$B_{s,d} \to \gamma\gamma$ decay in the model with one universal extra dimension

G. Devidze[a] *, A. Liparteliani[a] †, and Ulf-G. Meißner[b,c]

[a]Institute of High Energy Physics and Informatization,
9 University St., 0186 Tbilisi, Georgia

[b]Universität Bonn, Helmholtz-Institut für Strahlen- und Kernphysik (Theorie)
D-53115 Bonn, Germany

[c] Institut für Kernphysik (Theorie), Forschungszentrum Jülich, 52425 Jülich, Germany

We estimate the beyond the Standard Model (SM) contribution to the $B_{s,d} \to \gamma\gamma$ double radiative decay in the framework of the model with one universal extra dimension. This contribution gives a $\sim 3\,(6)\%$ enhancement of the branching ratio calculated in the SM for $B_{s\,(d)} \to \gamma\gamma$.

1. Introduction

It is known that in the Standard Model (SM) the double radiative decays of the $B_{s,d}$ mesons, $B_{s,d} \to \gamma\gamma$, first arise at the one loop level with the exchange of up-quarks and W-bosons in the loops [1,2,3,4,5]. The branching ratios for the above decays are of the order of $\sim 10^{-7}\,(10^{-9})$.

On the other hand there is the possibility to enhance the above mentioned decays in extended versions of the SM. In the papers [6,7] it was shown that in supersymmetric versions of the SM one could reach a branching ratio as large as $\mathrm{Br}(B_s \to \gamma\gamma) \sim 10^{-6}$ depending on the SUSY parameters. This enhancement was achieved mainly due to the exchange of charged scalar Higgs particles within the loop. There exists an analogous possibility in other exotic models as well for the scalar particle exchange inside the loop, which could potentially enhance this process. For example, the Appelquist, Chang and Dobresku (ACD) model with only one universal extra dimension [8] presents us with such an opportunity. One should note that in the above approach towers of charged Higgs particles arise as real objects with certain masses, not as fictitious (ghost) fields.

In this letter we aim to calculate the contributions from these real scalars to the $B_{s,d} \to \gamma\gamma$ decay. The article is organized as follows: in the section 2 some useful information about the ACD model, necessary for the calculations, is provided. Section 3 is devoted to the calculation of the pertinent amplitudes. In section 4, numerical estimates of the branching ratios are discussed.

2. Useful information on the structure of the ACD-model

In the Universal Extra Dimension (UED) scenarios all the fields presented in the SM live in extra dimensions, i.e. they are functions of all space-time coordinates. For bosonic fields one simply replaces all derivatives and fields in the SM lagrangian by their 5-dimensional counterparts. These are the $U(1)_Y$-and $SU(2)_L$-gauge fields as well as the $SU(3)_C$-gauge fields from the QCD -sector. The Higgs doublet is chosen to be even under P_5 (P_5 is a parity operator in the five dimensional space) and possesses a zero mode. Note that all zero modes remain massless before the Higgs mechanism is applied. In addition we should note that as a result of the action of the parity operator the fields receive additional masses $\sim n/R$ after dimensional reduction and transition to the four dimensional Lagrangians.

*Present address: Institut für Kernphysik, Forschungszentrum Jülich, 52425 Jülich, Germany
†Present address: Institut für Kernphysik, Forschungszentrum Jülich, 52425 Jülich, Germany

In the five dimensional ACD model the same procedure for gauge fixing is possible as in the models in which fermions are localized on the 4-dimensional subspace. With the gauge fixed, one can diagonalize the kinetic terms of the bosons and finally derive expressions for the propagators. Compared to the SM, there are additional Kaluza-Klein (KK) mass terms. As they are common to all fields, their contributions to the gauge boson mass matrix is proportional to the unity matrix. As a consequence, the electroweak angle remains the same for all KK-modes and is the ordinary Weinberg angle θ_W. Because of the KK-contribution to the mass matrix, charged and neutral Higgs components with $n \neq 0$ (n being the number of the KK-mode) no longer play the role of Goldstone bosons. Instead, they mix with $W_5^{\pm}$ and Z_5 to form, in addition to the Goldstone modes $G^0_{(n)}$ and $G^{\pm}_{(n)}$, three additional physical states $a^0_{(n)}$ and $a^{\pm}_{(n)}$. It is precisely the role of these additional charged physical states to double radiative neutral B-meson decays that is studied in this paper.

The Lagrangian responsible for the interaction of charged scalar KK towers $a^*_{(n)}$ with the ordinary down quarks reads

$$\mathcal{L} = \frac{g_2}{\sqrt{M_{(n)}}}\bar{Q}_{i(n)}(C_L^{(1)}P_L + C_L^{(1)})a^*_{(n)}d_j + \frac{g_2}{\sqrt{M_{(n)}}}\bar{U}_{i(n)}(C_L^{(2)}P_L + C_L^{(2)})a^*_{(n)}d_j \ , \qquad (1)$$

utilizing the following notations [9]:

$$\begin{aligned} C_L^{(1)} &= -m_3^{(i)}V_{ij}, \qquad C_L^{(2)} = m_4^{(i)}V_{ij}, \\ C_R^{(1)} &= Mm_3^{(i,j)}V_{ij}, \ \ C_R^{(2)} = -M_4^{(i,j)}V_{ij}, \\ M^2_{W(n)} &= m^2(a^*_{(n)}) = M_W^2 + \frac{n^2}{R^2}, \end{aligned} \qquad (2)$$

where V_{ij} are elements of the CKM matrix. The mass parameters in Eq.(2) are defined as

$$\begin{aligned} m_3^{(i)} &= -M_W c_{i(n)} + \frac{n}{R}\frac{m_i}{M_W}s_{i(n)}, \\ m_4^{(i)} &= M_W s_{i(n)} + \frac{n}{R}\frac{m_i}{M_W}c_{i(n)}, \\ M_3^{(i,j)} &= \frac{n}{R}\frac{m_j}{M_W}c_{i(n)}, \\ M_4^{(i,j)} &= \frac{n}{R}\frac{m_j}{M_W}s_{i(n)}. \end{aligned} \qquad (3)$$

Here, M_W and the masses of up (down)-quarks $m_i\,(m_j)$ in the right-hand-side of Eq.(3) are zero mode masses and the $c_{i(n)}$, $s_{i(n)}$ stand for the cos and sin of the fermions mixing angles, respectively,

$$\tan 2\alpha_{f(n)} = \frac{m_f}{n/R}, \quad n \geq 1 \ . \qquad (4)$$

The masses for the fermions are calculated as

$$m_{f(n)} = \sqrt{\frac{n^2}{R^2} + m_f^2}. \qquad (5)$$

In the phenomenological applications we use the restriction $n/R \geq 250\,\mathrm{GeV}$ and hence we assume that all the fermionic mixing angles except $\alpha_{t(n)}$ are equal zero.

3. Structure of $B_{s,d} \rightarrow \gamma\gamma$ in the ACD model with one extra dimension

The Feynman graphs, describing the contributions of scalar physical states to process under consideration, are shown in Fig.1.

The amplitude for the decay $B_{s,d} \rightarrow \gamma\gamma$ has the form

$$T(B \rightarrow \gamma\gamma) = \epsilon_1^{\mu}(k_1)\epsilon_2^{\nu}(k_2) \times [Ag_{\mu\nu} + iB\epsilon_{\mu\nu\alpha\beta}k_1^{\alpha}k_2^{\beta}]. \qquad (6)$$

This equation is correct after gauge fixing for the final photons which we have chosen as

$$\epsilon_1 \cdot k_1 = \epsilon_2 \cdot k_2 = \epsilon_1 \cdot k_2 = \epsilon_2 \cdot k_2 = 0, \qquad (7)$$

where ϵ_1 and ϵ_2 are photon polarization vectors, respectively. The condition Eq.(7) together with energy-momentum conservation leads to

$$\epsilon_i \cdot P = \epsilon_i \cdot p_1 = \epsilon_i \cdot p_2 = 0, \qquad (8)$$

where

$$P = k_1 + k_2 \quad \text{and} \quad p_1 = p_2 + k_1 + k_2. \qquad (9)$$

Let us write down some useful kinematical relations which are results of Eqs.(7,8) as well:

$$\begin{aligned} P \cdot p_1 &= m_b M_B, \quad P \cdot p_2 = -m_{s(d)}M_B, \\ P \cdot k_1 &= P \cdot k_2 = k_1 \cdot k_2 = \frac{1}{2}M_B^2, \\ p_1 \cdot p_2 &= -m_b m_{s(d)}, \\ p_1 \cdot k_1 &= p_1 \cdot k_2 = \frac{1}{2}m_b M_B, \\ p_2 \cdot k_1 &= p_2 \cdot k_2 = -\frac{1}{2}m_{s(d)}M_B. \end{aligned} \qquad (10)$$

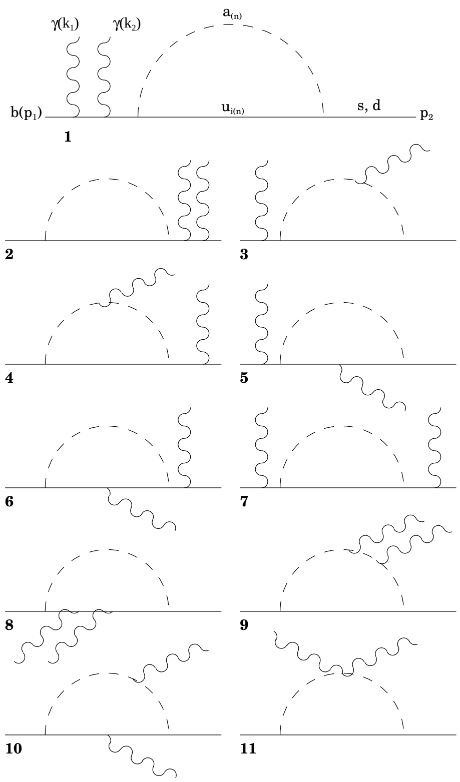

Figure 1. Double radiative B-meson decay $B_{s,d} \to \gamma\gamma$ in the theory with only one extra universal dimension (the dashed lines in the loops correspond to the charged KK towers $a^*_{(n)}$, while the solid lines in the loops are for the up-quark KK towers).

The total contributions into CP-even (A) and CP-odd (B) amplitudes from Eq.(6) are calculated as sums of the appropriate contributions of the diagrams in Fig.1 corresponding to a tower of scalar particle contributions in the ACD model with only one extra dimension. Let us note that we used the following formula for the hadronic matrix elements:

$$\langle 0|\bar{s}\,(\bar{d})\gamma_\mu\gamma_5 b|B(P)\rangle = -if_B P_\mu. \qquad (11)$$

Apart from one particle reducible (1PR) diagrams, one particle irreducible (1PI) ones contribute to the amplitudes, and hence, to their CP-even (A) and CP-odd (B) parts. We should note that each of the 1PI contributions is finite. Let us discuss these contributions in more details. In the SM only one 1PI diagram (one with the W-boson exchange in the loop, when both photons are emitted by virtual up-quarks) gives the contribution of the order of $\sim 1/M_W^2$. In the Ref.[10] it was observed that diagrams with light quark exchange contribute as $\sim 1/M_W^2$, while diagrams containing the heavy quarks are of order of $\sim 1/M_W^4$. In the ACD model the contributions of such diagrams are of the order of $\sim 1/M_W^4$ because the estimate for all KK-tower masses, including the ones exchanged in the loops, in our case are $M \geq 250\,\text{GeV}$. Likewise discussions show that all the 1PI diagrams existing in the ACD model also are of order $\sim 1/M_W^4$. Thus, the leading 1PI diagrams are negligible and we do not consider them.

4. Branching ratio for the $B \to \gamma\gamma$ decay

The total contributions to the $B \to \gamma\gamma$ decay amplitudes are:

$$\begin{aligned} A &= b\frac{m_b^3}{m_{s,(d)}}\Big\{\frac{n}{RM_W}m_3^{(i)}m_{i(n)}c_{i(n)}f_1(x_i) + \\ &\quad \big[(m_3^{(i)})^2 - \frac{n^2}{R^2M_W^2}m_b m_{s(d)}c^2_{i(n)}\big]\frac{1}{2}f_2(x_i)\Big\}, \\ B &= 2b\frac{m_b}{m_{s,(d)}}\Big\{\frac{n}{RM_W}m_3^{(i)}m_{i(n)}c_{i(n)}f_1(x_i) + \\ &\quad \big[(m_3^{(i)})^2 + \frac{n^2}{R^2M_W^2}m_b m_{s(d)}c^2_{i(n)}\big]\frac{1}{2}f_2(x_i)\Big\}, \end{aligned} \qquad (12)$$

where

$$\begin{aligned} b &= \frac{1}{4}\frac{i}{(4\pi)^2}e^2g_2^2f_B\frac{Q_d}{M^2_{W(n)}}\frac{V^*_{is(d)}V_{ib}}{m^2(a^*_{(n)})}, \\ f_1(x) &= \frac{-5x^2+8x-3+2(3x-2)\ln x}{6(1-x)^3}, \\ f_2(x) &= \frac{-2x^3-3x^2+6x-1+6x^2\ln x}{6(1-x)^4}, \\ x_i &= \frac{m^2(u_{i(n)})}{m^2(a^*_{(n)})}\ . \end{aligned} \qquad (13)$$

As it is obvious from Fig.1, the correct calculation assumes the inclusion of the crossed diagrams (not shown on fig.1). In the kinematics we use, cf. Eqs.(7-10), this leads to a factor 2 for all amplitudes, except for the one given by diagram 11. However, diagram 11 belongs to the class of 1PI diagrams. As it was stated above, one particle irreducible diagrams does not give leading contributions into process and therefore their contributions ($\sim 1/M_W^4$) are negligible comparing with that of the 1PR diagrams.

On the other hand, using the unitarity feature of the Kobayashi-Cabibbo-Maskawa matrix, the amplitude for double radiative B-meson decay can be rewritten as:

$$T = \sum_{i=u,c,t} \lambda_i T_i = \lambda_t \Big\{ T_t - T_c + \frac{\lambda_u}{\lambda_t}(T_u - T_c) \Big\} . \quad (14)$$

Let us note that we restricted ourselves by calculating the leading order terms of $\sim 1/M_W^2$ from the up-quark KK-towers. In this approximation it turns out that the $u_{(n)}$ and the $c_{(n)}$ towers have equal contributions. Therefore, the expressions for the amplitudes have a simpler form than before:

$$A = \lambda_t (A_{t(n)} - A_{c(n)}) \ , \qquad B = \lambda_t (B_{t(n)} - B_{c(n)}) \ . \quad (15)$$

Furthermore, it is easy to obtain from Eq.(6) the expression for the $B \to \gamma\gamma$ decay partial width:

$$\Gamma(B \to \gamma\gamma) = \frac{1}{32\pi M_B} \Big[4|A|^2 + \frac{1}{2} M_B^4 |B|^2 \Big] . \quad (16)$$

Now we are in the position to compare the ACD contribution to the decay with that of the SM. Namely, let us consider the ratio:

$$\frac{\Gamma(B_{s(d)} \to \gamma\gamma)_{\rm ACD}}{\Gamma(B_{s(d)} \to \gamma\gamma)_{\rm SM}} = \frac{24 n^2 M_W^6}{Q_d^2 R^2 M_{W(n)}^4 m^4(a^*_{(n)})} \times \Big\{ \frac{m_3^{(i)} m_{i(n)}}{M_W^2} c_{t(n)} f(x_{t(n)}) + \frac{n}{R M_W} f(x_{c(n)}) \Big\}^2 \Big/ \Big\{ 4\big(C(x_t) + \frac{23}{3}\big)^2 + 2\big(C(x_t) + \frac{23}{3} + 16 \frac{m_{s(d)}}{m_b}\big)^2 \Big\} \quad (17)$$

where

$$C(x) = \frac{22x^3 - 153x^2 + 159x - 46}{6(1-x)^3} + \frac{3(2-3x)}{(1-x)^4} \ln x , \qquad x_t = \frac{m_t^2}{M_W^2} . \quad (18)$$

Rough numerical estimates of Eq.(17) show that in case of B_s-meson decay we can get a difference from SM-result as much as $\sim$ 3%. The UED contribution to the $B_s \to \gamma\gamma$ is 3% of the SM estimate and increases the overal contribution (SM+UED) by 3%. The same difference for the case of $B_d \to \gamma\gamma$ is $\sim$ 6%.

The theoretical estimates of double radiative B-decays in the framework of the Standard Model, $\mathrm{Br}(B_s \to \gamma\gamma) \sim 10^{-7}$ and $\mathrm{Br}(B_d \to \gamma\gamma) \sim 10^{-9}$ along with the upper experimental limits [11] allows us to hope that in the not to far future these decays will be observed (say, by the BaBar or BELLE collaborations or at the CERN B-physics facility). We thus hope that not too much time will pass until the differences of $\sim$ 3%(6%) will be accessible for experimental analysis.

Acknowledgements

The research described in this publication was made possible in part by Award No. GEP1-33-25-TB-02 of the Georgian Research and Development Foundation (GRDF) and the U.S. Civilian Research & Development Foundation for the Independent States of the Former Soviet Union (CRDF). Part of this work have been done due to support of Deutscher Akademischer Austauchdienst (DAAD). G.D. and A.L. wish to express their great attitude to H. Ströher, N.N. Nikolaev, C. Hanhart, A. Kacharava, A. Wirzba for valuable discussions and help during our stay at IKP-FZJ as well as for excellent scientific environment and warm hospitality they provided for us. This research is part of the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RII3-CT-2004-506078.

REFERENCES

1. G. L. Lin, J. Liu and Y. P. Yao, Phys. Rev. D **42** (1990) 2314.
2. H. Simma and D. Wyler, Nucl. Phys. B **344** (1990) 283.

3. S. Herrlich and J. Kalinowski, Nucl. Phys. B **381** (1992) 501.
4. G. G. Devidze, G. R. Dzhibuti and A. G. Liparteliani, Nucl. Phys. B **468** (1996) 241.
5. G. G. Devidze, G. R. Dzhibuti and A. G. Liparteliani, Phys. Atom. Nucl. **59** (1996) 1948 [Yad. Fiz. **59** (1996) 2021].
6. G. G. Devidze and G. R. Dzhibuti, Phys. Lett. B **429** (1998) 48.
7. G. G. Devidze and G. R. Dzhibuti, Phys. Atom. Nucl. **63** (2000) 310 [Yad. Fiz. **63** (2000) 373].
8. T. Appelquist, H. C. Cheng and B. A. Dobrescu, Phys. Rev. D **64** (2001) 035002 [arXiv:hep-ph/0012100].
9. A. J. Buras, M. Spranger and A. Weiler, Nucl. Phys. B **660** (2003) 225 [arXiv:hep-ph/0212143].
10. M. K. Gaillard and B. W. Lee, Phys. Rev. D **10** (1974) 897.
11. Particle Data Group, http://pdg.lbl.gov.