Experimental Verification of Predicted Beam-Polarization Oscillations near a Spin Resonance

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The Chao matrix formalism allows analytic calculations of a beam's polarization behavior inside a spin resonance. We recently tested its prediction of polarization oscillations occurring in a stored beam of polarized particles near a spin resonance. Using a 1.85 GeV/c polarized deuteron beam stored in the COoler SYnchrotron, we swept a new rf solenoid's frequency rather rapidly through 400 Hz during 100 ms, while varying the distance between the sweep's end frequency and the central frequency of an rf-induced spin resonance. Our measurements of the deuteron's polarization near and inside the resonance agree with the Chao formalism's predicted oscillations.

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Polarized stored hadron and lepton beams often provide the best technique for studying the spin dependence of hadronic interactions in the 1 GeV/c to 1 TeV/c region. Polarized beam experiments at storage rings [1–5] need the ability to precisely control the beam's polarization. A stored beam's polarization can be manipulated in a well-controlled way by ramping an rf magnet's frequency through an rf-induced spin resonance.

The Froissart-Stora (F-S) formula [6] has been widely used to calculate a beam's polarization after crossing a spin resonance. However, it is valid only for a constant-rate linear crossing from far below to far above the resonance. Chao's new matrix formalism was proposed [7] to deal with conditions where the F-S formula is not valid; the Chao formalism can be used to calculate the spin dynamics anywhere inside a piecewise linear resonance crossing. An earlier experiment [8] at the COoler SYnchrotron (COSY) first tested the Chao formalism's predicted spin behavior by sweeping an rf dipole's frequency near or through an rf-induced spin resonance. That experiment suggested that a faster crossing rate was needed to test its striking prediction of large polarization oscillations near the resonance. This Letter describes a faster crossing rate experiment at COSY.

In an ideal flat circular storage ring or accelerator, with no horizontal magnetic fields, each particle's spin precesses around the vertical magnetic fields of the ring's bending dipoles. The spin tune ν_s, which is the number of spin precessions during one turn around the ring, is proportional to the particle's energy; $\nu_s = G\gamma$, where $G = (g-2)/2$ is its gyromagnetic anomaly and γ is its Lorentz energy factor. Horizontal rf magnetic fields can induce an rf spin resonance [6,9,10], which can be used to spin-manipulate the stored particles [11–16]. For deuterons, the rf-induced spin resonance's frequency is

$$f_r = f_c(k \pm G_d\gamma), \tag{1}$$

where f_c is the deuteron's circulation frequency, k is an integer, and $G_d = -0.142\,987$.

Ramping an rf magnet's frequency through a spin resonance with resonance strength $\mathcal{E}$ can rotate a stored beam's polarization. When its frequency is ramped at a constant rate, during a ramp time Δt, by a range Δf, from far below to far above a resonance, the Froissart-Stora equation [6] can relate the beam's initial vector polarization P_i and its polarization P_f after crossing the resonance,

$$P_f = P_i\left\{2\exp\left[-\frac{(\pi\mathcal{E}f_c)^2}{\Delta f/\Delta t}\right] - 1\right\}. \tag{2}$$

As discussed earlier [8], Chao's matrix formalism [7] for spin dynamics allows one to analytically solve the spin equation of motion near an isolated spin resonance, if its crossing can be expressed as a series of linear segments. Each segment must have a fixed or linearly changing distance between the spin tune $\nu_s = G\gamma$ and the resonance tune $\nu_r \equiv k \pm f_r/f_c$. After obtaining, for each segment, the time-dependent matrix describing a spinor's evolution in the segment, one multiplies these matrices sequentially to obtain the final polarization P_f.

We recently tested Chao's matrix formalism with the technique shown in Fig. 1 using our new rf solenoid; it is a 25-turn air-core water-cooled copper coil, of length 57.5 cm and average diameter 21 cm. Its inductance was 41 ± 3 μH, and its longitudinal rf magnetic field was about 1.25 mT at its center. It was part of an RLC resonant circuit, which operated near 917 kHz, typically at an rf voltage of 5.7 kV rms producing a longitudinal rf $\int B_{\rm rms} dl$ of 0.69 ± 0.05 T mm.

The other apparatus for this experiment, including the COSY storage ring [17–20], the EDDA polarimeter [21,22], the electron cooler [23], the low energy polarimeter [24], the injector cyclotron, and the polarized ion source [25–27] were shown in Fig. 4 of Ref. [8]. The beam from the polarized D^- ion source was accelerated by the cyclotron to 75.7 MeV and then strip-injected into COSY. The Low Energy Polarimeter measured the D^- beam's polarization before injection into COSY to monitor the cyclotron's and the ion source's stability.

The EDDA polarimeter [21,22] measured the beam's polarization in COSY. We reduced its systematic errors by repeatedly cycling the beam produced by the polarized deuteron ion source through five different vector P_V and tensor P_T vertical polarization states:

$$(P_V, P_T) = (0, 0), (+1, +1), (-\tfrac{1}{3}, -1), (-\tfrac{2}{3}, 0), (-1, +1).$$

The asymmetry measured in the (0, 0) spin state was subtracted from the other measured asymmetries, in each 20 ms time bin, to correct for detector efficiencies and beam motion asymmetries in the EDDA polarimeter.

In COSY, the deuterons' average circulation frequency f_c was 1.147 43 MHz at 1.85 GeV/c, where their Lorentz energy factor was $\gamma = 1.4046$. For these parameters, the spin tune $\nu_s = G_d\gamma$ was $-0.200\,84$. Thus, Eq. (1) implies that the $k = 1$ spin resonance's central frequency should be very near $f_r = (1 + G_d\gamma)f_c = 917.0$ kHz.

We measured the rf solenoid's strength $\mathcal{E}$ by measuring the polarization after ramping its frequency through the resonance with various ramp times Δt with its frequency range Δf and voltage fixed, as shown in Fig. 2. We then fit these data to Eq. (2), the Froissart-Stora equation [6], to obtain the measured value of $\mathcal{E}$.

To study the Chao formalism's predicted dependence on the beam's momentum spread $\Delta p/p$, we varied the 20.6 keV electron Cooler's on-time at injection. It cooled the deuterons' emittances both longitudinally and transversely for 15 or 25 s. The deuterons were then accelerated to 1.85 GeV/c. The rf acceleration cavity was off and shorted during COSY's flat top; thus, there were no synchrotron oscillations.

We tested the Chao formalism by ramping the rf solenoid's frequency over a range Δf, which started at $f_{\rm start}$ (well outside the rf spin resonance centered at f_r) and ended at $f_{\rm end}$ near or inside the resonance, as shown in Fig. 1. For each $f_{\rm end}$ data point, both Δf and the ramp time Δt were held fixed, at 400 Hz and 100 ms, respectively, while $f_{\rm end}$ was set to the values shown in Figs. 3 and 4. After $f_{\rm rf}$ reached $f_{\rm end}$, the rf solenoid was turned off abruptly (in a few μs) to preserve the vertical polarization at that instant. We then measured the deuterons' vector asymmetry in all five (P_V, P_T) states. The resulting final vector polarization P_f for each nonzero spin state is plotted vs $f_{\rm end}$ in Figs. 3 and 4.

We first calculated the Chao formalism's prediction for $\Delta p/p = 0$ by inserting, into Eqs. (4)–(9) of Ref. [8], our measured $\mathcal{E} = 1.06 \times 10^{-5}$ from Fig. 2, our Δf of 400 Hz, and our Δt of 100 ms. To take into account the beam's

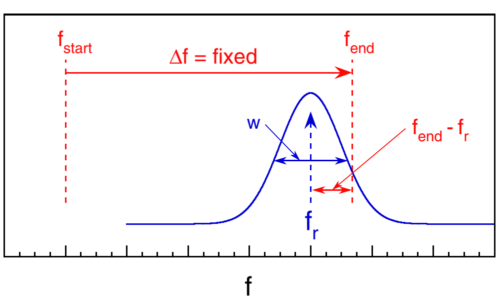

FIG. 1 (color online). Schematic of the Chao formalism test. The rf solenoid's frequency $f_{\rm rf}$ was ramped, by a fixed range Δf, for different distances between the ramp's end frequency $f_{\rm end}$ and the resonance's center f_r. The curve shows the resonance with a total (FWHM) width w.

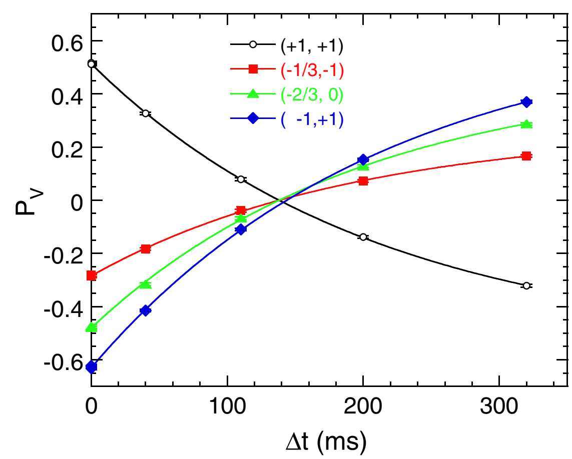

FIG. 2 (color online). Measured 1.85 GeV/c deuteron vector polarizations plotted vs rf-solenoid ramp time Δt with 15 s electron cooling for the four indicated spin states. Their averaged fit to Eq. (2) gave $\mathcal{E}$ of $(1.05 \pm 0.01) \times 10^{-5}$. The above Δt curve's frequency range Δf was 300 Hz. Averaging 13 similar Δt curves gave $\mathcal{E}$ of $(1.060 \pm 0.005) \times 10^{-5}$.

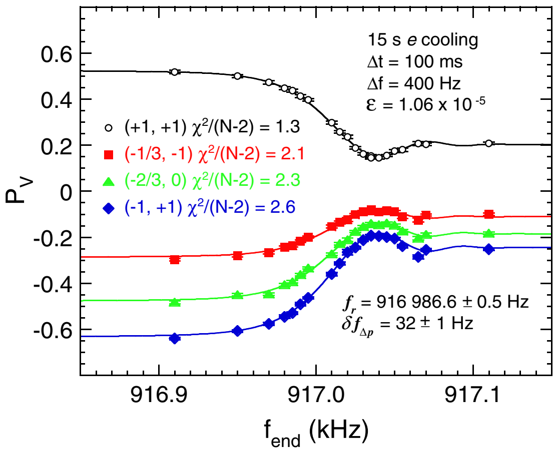

FIG. 3 (color online). Measured 1.85 GeV/c deuteron vector polarizations plotted vs rf-solenoid end frequency f_{end}. Its ramp time Δt was 100 ms; its frequency range Δf was 400 Hz, and its $\mathcal{E}$ was 1.06×10^{-5}. The Chao formalism fits, shown by lines, gave a resonance frequency f_r of $916\,986.6 \pm 0.5$ Hz and a Gaussian $\delta f_{\Delta p}$ of 32 ± 1 Hz FWHM. Only the statistical errors were used to calculate $\chi^2/(N-2)$; e cooling was on for 15 s.

momentum spread $\Delta p/p$, we next folded this result together with Gaussians representing different values of the beam's f_r spread, $\delta f_{\Delta p} \equiv \frac{df_r}{dp}\Delta p$, due to $\Delta p/p$. We then fit the data in Figs. 3 and 4 with f_r and $\delta f_{\Delta p}$ as the two free parameters. The Chao formalism fits are shown as solid lines for each nonzero spin state in each figure.

We calculated $\chi^2/(N-2)$ for each fit to compare its agreement with the data for each of the four nonzero spin states. Each χ^2 analysis included only the data's statistical errors and ignored systematic errors; nevertheless, all $\chi^2/(N-2)$ were near 1 despite the curves' complex shapes. The oscillations' positions and magnitudes are very sensitive to the values of f_r and $\delta f_{\Delta p}$, respectively.

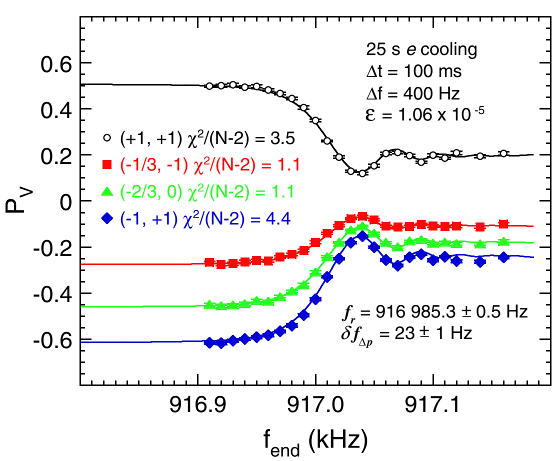

FIG. 4 (color online). Measured deuteron vector polarizations plotted vs f_{end} as in Fig. 3. With 25 s e cooling, the fit curves gave f_r of $916\,985.3 \pm 0.5$ Hz and $\delta f_{\Delta p}$ of 23 ± 1 Hz FWHM.

Minimizing the $\chi^2/(N-2)$ values in Fig. 3 gave f_r of $916\,986.6 \pm 0.5$ Hz and $\delta f_{\Delta p}$ of 32 ± 1 Hz for 15 s electron cooling. Minimizing $\chi^2/(N-2)$ for the 25 s electron cooling data in Fig. 4 gave an f_r of $916\,985.3 \pm 0.5$ Hz and $\delta f_{\Delta p}$ of 23 ± 1 Hz. The $\chi^2/(N-2)$ analyses in Figs. 3 and 4 both support the Chao formalism [7,8]. As predicted, the oscillation amplitude increased as $\delta f_{\Delta p}$ decreased.

Figure 5 compares the 25 s e-cooling data, shown in Fig. 4, with the prediction [7,8] for $\mathcal{E} = 1 \times 10^{-5}$. The fit with the measured $\mathcal{E} = (1.060 \pm 0.005) \times 10^{-5}$ agrees with the data in Fig. 5 far better than the prediction. This large difference clearly shows the strong sensitivity of such Δf sweeps to changes in $\mathcal{E}$.

For each cooling time, we also measured the final polarization P_f after the rf solenoid was run at many different fixed frequencies to independently determine the resonance's center f_r and its FWHM-width w. These data are plotted vs the rf frequency f_{rf} in Fig. 6, along with their fit values of f_r and w for the 15 and 25 s cooling times. The fit widths of 41 ± 1 and 29 ± 1 Hz for the 15 and 25 s cooling times, respectively, should be equal to the folding together of the resonance's natural width $2\mathcal{E}f_c = 24$ Hz and the f_r spreads, $\delta f_{\Delta p}$, obtained by fitting the data in Figs. 3 and 4. Adding this 24 Hz width in quadrature with the 32 and 23 Hz $\delta f_{\Delta p}$ values gives 40 ± 2 and 33 ± 2 Hz; these seem to be consistent with the 41 ± 1 and 29 ± 1 Hz from Fig. 6, for cooling times of 15 and 25 s, respectively.

For 25 s cooling, the f_r of $916\,985.3 \pm 0.3$ Hz from Fig. 4 and the f_r of $916\,990 \pm 10$ Hz from Fig. 6(b) differ by only 5 Hz. [The Fig. 4 and 6(b) data were obtained sequentially.] But, for 15 s cooling, with 102 hours between the Fig. 3 and 6(a) data runs, the f_r values are

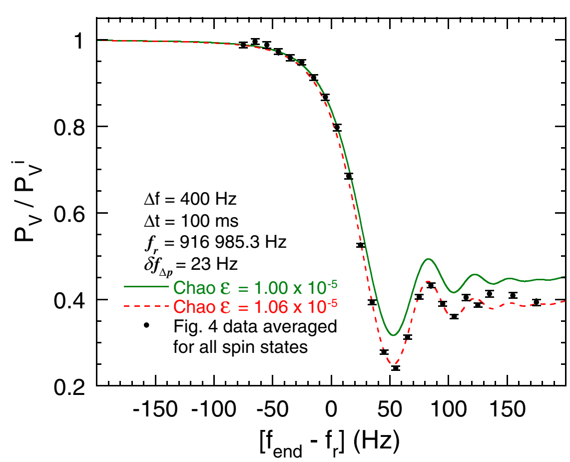

FIG. 5 (color online). Measured deuteron vector polarization ratios from Fig. 4, averaged for all four spin states, are plotted vs $f_{\mathrm{end}} - f_r$. With 25 s e cooling, f_r was $916\,985.3 \pm 0.5$ Hz and $\delta f_{\Delta p}$ was 23 ± 1 Hz FWHM. The solid line shows the prediction for $\mathcal{E}$ of 1.00×10^{-5} found in Fig. 9 of Ref. [8]. The dashed line shows the fit, from Fig. 4, for the solenoid's measured resonance strength $\mathcal{E}$ of $1.060 \pm 0.005 \times 10^{-5}$.

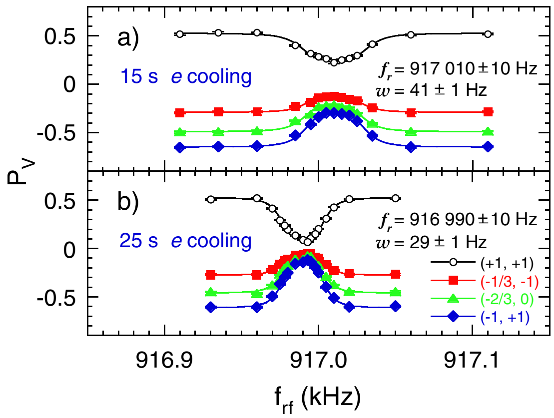

FIG. 6 (color online). Measured deuteron vector polarizations at 1.85 GeV/c plotted vs the rf-solenoid fixed frequency f_{rf}. Fits to 2nd-order Lorentzians give (a) with 15 s e cooling, $f_r = 917\,010 \pm 10$ Hz and $w = 41 \pm 1$ Hz FWHM; (b) $f_r = 916\,990 \pm 10$ Hz and $w = 29 \pm 1$ Hz FWHM with 25 s cooling. The data points' errors are purely statistical; the errors in f_r and w include estimates of systematic errors due to COSY's stability at the few Hz level.

$916\,986.6 \pm 0.3$ Hz and $917\,010 \pm 10$ Hz, respectively. This 23 Hz shift may be due to COSY's long-term stability.

In summary, we tested the Chao formalism's prediction of polarization oscillations when crossing an isolated spin resonance, in a region where the Froissart-Stora formula is not valid. Using 1.85 GeV/c vertically polarized deuterons stored in COSY, we ramped an rf solenoid's frequency through a range Δf ending near a spin resonance; the magnitudes of both Δf and the ramp time Δt were fixed, while we varied the ramp's distance from the resonance. The good fits to the precise data from our new type of experiment, shown in Figs. 3–5, clearly demonstrate the Chao formalism's [7,8] ability to obtain the values of f_r, $\delta f_{\Delta p}$, and $\mathcal{E}$ from the measured polarization's behavior inside a spin resonance.

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