

Charge independence studied in $NN \rightarrow d\pi$ reactions

H. Machner^{a,b}, J. A. Niskanen^c,

^a*Institut für Kernphysik, Forschungszentrum Jülich, 52425 Jülich, Germany*

^b*Fachbereich Physik, Universität Duisburg-Essen, Lotharstr. 1, 47048 Duisburg, Germany*

^c*Department of Physical Sciences, University of Helsinki, Finland*

Abstract

We review to what extent charge independence breaking (*CIB*) in isospin related reactions of the type $NN \rightarrow d\pi$ can or could be seen in existing data. In doing this we present fits to global and threshold cross sections including most recent data. Applying these we point out the probable impossibility to make model independent predictions even based solely on the different threshold energies and the well known $pp \rightarrow d\pi^+$. A possible discrepancy is seen between the $np \rightarrow d\pi^0$ and $pp \rightarrow d\pi^+$ data, which may require invoking explicitly isospin symmetry breaking interactions.

Key words: Pion production; isospin symmetry; charge independence breaking

PACS: 11.30.-j; 13.75.Cs; 24.80.+y

1 Introduction

It is well known that the isospin invariance as introduced originally by Heisenberg is only an approximate symmetry even in the case of strong interactions. While in nuclei it is relatively strongly broken, it is, however, very good on the level of nucleons, where the mirroring of neutrons and protons is related to charge symmetry (*CS*), a more accurate symmetry than charge independence (*CI*). In the framework of the meson theory of nuclear forces a classification of different isospin structures is given in Ref. [1]. After Weinberg's ground breaking work on the quark masses [2] it became clear that in addition to the

* Corresponding author

Email address: h.machner@fz-juelich.de (H. Machner).

Coulomb force, the difference between the light quark masses breaks isospin symmetry. This means that in QCD under exchange of the u and d quarks there will be physically observable changes. Therefore, measurements of the violation of the isospin symmetry might allow the deduction of this mass difference. Recent reviews related mainly to this charge symmetry breaking (*CSB*) are given in Refs. [3, 4]. For a recent general comprehensive review on meson production we refer to [5] and [6].

Isospin symmetry breaking effects are small and few dedicated experiments have been performed to study them in inelasticities. Nevertheless, in this article we attempt to draw together what is known about the relation between the two reactions

$$np \rightarrow \pi^0 d \quad (1)$$

and

$$pp \rightarrow \pi^+ d \quad (2)$$

paying special attention to their differences. It is readily clear that these are *not* related by the charge symmetry operation (a reflection of the isospin in the z or 0 direction), but by charge independence, a wider symmetry of arbitrary rotations in the isospin space.

As pointed out by Yang [7] already in 1952 isotopic spin invariance requires that the cross sections must be related by

$$\frac{d\sigma(pp \rightarrow \pi^+ d)}{d\sigma(np \rightarrow \pi^0 d)} = 2. \quad (3)$$

This can be shown by expressing the initial and final states in terms of eigenstates of the total isospin (spin summation implied)

$$\frac{d\sigma(pp \rightarrow \pi^+ d)}{d\sigma(np \rightarrow \pi^0 d)} = \frac{|\langle 1, 1 | S | 1, 1 \rangle|^2}{|1/\sqrt{2} \langle 1, 0 | S | 1, 0 \rangle + 1/\sqrt{2} \langle 1, 0 | S | 0, 0 \rangle|^2} \quad (4)$$

where S is the scattering matrix. If the isospin is conserved as required by charge symmetry, the second term in the denominator must vanish. Also, if the strong forces are independent of the third component of isospin, i.e. charge independent, then

$$\langle 1, 1 | S | 1, 1 \rangle = \langle 1, 0 | S | 1, 0 \rangle, \quad (5)$$

and the amplitudes in the numerator and denominator cancel leaving the factor of two.

CSB requires typically a vector operator (tensor of rank one) in the isospin space as explicitly seen in the classification of two nucleon forces in Ref. [1], i.e. the isospin structures $(\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)_0$ (class III) or $(\boldsymbol{\tau}_1 - \boldsymbol{\tau}_2)_0$ or $(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_0$ (class IV). Here the latter operators necessarily change the isospin and thus also in the $np \rightarrow d\pi^0$ reactions (and only there) can mix isospin zero initial state in the production amplitudes. This causes an asymmetry of the cross section

about 90° in the centre of mass system. Otherwise the cross section would be symmetric [8]. A measurement of the charge symmetry breaking asymmetry in the meson production reaction $np \rightarrow \pi^0 d$ close to threshold has been published recently [9].

Charge independence is a broader symmetry encompassing any combination of charges, *i.e.* a symmetry with respect to arbitrary rotations in the isospin space. Isotensor class II forces of Ref. [1] ($3\tau_{10}\tau_{20} - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$), while respecting charge symmetry, do violate charge independence in the two nucleon case, and similarly again Eq. (5) need not be valid in meson production either. A deviation from the factor of two would be a sign and a measure of violation of isotopic invariance. It should be noted that in the present case and in NN scattering the isotensor operator cannot change the isospin, which has to be unity in both the initial and final state. Therefore, the spin-spatial symmetries would be the same as in the isospin symmetric case, contrary to charge symmetry breaking.

Unfortunately even less is available for charge independence breaking (*CIB*) than *CSB* in meson production, although, in principle, this should be significantly larger than *CSB*, a few % vs. a few thousandths. To our knowledge, only the unsuccessful attempt [10] to see *CSB* reports possible marginal *CIB* in the angular dependence at 795 MeV. The experimental difficulties here stem from two origins: Firstly, there is not a "null experiment" nor such a clear forward-backward asymmetry signal as in *CSB*. Secondly, these measurements require clearly a comparison of separate experiments, one with neutron beams, prone to meet normalisation problems.

However, further motivation to study isospin symmetry breaking for the above reactions (1) and (2) is given by the recently measured, somewhat related more complicated reactions

$$pd \rightarrow \pi^0 \text{}^3\text{He} \quad (6)$$

and

$$pd \rightarrow \pi^+ \text{}^3\text{H}. \quad (7)$$

These reactions have the advantage that they can be measured simultaneously and thus their ratio does not suffer from normalisation uncertainties. Indicative of isospin symmetry breaking, the excitation function of the ratio between the differential cross sections for maximal momentum transfer to the pion showed deviation from the isospin ratio two when passing the $pd \rightarrow \eta^3\text{He}$ threshold [11]. Full angular distributions for both reactions at lower energies were reported in Refs. [12] and [13]. There the angular distributions also show a slight but systematic difference. However, the origin of this difference could be different nuclear wave functions (possibly due to the Coulomb force) of ${}^3\text{H}$ and ${}^3\text{He}$ rather than *CIB* production mechanisms. It would be interesting to see if and how isospin symmetry breaking would persist in the more basic and theoretically cleaner reactions (1) and (2) without this distortion.

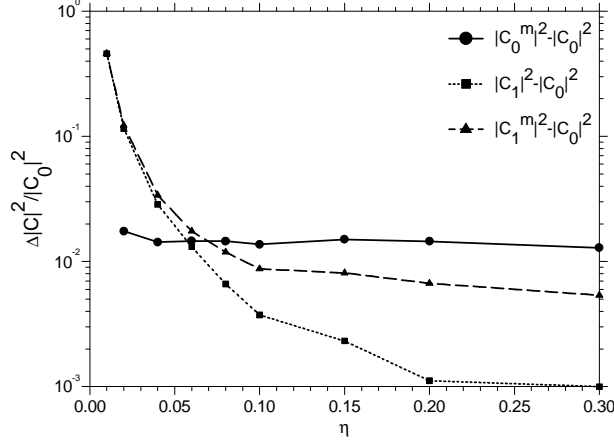


Fig. 1. Relative differences between the Gamow factors (see Ref. [16]).

The basic theoretical arguments for handling the data are presented in the next Section and then the results of the analyses in Section 3.

2 Coulomb corrections and kinematic considerations

Differential as well as total cross sections of the two reactions obviously differ because of the mass differences of the nucleons in the entrance channel and of the pions in the exit channel, and also due to the presence of the Coulomb force in the reaction (7). Furthermore, charge dependent forces may be involved in the reactions. Typically these would arise from charged vs. neutral meson mass differences in meson exchanges. Charge asymmetry, in turn, can get contributions from nucleon mass differences, meson mixing [14], magnetic interactions and explicitly isospin symmetry violating pion-nucleon rescattering [15].

In comparisons of the reactions (1) and (2) we shall first study the second effect. A common approach to correct for the Coulomb effect is to apply the s -wave Gamow penetration factor

$$|C_0(\xi)|^2 = \frac{2\pi\xi}{e^{2\pi\xi} - 1} \quad \text{with} \quad \xi = \frac{\alpha m_{\text{red}} c^2}{\hbar c q} \approx \frac{0.0068}{\eta} \quad (8)$$

to the cross sections. Here q is the centre-of-mass momentum of the pion and $\eta = q/m_{\pi^+}$.¹

¹ One should note that the final state dependence of the partial wave amplitudes is through the pion momentum q and that in η the charged pion mass acts only as a unit and should be kept the same in both reactions (1) and (2). Use of different scaling masses would cause an artificial effect based on the choice of units. Therefore, the use of "natural" mass m_{π^0} in reaction (1) is an unfortunate choice.

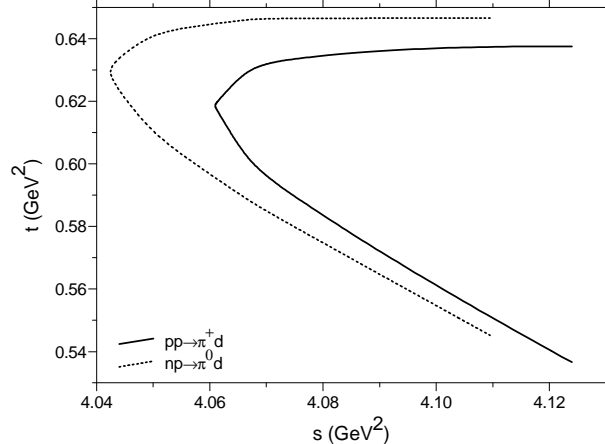


Fig. 2. Kinematical limits of the two reactions. t_{max} and t_{min} are shown as function of s .

Niskanen and Vestama [16] have calculated more generally such factors under various assumptions: for a point-like charge distribution with pion angular momentum l ($|C_l|^2$) and for an extended charge distribution ($|C_l^e|^2$) applying the Reid potential [17] to obtain the deuteron wave function. It was seen that the extended and point sources gave very similar corrections so that the use of an extended source is not really warranted with the experimental accuracy expected in near future. In Fig. 1 we show the relative difference of the Gamow factors for p -waves with extended charge $|C_1^e|^2$ and s -waves $|C_0^e|^2$ relative to $|C_0|^2$ as a function of the dimensionless centre of mass pion momentum $\eta = p_\pi/m_{\pi^+}$. The first difference is large only in a range for very small momenta $\eta \leq 0.05$, where p -wave cross section is negligibly small, and then the difference between the two factors decreases below 2%. This is the order of magnitude of precision that can possibly be obtained.

Justified by these results in removing the Coulomb, for the sake of simplicity, we will make use of only $|C_0^e|^2 \approx |C_1^e|^2 \approx 1.008 |C_0|^2$, since a separation into different partial waves is far from straightforward [16, 18]. As can be seen from Fig. 1 in relative angular dependence this average factor cancels off to a good approximation for extended sources, since $|C_1^e|^2$ and $|C_0^e|^2$ are within 0.5% from each other. The similar factor $|C_0|^2$ is also applied for the pp initial state giving a normalisation effect of 1-2%.

We proceed now to study the effect of kinematical differences due to different masses. The differential cross section $d\sigma/dt$ depends on the Mandelstam variables s and t . Here we use t as the four momentum transfer squared from the projectile to the pion. In Fig. 2 the kinematical limits, i. e. maximal and minimal values of t as functions of s are given for the two reactions (1) and (2).

In addition to the different threshold energies $\sqrt{s_0}$ also for given CMS excess

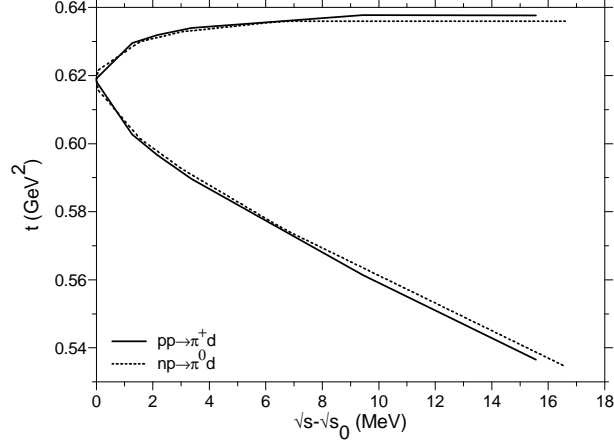


Fig. 3. Kinematical limits as function of the excess energy $\epsilon = \sqrt{s} - \sqrt{s_0}$. The t -scale is shifted for reaction (1) by -0.01065 GeV^2 .

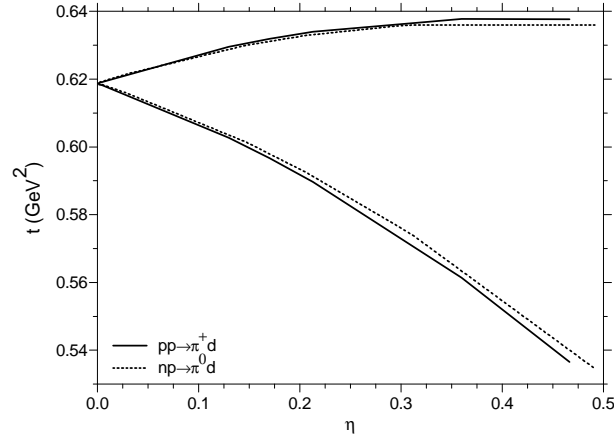


Fig. 4. Same as Fig. 3 but as function of the dimensionless pion momentum $\eta = q/m_{\pi^+}$.

energies $\epsilon = \sqrt{s} - \sqrt{s_0}$ the t values are shifted to a good approximation by a constant 0.01065 GeV^2 as can be seen from Fig. 3, where the limits for the reaction (1) have been brought on top of the reaction (2). As a function of the final state momentum the rounded bottom of the lopsided parabola gets a V shape in Fig. 4.

One may well argue that a large part of the difference between the two reactions comes, in addition to the Coulomb effects already described, simply from the different thresholds. This means that for a given final state momentum (the decisive dependence close to thresholds) the incident kinetic energy is slightly different. This gives a trivial effect in the phase space factor P (with an obvious notation for the energies and momenta p^* in the centre-of-mass system, $N = p$ or n , and including also the spin factor $1/4$)

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{1}{4(2\pi)^2\hbar^4} \underbrace{\frac{p_\pi^* E_N E_p E_d E_\pi}{p_N^* s}}_{P_{Np}} \underbrace{\sum_{\mu SM} |\langle \psi_d^\mu | H^\pi | \phi^{SM} \rangle|^2}_{R_{Np}} \quad (9)$$

$$\equiv \underbrace{\hspace{10em}}_{P_{Np}} \underbrace{\hspace{10em}}_{R_{Np}} \quad (10)$$

present in cross sections, but it influences also the transition matrices in R themselves. In particular, due to the lower threshold as a function of the final state momentum the amplitudes for $np \rightarrow \pi^0 d$ should lag behind $pp \rightarrow \pi^+ d$ in reaching and passing e.g. the Δ resonance. (One should note that the mass difference of the nucleons does not directly affect the proximity of the Δ , since similar mass differences would be present in that, too.)

In Ref. [16] changes in the total and differential cross section were expressed directly by differences in both the phase space and spin amplitudes as

$$\delta\sigma \equiv 2\sigma(np) - \sigma(pp) = \frac{P_{np} + P_{pp}}{2}(R_{np} - R_{pp}) + (P_{np} - P_{pp})\frac{R_{np} + R_{pp}}{2}. \quad (11)$$

Results for the relative (integrated) differences in P and R were given, for the latter by a model calculation.

One might choose another path aiming at a model independent prediction for the difference based on existing data. First dividing the trivial phase space factor off directly from the empirical data as well as the Gamow factors (8) one can study the energy dependence of the more interesting R for equal pion momenta in the two reactions. Then one would get from

$$\delta R \equiv R_{np} - R_{pp} = \frac{dR_{pp}}{dE_i}(E_i(np) - E_i(pp)) \quad (12)$$

a prediction for the dynamical difference between the reactions (1) and (2) from data on only the latter. However, if the quantity R_{pp} is given as a function of η like usual for the cross section, a singular $d\eta/dE_i$ arises. Consequently this product close to threshold is too large for the first order estimate to be useful.

It would now be tempting to think that deviations from this prediction would, in principle, be a direct indication of charge dependent forces. However, this expectation is based on the kinematic relation between E_i and η used to connect the two different reactions. It still leaves untouched the possible dynamical difference in the initial states due to the change of the energy but without any change in the final momentum. This will be seen later in Fig. 7.

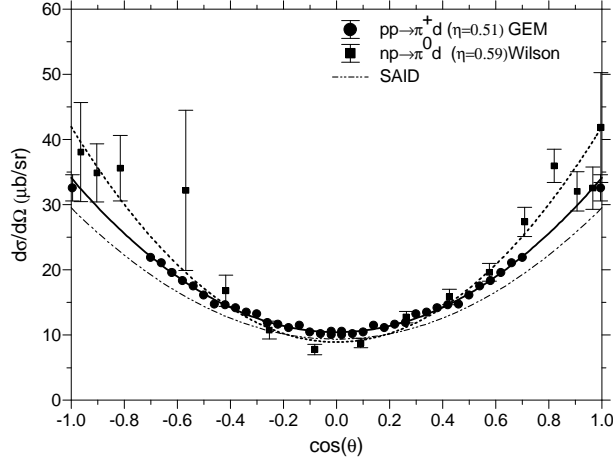


Fig. 5. Comparison of differential cross sections for reaction(1) shown as squares and for reaction (2) shown as dots. The solid and long dashed curves are parameterisations discussed in the text. The short dashed curve is the SAID-fit [21] at $\eta = 0.51$.

3 Data Analysis

We are now ready to compare data for the two reactions of interest. Unfortunately there are no differential cross sections for the same η or ϵ and the same cm emission angle. Nevertheless, in order to stress this shortage we compare two angular distributions, at reasonably similar energies. Fig. 5 shows the centre-of-mass differential cross sections from the data of Wilson et al. for the reaction (1) [19] together with those for the reaction (2) from the GEM collaboration at COSY [20] (reflected about $\cos \theta = 0$).

The differential cross sections can be expanded in terms of Legendre polynomials $P_l(\cos \theta)$ [22]

$$4\pi \frac{d\sigma(\eta, \cos \theta)}{d\Omega} = A_0(\eta)P_0(\cos \theta) + A_2(\eta)P_2(\cos \theta) \quad (13)$$

$$= \sigma(\eta) [1 + a(\eta)P_2(\cos \theta)], \quad (14)$$

with the indicated normalisation

$$\sigma(\eta) = A_0(\eta) \quad (15)$$

and the anisotropy

$$a(\eta) = A_2(\eta)/A_0(\eta). \quad (16)$$

Eq. (13) is valid for both the reaction $pp \rightarrow \pi^+d$ and for reaction $np \rightarrow \pi^0d$ with the same coefficients, if charge independence is obeyed (apart from the normalisation factor 2). As an example this functional dependence is fitted

Table 1

Uncertainty of the fitted parameters Eq. (13) to the data shown in Fig. 5.

reaction	data	η	$\Delta A_0/A_0$	$\Delta A_2/A_2$	$\Delta a/a$
$np \rightarrow \pi^0 d$	[19]	0.59	0.027	0.064	0.048
$pp \rightarrow \pi^+ d$	[20]	0.51	0.0076	0.023	0.018

to the data in Fig. 5. The uncertainties of the fit parameters are compiled in Table 1.

The error in the charged total cross section would be small enough to see isospin breaking effects, while that for the np reaction is of the same order as those effects computed in Ref. [16]. Furthermore, one should note that the np data are not absolute in their normalisation. The error in A_2 is still significantly larger. However, interestingly the relative error of the anisotropy in the form (14) is smaller than what would be obtained from the form (13). Apparently this reflects the fact that normalisation uncertainties present in $A_2(\eta)$ are not present in the relative $a(\eta)$ making the latter a preferred form to fit. Of course, the actually obtained fitting result is the same in both. The anisotropy will be discussed in more detail further down. It is also of interest to note that the SAID phase shift analysis [21], which does not include the data from Ref. [20], is significantly different.

In the comparison (Fig. 5 and Table 1) we have not applied the corrections discussed above, because of the differences between the two data sets. It is evident that the precision and quality of cross sections in proton-proton induced reactions is far superior to those from neutron-proton induced reactions, where high quality data are still lacking. Therefore, in the following considerations we do not use individual data sets but rely rather on more global parameterisations of almost all available data as functions of the pion angle and momentum.

We now consider the total cross section more globally as function of η . In Fig. 6 we show almost the world data set of cross sections for $pp \rightarrow \pi^+ d$ (Refs. [20, 23–41]) After a steep rise the cross section falls down after the maximum which corresponds to the centre of the Δ resonance. A global fit performed by Ritchie [42] underestimates the new near threshold data. Therefore we have performed a new fit to the data body after removing the phase space and the Coulomb effects. To get a reasonable fit to the partially contradictory data it was found necessary to make some choices concerning their inclusion. The following data were omitted in the fit: The data from Ref. [43] (see the discussion below). Two points from Ref. [28], one point from Ref. [44] in the interval $0.661 \leq \eta \leq 0.735$, and the lowest point from Ref. [27].

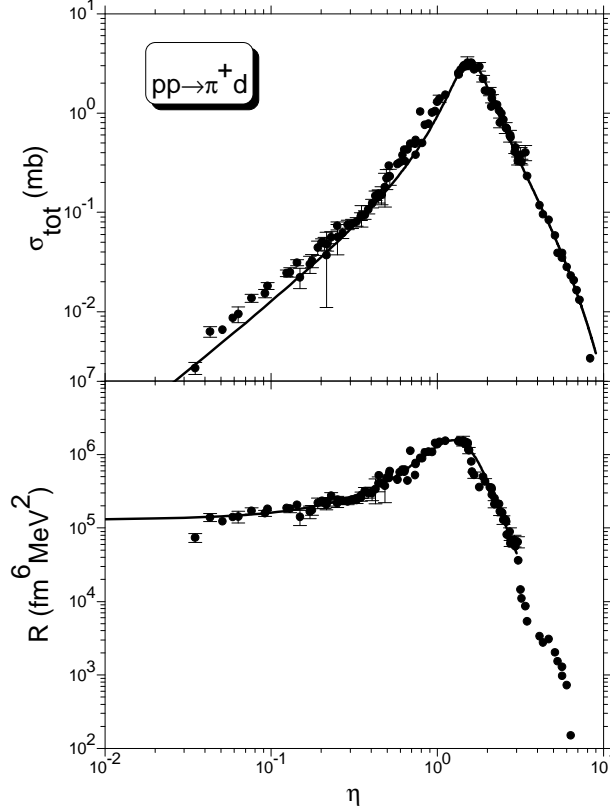


Fig. 6. Upper part: The total cross section for $pp \rightarrow \pi^+ d$ as function of the dimensionless pion centre of mass momentum. Lower part: Same as the upper part but for the angle integrated sum of the squared matrix elements, R_{pp} (Coulomb corrected). The solid curves are fits which are discussed in the text.

Table 2

Fitted parameters of Eq. (17) to the total cross sections.

$a(\text{mb})$	b	c	d	e
0.17 ± 0.03	1.34 ± 0.06	1.77 ± 0.04	0.38 ± 0.02	0.096 ± 0.02

We find the function

$$\sigma = \frac{a\eta^b}{(c - e^{d\eta})^2 + e} \quad (17)$$

to account for the raw cross section data. This fit is included in the upper panel of Fig. 6 and the fit parameters are given in Table 2.

Also shown in Fig. 6 are the Coulomb corrected angle integrated matrix elements. These can be excellently fitted by the squared Lorentz function

$$R_{pp} = \left[\frac{b_1 b_3^2}{(b_2 - \eta)^2 + b_3^2} \right]^2 \quad (18)$$

where b_2 sets the resonance position and b_3 the half-width. The corresponding curve is included in the figure and the fit parameters are given in Table 3. We

Table 3

Fitted parameters of Eq. (18) to the angle integrated and Coulomb corrected matrix elements.

b_1 (fm ³ MeV)	b_2	b_3
1252.347 ± 11.232	1.2503 ± 0.0064	0.7922 ± 0.0108

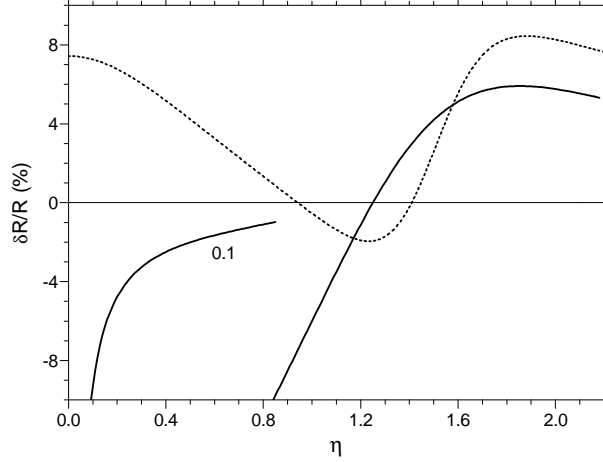


Fig. 7. Predictions for the relative difference of the total cross sections of the reactions (1) and (2). The dashed curve is the model prediction of Ref. ([16]) for $\delta R/R_{av}$, while the solid curve is based on Eq. (12) with R_{pp} .

also tried adding a constant and η -dependent background, but the quality of the fit remained essentially the same and the additional terms were consistent with zero.

Although the above form (especially with background terms) may look superficially like a theoretically motivated squared amplitude, this is not in reality the case. Since different spin amplitudes do not mix in the cross section, odd pion waves (starting with the p waves) should have a separate squared amplitude with overall threshold behaviour η^2 (and the $\Delta(1232)$ resonance) and even waves their own with a constant threshold behaviour (to which even powers of η would be added at the amplitude level). The above form is only justified as an attempt to fit the resonant structure with a simple function and it is, indeed, remarkable that nearly all total cross section data can be fitted with just three parameters. One may note that this form is not even analytically of the correct form, which should be a function of η^2 . With functions depending on only this we were not able to get fits of comparable quality even by increasing the number of parameters. The data appear to require a linear term in η in particular in the neighbourhood of $\eta \approx 1$. Also theoretical treatments have had trouble in giving enough cross section in this region without destroying agreement elsewhere. With the threshold behaviour given by this form (i.e. $dR/d\eta \neq 0$ as $\eta \rightarrow 0$) the prediction (12) becomes excessively large as can be seen from Fig. 7.

In Eq. (12) the above functional form is used to make a prediction for the difference between the reactions (1) and (2). This is shown together with the model prediction of Ref. [16] in Fig. 12. The agreement between the two is quite reasonable at higher energies. Considering the discussion of the weak model dependence in that reference the difference is significant. However, it cannot be attributed to charge dependent interactions (both are independent of those). As discussed in the previous section, the present prediction is based on the relation between the initial energy and final pion momentum (albeit with different thresholds) employing the known pp data. Possible difference in the initial state at different energies does not come automatically except through the final momentum dependence. This difference is most pronounced for small values of η . Even with the correct threshold dependence the uniformly increasing matrix elements should predict smaller np than pp cross sections. However, the converse is seen in the model prediction (dashed curve). Apparently, *if* one could have a varying initial nucleon energy with a constant final momentum, the matrix elements would decrease with the energy. This trend is overcome only in the region of the sharp rise of the Δ -resonance effect, where the np reaction lags behind and the cross section becomes smaller than that of the pp initiated reaction.

We now compare the angle integrated matrix elements as function of η in the threshold region where there are rather precise data for both reactions. Here the cross sections for $np \rightarrow \pi^0 d$ are taken from Ref. [45]. They are in the threshold range up to $\eta = 0.32$. In this range one should be able to apply the low energy expansion

$$R = \alpha_0(1 + \alpha_1\eta^2), \quad (19)$$

where the first term is predominantly s -wave and the second p -wave [18]. The Coulomb corrected cross sections for $pp \rightarrow \pi^+ d$ are from Refs. [20, 23–27]. Here we have restricted the range to $\eta \leq 0.5$ to be well-matched. It is worth mentioning that the cross sections from Refs. [27] and [26] agree nicely with each other when the same Coulomb correction is applied. The Coulomb correction used in Ref. [27] is somewhat too large (see the discussion in Ref. [16]). Here we have started with the uncorrected cross sections from Ref. [46] and applied the corrections discussed above. The deduced matrix elements are shown in Fig. 8 and the corresponding fit parameters in Table 4. Those for $np \rightarrow \pi^0 d$ reactions are multiplied by the isospin factor of two. Fits with Eq. (19) are shown as error bands.

The excitation functions for the two reactions clearly differ from each other with respect to magnitude and shape. Applying the same corrections as for the data we have also extracted the matrix elements from an earlier fit [42] to data, which appeared before the advent of new data in the threshold region [25–27]. This is also shown in the figure, where the different slope to the data shows the

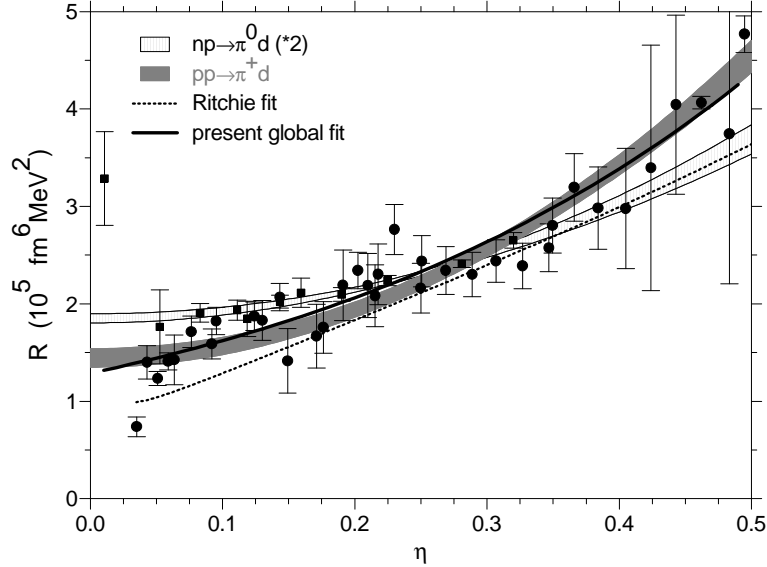


Fig. 8. Fit of the low energy function (19) to the sum of squared matrix elements extracted from the data. The error band represents the confidence interval on the 95% level. Data for reaction $np \rightarrow \pi^0 d$, corrected by the isospin factor of 2, are shown as squares, those for reaction $pp \rightarrow \pi^+ d$ as dots. Also shown is the result for the fit from Ref. [42].

importance of the new threshold data. Following Feynman’s last point rule² in fitting Eq. (19) to the data we have excluded one point from Ref. [23] which is far from the band of the other data. The same is true for the points with smallest η from Refs. [27] and [45] (see Fig. 8).

Differing from other $np \rightarrow \pi^0 d$ measurements, the normalisation performed by Hutcheon et al. [45] is to the simultaneously measured $np \rightarrow pn$ scattering cross sections. The absolute value of this reaction was taken from the SAID phase shift analysis [48]. The other measurements normalized their counting rate to $pp \rightarrow \pi^+ d$ cross sections. However, even the $np \rightarrow pn$ data might not be completely independent of an assumed isospin symmetry (see Ref. [49] and Ref. [50]). If that would be so we would expect two bands running in parallel with only different normalisations. Unfortunately at the present time we are not aware of such np reaction data which do not suffer from normalisation problems to such an extent as to make comparisons meaningless.

In spite of the above normalisation problems it should be noted that at threshold $R_{np} > R_{pp}$ as predicted by Ref. [16] (the dashed curve in Fig. 7). However, the experimental difference is larger and goes through zero much faster than the prediction. This difference of the only comparable data shown in Fig. 8 can be further elaborated into the form of the prediction of Ref. ([16] in Fig. 9.

² According to Feynman [47] ”there is a principle that a point on the edge of the range of the data—the last point—isn’t very good, because if it was, they’d have another point further along”.

Table 4

Parameters from fits of Eq. (19) to the matrix elements in the close to threshold area.

reaction	α_0 (fm ⁶ MeV ²)	α_1
$np \rightarrow \pi^0 d$	$(185 \pm 2) \times 10^3$	3.97 ± 0.22
$pp \rightarrow \pi^+ d$	$(144.5 \pm 4.7) \times 10^3$	8.57 ± 0.52

Quite clearly the disagreement between experiment and theory cannot be removed even by renormalising the np cross sections. In the figure this has been done by moderate factors 0.9 and 1.1. This discrepancy might possibly be an indication of an effect of charge dependent forces and apparently deserves a more dedicated study.

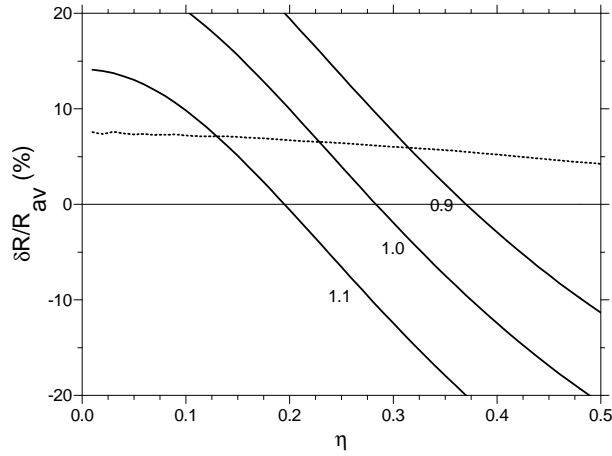


Fig. 9. Comparison of the relative change between the model results of Ref. [16] (dashed curve) and the fit of Eq. 19 (solid curves) directly and with $\alpha_0(np)$ renormalised by factors 0.9 and 1.1.

To avoid problems with the normalisation we now study the relative anisotropy $a(\eta)$, instead. The nice feature of studying this quantity is that the phase space correction and the isospin factor cancel exactly. As seen below, to a good approximation this is true for the Coulomb correction as well. However, before discussing the data we should first have a more detailed look into the meaning of this observable.

Due to parity conservation always

$$L(np) = l_\pi \pm 1, \quad (20)$$

so that even isospin conserving pion waves are associated with initial odd-triplets and *vice versa* odd pion waves with even-singlet states, while for isospin breaking (*CSB*) amplitudes the converse is valid³. With the interference of

³ Ref. [51] quotes additionally a superfluous and incorrect condition $J = L(np) \pm 1$

opposite parities the contribution from *CSB* isospin mixing is odd in $\cos \theta$ and does not appear in the expansion with even polynomials, if both forward and backward angular ranges are represented with equal weights. Charge dependence due to isotensor forces does not mix these parities, so for this study it is sufficient and consistent to consider only angular dependencies symmetric about 90° .

The differential cross section can be expanded in terms of the lowest partial waves a_J (here up to the p -waves) as

$$4\pi \frac{d\sigma(\eta, \cos \theta)}{d\Omega} = \frac{1}{4} \left[(|a_0|^2 + |a_1|^2 + |a_2|^2) + (|a_2|^2 - 2\sqrt{2}\Re(a_0 a_2^*)) P_2(\cos \theta) \right]. \quad (21)$$

By comparison with Eq. (13) one identifies now $1/4 (|a_0|^2 + |a_1|^2 + |a_2|^2)$ as the total cross section and $1/4 (|a_2|^2 - 2\sqrt{2}\Re(a_0 a_2^*))$ as A_2 . Here the $^1S_0 \rightarrow p$ amplitude a_0 is significantly smaller than the isotropic s -wave a_1 or the dominant p -wave $^1D_2 \rightarrow p$ enhanced by the Δ excitation with increasing energy [52]. In the case of a vanishing amplitude a_0 , $A_2(\eta)$ would be exclusively of a single p -wave without phase shift dependent interference. Also for a nonvanishing a_0 , if the same Coulomb phase is associated with the both p -wave amplitudes a_0 and a_2 , it will cancel off at this level of expansion and the result would be formally the same for both reactions.

In order to be more sensitive to the p -wave we have to extend the momentum range. However, then also higher Legendre-polynomials and higher partial waves have to be considered in fitting the angular distributions and possible interference effects involving different Coulomb phases can arise, first the d -waves with the s -wave pions. Their partial wave dependencies will not be discussed here. For the general relation between the Legendre-coefficients and higher partial wave amplitudes (also including many spin observables) see Ref. [53]. Again both reactions should be well matched in η . In the upper end of energies lack of data from $np \rightarrow \pi^0 d$ above $\eta = 2$ limits the range of comparison with $pp \rightarrow \pi^+ d$.

The data for reaction (2) are from Refs. [20, 24–27, 31, 34–36, 54]. The data from [32] and [55] were excluded, since they are far above the other data. The data from the time reversed reaction from [43] were also excluded from the fit because of the large spread within this data set (see the discussion in Ref. [20]). In the case of reaction (1) only the data from Refs. [45, 51, 56] were taken into account. Those from [19] and [57] were excluded on similar grounds as in the case of the charged channel. Although the error bars are small (see also Table 1) there seem to be systematical uncertainties. This is demonstrated in Fig. 10.

for initial triplets, which would prevent e.g. pion s -wave production.

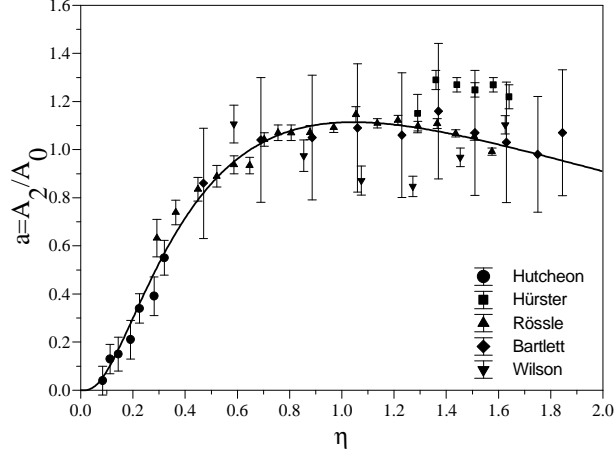


Fig. 10. The ratio $a = A_2/A_0$ for the reaction $np \rightarrow \pi^0 d$ as function of $\eta = p_\pi/m_\pi$. The solid curve is a fit as discussed in the text.

Table 5

Parameters obtained from fitting Eq. (22) to the anisotropies.

reaction	α	β	γ
$np \rightarrow \pi^0 d$	1.114 ± 0.007	1.044 ± 0.017	1.020 ± 0.029
$pp \rightarrow \pi^+ d$	1.111 ± 0.007	1.003 ± 0.009	0.926 ± 0.020

In order to make a comparison between the two reactions possible, we have fitted to data a function which describes the dependence very well, although it is not motivated by any theory. The function is

$$a(\eta) = \alpha \exp \left\{ -\frac{1}{2} \left[\ln \left(\frac{\eta}{\beta} \right) / \gamma \right]^2 \right\}. \quad (22)$$

The fitted parameters are compiled in Table 5. While the values for α agree with each other, the values for β and γ are slightly different. This also reflects in slightly different momentum dependencies of the anisotropy as shown in Fig. 11.

In order to study further the deviations of the anisotropy for the two reactions from each other, we plot the difference of the anisotropies in Fig. 12. Also the uncertainty is shown. We can state that a_{pp} is larger than a_{np} in the vicinity of the threshold ($\eta < 0.5$) and smaller for the range above, although the error bar is large.

4 Discussion

We have discussed the total and differential cross sections of the reactions $np \rightarrow \pi^0 d$ and $pp \rightarrow \pi^+ d$ as functions of the pion momentum in order to allow

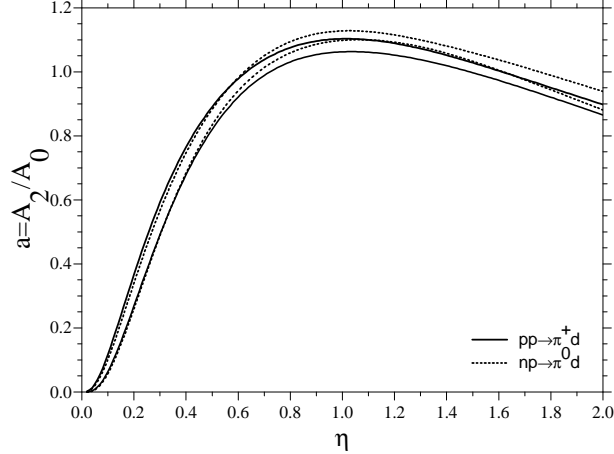


Fig. 11. The

ropies.

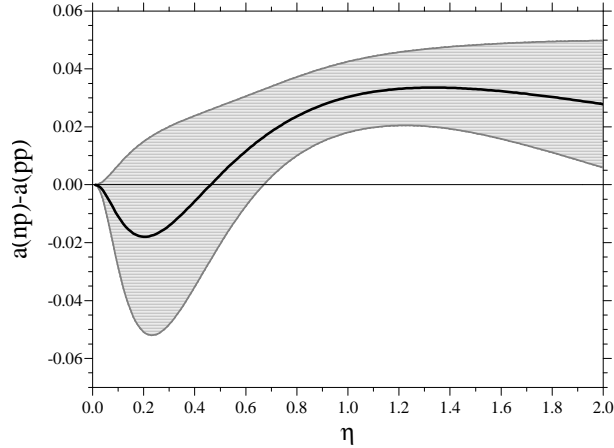


Fig. 12. The difference of the anisotropy for $np \rightarrow \pi^0 d$ and $pp \rightarrow \pi^+ d$ (thick curve). The confidence interval (on 95% level) is indicated by the shaded area.

for a comparison with respect to isospin symmetry. This takes into account automatically the different thresholds but with the cost of having different initial kinetic energies. However, this difference can be dealt with easily to first order once Coulomb effects have been corrected for and the slight difference in the phase space. It is found that the differential cross section data at individual energies for the reaction $np \rightarrow \pi^0 d$ do not have the quality to make such a comparison useful.

Having first made a global fit of the total cross sections and the associated effective matrix elements for future use, we then compared a number of total cross sections in the threshold region. This limitation is enforced by the lack of higher energy absolute cross sections of the $np \rightarrow \pi^0 d$ reaction. The result is that very close to threshold $2\sigma(np \rightarrow \pi^0 d) > \sigma(pp \rightarrow \pi^+ d)$, contrary to a simple "model independent" expectation (12) but in agreement with a theoretical anticipation [16]. It was not possible to get both the size and steepness of the difference as a function of the final pion momentum to agree with the

charge independent theory.

The cross section in the interval studied is mainly due to the transition ${}^3P_1 \rightarrow {}^3S_1s$, which dominates the outgoing s -wave with the partial wave amplitude a_1 , and ${}^1D_2 \rightarrow {}^3S_1p$ in the p -wave amplitude a_2 . There is also the externally indistinguishable contribution from the deuteron D state, so the shorter notation ${}^{2S+1}L_J \rightarrow l_\pi$ is well justified. Apparently the threshold normalisation is dictated by the s -wave amplitude (which yields the size order of the two cross sections dependent on the initial energy), while the p -wave determines the slope and thus the zero crossing point of the difference. Simply renormalising the np cross section has opposite effects in the threshold difference and zero crossing in a comparison of theory and experiment and is not a solution to their discrepancy.

We then compared the relative anisotropy for both reactions, where phase space corrections are not necessary and also the Coulomb effects are minute. It is mainly due to the outgoing p -wave with the Δ enhanced transitions ${}^1D_2 \rightarrow p$. The comparison of the anisotropy for both reactions shows that $a(np \rightarrow \pi^0d) > a(pp \rightarrow \pi^+d)$ although the uncertainty is large. The reason for this is again the poor quality of the np data. As an example we mention the overlap region around $\eta \approx 0.35$ between the two data sets [45] and [56], which differ strongly, as is shown in Fig. 10. The problem of energy resolution of a neutron beam and the absolute calibration of its intensity may be overcome by changing the incident channel at the cost of increasing the number of participating nucleons. A quasi-free reaction $pd \rightarrow \pi^0dp_s$ with a spectator proton p_s was recently shown to be feasible in a storage ring with a cluster target [58]. Also the reaction $dp \rightarrow \pi^0dp_s$ with a fast spectator proton may be considered.

5 Acknowledgement

This work was supported in part by the Academy of Finland (number 211592) and DAAD (Germany, number DB000379) exchange grant. We thank C. Hanhart and K. Kilian for useful discussions.

References

- [1] E. M. Henley, G. A. Miller: (1979) in M. Rho, D. Wilkinson (editors), *Mesons in Nuclei*, North Holland, volume I.
- [2] S. Weinberg: (1977) in *A Festschrift for I. I.Rabi*, New York Academy of Sciences, New York.

- [3] G. A. Miller, B. M. K. Nefkens, I. Slaus: Phys. Rep., **194** (1990) 1.
- [4] G. A. Miller, W. T. H. van Oers: (1995) in W. C. Haxton, E. M. Henley (editors), Symmetries and Fundamental Interactions in Nuclei, World Scientific, Singapore.
- [5] H. Machner, J. Haidenbauer: J. Phys., **G 25** (1998) R231.
- [6] C. Hanhart: Phys. Rep., **397** (2004) 155.
- [7] C. N. Yang: unpublished, (1952).
- [8] S. Barshay, G. M. Temmer: Phys. Rev. Lett., **12** (1964) 728.
- [9] A. K. Opper et al.: Phys. Rev. Lett., **91** (2003) 212302.
- [10] C. L. Hollas et al.: Phys. Rev., **C 24** (1981) 1561.
- [11] M. Abdel-Bary et al.: Phys. Rev., **C 68** (2003) 021603 R.
- [12] M. Betigeri et al.: Nucl. Phys., **A 690** (2001) 473.
- [13] S. Abdel-Samad et al.: Phys. Lett., **B 553** (2003) 32.
- [14] J. A. Niskanen: Few-Body Systems, **26** (1999) 241.
- [15] U. van Kolck, J. A. Niskanen, G. A. Miller: Phys. Lett., **B 493** (2000) 65.
- [16] J. A. Niskanen, M. Vestama: Phys. Lett., **B 374** (1997) 253.
- [17] R. Reid: Ann. Phys. (N.Y.), **50** (1968) 411.
- [18] H. Machner: Nucl. Phys., **A 633** (1998) 341.
- [19] S. S. Wilson et al.: Nucl. Phys., **B 33** (1971) 253.
- [20] M. Betigeri et al.: Phys. Rev., **C 63** (2001) 044011.
- [21] C. H. Oh et al.: Phys. Rev., **C 56** (1997) 635.
- [22] J. A. Niskanen: (1980) in Proc. 5th Int. Symp. on Polarization Phenomena in Nuclear Physics, Santa Fe, Aug. 1980, AIP Conf. Proc. **69**, page 77.
- [23] C. M. Rose: Phys. Rev., **154** (1967) 1305.
- [24] B. G. Ritchie et al.: Phys. Rev. Lett., **66** (1991) 568.
- [25] M. Drochner et al.: Phys. Rev. Lett., **77** (1996) 454.
- [26] M. Drochner et al.: Nucl. Phys., **A 643** (1998) 55.
- [27] C. Heimberg et al.: Phys. Rev. Lett., **77** (1996) 1012.
- [28] S. I. Gogolev et al.: Phys. Lett., **B 300** (1993) 24.
- [29] B. M. Preedom et al.: Phys. Lett., **B 64** (1976) 11.
- [30] D. Axen et al.: Nucl. Phys., **A 256** (1976) 387.

- [31] D. Aebischer et al.: Nucl. Phys., **B 108** (214).
- [32] C. L. Dolnick et al.: Nucl. Phys., **B 22** (1970) 461.
- [33] F. Shimizu, et al.: Nucl. Phys., **A 386** (1982) 571.
- [34] J. Hoftietzer et al.: Phys. Lett., **B 100** (1981) 462.
- [35] C. Richard-Serre et al.: Nucl. Phys., **B 20** (1979) 413.
- [36] J. Boswell: Phys. Rev., **C 25** (1982) 2540.
- [37] B. Mayer et al.: Nucl. Phys., **A 437** (1985) 630.
- [38] H. Nann et al.: Phys. Lett., **B 88** (1979) 257.
- [39] M. Ja Borkowski et al.: **G 11** (1985) 69.
- [40] J. Yonnet et al.: Nucl. Phys., **A 562** (1993) 352.
- [41] H. L. Anderson: Phys. Rev., **D 9** (1974) 580.
- [42] B. G. Ritchie: Phys. Rev., **C 44** (1991) 533.
- [43] E. A. Pasyuk et al.: Phys. Rev., **C 55** (1997) 1026.
- [44] B. G. Ritchie et al.: Phys. Rev., **C 22** (1981) 552.
- [45] D. A. Hutcheon et al.: Phys. Rev. Lett., **64** (1990) 176.
- [46] C. Heimberg: (1997) Ph.D. thesis, North Western University.
- [47] R. P. Feynman: (1986) p. 233, Surely you're joking, Mr. Feynman!, Bantam Books, Toronto, New York, London, Sidney, Auckland.
- [48] R. A. Arndt et al.: Phys. Rev., **D 28** (1983) 97.
- [49] J. Blomgren, N. Olsson: Phys. Rev. Lett., **83** (1999) 1692.
- [50] M. Drochner et al.: Phys. Rev. Lett., **83** (1999) 1693.
- [51] D. F. Bartlett, F. Friedburg, K. Goulios, I. S. Hammerman, D. F. Hutchinson: Phys. Rev., **D 1** (1970) 1984.
- [52] D. V. Bugg, A. Hasan, R. L. Shypit: Nucl. Phys., **A 477** (1988) 546.
- [53] B. Blankleider, I. J. Afnan: Phys. Rev., **C 31** (1985) 1380.
- [54] E. Gülmez et al.: Nucl. Phys., **A 551** (1993) 621.
- [55] J. H. Norem: Nucl. Phys., **B 33** (1971) 512.
- [56] E. Rössle, et al.: (1981) in R. D. Bent (editor), Pion Production and Absorption in Nuclei, AIP Conf. Proc. **47**, page 171.
- [57] W. Hürster et al.: Phys. Lett., **B 91** (1980) 214.
- [58] R. Bilger at al.: **A 457** (2001) 64.