

Measurement of the Slope Parameter for the $\eta \rightarrow 3\pi^0$ Decay in the $pp \rightarrow pp\eta$ Reaction

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The CELSIUS/WASA setup is used to measure the $3\pi^0$ decay of η mesons produced in pp interactions with beam kinetic energies of 1.36 and 1.45 GeV. The efficiency-corrected Dalitz plot and density distributions for this decay are shown, together with a fit of the quadratic slope parameter α yielding $\alpha = -0.026 \pm 0.010(stat) \pm 0.010(syst)$. This value is compared to recent experimental results and theoretical predictions.

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I. INTRODUCTION

Isospin violating decays into three pions are two of the most probable η meson decay modes: $BR(\eta \rightarrow 3\pi^0) = 32.5\%$ and $BR(\eta \rightarrow \pi^0\pi^+\pi^-) = 22.6\%$ [1]. It appears that contribution of electromagnetic processes is suppressed [2, 3] and the decays are driven by an isospin violating term of the QCD Lagrangian proportional to $m_d - m_u$. The partial width of the $\eta \rightarrow \pi^0\pi^+\pi^-$ decay calculated using current algebra (PCAC) is 66 eV [4], much below the experimental value 294 ± 16 eV [1]. Second order contributions in the low energy expansion of the QCD Lagrangian (Chiral Perturbation Theory – CHPT) were calculated by Gasser and Leutwyler increasing the result to 160 eV [5]. The big change with respect to the first order calculations implies the importance of $\pi\pi$ interaction in the final state. An elegant method of including the interaction up to higher orders is provided by dispersion relations. There are two calculations using this technique [6, 7] but employing different formalism. They lead consistently to an enhancement of the decay rate by about 14%. A free parameter in this approach is the value of the so called *subtraction point* and it was

constrained using CHPT calculations.

The decay width can be expressed in the factorized form:

$$\Gamma = \left(\frac{Q_D}{Q}\right)^4 \bar{\Gamma} \quad (1)$$

where the dependence on the $m_d - m_u$ is contained only in the Q term:

$$\frac{1}{Q^2} = \frac{m_d^2 - m_u^2}{m_s^2 - \frac{1}{4}(m_d + m_u)^2}. \quad (2)$$

Normally the calculations of Γ are performed using the Dashen theorem [8] where $Q = Q_D \equiv 24.1$. Since Γ is sensitive to the exact value of Q , it was suggested that the decay might provide precise constrain for the light quark mass ratios [9]. Namely Q determines the major axis of the ellipse in the $m_u/m_s, m_d/m_s$ plane. One important prerequisite is the reliability of the $\bar{\Gamma}$ calculations. The calculations can be tested by comparing the ratio $\Gamma(\eta \rightarrow 3\pi^0)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ and the density distributions in the respective Dalitz plots with experiment. Recently Q was derived from preliminary KLOE data on $\eta \rightarrow \pi^+\pi^-\pi^0$ decay [10, 11], by determining the subtraction point within the dispersion relation approach of [7], yielding a Q value 22.1 [12].

This letter describes an exclusive measurement of kinematical distributions of the π^0 's from the $\eta \rightarrow 3\pi^0$ chan-

*Deceased

TABLE I: Experimental and theoretical results for the slope parameter α

α	Comment	Ref.
$-0.052 \pm 0.017(\text{stat}) \pm 0.010(\text{syst})$	Exp (CBarrel)	[18]
-0.031 ± 0.004	Exp (CBall)	[19]
$-0.013 \pm 0.004(\text{stat}) \pm 0.005(\text{syst})$	Exp (KLOE)	[10]
0	PCAC	[4]
+0.015	CHPT, 1loop	[5, 20]
-0.007...-0.014	CHPT+dispersive	[7]
-0.007	UCHPT	[13]
-0.031 ± 0.003	UCHPT/fit	[21]

nel. The phase space of the decay can be represented by a Dalitz plot employing symmetrized variables as shown in Fig. 1(left). The amplitude can then be expressed in terms of polar coordinates ρ and ϕ , where due to symmetry ϕ is restricted to $0 < \phi < 60^\circ$. Instead of ρ a normalized dimensionless variable $0 \leq z \leq 1$ is used:

$$z \equiv \frac{\rho^2}{\rho_{max}^2} = \frac{2}{3} \sum_{i=1}^3 \left(\frac{T_i - \langle T \rangle}{\langle T \rangle} \right)^2. \quad (3)$$

where T_i is kinetic energy of a π° in the η rest frame and $\langle T \rangle$ is the average kinetic energy.

The slope parameter, α , is defined as a coefficient in the leading term of the decay amplitude expansion in z :

$$|A(z, \phi)|^2 = c_0(1 + 2\alpha z). \quad (4)$$

The situation for the α measurement is unsettled with inconsistent experimental results and disagreement with CHPT predictions (Table I). In lowest order CHPT $\alpha = 0$. The deviation is caused by the $\pi^\circ\pi^\circ$ interactions. All calculations beyond one loop predict a negative value of α . Dispersion relations [7] and U(3) chiral effective field theory with rescattering [13] give consistently a low value for the $|\alpha|$ of about 0.007. Most recent calculations with U(3) effective Lagrangian and Bethe-Salpeter equations include more general rescattering graphs and parameters are fitted to the scattering data for the pseudoscalar mesons [14]. The calculations are able to explain the large experimental values for $|\alpha|$. However in that approach a change of the Q value can be compensated by other fit parameters and could even be equal to Q_D .

A special feature of the $\eta \rightarrow 3\pi^\circ$ decay is that the physical region extends below threshold of the $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ reaction. Therefore a cusp structure should be visible in invariant mass distribution of two π^0 around $2m_{\pi^\pm}$ (dashed lines in Fig. 1(left)), in analogy to the recent observation in $K^+ \rightarrow \pi^+\pi^0\pi^0$ decay [15, 16, 17]. The exact parameters of the cusp in the $K^+ \rightarrow \pi^+\pi^0\pi^0$ decay provide one of most precise determinations of the $\pi\pi$ scattering length ($a_2 - a_0$).

The experimental determination of the α requires that background contribution to the η production process and other systematic errors are well under control. All recent

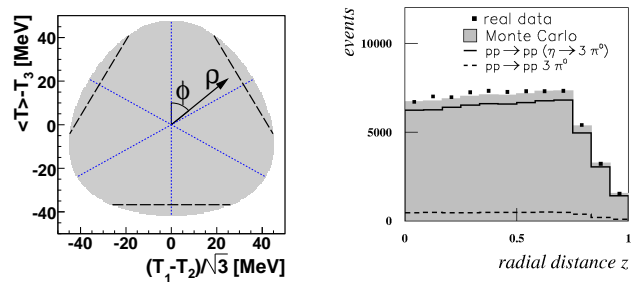


FIG. 1: (Left) Symmetrized Dalitz plot for $\eta \rightarrow 3\pi^\circ$ decay. (Right) radial density distribution (right) for the MC (shaded area) and experimental data (closed symbols).

experiments on the decay $\eta \rightarrow 3\pi^\circ$ rely solely on photon detection with close to 4π electromagnetic calorimeters. The Crystal Barrel experiment used the final state with 10γ ($\bar{p}p \rightarrow \eta 2\pi^\circ$) [18], Crystal Ball used the reaction $\pi^-p \rightarrow n\eta$ where only the 6γ from the decay were measured [19]. KLOE [10] used the decay $\phi \rightarrow \eta\gamma$ as a source of the η mesons leading to 7γ in the final state. The α values differ by more than three standard deviations between the experiments. We have performed an independent measurement using the reaction $pp \rightarrow pp\eta$ [22]. The main advantage is that the excellent missing mass resolution of the forward going pp pair can be used to tag the process.

II. MEASUREMENT AND DATA REDUCTION

The analysis is based on data taken with the WASA detector at CELSIUS [23] using a pellet target system that provides small ($30 \mu\text{m}$ diameter) hydrogen pellets that cross the proton beam of nominal 1.36 GeV and 1.45 GeV kinetic energy (the corresponding center of mass excess energies are 41 MeV and 75 MeV respectively). The WASA detector system comprises a multilayered forward detector (FD) for the measurement of charged particles emerging in the scattering angle range of 2.5° - 18° , and a central detector (CD) composed of an electromagnetic calorimeter of 1012 CsI(Na) crystals and a drift chamber/solenoid combination for the measurement of particles from η decays in the angular range of 20° - 140° .

The basic criteria for the selection of the final $\eta \rightarrow 3\pi^\circ$ state are (i) two protons detected in the FD and (ii) six γ hit clusters (from $\pi^\circ \rightarrow \gamma\gamma$) detected in the CD as neutrals with a minimum of 20 MeV deposited energy. The experimental proton-proton missing mass distribution can be compared to Monte Carlo (MC) simulated data for the reaction channels $pp \rightarrow pp\pi^0\pi^0\pi^0$, $pp \rightarrow pp\eta$, and $pp \rightarrow pp\pi^0\pi^0$. The procedure is described in detail in [22], the result for 1.36 GeV kinetic energy is shown in Fig. 2. The good agreement in shape that is obtained also for other kinematical proton variables and of the absolute cross sections for the η channel with existing

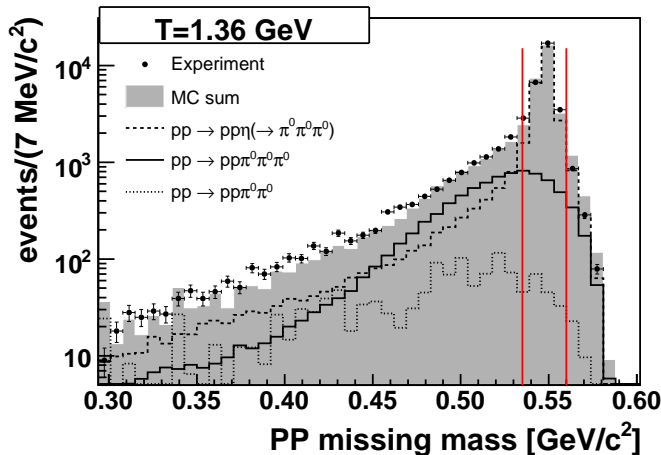


FIG. 2: Experimental pp missing mass distribution for reconstructed $pp\pi^0\pi^0\pi^0$ final states with a fit of the MC distributions for $2\pi^0$, $3\pi^0$ and η production (from [22]). The vertical lines indicate the η selection.

data, demonstrates that the detector and its efficiency are well under control. The missing mass resolution of the FD (5 MeV/ c^2 FWHM at 1.36 GeV) therefore allows selection of the $pp \rightarrow pp\eta$ reaction (the condition $535 \text{ MeV}/c^2 < MM_{pp} < 560 \text{ MeV}/c^2$ was applied) such that the remaining background, mostly from direct $pp \rightarrow pp3\pi$ reaction, is about 5%.

All possible combinations (15) of the 6 reconstructed gammas to form $3\pi^0 \rightarrow \gamma\gamma$ pairs are sorted by means of a parameter $\chi^2 = \sum_{i=1}^3 (IM_i - m_{\pi^0})^2 / \sigma_i^2$, where IM_i is the invariant mass of the i -th pair of a combination, and σ_i the resolution. At most two combinations with the lowest χ^2 are selected for a kinematical fit of the full event with 8 constraints: four-momentum conservation, the $\pi^0 \rightarrow \gamma\gamma$ constraints and the $\eta \rightarrow 3\pi^0$ constraint. A cut on the χ^2 of the most probable combination is applied to further suppress background and to increase the combinatorial purity. Three different data sets were individually analyzed, the combined result is based on 75000 events in the Dalitz plot after all cuts.

III. DALITZ PLOT AND SLOPE α

The reconstructed experimental z distribution is shown in Fig. 1(right) together with MC simulation including $pp \rightarrow pp\eta$ and $pp \rightarrow pp\pi^0\pi^0\pi^0$ reactions. The detector response function for the reconstructed z was determined from MC: it has a gaussian distribution with a standard deviation (RMS) 0.067. The resolution in the $\pi^0\pi^0$ invariant mass ($M_{\pi\pi}$) is approximately constant and equal to 6 MeV/ c^2 (RMS).

The acceptance corrected $|A|^2$ dependence on the z variable is obtained by dividing the measured distribution by the MC prediction with α set to 0. A fit of the function (4) is performed to extract the parameter

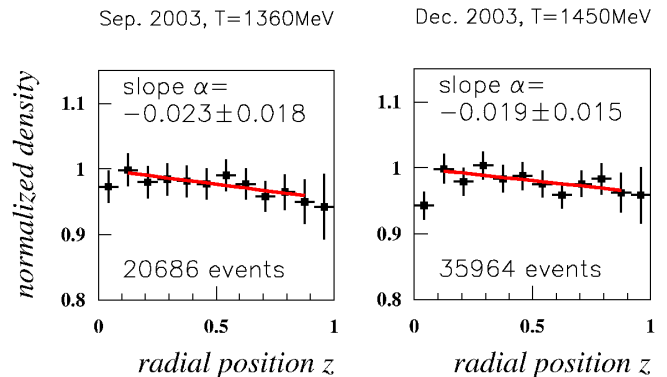


FIG. 3: Extracted experimental $|A|^2/c_0$ dependence on the z variable for a subset of the $T = 1.36$ GeV (left) and for all 1.45 GeV (right) data together with a fit of the slope parameter α . Only statistical errors are shown.

TABLE II: Statistical errors of α fitted for data subsets obtained by parameter variation for selection criteria and reconstruction procedure.

Condition	$\alpha \pm \sigma_{stat}$	events
All data, $0.1 \leq z \leq 0.9$	-0.026 ± 0.010	74700
All data, $0.0 \leq z \leq 0.9$	-0.014 ± 0.009	74700
Analysis of subsets:		
Subset I $T = 1.36$ GeV	-0.023 ± 0.018	20700
Subset II $T = 1.36$ GeV	-0.041 ± 0.018	18000
Subset $T = 1.45$ GeV	-0.019 ± 0.015	36000
Subset I, variation of MM_{pp} cut:		
$0.530 \leq MM_{pp} \leq 0.575$	-0.019 ± 0.018	21300
$0.535 \leq MM_{pp} \leq 0.560$	-0.023 ± 0.018	20700
$0.540 \leq MM_{pp} \leq 0.555$	-0.025 ± 0.018	18800
Subset I, variation of χ^2_{kinfit} cut:		
$\chi^2_{kinfit} \leq 999$	-0.023 ± 0.015	27900
$\chi^2_{kinfit} \leq 50$	-0.021 ± 0.017	23600
$\chi^2_{kinfit} \leq 30$	-0.023 ± 0.018	20700
$\chi^2_{kinfit} \leq 15$	-0.021 ± 0.020	15200
$\chi^2_{kinfit} \leq 15, \chi^2_{second}/\chi^2_{first} \geq 1.2$	-0.038 ± 0.024	10800
Subset I, alternative parameterization of kin. fit	-0.023 ± 0.019	17000

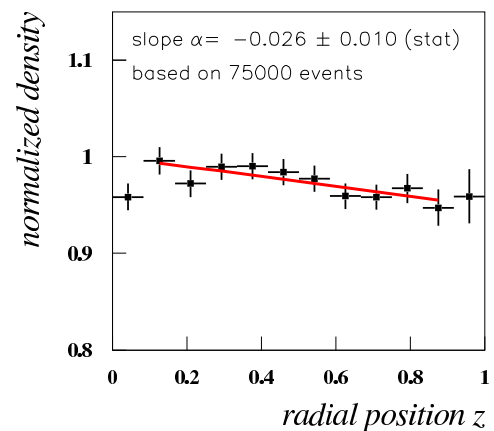


FIG. 4: Extracted experimental $|A|^2/c_0$ dependence on the z variable for all data together with a linear fit to of the slope parameter α . Only statistical errors are shown.

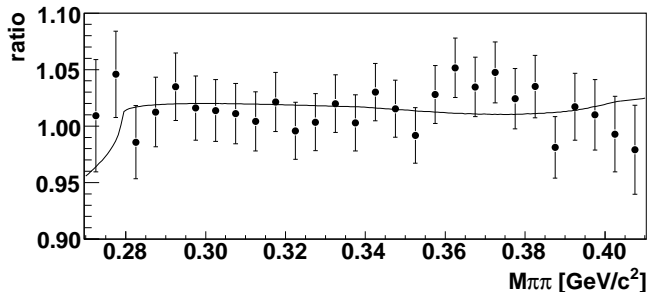


FIG. 5: Ratio of the $M_{\pi\pi}$ distribution to the MC predictions with $\alpha = 0$: points – data; line – CHPT calculations from reference [16].

α . The MC data are normalized to obtain $c_0 = 1$. For the final result $\alpha = -0.026 \pm 0.01(stat) \pm 0.01(syst)$ the first bin was excluded due to large systematic uncertainty and the last bin due to low statistics. The estimate of the systematic errors is obtained by variation of all essential cuts applied in the reconstruction: χ^2 for the kinematic fit, the combinatorial purity of the sample, the missing mass and the z range. For each subsample a fit to α was performed: an example is shown in Fig. 3 and the summary is given in Table II. The overall systematical error was obtained by comparing central values of the fits for the subsamples and it takes into account the influence of

the first bin for the α value [24].

We have also performed a search for a cusp structure in the invariant mass distribution of the two π^0 (Fig. 5). An eventual observation would require at least two orders of magnitude larger data sample.

The presented result for α is limited by the available statistics which determines also attainable systematical accuracy (due to the size of the subsamples used for the tests). Within the errors it is compatible both with the result from Crystal Ball [19], based on 10^6 events, and with the preliminary KLOE result [10] listed in Table I. The reconstruction methods developed for the purpose of this experiment are now being used in analysis of recently collected data from the followup experiment with WASA detector located at COSY ring (FZ Juelich) [25] with much larger statistics.

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