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Towards an understanding of the light scalar mesons

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Abstract. Although studied for many years the nature of the light scalar mesons remains controversial. Here we shall present a method, applicable for s-wave states located close to a threshold, that allows one to quantify the molecular part of a given state. When applied to the $f_0(980)$ a dominance of the molecular component is found. In the second part we show that requirements of field theoretic consistency and chiral symmetry, when applied to the scattering of light pseudo–scalars, naturally lead to the appearance of dynamical poles in the scalar sector. A program is proposed on how to further investigate experimentally the mixing between these dynamical states and possible genuine quark states.

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1 Introduction

At first glance it comes as a surprise that the lowest scalar excitations of QCD are still not fully understood. First, there has been a long debate on how many scalar states there are below 1 GeV. Besides the well established isovector $a_0(980)$ and the isoscalar $f_0(980)$ there was experimental as well as theoretical evidence for two more states, namely the $f_0(600)$ — often called the σ meson — and the isodoublet κ. Together these states could fill the lowest scalar nonet.

Although recent efforts in dispersion theory in combination with chiral perturbation theory unambiguously determined the existence as well as the position of the σ–pole [1], the discussion of the very nature of this state and its relatives is far from settled. Analyses can be found in the literature that identify these structures with conventional $q\overline{q}$ states (see e.g. Refs. [2]) — in some analyses with a sizable admixture from the continuum [3,4] — compact $qq-\bar{q}\bar{q}$ states [5,6] or loosely bound $\bar{K}K$ molecules [7,8].

What is clearly called for is a study that identifies those cases when the nature of a particular state can be read off from an experimental observable. Based on an old proposal by Weinberg [9], a first step in this direction was taken in Ref. [12] — specifically, the original argument was extended to also allow for the presence of inelasticities. The conditions where this method can be applied were found to be

- the state must be an s–wave with respect to the continuum states[1];
- the binding energy ϵ must be *much* smaller than any intrinsic scale of the problem;
- any inelastic threshold must be 'far away' (in units of the binding energy) from the elastic threshold of interest.

It was the central finding of Ref. [12] that also for inelastic interactions the value of the effective coupling of a resonance to the continuum state of interest[2] is a direct measure of the molecular component; especially, its value gets maximum (up to higher order corrections) in the case of a pure molecule.

For the derivation of the above mentioned result we refer to Ref. [12]. In this brief note we will focus more on a discussion of why this works and how to further exploit this insight. Thus, in the next section, the role of the effective coupling will be discussed and the scheme will be applied to the $f_0(980)$. In section 3 we argue that a prominent molecular structure of the light scalar mesons emerges quite naturally from the properties of the meson–meson scattering amplitude near threshold controlled by chiral symmetry. In section 4 we shall briefly comment on possible further experiments to investigate the mixing of the light scalar mesons with the (heavier) quark states.

2 How does this work?

For simplicity let us focus on a situation where two spin zero mesons of mass m couple to a single, isolated resonance state. The possible presence of inelasticity will be discussed later. The two point function $g(s)$ for the resonance state may then be written as

$$g(s) = \frac{1}{s - M_0^2 - i\Sigma(s)} + \text{regular terms}$$

[1] Not to be obscured with the quantum numbers with respect to the quark constituents.

[2] To be more specific: what is meant is the corresponding residuum at the resonance pole.

$$= \frac{Z}{s - M^2} + \text{regular terms} \qquad (1)$$

where M_0 (M^2), Σ, Z, denote the bare (physical) mass, the self energy, and the wave function renormalization of the state of interest. It is straightforward to show that (for a non–relativistic system) the Z factor measures the bare state admixture of the physical state [9,12]. Thus, $Z = 0$ ($Z = 1$) corresponds to a pure molecule (compact state).

By assumption the binding energy is much smaller than any intrinsic scale of the problem — assumed especially to be smaller that the inverse of the range of forces. In this situation the vertex that couples the state of interest to the continuum can be safely assumed to be point–like and the self energy is just the standard scalar loop function times a strength parameter that we will call G. One then easily derives

$$\frac{1}{Z} - 1 = \frac{G}{4}\sqrt{\frac{m}{\epsilon}} + \mathcal{O}(\epsilon R) \ , \qquad (2)$$

where R denotes the range of forces. Weinberg applied this to the deuteron, where $R \sim 1/m_\pi$. For the scalar mesons, on the contrary, we look at the scattering of two pseudo–scalar mesons. Thus the lightest particle that can be exchanged in the t–channel is the ρ meson from which we get $R \sim 0.25$ fm, which is of the order of the extension of conventional mesons. Under the conditions assumed the $1/\sqrt{\epsilon}$ term should dominate the right hand side of Eq. (2) thus allowing one to express the effective coupling G in terms of Z, which 'measures' the nature of the state. On the other hand the effective coupling ZG is — in principle — a measurable quantity: it is the residue of the scattering matrix at the resonance pole and it can be related to the scattering length and the effective range for the meson–meson scattering. This follows directly from matching the expression for the meson–meson scattering matrix—$Gg(s)$—to the corresponding effective range expansion

$$\begin{aligned} Gg(s) &= \frac{G}{s - M^2 + iMG\sqrt{s - 4m^2}/2} \\ &= \frac{1}{2m}\left(\frac{G/2}{\epsilon + k^2/m + G/2(ik + \sqrt{m\epsilon})}\right) + \mathcal{O}\left(\frac{k^2}{m^2}\right) \\ &= -\frac{1}{2m}\left(\frac{1}{1/a + r/2\,k^2 - ik}\right) + \mathcal{O}\left(\frac{k^2}{m^2}\right) \ , \end{aligned} \qquad (3)$$

which leads (up to higher orders) to

$$-\frac{1}{a} = \frac{2\epsilon}{G} + \sqrt{m\epsilon} \ , \quad -\frac{1}{2}r = \frac{2}{Gm} \ . \qquad (4)$$

Using Eq. (2) one may equivalently express a and r directly in terms of Z.

Note that the coupling G controls the relative importance of the term non–analytic in s to that analytic in s. Since the only source of a non–analyticity are the unitarity cuts, this is exactly what controls the amount of molecular admixture. For a recent discussion of s-wave thresholds on amplitudes we refer to Ref. [11] and references therein. We shall come back to this below.

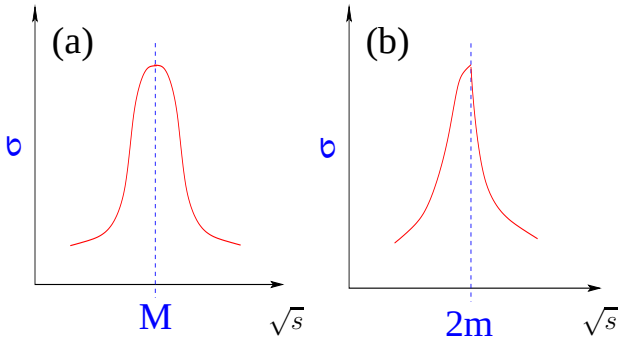

Fig. 1. Typical resonance signals for a genuine state (a) and a dominant molecular state (b).

For later use it is convenient to introduce g_{eff}, defined as

$$\begin{aligned} \frac{g_{eff}^2}{4\pi} &= Z 8m^2 G \\ &= 32(1 - Z)m\sqrt{\epsilon m} \leq 32m\sqrt{\epsilon m} \ . \end{aligned}$$

It is this effective coupling that controls the resonance coupling in inelastic reactions like $\phi \to \gamma\pi\pi$. This reaction will be discussed in more detail below. It is important to observe that the formalism sketched here gives a results that is intuitively clear: the larger the effective coupling of the physical resonance to the continuum the larger the probability to find the continuum state in the physical state or, stated differently, the larger its molecular component. Eq. (5), however, makes an even stronger statement: the maximum coupling is constrained from above and the value of this maximal coupling is controlled solely by the binding energy. Below we will call the model with $g_{eff}^2/4\pi = 32m\sqrt{\epsilon m}$ the *naive molecular model*.

What changes now if we introduce an inelastic channel? By assumption this new channel is not allowed to introduce any new small scale into the problem. Thus we may assume that the inelastic threshold is, when measured in units of the binding energy, very far away. Then its leading effect is to introduce a constant (or at most weakly energy dependent) imaginary part $i\Gamma$ to the denominator of $g(s)$, or — equivalently — to the self energy.

$$g(s) = \frac{1}{s - M^2 + iMG\sqrt{s - 4m^2}/2 + i\Gamma} \ . \qquad (5)$$

Thus, the appearance of Γ does not change the relative importance of the $s - M^2$ piece and the $iMg\sqrt{s - 4m^2}/2$ piece, and consequently G still measures the amount of molecular admixture with respect to the elastic channel [12]. A discussion on the subleading corrections is provided in Ref. [13].

Eq. (5) is nothing but the standard Flatté form used for the parameterization of resonance signals near thresholds [14]. As stated above, for a state with a negligible continuum admixture, the term linear in G of Eq. (5) can be neglected and the resulting distribution for the resonance is that of a standard Breit–Wigner. This is sketched in

Fig. 1a. On the other hand, if the state is predominantly a molecule, it is the G–term that controls the dynamics near the meson–meson threshold and the variation of s in the first term may be neglected. The resonance signal then produces a very pronounced cusp structure, for which a typical case is shown in Fig. 1b. (Many different shapes are possible; see the discussion in Ref. [14].) It therefore appears a straight forward task to use measured Flatté distributions to extract G and deduce from this the molecular admixture. Unfortunately, due to a scale invariance of the amplitude, an extraction of the absolute value of G directly from the mass distributions is very difficult. (See Ref. [15] for a detailed discussion.) As we shall see, inelastic processes might be more useful here.

Thus, qualitatively one comes to the conclusion that the more distorted a Breit–Wigner distribution gets through the opening of a production threshold the more molecular component there is in the corresponding resonance. In Ref. [12] we tried to put this on more quantitative grounds. Thus the strongly asymmetric mass distribution seen for the $f_0(980)$ by the BES collaboration [16] in the reaction $J/\Psi \to \phi\pi\pi$ already provides strong evidence for a predominantly molecular composition of the f_0.

3 The physics of light scalar mesons

It is surprising that the lowest scalar excitations in QCD seem to be of quite complicated structure — even more complicated than the higher excited states, where we assume to find the $\bar{q}q$ states. In this section we will argue that the appearance of low–lying dynamical states is a natural consequence of chiral symmetry. Note, here 'natural' should not be confused with necessary. In this context see discussion in Ref. [17].

The observation essential for our argument is the energy dependence of the $\pi\pi$ scattering amplitude near its threshold. It reads in the scalar–isoscalar channel [18]

$$V_{\pi\pi} = (s - m_\pi^2/2)/f_\pi^2 \ . \tag{6}$$

Corrections to this expression can be calculated using chiral perturbation theory in a controlled way [19].

One may ask if such an energy dependence near threshold can emerge solely from an s–channel resonance. The answer is "no" for two reasons and may be read off the corresponding scattering matrix directly. The scattering potential for scattering through a resonance has the form

$$V_R = G_R^2/(s - m^2) \ , \tag{7}$$

thus, matching to $V_{\pi\pi}$ would give first of all a relation between the masses scalar resonances and f_π that looks very counter intuitive; but, even more importantly, for V_R to employ the energy dependence of $V_{\pi\pi}$ calls for

$$G_R \propto \sqrt{s - m_\pi^2/2} \ . \tag{8}$$

In order to give the correct energy dependence near threshold the amplitude must contain an unphysical branch point. Clearly, in elastic $\pi\pi$ scattering this will not show up, because only G_R^2 appears. Production amplitudes, however, are linear in G_R and thus one would start to notice the unphysical cut in calculations based on Eq. (7). Although not allowed by analyticity, several works use the vertex function of Eq. (8) (see e.g. Ref. [22]) or similar ones (see e.g. Ref. [23]).

To cure this problem one option would be to use the linear sigma model. There a four pion contact term is present in addition to the σ pole diagrams and a threshold amplitude of the form of 6 emerges naturally. Then one finds that the contact term as well as the t– and u–channel exchanges play a prominent role in the dynamics below 1 GeV [20]. An alternative and theoretically more appealing approach (see discussion in Ref. [21]) is chiral perturbation theory. There only pions (and in the SU(3) extension also kaons and eta mesons) appear as dynamical fields that interact through contact interactions constructed consistent with chiral symmetry. The corresponding leading order Lagrangian automatically gives a potential of the form of Eq. (6) — see Ref. [19] and references therein. In this scheme there is a strongly energy dependent, non–resonant π-π interaction to be included in the theory, whose s–dependence is so strong that the tree–level potential of Eq. (6) hits the unitarity bound already at quite low energies. Or, stated differently, a properly unitarized amplitude will naturally employ a pole in the complex plane to prevent the amplitude from growing beyond what is allowed by unitarity. This pole should be identified with the $f_0(600)$ — the σ meson. This picture was introduced in Refs. [21,6] and was further supported by studies within unitarized chiral perturbation theory [24]. For a recent review see Ref. [25].

So far the argument was given for $\pi\pi$ scattering only, however studies within both unitarized chiral perturbation theory discussed above as well as phenomenological models [26] show that the same holds for the scattering of all the pseudo–Goldstone bosons with each other. As a consequence, in addition to the lowest pole in the $\pi\pi$ channel, coupling to the $\bar{K}K$ system leads to a pole close to the $\bar{K}K$ threshold, identified with the $f_0(980)$. Also in the $\pi\eta$-$\bar{K}K$ coupled system a pole appears close to the $\bar{K}K$ threshold, interpreted as $a_0(980)$ and in the πK channel the κ pole appears.

To summarize this part, we find as a natural consequence of the analytic properties of the scattering amplitude for the scattering of the ground state pseudo–scalar mesons with each other that dynamical poles with scalar quantum numbers are produced. This insight is fully in line with the experimental evidence that the $f_0(980)$ is predominantly of dynamical origin. What remains to be seen is the mixing pattern of those molecular states with the non–molecular states. In the rest of this presentation we shall argue that exploiting the matrix element for scalars coupling to a photon and a vector meson in various kinematic regimes is what should provide important information in this direction.

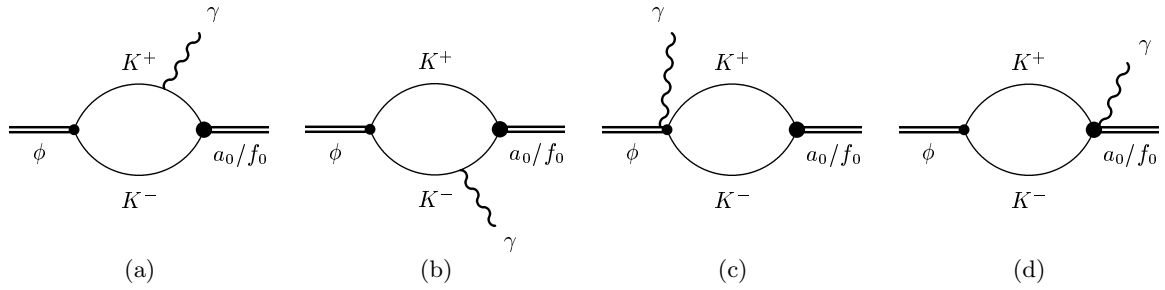

Fig. 2. Kaon loop contributions to the radiative decay amplitude.

4 $\phi \to \gamma\pi\pi$ and possible further experiments

In recent years a lot of experimental as well as theoretical effort went into studies of the reactions $\phi \to \gamma\pi\pi$ and $\phi \to \gamma\pi\eta$, both believed to shed light on the nature of the light scalar mesons f_0 and a_0, respectively. Following the reasoning of Ref. [27] and that of the previous sections, in this section we shall argue that in these reactions the molecular component of the scalar mesons was measured. In addition we shall show that looking at the same matrix elements for different masses of the vector meson will allow one to study the mixing of this molecular component with possible compact states.

There is strong experimental evidence that the reaction $\phi \to \gamma\pi\pi$ in the upper part of the available phase space runs predominantly through a kaon loop followed by a resonance formation into the $f_0(980)$. The kaon loop dominance was observed by Achasov and Kiselev (see Ref. [28] and references therein). The reason why one can in the reaction $\phi \to \gamma\pi\pi$ almost 'see' the kaon loops in the spectrum is the following: gauge invariance demands the transition amplitude for $\phi \to \gamma\pi\pi$ to vanish for vanishing values of ω — the energy of the outgoing photon. As a consequence the ϕ decay rate has to scale as ω^3 at the upper end of phase space. On the other hand, the experimental spectrum of Refs. [29,30] is (almost) identical to that expected from an undistorted f_0 spectral function. What looks contradictory at first glance is quite natural, since the $\bar{K}K$ threshold is very close to the mass of both the ϕ and the f_0. Consequently there is a pronounced cusp structure in the kaon loop that effectively compensates the mentioned ω^3 suppression at the upper end of phase space.

In the previous section we argued that the effective coupling of the scalar mesons to kaons is a measure for the importance of the molecular component of the $f_0(980)$. We now see that the kaon loop dominates the transition rate $\phi \to \gamma\pi\pi$. We therefore have to conclude that the radiative decay of the ϕ into a pion pair measures the molecular component of the scalar meson. Even more, we may use the naive molecular model introduced above to estimate the rate for $\phi \to \gamma\pi\pi$ based on Eq. (5) with $Z = 0$ and a typical value of $\epsilon = 10$ MeV. Then we get [27] $\Gamma \sim 0.6$ keV to be compared to the experimental value of 0.4 keV [30]. This we interpret as strong evidence in favor of a prominent molecular structure of the f_0. This conclusion is in line with the results of calculations within the unitarized chiral perturbation theory for the ϕ radiative decay [31, 32,33].

It should be mentioned that the interpretation of the data for $\phi \to \gamma\pi\pi$ is still controversial and the above picture is not yet fully accepted. In Ref. [34] it is claimed that there should be a strong suppression of the $\phi \to \gamma f_0/a_0$ branching ratio for the scalars in case they are loosely bound molecules as compared to point–like scalars that correspond to compact quark states, (10^{-5} *vs* 10^{-4}). A study by Achasov et al. [35], where the finite width of scalars was taken into account, arrived at the same conclusion. Thus, the authors of [34] and [35] stress that data for this branching ratio should allow to prove or rule out the molecular model of the scalars. However, this conclusion is based on a confusion between the notion of a wave function and that of a vertex function. Indeed, the relevant scale in the spatial extension of a wave function of a molecule is indeed the binding energy: the smaller the binding the larger the extension. What enters, however, in the loops depicted in Fig. 2 is not the wave function but the vertex function, whose scale is set by the range of forces — for a molecule made of two pseud scalars this is of the order of the mass inverse of the ρ meson, much smaller than what derives from the binding energy. The wave function, on the other hand, is proportional to the vertex function times the two–meson propagator (see Eq. (22) in Ref. [36]). This very propagator, however, is already included explicitly in the integral for the kaon loop[3].

Recently an attempt was made to set up an analysis for the data on $\phi \to \gamma\pi\pi$ that avoids the use of the kaon loops [39]. The authors parameterize the $\phi \to \gamma\pi\pi$ amplitude as a polynomial in s and succeeded in fitting the ϕ–decay spectra once a sufficient number of terms was included in the polynomial. This one might interpret as an indication that the kaon loops are not of as high relevance as indicated in the previous paragraphs. However, a polynomial in s will not have the non–analytic pieces that are provided by the kaon loops. A truly model–independent analysis would therefore include both the mentioned polynomial as well as the kaon loop. A fit then needs to decide how much loop is really necessary. Given the comments above it is likely that such a fit will call for a quite small contribution from the terms analytic in s.

[3] For more details on this discussion see the comment on Ref. [27] in Ref. [37] and our reply to this in Ref. [38].

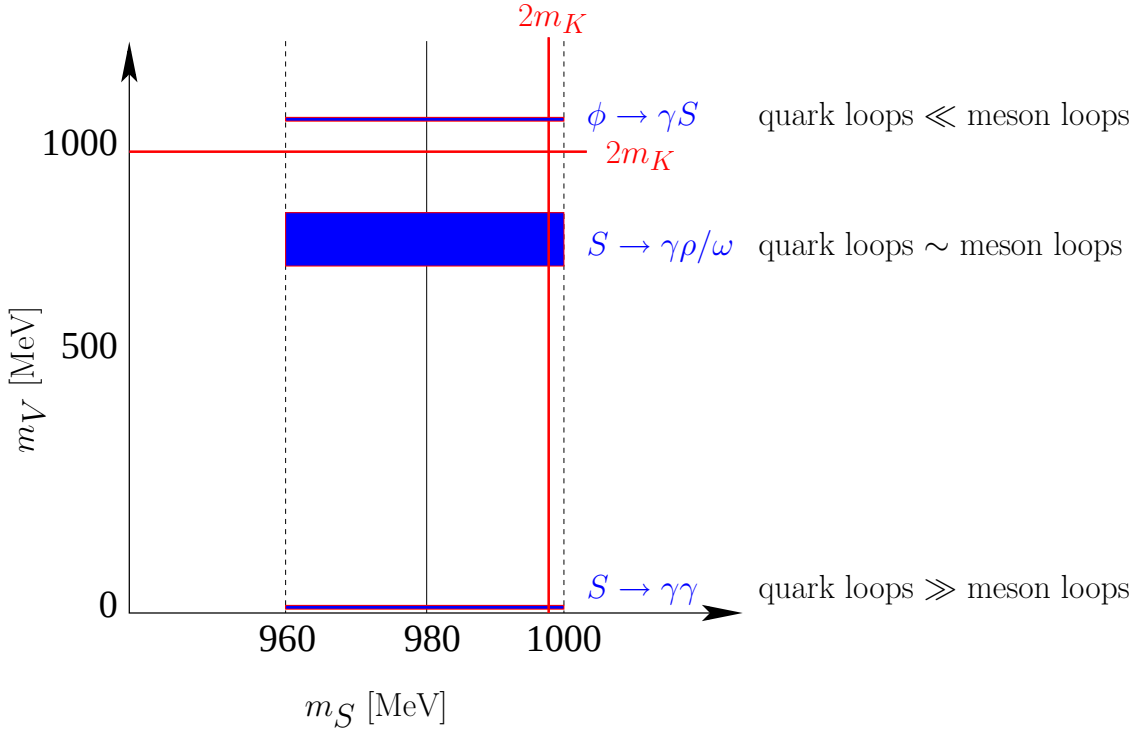

Fig. 3. *Illustration of various kinematical regimes probed by the decays involving scalars.*

Thus far we have argued that the ϕ radiative decay measures the molecular component of the scalar mesons. What remaines unanswered so far is the amount of mixing of molecular states with genuine quark states. As argued in Ref. [27], further insight into this issue can be gained from a systematic study of decays of scalar mesons into a photon and a vector meson. The physics behind this is quite easy: the reason why the ϕ radiative decays have large kaon loop contributions is the proximity of the $\bar{K}K$ threshold to both the mass of the ϕ as well as of the scalars of interest here. Looking at the radiative decays of the scalars allows us to study the same matrix element in different kinematics: we can now change the mass of the vector meson away from that of the ϕ to that of the ω and ρ, or even to $m = 0$ for the decay $s \to \gamma\gamma$. As explained above, the unitarity cut related to the $\bar{K}K$ threshold introduces a significant energy dependence to the kaon loop. It gets smaller the further we move away from the threshold. On the other hand, quark loops do not feel the kaon threshold and therefore a much weaker energy dependence is expected for these. This means, that as we move from the the $\phi s\gamma$ vertex to the $(\rho\omega)s\gamma$, and eventually the $\gamma s\gamma$ vertex, it should be straightforward to disentangle the kaon loop part of the matrix element from the quark lines. This logic is sketched in Fig. 3[4].

In oder to be more quantitative, particular models need to be employed for the quark loop contributions. For details we refer to Ref. [27]. It is important to stress that already the naive molecular model, as sketched in section 2, gives values for the decays $s \to \gamma\gamma$ that are of the right order of magnitude.

5 Summary

To summarize, we argued that the available data — especially that on the ϕ radiative decays — are compatible with a model that assumes the f_0 to be predominantly of molecular nature. In order to determine the mixing scheme of this molecule and its $SU(3)$ relatives with possible quark states a series of experiments that studies the radiative decays of the scalars was proposed.

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[4] Whenever quark loops and meson loops are considered simultaneously there is the possibility of double counting. However, here this is not the case since the relavant meson loops are finite.

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