the leading few–body correction although very different wave functions were employed (see discussion in Ref. [1]) [1] . Stated differently, we will show that the contact term that necessarily arises at next-to-leading order (NLO), when pions are treated perturbatively in the wave function, can be calculated once the pion exchange is included non–perturbatively in the wave function. This was already conjectured in Ref. [5], but not shown explicitly.

Thus the main goal of our study is to investigate the regulator dependence of the leading few–body correction. Since we are going to employ wave functions that contain non–perturbative pion contributions, this study can only be performed numerically. We will use deuteron wave functions that were constructed for cut–offs that vary over a wide range (Λ = 2–20 fm^{-1} = 400–4000 MeV). The procedure of their construction is described in Ref. [8] and will be briefly reviewed below. Already in Ref. [1] a mild cut–off dependence was reported for calculations using wave functions with non–perturbative pions, when the regulator was changed from 500 to 600 MeV. This might either be because of the absence of the logarithmic divergence due to the wave functions used or simply because the coefficient in front of the logarithm is accidentally small. Due to the large range of variation of cut–off values used here we are in the position to answer this question: we will show that there is no sign of a logarithmic regulator dependence of the results as soon as the complete wave functions are used. The consequences of this observation will be discussed in the final section.

In Ref. [9,10], it was stressed that care has to be taken when calculating pion reactions on nuclei. There it was shown that a subtle cancellation pattern exits between contributions from loops in one–body and few–body operators. This has the effect that the static pion exchange is an excellent approximation to the exact result for the leading few–body corrections to π-d scattering. Therefore here we will focus on the static exchange only.

2. In our investigation we use the wave functions constructed as outlined in

[1] Please note that in Ref. [6], it was shown that in the deuteron channel pions should not be treated perturbatively.

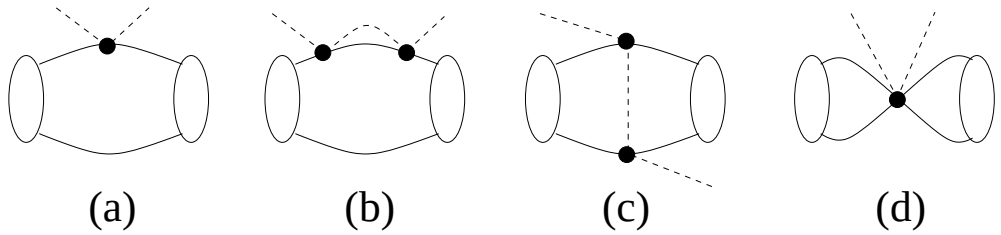

Fig. 1. Typical contributions to πd scattering. Diagram (a) and (b) show the tree level and the one–loop contribution to the one–body term, the pion rescattering contribution is depicted by (c) and diagram (d) shows a two–nucleon contact term. In this figure solid (dashed) lines denote nucleons (pions) and ellipses the deuteron wave function.

Ref. [8]. They emerge as a solution of the Schrödinger equation

$$\Psi_\Lambda^\pi(p) = G(p) \int d^3p' V(p,p') f_\Lambda(p,p') \Psi_\Lambda^\pi(p') \ , \tag{1}$$

where $G(p) = (-\epsilon - p^2/M)^{-1}$ denotes the two–nucleon propagator with ϵ and M for the deuteron binding energy and the nucleon mass, respectively. The leading order potential $V(p,p')$ comprises contributions from both the one–pion exchange as well as a contact term as depicted in Fig. 2 (see [7]). As regulator function we use

$$f_\Lambda(p,p') = \exp\left(\frac{p^4 + p'^{\,4}}{\Lambda^4}\right) \ . \tag{2}$$

For a given value of the regulator Λ the only free parameter is C—the strength of the contact term as depicted in Fig. 2(b). For this study, this parameter was adjusted such that the deuteron binding energy was reproduced, to exclude any dependence of the results on an incorrect asymptotic behavior of the deuteron wave function. We checked that the description of the phase shifts in the ^{3}S$_1$-^{3}D$_1$ channel is comparable to the one obtained in [8]. This numerical study can only be conclusive, when we cover a wide range of cut–offs. We decided to use values of Λ between 2 and 20 fm^{-1} (400–4000 MeV). This range starts below the chiral symmetry breaking scale of $\Lambda_\chi \approx 1000 - 1200$ MeV and extends to values larger by a factor of 4. In this range, we also observe the appearance of spurious bound states in the ^{3}S$_1$-^{3}D$_1$ channel. However, their energies are large and, therefore, these bound states should not affect any low energy physics.

For comparison we also prepared a series of wave functions from only contact NN interactions (thus omitting diagram 2(a) in the potential). These wave functions are denoted by $\Psi_\Lambda^{no\ \pi}(p)$ in what follows. Again we impose a regulator as given in Eq. (2) in the Schrödinger equation.

For completeness, we summarize the binding energy results and some wave function properties for both series of wave functions in Table 1 and 2.

Λ	E_0	T	P_D	A_S	η	r_d	Q_d
2	2.225	28.91	5.24	0.839	0.030	1.889	0.3005
4	2.225	45.48	8.23	0.866	0.027	1.933	0.2827
6	2.224	62.33	6.94	0.866	0.025	1.932	0.2704
8	2.225	75.95	6.76	0.864	0.026	1.926	0.2676
12	2.227	85.80	7.14	0.864	0.026	1.925	0.2675
16.5	2.214	102.50	7.08	0.862	0.026	1.929	0.2676
20	2.210	115.07	7.07	0.861	0.026	1.929	0.2675
Expt.	2.225	—	—	0.8846	0.0256	1.9671	0.2859

Table 1
Summary of some deuteron properties obtained from Ψ^{π}_{Λ}—the wave functions with non–perturbative one–pion exchange for various cutoffs. Here, the cut-off Λ is given in fm^{-1}, the binding energy and kinetic energy E_0 and T in MeV, the asymptotic S-state normalization A_S is in fm$^{-1/2}$, the point nucleon radius in fm, and the quadrupol moment in fm^2. η is the ratio of the asymptotic S- and D-state normalization.

Λ	E_0	T	A_S	r_d
2	2.225	32.60	0.76	1.728
4	2.225	69.59	0.72	1.617
6	2.225	106.76	0.71	1.585
8	2.225	143.99	0.70	1.569
12	2.225	218.51	0.69	1.555
16	2.225	293.04	0.69	1.547
20	2.225	367.59	0.69	1.543
Expt.	2.225	—	0.8846	1.9671

Table 2
Summary of deuteron properties obtained from the wave functions $\Psi^{no\ \pi}_{\Lambda}$—where only a contact interaction was used in the potential—for various cutoffs. The notation is the same as in Table 1.

3. Let us now turn to the calculation of the leading few–body correction to the πd scattering length. As stated in the introduction we will exclusively focus on the static contribution. The corresponding expression reads

$$a^{(static)} = -\xi \int d^3p d^3q \Psi^{\kappa}_{\Lambda}(\vec{p}-\vec{q})^{\dagger} \frac{1}{\vec{q}^{\,2}} \Psi^{\kappa}_{\Lambda}(\vec{p}) \ , \qquad (3)$$

where $\xi = m_\pi^2/(32\pi^4 f_\pi^4 (1+m_\pi/(2M_N)))$. Clearly, no physical quantity can be regulator dependent. Naïve dimensional analysis does not require a two–

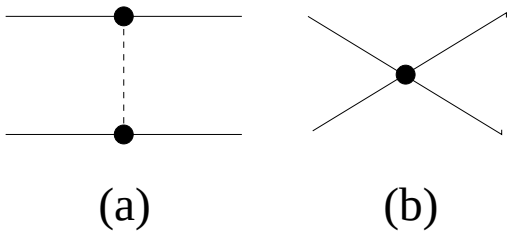

Fig. 2. Contributions to the NN potential at leading order: the one pion exchange (a) and a momentum independent contact term (b).

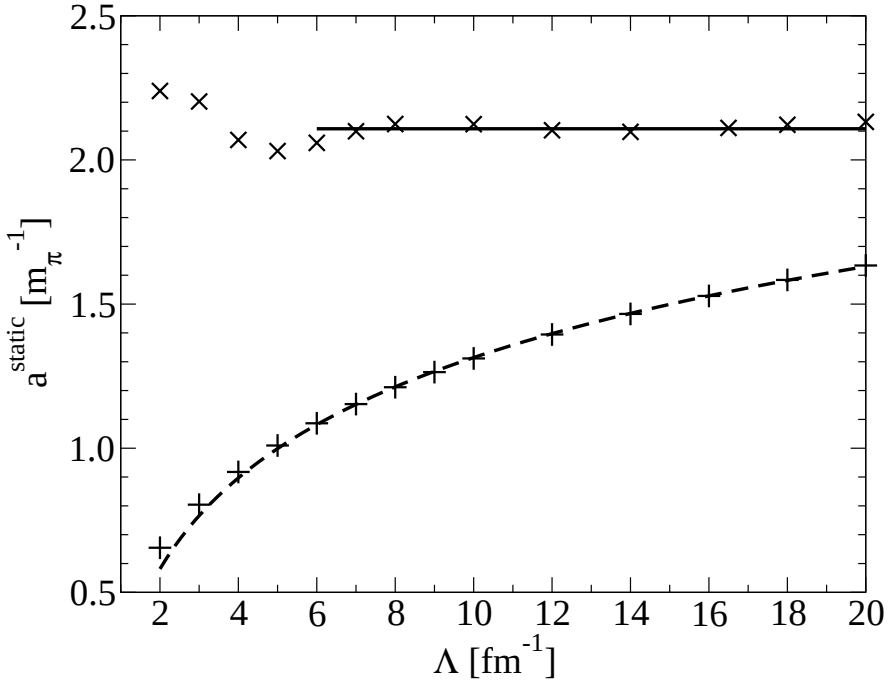

Fig. 3. Results for the Λ dependence of the leading few–body correction from Eq. (3). The $'x'$ symbols show the results of the numerical evaluation of the integral using the full wave functions Ψ^{π}_{Λ}, whereas the $'+'$ symbols shows the result for wave functions with only point interactions $\Psi^{no\ \pi}_{\Lambda}$, downscaled by a factor of 4. The dashed line is a fit to the latter of the form $A + B\ln(\Lambda)$, as described in the text. The solid line shows the fit of a constant to the former for values of the cut–off larger than 6 fm^{-1}.

body counter term in the same order as this first three–body correction. Thus, studying the Λ dependence of the given integral tells, whether a counter term of naïvely higher orders is needed in conjuction with this few–body correction. We evaluated numerically the integral in Eq. (3) using both, the wave functions from the full LO NN potential ($\kappa = \pi$) and those from only the point interaction ($\kappa = no\ \pi$), as described in the previous section.

The results are shown in Fig. 3. Here the x symbols emerged from the cal-

culation with the wave functions Ψ_Λ^π—where the non–perturbative pion exchange was included—whereas the plus symbols stem from the calculation using $\Psi_\Lambda^{no\ \pi}$—that does not have any pion exchange in the wave function. For the latter, our results clearly show the $\ln(\Lambda)$ behavior in accordance with the findings of Refs. [3,4,5]. To show that explicitly, we fitted a logarithm to our results, which is also shown in the figure. The perfect agreement shows that we can recover the previous results numerically in our cut–off range. However, the calculation using the full wave functions shows almost no regulator dependence at all for cut-offs above Λ_χ [2]. Thus, as soon as the pion exchange is included non–perturbatively in the wave functions, no counter term is needed at the order of the leading few–body corrections to absorb the regulator dependence of the pion exchange contribution. For a numerical comparison with previous work, we fitted a constant to our results for $\Lambda \geq 6$ fm^{-1}. In this way, we obtain for the static three-body contribution $a^{static} = 2.12$ m$_\pi^{-1}$, which is in good agreement with the previous calculations [1].

We also checked that there is no unnatural enhancement of the leading $\pi NN \to \pi NN$ counter term due to the wave function at small distances: the contribution of this term to the scattering length—from an explicit evaluation of diagram (d) in Fig. 1—was in line with the counting, when the transition operator was assumed to be of natural strength [11]. The same observation was also made in Ref. [4]. This shows that as soon as non–perturbative pions are included in the construction of the wave function, the wave function at the origin assumes natural values. Combining this finding with the observation made above that there is no regulator dependence of the leading few–body correction, there is no reason to change the original Weinberg counting.

We conjecture that the full wave function is driven not only by the the binding momentum of the deuteron, as $\Psi_\Lambda^{no\ \pi}$, but also by a second scale, probably m_π or f_π. The numerical calculation does not easily allow to identify such a scale in the wave functions. For illustration, we compare the two types of wave functions in r-space in Fig. 4. For Ψ_Λ^π, we see that the wave functions converge for radii larger than $R = 0.7$ fm. For smaller r, the wave functions are not unique in our range of cutoffs and show different numbers of nodes coming from the spurious bound states. They remain, however, small for these distances. This is different for the point–like wave functions, which, for $r = 0$ increase linearly with the cut-off. We conclude that the finite range of the pion–exchange regularizes the wave functions in the sense that they remain small for small distances independent of the cut–off and oscillate. Both effects reduce the contribution of the small distance behavior to the scattering length. A more systematic insight would obviously be highly welcome.

[2] Note that we observe a mild regulator dependence of the integral for regulators below Λ_χ, in line with the findings reported in Ref. [1].

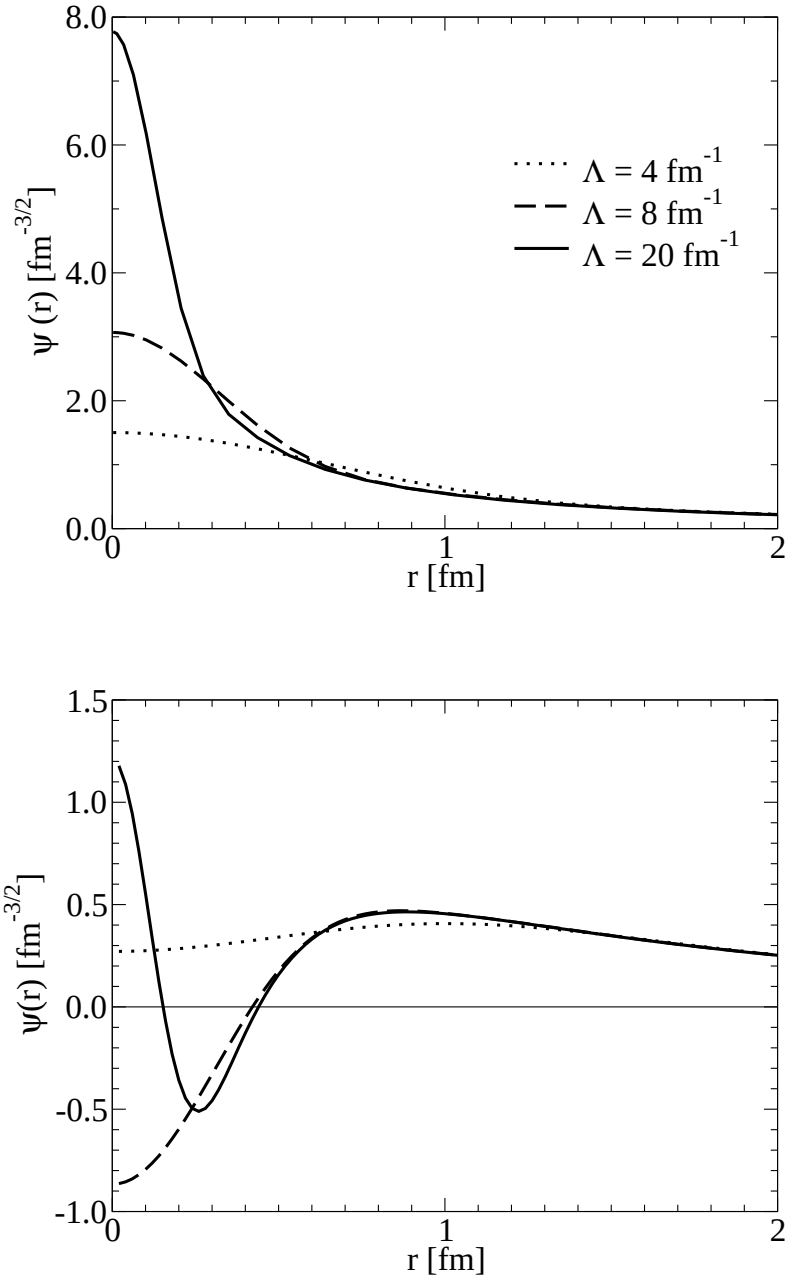

Fig. 4. The upper panel shows $\Psi_\Lambda^{no\ \pi}$ and the lower one the s-wave of Ψ_Λ^π for various values of Λ. Note the different scales of the figures.

4. To summarize, as soon as a finite range potential is used in the construction of the deuteron wave functions, the matrix element for the leading few–body correction to the π-d scattering length gets independent of the regulator. In particular, the logarithmic divergence that emerges in case of wave functions from a point–like NN potential disappears. We have demonstrated this in a numerical study using wave functions constructed for a large variety of cut–

offs. Our numerical results extend to cut-offs, which are much larger than the chiral symmetry breaking scale. For the same range of cut-offs, we numerically observe the expected logarithmic divergence for the case of $\psi^{no\ \pi}$. Therefore, we are confident that the range of cut-offs is large enough to take our result as a prove of evidence that no counter term is needed at the same order where the leading few–body correction appears. This is the precondition to allow for a high accuracy extraction of the π-n scattering length from π-d data.

Please note that the present discussion has large implications also for the investigations of other reactions. Based on our findings the calculations for $\gamma d \to \pi^0 d$ [12,13], $\pi^3\mathrm{He} \to \pi^3\mathrm{He}$ [14], $\pi^- d \to \gamma nn$ [15], and $\gamma d \to \pi^+ nn$ [10] indeed were performed with the accuracy as given in the publications.

Finally, we note that the observation, that the one–pion exchange, which leads to a finite range interaction, changes the divergence structure of the theory, is not unique to π-d scattering. The probably most famous other example in this context is the three–nucleon system in the spin 1/2 channel: as long as only point–like two body interactions are included, a three body counter term needs to be promoted to leading order [16]. However, as soon as there is a non–perturbative pion–exchange included, this is no longer necessary [8].

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References

[1] S. R. Beane, V. Bernard, E. Epelbaum, U.-G. Meißner and D. R. Phillips, Nucl. Phys. A **720** (2003) 399. [arXiv:hep-ph/0206219].

[2] S. Weinberg, Phys. Lett. B **295** (1992) 114.

[3] B. Borasoy and H. W. Griesshammer, Int. J. of Mod. Phys. **E** 12, 65 (2003) [arXiv:nucl-th/0105048].

[4] U.-G. Meißner, U. Raha and A. Rusetsky, Eur. Phys. J. C **41**, 213 (2005) [arXiv:nucl-th/0501073]; erratum to be published.

[5] S. R. Beane and M. J. Savage, Nucl. Phys. A **717** (2003) 104 [arXiv:nucl-th/0204046].

[6] S. Fleming, T. Mehen and I. W. Stewart, Nucl. Phys. A **677** (2000) 313 [arXiv:nucl-th/9911001].

[7] S. Weinberg, Nucl. Phys. B **363**, 3 (1991).

[8] A. Nogga, R. G. E. Timmermans and U. van Kolck, Phys. Rev. C in press [arXiv:nucl-th/0506005].

[9] V. Baru, C. Hanhart, A. E. Kudryavtsev and U. G. Meißner, Phys. Lett. B **589**, 118 (2004) [arXiv:nucl-th/0402027].

[10] V. Lensky, V. Baru, J. Haidenbauer, C. Hanhart, A. E. Kudryavtsev and U. G. Meißner, arXiv:nucl-th/0505039.

[11] J. Friar, Few–Body Systems **22**, 161 (1997).

[12] S. R. Beane, V. Bernard, T. S. H. Lee, U.-G. Meißner and U. van Kolck, Nucl. Phys. A **618**, 381 (1997) [arXiv:hep-ph/9702226].

[13] H. Krebs, V. Bernard and U. G. Meißner, Eur. Phys. J. A **22** (2004) 503 [arXiv:nucl-th/0405006].

[14] V. Baru, J. Haidenbauer, C. Hanhart and J. A. Niskanen, Eur. Phys. J. A **16** (2003) 437.

[15] A. Gardestig and D. R. Phillips, "Using chiral perturbation theory to extract the neutron-neutron scattering length from π-d $\to$ nn γ," arXiv:nucl-th/0501049.

[16] P. F. Bedaque, H. W. Hammer and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999) [arXiv:nucl-th/9809025].