

ω - ϕ mixing in chiral perturbation theory

Ayse Kucurkarslan¹, and Ulf-G. Meißner^{2,3}

¹ *Canakkale 18 Mart University, 17020 Canakkale, Turkey*

² *Helmholtz-Institut für Strahlen- und Kernphysik (Theorie),
Universität Bonn, Nußallee 14-16, D-53115 Bonn, Germany*

³ *Institut für Kernphysik (Theorie), Forschungszentrum Jülich, D-52425 Jülich, Germany.*

We investigate ω - ϕ meson mixing to leading order in chiral perturbation theory utilizing the antisymmetric tensor field formulation. We update the quark mass ratio R from ρ - ω mixing, $R = 42 \pm 4$.

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1. The mixing between the vector mesons ω and ϕ plays an important role in the understanding of OZI violation and SU(3) breaking in QCD. In QCD, this mixing is entirely generated by the light quark mass differences. In addition, QED effects by photon exchange lead to a further mixing contribution. In the framework of chiral perturbation theory with explicit vector fields the issues related to ρ - ω mixing were worked out by Urech [1]. In this short note, we use the same framework to calculate ω - ϕ mixing and also to discuss the momentum dependence of the mixing amplitude. This might be of relevance for the theoretical studies trying to explain the recently measured large ϕ -meson photo-production cross section of nuclei in the non-perturbative regime of QCD [2].

2. We first give a brief reminder of ω - ϕ mixing in the standard quark model picture. Since there is some small OZI violation, one has an admixture of light quarks in the ϕ meson wave function. The ϕ and ω mesons are a mixture of the SU(3) singlet ω_0 and the octet ω_8 states,

$$\phi = \cos\theta_V \omega_8 - \sin\theta_V \omega_0, \quad (1)$$

$$\omega = \sin\theta_V \omega_8 + \cos\theta_V \omega_0, \quad (2)$$

where

$$\omega_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}, \quad (3)$$

$$\omega_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}. \quad (4)$$

The ϕ and ω wave functions are then given by

$$\begin{aligned} \phi &= (u\bar{u} + d\bar{d}) \left(\frac{1}{\sqrt{6}} \cos\theta_V - \frac{1}{\sqrt{3}} \sin\theta_V \right) \\ &\quad - s\bar{s} \left(\frac{2}{\sqrt{6}} \cos\theta_V + \frac{1}{\sqrt{3}} \sin\theta_V \right), \end{aligned} \quad (5)$$

$$\begin{aligned} \omega &= (u\bar{u} + d\bar{d}) \left(\frac{1}{\sqrt{6}} \sin\theta_V + \frac{1}{\sqrt{3}} \cos\theta_V \right) \\ &\quad + s\bar{s} \left(-\frac{2}{\sqrt{6}} \sin\theta_V + \frac{1}{\sqrt{3}} \cos\theta_V \right). \end{aligned} \quad (6)$$

The strict OZI limit corresponds to the ideal mixing angle

$$\tan\theta_V = \frac{1}{\sqrt{2}} \rightarrow \theta_V = 35.3^\circ, \quad (7)$$

and the ideal ϕ and ω states are $\omega^{\text{ideal}} = 2\omega_0/\sqrt{6} + \omega_8/\sqrt{3}$ and $\phi^{\text{ideal}} = \omega_0/\sqrt{3} - 2\omega_8/\sqrt{6}$. It is further instructive to rewrite the ϕ and ω wave functions as follows

$$\phi = \sin\varphi_V \left(\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \right) - \cos\varphi_V(s\bar{s}) \quad (8)$$

$$\omega = \sin\varphi_V(s\bar{s}) + \cos\varphi_V \left(\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \right), \quad (9)$$

where $\varphi_V = \theta - \theta_V$. The physical mixing angle θ can e.g. be determined from the masses of the mesons in the vector meson nonet differs from the ideal mixing angle. Using the quadratic Gell-Mann-Okubo mass formula, the physical mixing angle of the vector mesons can be obtained as $\theta = 39^\circ$, close to the ideal one: $\varphi_V \simeq 39^\circ - 35.3^\circ = 3.7^\circ$. Note further that the wave function of ϕ meson can also be written as

$$\phi(1020) = s\bar{s} + \varepsilon_{\phi\omega}(u\bar{u} + d\bar{d})/\sqrt{2} \quad (10)$$

where $\varepsilon_{\phi\omega}$ is the mixing parameter, $|\varepsilon_{\phi\omega}| \ll 1$, which describes the $\phi\omega$ mixing amplitude. In what follows, we will determine the mixing amplitude and angle from vector meson decays.

3. For our analysis, we use the vector meson chiral effective Lagrangian presented in [3] and extended in [1]. To construct the pertinent Lagrangian, we introduce the antisymmetric tensor field $V_{\mu\nu}$ to parameterize the octet of the spin-1 vector mesons

$$V_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*-} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*-0} \\ K^{*-} & K^{*0} & \phi \end{pmatrix}_{\mu\nu}. \quad (11)$$

To include the ϕ (the singlet field), we extend the SU(3) representation to U(3) and substitute

$$V_{\mu\nu} \rightarrow V_{\mu\nu} + (\omega_0)_{\mu\nu} \frac{I_3}{\sqrt{3}}, \quad (12)$$

where ω_0 is the lightest singlet vector resonance and I_3 is the unit matrix in three dimensions. For the analysis of vector meson mixing, we need the effective Lagrangian to leading order. The strong contribution to vector-meson

mixing stems from the terms quadratic in the vector meson fields, i.e. the kinetic part of the Lagrangian for the ω and ϕ mesons. In the chiral limit, it takes the form (from now on, we only display the terms needed for our discussion)

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & -\frac{1}{2}\partial^\lambda\phi_{\lambda\mu}\partial_\nu\phi^{\nu\mu} + \frac{1}{4}M_\phi^2\phi_{\mu\nu}\phi^{\mu\nu} \\ & -\frac{1}{2}\partial^\lambda\omega_{\lambda\mu}\partial_\nu\omega^{\nu\mu} + \frac{1}{4}M_\omega^2\omega_{\mu\nu}\omega^{\mu\nu}. \end{aligned} \quad (13)$$

The part of \mathcal{L}_{kin} that contains the $\phi - \omega$ mixing is

$$\mathcal{L}_{\phi\omega} = \sqrt{2}M_\rho(\hat{m} - m_s)\phi_{\mu\nu}\omega^{\mu\nu}, \quad (14)$$

where we have identified the octet mass in the chiral limit with the ρ mass as also done in [1]. We further need the interaction Lagrangian,

$$\mathcal{L}_2^V = \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu \rangle + \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu} \rangle \quad (15)$$

where

$$\begin{aligned} u_\mu &= iu^\dagger D_\mu U u^\dagger = u_\mu^\dagger, \\ f_+^{\mu\nu} &= uF^{\mu\nu}u^\dagger + u^\dagger F^{\mu\nu}u, \\ F^{\mu\nu} &= eQ(\partial^\mu A^\nu - \partial^\nu A^\mu), \end{aligned} \quad (16)$$

with Q the quark charge matrix, $Q = \text{diag}(2, -1, -1)/3$, $U = u^2$ collects the Goldstone boson octet and A_μ is the photon field. The first term in Eq. (15) generates the vector meson couplings to two pion which is needed for the calculation of the strong decay channels. The electromagnetic contribution to the vector meson mixing stems from the vector meson conversion to the photon field and its reconversion into another neutral vector meson. These terms are generated by the second term in Eq. (15). Expanding this, the direct couplings of the neutral vector meson fields (ρ^0 , ω , and ϕ) to the photon fields are given by

$$\begin{aligned} \mathcal{L}_{\rho\gamma} &= -\frac{e}{2}F_V\rho_{\mu\nu}^0(\partial^\mu A^\nu - \partial^\nu A^\mu), \\ \mathcal{L}_{\omega\gamma} &= -\frac{e}{6}F_V\omega_{\mu\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu), \\ \mathcal{L}_{\phi\gamma} &= \frac{e}{3\sqrt{2}}F_V\phi_{\mu\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu). \end{aligned} \quad (17)$$

The last two equations in Eqs. (17) will lead to $\phi - \omega$ mixing through the transition process $\phi - \gamma - \omega$. The corresponding part of the Lagrangian takes the form

$$\mathcal{L}_{\phi\omega} = \frac{\sqrt{2}}{9}e^2F_V^2\phi_{\mu\nu}\omega^{\mu\nu} \quad (18)$$

Putting pieces together, the Lagrangian relevant for $\phi - \omega$ mixing can be written as (using Eqs. (14,18))

$$\mathcal{L}_{\phi\omega} = \Theta_{\phi\omega}\phi_{\mu\nu}\omega^{\mu\nu} \quad (19)$$

where the mixing angle $\Theta_{\phi\omega}$ has a strong and an electromagnetic piece,

$$\mathcal{L}_2^{\phi\omega} = \left(\sqrt{2}M_\rho(\hat{m} - m_s) + \frac{\sqrt{2}}{9}e^2F_V^2 \right) \phi_{\mu\nu}\omega^{\mu\nu}. \quad (20)$$

We remark that we can use here the average light quark mass $\hat{m} = (m_u + m_d)/2$ since only its relative size compared to the strange quark mass is of relevance. Note that using the lowest order expressions for the quark mass expansion of the Goldstone boson masses, we can rewrite the first term in Eq. (20) stemming from the quark mass differences entirely in terms of Goldstone boson masses. Further, we have employed quark counting rules for the vector meson masses to leading order in the quark masses. This leads to $M_V \simeq B_0/2$ (with $B_0 = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2$) and lifts the apparent conflict with renormalization group invariance of Eq. (20) (for details, see [1]). Note further that the on-shell mixing amplitude $\Theta_{\phi\omega}$ is related to the parameter $\varepsilon_{\phi\omega}$ introduced in Eq. (10) via

$$\varepsilon_{\phi\omega} = \frac{\Theta_{\phi\omega}}{M_\phi^2 - M_\omega^2}. \quad (21)$$

4. To evaluate the mixing amplitude, we consider the Fourier transform of the two-point function in the tensor field notation. It has the form

$$\begin{aligned} & i \int d^4x e^{ikx} \langle 0|T\phi_{\mu\nu}(x)\omega_{\rho\sigma}(0)e^{i\int d^4y\{\mathcal{L}_2^V + \mathcal{L}_2^{\phi\omega}\}}|0\rangle \\ &= \frac{2\sqrt{2}M_\rho(\hat{m} - m_s)}{M_\phi^2 M_\omega^2} \times \left\{ G_{\mu\nu\rho\sigma} \right. \\ &+ \left. \left[\frac{1}{M_\phi^2 - k^2} + \frac{M_\phi^2}{(M_\phi^2 - k^2)(M_\omega^2 - k^2)} \right] P_{\mu\nu\rho\sigma} \right\} \\ &+ \frac{\sqrt{2}}{9}e^2F_V^2 \frac{1}{(M_\phi^2 - k^2)(M_\omega^2 - k^2)k^2} P_{\mu\nu\rho\sigma}, \end{aligned} \quad (22)$$

where

$$\begin{aligned} G_{\mu\nu\rho\sigma} &= g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}, \\ P_{\mu\nu\rho\sigma} &= g_{\mu\rho}k_\nu k_\sigma - g_{\mu\sigma}k_\nu k_\rho - g_{\nu\rho}k_\mu k_\sigma + g_{\nu\sigma}k_\mu k_\rho. \end{aligned}$$

The on-shell amplitude is obtained in two steps. First, we must collect some formula from Ref. [1] for $\rho - \omega$ mixing. For that, consider the decay width of the process $\omega \rightarrow \rho^0 \rightarrow \pi^+\pi^-$. It is expressed as

$$\begin{aligned} \Gamma(\omega \rightarrow \pi^+\pi^-) &= \left| \frac{\Theta_{\rho\omega}}{M_\omega^2 - M_\rho^2 - i(M_\omega\Gamma_\omega - M_\rho\Gamma_\rho)} \right|^2 \\ &\times \Gamma(\rho^0 \rightarrow \pi^+\pi^-), \end{aligned} \quad (23)$$

and the decay width of $\rho^0 \rightarrow \pi^+\pi^-$ has been calculated using the antisymmetric tensor field Lagrangian in Ref. [3],

$$\Gamma(\rho^0 \rightarrow \pi^+\pi^-) = \frac{1}{48\pi} \frac{G_V^2 M_\rho^3}{F_0^4} \left(1 - \frac{4M_\pi^2}{M_\rho^2} \right)^{\frac{3}{2}}, \quad (24)$$

with F_0 the pion decay constant in the chiral limit. In the numerical analysis, we will identify this with the physical value of the pion decay constant, $F_0 = F_\pi = 92.4 \text{ MeV}$. We further have [1]

$$\Theta_{\rho\omega} = 2M_\rho(m_u - m_d) + \frac{1}{3}e^2 F_V^2. \quad (25)$$

Of course, here we need to take care of the up and down quark mass difference since otherwise there would be no strong $\rho - \omega$ mixing. Next we consider the decay width of the decay $\phi \rightarrow \pi^+\pi^-$, we find

$$\Gamma(\phi \rightarrow \pi^+\pi^-) = \left| \frac{\Theta_{\phi\omega}}{M_\phi^2 - M_\omega^2 - i(M_\phi\Gamma_\phi - M_\omega\Gamma_\omega)} \right|^2 \times \Gamma(\omega \rightarrow \pi^+\pi^-), \quad (26)$$

where

$$\Theta_{\phi\omega} = \sqrt{2}M_\rho \left[(\hat{m} - m_s) + e^2 \frac{F_V^2}{9M_\rho} \right]. \quad (27)$$

5. We are now in the position to analyse the vector-meson mixings. We use the following values $M_\rho = 775.8 \pm 0.5 \text{ MeV}$, $\Gamma_\rho = 150.3 \pm 1.6 \text{ MeV}$, $M_\omega = 782.59 \pm 0.11 \text{ MeV}$, $\Gamma_\omega = 8.49 \pm 0.08 \text{ MeV}$, $M_\phi = 1019.456 \pm 0.020 \text{ MeV}$, $\Gamma_\phi = 4.26 \pm 0.05 \text{ MeV}$, $\text{BR}(\phi \rightarrow \pi^+\pi^-) = (7.3 \pm 1.3) \times 10^{-5}$ from [4] and $\text{BR}(\omega \rightarrow \pi^+\pi^-) = (1.30 \pm 0.24 \pm 0.05)\%$ from [5]. This gives for the on-shell mixing amplitude

$$\Theta_{\rho\omega} = (-3.75 \pm 0.35 \pm 0.07) \times 10^{-3} \text{ GeV}^2. \quad (28)$$

The uncertainty in the branching ratio of the process $\omega \rightarrow \pi^+\pi^-$ causes the error in the value of $\Theta_{\rho\omega}$ amplitude. Furthermore, we obtain values of $\Theta_{\phi\omega}$ that depend on $\Theta_{\rho\omega}$ (cf. Eqs. (23,26)):

$$\Theta_{\phi\omega} = (25.34 \pm 2.39) \times 10^{-3} \text{ GeV}^2 \quad (29)$$

Note that if we substitute $M_\omega^2 - M_\rho^2 - i(M_\omega\Gamma_\omega - M_\rho\Gamma_\rho)$ with the dominant term $iM_\rho\Gamma_\rho$ in Eq. (23), the $\rho - \omega$ mixing amplitude is $\Theta_{\rho\omega} = (-3.96 \pm 0.37 \pm 0.08) \times 10^{-3} \text{ GeV}^2$ and the value affects the $\phi - \omega$ mixing amplitude as follows: $\Theta_{\phi\omega} = (24.03 \pm 2.27) \times 10^{-3} \text{ GeV}^2$. In Table 1 we have collected some values for the $\rho - \omega$ mixing amplitude [6, 7, 8] (as extracted from the pion vector form factor) and the resulting $\omega - \phi$ mixing deduced from Eqs. (23,26). Our result of the $\rho - \omega$ mixing amplitude is in good agreement with these values. For further studies of $\rho - \omega$ and $\rho - \omega - \phi$ mixing see [9] and [10], respectively. Tab. 1 also includes the values of the $\phi - \omega$ amplitude which is calculated using the values of the $\rho - \omega$ mixing amplitude. These values of the amplitude $\Theta_{\phi\omega}$ (including ours) are consistent with the range $20.00 \dots 29.00 \times 10^{-3} \text{ GeV}^2$ given in [11, 12]. We can also give the magnitude of the $\phi - \omega$ mixing parameter defined in Eq.(21) and the deviation from the ideal mixing angle,

$$\varepsilon = 0.059 \pm 0.005, \quad \varphi_V = 3.4 \pm 0.3^\circ, \quad (30)$$

TABLE I: Values of the $\phi - \omega$ mixing amplitude that depends on the $\rho - \omega$ mixing amplitude for various values from the literature in comparison to this work.

Ref.	$\Theta_{\rho\omega} [\times 10^{-3} \text{ GeV}^2]$	$\Theta_{\phi\omega} [\times 10^{-3} \text{ GeV}^2]$
this work	-3.75 ± 0.36	25.34 ± 2.39
[1]	-3.91 ± 0.30	24.31 ± 1.88
[6]	-4.42 ± 0.60	21.03 ± 2.84
[7]	-3.74 ± 0.30 -4.23 ± 0.68 -3.67 ± 0.30	25.41 ± 2.05 22.47 ± 3.70 25.90 ± 2.13
[8]	-3.97 ± 0.20	23.94 ± 1.20

consistent with the findings e.g. in Refs. [13, 14].

From our analysis of $\rho - \omega$ mixing, we can update the value of the quark mass ratio $R = (m_s - \hat{m})/(m_d - m_u)$ using the formalism developed in Ref. [15]. We find

$$R = 42 \pm 4, \quad (31)$$

where the uncertainty has been estimated in a similar fashion as in [1].

6. Finally, we consider the off-shell behavior of the two-point function, that allows to describe the momentum-dependence of the $\phi - \omega$ mixing. Using the definition of the vector mesons in the tensor representation

$$V_\mu = \frac{1}{M_V} \partial^\nu V_{\mu\nu}, \quad (32)$$

we obtain the following expression

$$i \int d^4x e^{ikx} \langle 0 | T \phi_\mu(x) \omega_\nu(0) e^{i \int d^4y \{ \mathcal{L}_2^V + \mathcal{L}_2^{\phi\omega} \}} | 0 \rangle = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{\Theta(k^2)}{(M_\phi^2 - k^2)(M_\omega^2 - k^2)}, \quad (33)$$

with

$$\Theta(k^2) = \left[2\sqrt{2}M_\rho(\hat{m} - m_s) + \frac{\sqrt{2}}{9}e^2 F_V^2 \right] \frac{k^2}{M_\phi M_\omega}. \quad (34)$$

This result is similar to the one of Urech [1] for the momentum dependence of the $\rho - \omega$ mixing amplitude. Using similar arguments than given in that paper, one concludes that the momentum dependence of the amplitude for nucleon-nucleon scattering with resonance exchange and $\omega - \phi$ mixing is more complicated than the one of $\Theta(k^2)$ (we refrain from giving the pertinent formulae here).

7. In this note, we have considered $\omega - \phi$ mixing to leading order in chiral perturbation theory with vector mesons and discussed some implications. In the future, one

should consider loop corrections to these results utilizing either the heavy vector meson chiral Lagrangian developed in [16] or the infrared regularization scheme for spin-1 fields presented in [17]. Note that some of these issues were already addressed in [18, 19].

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