HISKP-TH-06/24, FZJ-IKP(TH)-2006-23

On the extraction of the quark mass ratio $(m_d - m_u)/m_s$ from $\Gamma(\eta' \to \pi^0\pi^+\pi^-)/\Gamma(\eta' \to \eta\pi^+\pi^-)$

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Abstract

The claim that the light quark mass ratio $(m_d - m_u)/m_s$ can be extracted from the decay width ratio $\Gamma(\eta' \to \pi^0\pi^+\pi^-)/\Gamma(\eta' \to \eta\pi^+\pi^-)$ is critically investigated within a U(3) chiral unitary framework. The influence of the recent VES data on the $\eta' \to \eta\pi^+\pi^-$ decay is also discussed.

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1 Introduction

The light quark masses m_u, m_d, m_s are fundamental parameters of Quantum Chromo Dynamics and ought to be constrained as accurately as possible. The determination of the light quark mass ratios has been the goal of a variety of investigations in low-energy hadron physics, see e.g. [1–5]. Of particular interest is the quark mass difference $m_d - m_u$ which induces isospin breaking in QCD. Moreover, the possibility $m_u = 0$ would provide an explanation for the strong CP problem.

An accurate way of extracting $m_d - m_u$ is given by the isospin-violating decays $\eta, \eta' \to \pi^0\pi^+\pi^-$ and $\eta, \eta' \to 3\pi^0$. While for most processes isospin-violation of the strong interactions is masked by electromagnetic effects, these corrections are expected to be small for the three pion decays of the η and η' (Sutherland's theorem) [6] which has been confirmed in an effective Lagrangian framework [7]. Neglecting electromagnetic corrections the decay amplitudes are directly proportional to $m_d - m_u$.

For this reason, it has been claimed in [8] that the branching ratio $r = \Gamma(\eta' \to \pi^0\pi^+\pi^-)/\Gamma(\eta' \to \eta\pi^+\pi^-)$ can be utilized in a very simple manner to extract the light quark mass difference $m_d - m_u$. To this aim, it is assumed that

a) the amplitude $A(\eta' \to \pi^0\pi^+\pi^-)$ is determined by the corresponding amplitude $A(\eta' \to \eta\pi^+\pi^-)$ via

$$A(\eta' \to \pi^0\pi^+\pi^-) = \epsilon\; A(\eta' \to \eta\pi^+\pi^-) \qquad (1)$$

with $\epsilon = (\sqrt{3}/4)\,(m_d - m_u)/(m_s - \hat{m})$ the π^0-η mixing angle and $\hat{m} = (m_d + m_u)/2$. (Note that in [8] the difference $m_s - \hat{m}$ has been approximated by m_s in the denominator of ϵ.) Eq. (1) implies that the decay $\eta' \to \pi^0\pi^+\pi^-$ proceeds entirely via $\eta' \to \eta\pi^+\pi^-$ followed by π^0-η mixing.

b) both amplitudes are *"essentially constant"* over phase space (see the remark in front of Eq. (19) of Ref. [8]).

Based on these two assumptions one arrives at the relation

$$r = \frac{\Gamma(\eta' \to \pi^0\pi^+\pi^-)}{\Gamma(\eta' \to \eta\pi^+\pi^-)} \simeq (16.8)\frac{3}{16}\left(\frac{m_d - m_u}{m_s}\right)^2, \qquad (2)$$

where the factor 16.8 represents the phase space ratio. Comparison with experimental data—for which, so far, only an upper limit exists—would then lead to a prediction for the quark mass ratio $(m_d - m_u)/(m_s - \hat{m}) \simeq (m_d - m_u)/m_s$. The purpose of the present work is to critically examine these two assumptions which lead to the simple relation in Eq. (2). Such an investigation is very timely in view of the recent and ongoing experimental activities on η and η' decays at the WASA facility at COSY [9], MAMI-C [10], KLOE at DAΦNE [11] and by the VES Collaboration [12, 13].

An appropriate theoretical framework to investigate low-energy hadronic physics is provided by chiral perturbation theory (ChPT) [14], the effective field theory of QCD. In ChPT Green functions are expanded perturbatively in powers of Goldstone boson masses and small momenta. However, final-state interactions in $\eta \to 3\pi$ have been shown to be substantial both in a complete one-loop calculation in SU(3) ChPT [15] and using a dispersive framework [16, 17]. It is hence important to include final-state interactions in a non-perturbative fashion.

In η' decays final-state interactions are expected to be even more important due to larger phase space and the presence of nearby resonances. In this investigation, we apply the framework of U(3) chiral effective field theory in combination with a relativistic coupled-channels approach developed in [18, 19] in order to calculate the ratio r. Final-state interactions are included by deriving s- and p-wave interaction kernels for meson-meson scattering from the chiral effective Lagrangian and iterating them in a Bethe-Salpeter equation. The infinite iteration of the chiral effective potentials generates resonances dynamically. Very good overall agreement with currently available data on η, η' decay widths and spectral shapes has been achieved in [18, 19].

In the next section, we will investigate in our approach if both the assumptions "a" and "b" are justified. The inclusion of the recent VES data [13] which provide higher statistics on the spectral shape of $\eta' \to \eta\pi^+\pi^-$ than previous experiments is studied in Sec. 3, while Sec. 4 contains our conclusions.

2 Validity of the assumptions

In the following, we will work with the double quark mass ratio Q^2

$$Q^2 = \frac{m_s - \hat{m}}{m_d - m_u} \frac{m_s + \hat{m}}{m_d + m_u} \tag{3}$$

instead of the mixing angle ϵ, since the Kaplan-Manohar reparametrization invariance [20] of the chiral effective Lagrangian is respected by Q^2 up to chiral order $\mathcal{O}(p^4)$, whereas ϵ receives corrections already at $\mathcal{O}(p^2)$. Hence, it is preferable to employ Q in phenomenological analyses in order to suppress the ambiguity stemming from this reparametrization invariance.

Following Dashen's theorem which asserts equal electromagnetic corrections for pion and kaon masses at leading chiral order [21], Q^2 can be expressed in terms of physical meson masses

$$Q^2_{\text{Dashen}} = \frac{m_K^2}{m_\pi^2} \, \frac{m_K^2 - m_\pi^2}{m_{K^0}^2 - m_{K^\pm}^2 + m_{\pi^\pm}^2 - m_{\pi^0}^2} = (24.1)^2 \, . \tag{4}$$

However, there are various investigations on the size of violations to Dashen's theorem with (partially contradictory) results for Q in the range of about $21 \ldots 24$ [22]. The 3π decays of η and η' provide thus a good opportunity to pin down the value of the double quark mass ratio Q^2 [16, 23].

We will first investigate the validity of assumption "a", i.e. we assume that the decay $\eta' \to \pi^0\pi^+\pi^-$ proceeds entirely via $\eta' \to \eta\pi^+\pi^-$ followed by π^0-η mixing. This implies for the neutral decay $A(\eta' \to 3\pi^0) = 3\epsilon\, A(\eta' \to \eta\pi^0\pi^0)$. Employing the amplitudes $A(\eta' \to \eta\, 2\pi)$ from the approach advocated in [19] — which are in very good agreement with the data given in [24] — one can thus predict the decay amplitudes for $A(\eta' \to 3\pi)$ and calculate both the decay widths $\Gamma(\eta' \to \pi^0\pi^+\pi^-)$, $\Gamma(\eta' \to 3\pi^0)$ and the branching ratios r and $r_2 = \Gamma(\eta' \to 3\pi^0)/\Gamma(\eta' \to \eta\pi^0\pi^0)$.

In [19] a least-squares fit to meson-meson scattering phase shifts and η, η' decays has been performed. One observes four different classes of fits, i.e. clusters, which describe these data equally well, but differ in their predictions for yet unmeasured quantities such as the $\eta' \to \pi^0\pi^+\pi^-$ decay width (for which there exists only a weak upper limit) and the Dalitz slope parameters of $\eta' \to 3\pi$. In fact, as we will see in the next section, inclusion of the recent VES data for $\eta' \to \eta\pi^+\pi^-$ [13] reduces the number of fit clusters to one, but in the current section we

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Exp.
$\Gamma(\eta' \to \pi^0\pi^+\pi^-)$ [eV]	69 ± 12	73 ± 9	141 ± 44	141 ± 26	< 3800
r [%]	0.09 ± 0.02	0.09 ± 0.02	0.17 ± 0.06	0.17 ± 0.03	< 4.1
$\Gamma(\eta' \to 3\pi^0)$ [eV]	116 ± 22	120 ± 16	217 ± 67	217 ± 40	315 ± 78
r_2 [%]	0.26 ± 0.05	0.26 ± 0.04	0.47 ± 0.15	0.47 ± 0.08	0.74 ± 0.12

Table 1: Decay widths and branching ratios in the chiral unitary approach [19] employing assumption "a".

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Exp.
$\Gamma(\eta' \to \pi^0\pi^+\pi^-)$ [eV]	155 ± 7	155 ± 7	153 ± 7	154 ± 5	< 3800
r [%]	0.19	0.19	0.19	0.19	< 4.1
$\Gamma(\eta' \to 3\pi^0)$ [eV]	238 ± 11	239 ± 10	237 ± 11	239 ± 6	315 ± 78
r_2 [%]	0.52	0.52	0.52	0.52	0.74 ± 0.12

Table 2: Decay widths and branching ratios in the chiral unitary approach employing assumptions "a" and "b". Since in this case the branching ratios only depend on phase space and Q, they do not have an error bar.

take the data given in [24]. We employ furthermore the upper limit of 1.75 % for the branching fraction of $\eta' \to \pi^0\pi^+\pi^-$ as measured by the VES collaboration [12] which is significantly lower than the previous upper limit of 5 % [24]. This tighter bound translates to an upper limit of 3.8 keV for the partial decay width and reduces the upper limit for r from 10 % (as quoted by the PDG) to 4.1 %. The pertinent results for the four fit clusters of [19] are well below these new upper limits and can be utilized without modification. As already reported in [19], the fit to the data does not allow for conclusions on the size of violations to Dashen's theorem since Q is treated as an input parameter and variations of Q in the range of 20 . . . 24 lead to equally good fits within our approach. Hence, we will set $Q = 24.1$ in our calculations—the value predicted by Dashen's theorem. The results for the branching ratios obtained by employing assumption "a" and $Q = 24.1$ are shown in Table 1. The ratios are obtained by explicitly performing the integration of the amplitudes over phase space. Obviously, assumption "a" is not justified—at least for the neutral decay where, in particular, clusters 1 and 2 are in clear disagreement with experiment.

Next, we employ in addition assumption "b". This is achieved by averaging the $\eta' \to \eta\, 2\pi$ amplitudes over phase space which are then employed for $\eta' \to 3\pi$ by means of assumption "a". The results are displayed in Table 2. One observes that for clusters 1 and 2 the decay widths into 3π and hence the ratios r, r_2 increase, while the changes for clusters 3 and 4 are rather moderate. However, recall that the Dalitz plot parameters of the approach [19] clearly indicate that the assumption of a constant amplitude is not justified for $\eta' \to \pi^0\pi^+\pi^-$, particularly for clusters 3 and 4. The partial compensation of the effects of assumption "a" in clusters 1, 2 and the moderate changes in clusters 3, 4 are therefore purely accidental.

We conclude that both assumptions "a" and "b" are not justified. This is further substantiated by comparison of r and r_2 in Tab. 2 with the respective values from the full chiral unitary approach shown in Tab. 3. The values are in clear disagreement and, hence, both assumptions are not appropriate—at least within the chiral unitary approach.

Finally, we would like to investigate the differences which result if assumption "a" is replaced

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Exp.
$\Gamma(\eta' \to \pi^0\pi^+\pi^-)$ [eV]	470 ± 200	520 ± 200	740 ± 420	620 ± 180	< 3800
r [%]	0.58 ± 0.24	0.66 ± 0.27	0.92 ± 0.52	0.77 ± 0.21	< 4.1
$\Gamma(\eta' \to 3\pi^0)$ [eV]	331 ± 24	326 ± 28	330 ± 33	336 ± 21	315 ± 78
r_2 [%]	0.73 ± 0.06	0.72 ± 0.06	0.71 ± 0.07	0.73 ± 0.05	0.74 ± 0.12

Table 3: Decay widths and branching ratios in the chiral unitary approach [19].

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Exp.
$\Gamma(\eta' \to \pi^0\pi^+\pi^-)$ [eV]	2450 ± 1930	1720 ± 1160	260 ± 260	290 ± 290	< 3800
r [%]	2.96 ± 2.30	2.10 ± 1.40	0.34 ± 0.34	0.37 ± 0.37	< 4.1
$\Gamma(\eta' \to 3\pi^0)$ [eV]	1080 ± 840	800 ± 550	120 ± 120	120 ± 120	315 ± 78
r_2 [%]	2.34 ± 1.79	1.73 ± 1.19	0.28 ± 0.28	0.28 ± 0.28	0.74 ± 0.12

Table 4: Decay widths and branching ratios in the chiral unitary approach if isospin-breaking takes place solely via π^0-η' mixing. For the fits of Clusters 3 and 4 this mixing angle can actually become zero leading to vanishing decay widths $\Gamma(\eta' \to 3\pi)$ and branching ratios r, r_2.

by the decay mechanism where $\eta' \to 3\pi$ occurs due to π^0-η' mixing followed by a (virtual) transition $\pi^0 \to 3\pi$. Employing the relation $A(\eta' \to 3\pi) = \epsilon'\, A(\pi^0 \to 3\pi)$ with ϵ' being the π^0-η' mixing angle [18] we find the values shown in Table 4. Assuming the $\eta' \to 3\pi$ decays to proceed via this mechanism introduces a huge uncertainty and leads to different ratios r and r_2. This underlines the observation that the decays $\eta' \to 3\pi$ cannot be expected to simply proceed either via π^0-η or π^0-η' mixing. In particular, the isospin-breaking transition due to the quark mass difference $m_d - m_u$ cannot be completely assigned to π^0-η mixing as done in assumption "a". Despite its appealing simplicity, the crude estimate given in Eq. (2) is certainly not suited to precisely determine the double quark mass ratio Q^2. In fact, even at leading chiral order the $\eta' \to 3\pi$ decay amplitude is not entirely due to π^0-η mixing. There is also a contribution from an isospin-violating $\eta' 3\pi$-vertex from the explicit chiral symmetry breaking part of the Lagrangian at second chiral order, see e.g. Ref. [18].

On the other hand, employing the chiral unitary approach of [19] does not lead to a conclusive extraction of Q due to the present experimental situation. From a fit to the data in [24] supplemented by Dashen's theorem one obtains the decay width ratio $r = (0.35 \ldots 1.5)\%$ which is larger than the value of 0.18% quoted in [8]. Note also that there is a tendency to even larger values of r if Q is lowered, e.g., for $Q = 22$ we obtain the range $r = (0.4 \ldots 2.8)\%$.

3 Inclusion of the VES data for $\eta' \to \eta\pi^+\pi^-$

In this section we study the changes in our results that occur if the recent VES data on the spectral shape of $\eta' \to \eta\pi^+\pi^-$ [13] are taken into account. Note that the most recent analysis of the VES Collaboration [13] has not yet been included in Ref. [24]. The VES data have much higher statistics on the Dalitz slope parameters than previous experiments and by including them in the fit we obtain the results shown in Tab. 5. Since the amplitudes for $\eta' \to \eta\pi^+\pi^-$ and $\eta' \to \eta\pi^0\pi^0$ are equal in the isospin limit and deviations are thus isospin-breaking and small in our approach, we only include the leading Dalitz parameter a of $\eta' \to \eta\pi^0\pi^0$ [25] and omit the higher ones which are—assuming only small isospin-violating contributions—not quite

$\eta' \to \eta\pi^+\pi^-$			
	a	b	c
Theory	-0.109	-0.087	-0.036
Exp.	-0.127 ± 0.024	-0.106 ± 0.042	-0.082 ± 0.025
$\eta' \to \eta\pi^0\pi^0$			
	a	b	c
Theory	-0.123	-0.104	-0.041
Exp.	-0.116 ± 0.026		

Table 5: Results for the Dalitz plot parameters of $\eta' \to \eta\pi\pi$ if the VES data are included in the fit.

	Theory	Exp.
$\Gamma(\eta' \to \pi^0\pi^+\pi^-)$ [eV]	3120	< 3800
r [%]	3.9	< 4.1
$\Gamma(\eta' \to 3\pi^0)$ [eV]	330	315 ± 78
r_2 [%]	0.73	0.74 ± 0.12

Table 6: Decay widths and branching ratios if the VES data are taken into account.

compatible with the new results of the VES experiment for $\eta' \to \eta\pi^+\pi^-$. Our results are in good agreement with the Dalitz plot parameters extracted from the VES experiment. In Tab. 5 only the best least-squares fit is shown which is sufficient to discuss the qualitative changes of the results compared to those of Sec. 2. Note also that we have supplemented our fitting routine by a conjugate gradient method [26] and hence the numerical values have improved with respect to [19].

Our results for the $\eta' \to 3\pi$ decay widths and width ratios are displayed in Tab. 6. It is important to emphasize that the inclusion of the VES data reduces the number of fit clusters to one and we observe indeed one global minimum. There is, however, a strong tendency of the fits towards the upper limit $\Gamma(\eta' \to \pi^0\pi^+\pi^-) < 3.8\,\mathrm{keV}$ and, in fact, slightly improved fits with a smaller χ^2 value can be obtained if this upper limit is omitted. (In this case, the best overall fit leads to the width $\Gamma(\eta' \to \pi^0\pi^+\pi^-) = 5.73\,\mathrm{keV}$.) The result for the width ratio, $r = 3.9\%$, has thus increased if the VES data are taken into account. Furthermore, the amplitude $A(\eta' \to \pi^0\pi^+\pi^-)$ fluctuates strongly over phase space with slope parameters which can be more than one order of magnitude larger in size than those obtained in [19]. The Dalitz plot distribution $|A(\eta' \to \pi^0\pi^+\pi^-)|^2$ can therefore not be properly described by a low-order polynomial in the usual expansion variables x and y, see [19] for definitions, and it can definitely not be assumed to be constant over phase space.

The reason for both the large decay width $\Gamma(\eta' \to \pi^0\pi^+\pi^-)$ and the strong fluctuations over phase space are mainly due to a large contribution from isospin $I = 1$ p-waves in the final-state interactions of the decay. While for $I = 1$ p-waves the uncharged two-particle channels are C-even and, due to C-invariance, do not couple to C-odd channels related to the $\rho^0(770)$ as already pointed out in [19], the coupling of charged channels to the $\rho^\pm(770)$ is not forbidden. In fact, an important feature of the fits including the VES data compared to those without these is the large enhancement of the $\eta'\pi^\pm \to \pi^0\pi^\pm$ coupling which also determines the importance of the $\rho^\pm(770)$ in this decay. The pertinent Dalitz plot is shown in Fig. 1 and exhibits signatures

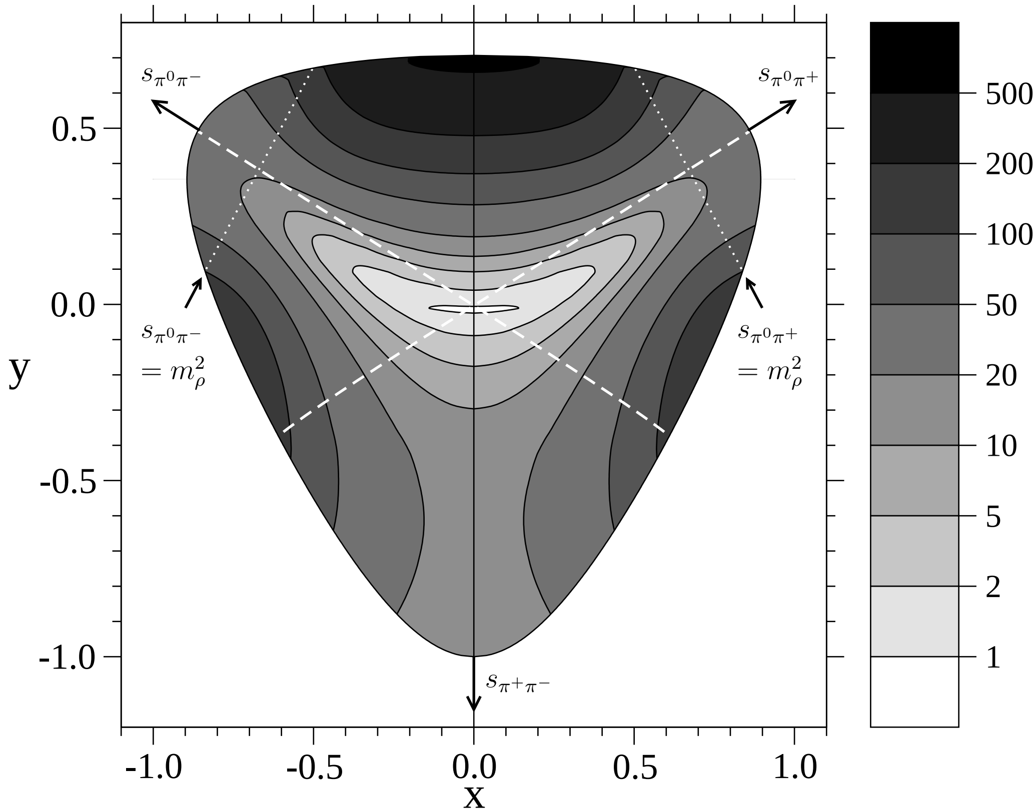

Figure 1: Dalitz plot distribution $|A(\eta' \to \pi^0\pi^+\pi^-)|^2$ of the best overall fit including the VES data for $\eta' \to \eta\pi^+\pi^-$ [13]. The distribution is normalized to unity at $x = y = 0$ (see [19] for definitions of x, y). The p-wave contributions to $\pi^0\pi^+$ ($\pi^0\pi^-$) rescattering vanish on the rising (falling) dashed line and the invariant energies associated with the $\rho^\pm(770)$ are indicated by the dotted lines.

of the $\rho^\pm(770)$. Note, however, that these resonances do not appear as bands of increased amplitude at fixed two-particle energies (dotted lines in Fig. 1), since the p-wave contributions have a kinematical zero in the middle of these bands as indicated in Fig. 1 (dashed lines). Thus the amplitude only peaks at the edge of the Dalitz plot. Moreover, due to the symmetry of the amplitude under $\pi^+ \leftrightarrow \pi^-$ exchange (C-invariance) the ρ^+, ρ^- peaks interfere constructively on the symmetry axis producing a pronounced peak structure at the top of the Dalitz plot, where the invariant mass of the $\pi^+\pi^-$ system is minimal. These features of the Dalitz plot of a pseudoscalar meson decaying into three pions have been pointed out long ago in [27].

4 Conclusions

In this work, we have critically investigated the claim of Ref. [8] that the light quark mass ratio $(m_d - m_u)/(m_s - \hat{m})$ can be extracted from the decay width ratio $r = \Gamma(\eta' \to \pi^0\pi^+\pi^-)/\Gamma(\eta' \to \eta\pi^+\pi^-)$. In order to study this issue we have employed a U(3) chiral unitary framework developed in [18,19] which is in very good agreement with the η, η' data on widths and spectral shapes. Our results clearly indicate that the two underlying assumptions of [8] in order to arrive at a relation between r and $(m_d - m_u)/(m_s - \hat{m})$, i.e., that a) the decay $\eta' \to \pi^0\pi^+\pi^-$

proceeds entirely via the decay $\eta' \to \eta\pi^+\pi^-$ followed by π^0-η mixing and that b) the decay amplitudes are constant over phase space, are not justified at all. The results from the full chiral unitary approach are in plain disagreement with these two assumptions. Moreover, the present experimental situation which is used as input to fit the parameters of the chiral unitary approach does not allow for a precise determination of the double quark mass ratio Q^2 from r.

Inclusion of the recent VES data on the $\eta' \to \eta\pi^+\pi^-$ spectral shape reduces the uncertainty of the fit results to some extent. In this case, the overall fit to η, η' data yields for $\eta' \to \pi^0\pi^+\pi^-$ a large contribution from the isospin $I = 1$ p-wave in the final-state interactions which can be attributed to a large coupling to the $\rho^\pm(770)$ resonances while contributions related to the $\rho^0(770)$ are forbidden by C-invariance. More precise data on η and η' decays are needed in order to eventually clarify this issue. An improvement of the experimental situation is foreseen in the near future due to the upcoming data from WASA at COSY [9], MAMI-C [10] and KLOE at DAΦNE [11].

Acknowledgments

We thank Heiri Leutwyler for reading the manuscript and useful comments. This research is part of the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RII3-CT-2004-506078. Work supported in part by DFG (SFB/TR 16, "Subnuclear Structure of Matter", and BO 1481/6-1).

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