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JÜLICH FORSCHUNGSZENTRUM

Physics of High Intensity Laser-Plasma Interactions

7th Asian Summer School and Symposium on Laser-Plasma Acceleration and Radiation
Shanghai, 17-23 July 2016 | Paul Gibbon

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Course outline

- Lecture 1: Introduction – Definitions and Thresholds
- Lecture 2: Interaction with Underdense Plasmas
- Lecture 3: Interaction with Solids

Lecture 1: Introduction

What is a plasma?

Simple definition: a *quasi-neutral* gas of charged particles showing *collective behaviour*.

Quasi-neutrality: number densities of electrons, n_e, and ions, n_i, with charge state Z are *locally balanced*:

$$n_e \simeq Z n_i. \tag{1}$$

Collective behaviour: long range of Coulomb potential ($1/r$) leads to nonlocal influence of disturbances in equilibrium.

Macroscopic fields usually dominate over microscopic fluctuations, e.g.:

$$\rho = e(Z n_i - n_e) \Rightarrow \nabla . \boldsymbol{E} = \rho / \varepsilon_0$$

Where are plasmas found?

1. cosmos (99% of visible universe):
   - interstellar medium (ISM)
   - stars
   - jets
2. ionosphere:
   - $\leq$ 50 km = 10 Earth-radii
   - long-wave radio
3. Earth:
   - fusion devices
   - street lighting
   - plasma torches
   - discharges - lightning
   - *plasma accelerators and radiation sources*!

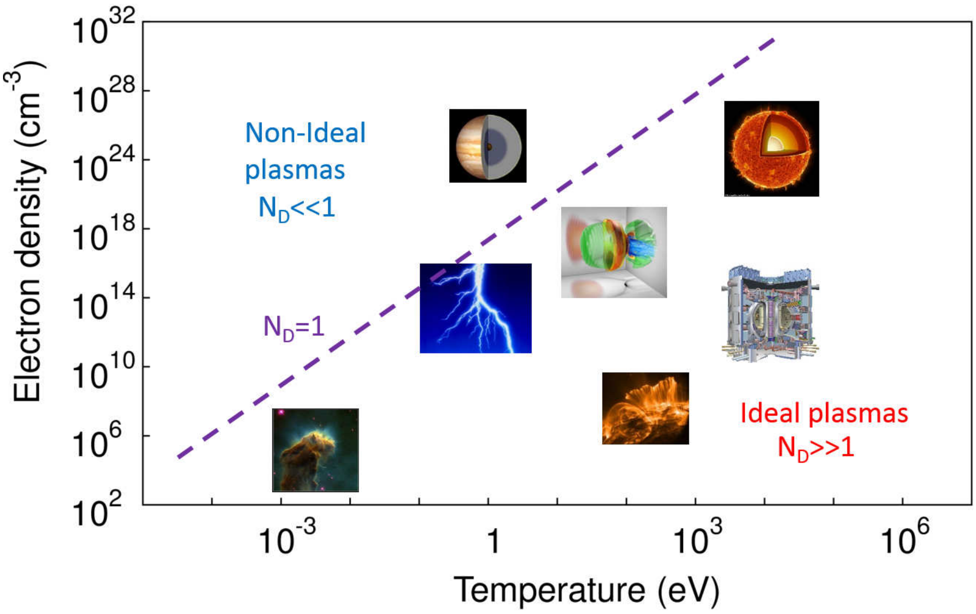
Plasma properties

Type	Electron density n_e (cm^{-3})	Temperature T_e (eV^*)
Stars	10^{26}	2×10^3
Laser fusion	10^{25}	3×10^3
Magnetic fusion	10^{15}	10^3
Laser-produced	$10^{18} - 10^{24}$	$10 - 10^3$
Discharges	10^{12}	1-10
Ionosphere	10^6	0.1
ISM	1	10^{-2}

Table 1: Densities and temperatures of various plasma forms

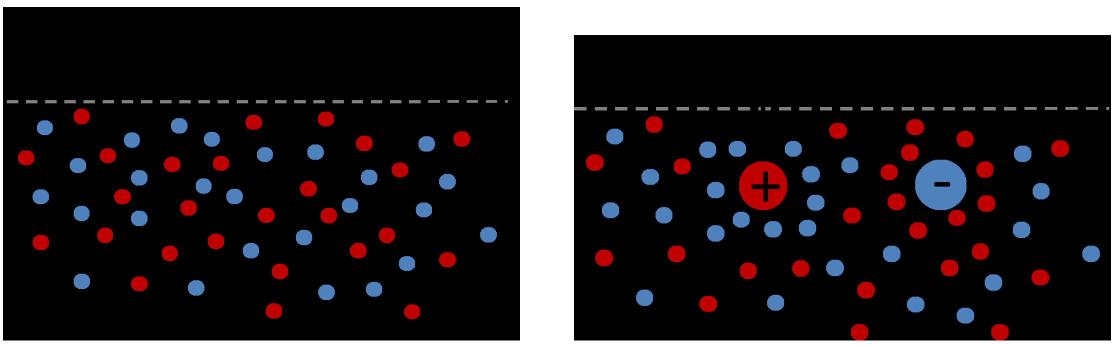
* 1eV $\equiv$ 11600K

Plasma classification



N_D characterises plasma 'collectiveness' – see Eq.(7)

Debye shielding



What is the potential $\phi(r)$ of an ion (or positively charged sphere) immersed in a plasma?

Debye shielding (2): ions vs electrons

For equal ion and electron temperatures ($T_e = T_i$), we have:

$$\frac{1}{2}m_e v_e^2 = \frac{1}{2}m_i v_i^2 = \frac{3}{2}k_B T_e \qquad (2)$$

Therefore,

$$\frac{v_i}{v_e} = \left(\frac{m_e}{m_i}\right)^{1/2} = \left(\frac{m_e}{Am_p}\right)^{1/2} = \frac{1}{43} \qquad \text{(hydrogen, Z=A=1)}$$

Ions are almost stationary on electron timescale!
To a good approximation, we can often write:

$$n_i \simeq n_0,$$

where the material (eg gas) number density, $n_0 = N_A\rho_m/A$;
$N_A =$ Avogadro number, $\rho_m =$ mass density.

Debye shielding (3)

In thermal equilibrium, the electron density follows a Boltzmann distribution*:

$$n_e = n_i \exp(e\phi/k_B T_e) \qquad (3)$$

where n_i is the ion density and k_B is the Boltzmann constant.

From Gauss' law (Poisson's equation):

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0} = -\frac{e}{\varepsilon_0}(n_i - n_e) \qquad (4)$$

* See, eg: F. F. Chen, p. 9

Debye shielding (4)

Combining (4) with (3) in spherical geometry[a] and requiring $\phi \to 0$ at $r = \infty$, we obtain a solution:

Exercise

$$\phi_D = \frac{1}{4\pi\varepsilon_0} \frac{e^{-r/\lambda_D}}{r}. \qquad (5)$$

with

Debye length

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e}\right)^{1/2} \simeq 743 \left(\frac{T_e}{\mathrm{eV}}\right)^{1/2} \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{-1/2} \mathrm{cm} \qquad (6)$$

[a] $\nabla^2 \to \frac{1}{r^2}\frac{d}{dr}(r^2 \frac{d\phi}{dr})$

Debye sphere

An **ideal plasma** has many particles per Debye sphere:

$$N_D \equiv n_e \frac{4\pi}{3} \lambda_D^3 \gg 1. \qquad (7)$$

$\Rightarrow$ Prerequisite for collective behaviour.

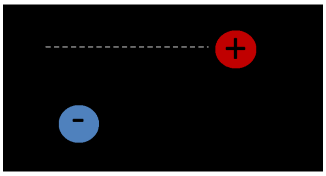
Alternatively, can define **plasma parameter**:

$$g \equiv \frac{1}{n_e \lambda_D^3}$$

Classical plasma theory based on assumption that $g \ll 1$, which also implies dominance of collective effects over collisions between particles.

Collisions in plasmas

At the other extreme, where $N_D \leq 1$, screening effects are reduced and collisions dominate. A quantitative measure of this is the



Electron-ion collision rate

$$\nu_{ei} = \frac{\pi^{\frac{3}{2}} n_e Z e^4 \ln \Lambda}{2^{\frac{1}{2}} (4\pi\varepsilon_0)^2 m_e^2 v_{te}^3} \mathrm{s}^{-1}$$

$$\simeq 2.91 \times 10^{-6} Z n_e T_e^{-3/2} \ln \Lambda \ \mathrm{s}^{-1} \quad (8)$$

Collision frequency: details

$$\nu_{ei} = \frac{\pi^{\frac{3}{2}} n_e Z e^4 \ln \Lambda}{2^{\frac{1}{2}} (4\pi\varepsilon_0)^2 m_e^2 v_{te}^3} \mathrm{s}^{-1}$$

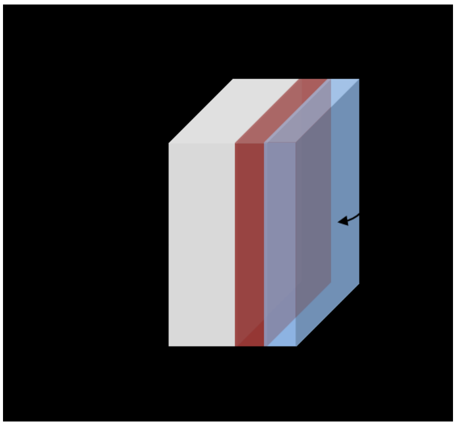
$v_{te} \equiv \sqrt{k_B T_e / m_e}$, electron thermal velocity
Z = number of free electrons per atom (ionization degree)
n_e = electron density in cm^{-3}
T_e = electron temperature in eV
$\ln \Lambda \sim O(2 \to 10)$ is the **Coulomb logarithm**. Can show that

$$\frac{\nu_{ei}}{\omega_p} \simeq \frac{Z \ln \Lambda}{10 N_D} \quad (9)$$

with

$$\Lambda = \frac{b_{\max}}{b_{\min}} = \lambda_D . \frac{k_B T_e}{Z e^2} \simeq 9 N_D / Z$$

Plasma oscillations: capacitor model



Consider electron layer displaced from plasma slab by length δ. This creates two 'capacitor' plates with surface charge $\sigma = \pm e n_e \delta$, resulting in an electric field:

$$\boldsymbol{E} = \frac{\sigma}{\varepsilon_0} = \frac{e n_e \delta}{\varepsilon_0}$$

Capacitor model (2)

The electron layer is accelerated back towards the slab by this restoring force according to:

$$m_e \frac{dv}{dt} = -m_e \frac{d^2\delta}{dt^2} = -eE = \frac{e^2 n_e \delta}{\varepsilon_0}$$

Or:

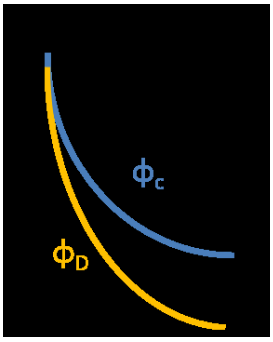
$$\frac{d^2\delta}{dt^2} + \omega_p^2 \delta = 0,$$

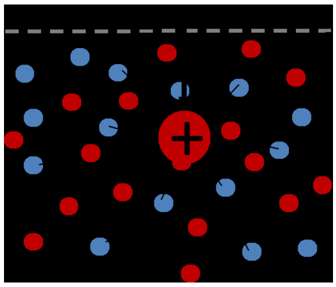
where

Electron plasma frequency

$$\omega_p \equiv \left(\frac{e^2 n_e}{\varepsilon_0 m_e}\right)^{1/2} \simeq 5.6 \times 10^4 \left(\frac{n_e}{\text{cm}^{-3}}\right)^{1/2} \text{s}^{-1}. \quad (10)$$

Response time to create Debye sheath





For a plasma with temperature T_e (and thermal velocity $v_{te} \equiv \sqrt{k_B T_e / m_e}$), one can also define a characteristic *reponse time* to recover quasi-neutrality:

$$t_D \simeq \frac{\lambda_D}{v_{te}} = \left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e} \cdot \frac{m}{k_B T_e} \right)^{1/2} = \omega_p^{-1}.$$

Plasma creation: field ionization

At the Bohr radius

$$a_B = \frac{\hbar^2}{me^2} = 5.3 \times 10^{-9} \text{ cm},$$

the electric field strength is:

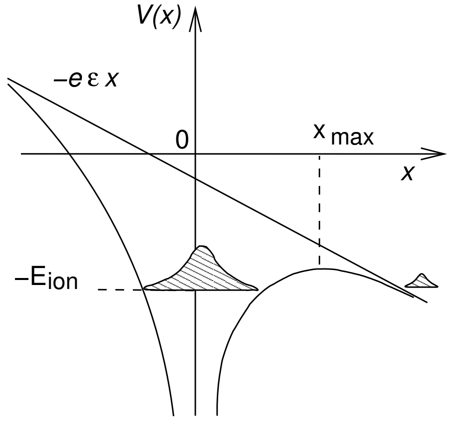
$$\begin{aligned} E_a &= \frac{e}{4\pi\varepsilon_0 a_B^2} \\ &\simeq 5.1 \times 10^9 \text{ Vm}^{-1}. \end{aligned} \tag{11}$$

This leads to the **atomic intensity**:

$$\begin{aligned} I_a &= \frac{\varepsilon_0 c E_a^2}{2} \\ &\simeq 3.51 \times 10^{16} \text{ Wcm}^{-2}. \end{aligned} \tag{12}$$

A laser intensity of $I_L > I_a$ will *guarantee ionization* for any target material, though in fact this can occur well below this threshold value (eg: $\sim 10^{14}$ Wcm^{-2} for hydrogen) via *multiphoton* effects .

Tunnelling ionization: barrier suppression model



Potential barrier tipped below ionization energy $E_{\rm ion}$ by external electric field ε

- Hydrogen: $Z = 1$

$$E_{\rm ion} = E_h = \frac{e^2}{2a_B} = 13.61 \text{ eV}$$

- Critical field for hydrogen:

$$\varepsilon_c = \frac{E_h^2}{4e^3} = \frac{e}{16a_B^2} = \frac{E_a}{16}$$

Appearance intensity of hydrogen ions

$$I_{\rm app} = \frac{I_a}{256} \simeq 1.4 \times 10^{14} \text{ Wcm}^{-2} \qquad (13)$$

Ionized gases: when is plasma response important?

Simultaneous field ionization of many atoms produces a plasma with electron density n_e, temperature $T_e \sim 1 - 10$ eV. *Collective effects* important if

$$\omega_p \tau_{\rm interaction} > 1$$

Example (Gas jet)

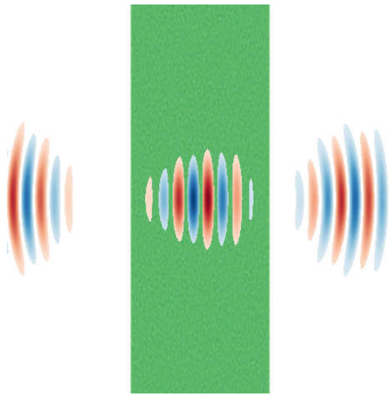
$\tau_{\rm int} = 100$ fs, $n_e = 10^{17}$ cm^{-3} $\rightarrow \omega_p \tau_{\rm int} = 1.8$
Typical gas jets: $P \sim 1$bar; $n_e = 10^{18} - 10^{19}$ cm^{-3}
Recall that from Eq.14, critical density for glass laser $n_c(1\mu) = 10^{21}$ cm^{-3}. Gas-jet plasmas are therefore *underdense*, since $\omega^2/\omega_p^2 = n_e/n_c \ll 1$.

Exploit plasma effects for nonlinear refractive properties and high electric/magnetic fields, namely: for **particle acceleration**, or source of **short-wavelength radiation**.

Plasma response time ω_p^{-1} dictates type of interaction with external fields

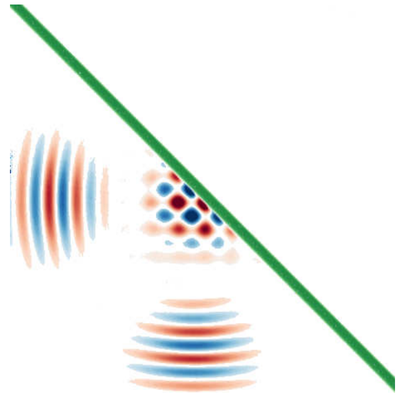
Underdense plasma, $\omega > \omega_p$:

- slow plasma response
- nonlinear refractive medium



Overdense plasma, $\omega < \omega_p$:

- radiation shielded out
- mirror-like optics



The critical density

To make this more quantitative, consider ratio:

$$\frac{\omega_p^2}{\omega^2} = \frac{e^2 n_e}{\varepsilon_0 m_e} \cdot \frac{\lambda^2}{4\pi^2 c^2}.$$

Setting this to unity defines the wavelength for which $n_e = n_c$, or

Critical density

$$n_c \simeq 10^{21} \lambda_\mu^{-2} \text{ cm}^{-3} \quad (14)$$

above which radiation with wavelengths $\lambda > \lambda_\mu$ will be reflected.
cf: radio waves from ionosphere.

Relativistic field strengths

Classical equation of motion for an electron exposed to a linearly polarized laser field $\mathbf{E} = \hat{y} E_0 \sin \omega t$:

$$\frac{dv}{dt} \simeq \frac{-eE_0}{m_e} \sin \omega t$$

$$\rightarrow v = \frac{eE_0}{m_e \omega} \cos \omega t = v_{\rm os} \cos \omega t \qquad (15)$$

Dimensionless oscillation amplitude, or 'quiver' velocity:

$$a_0 \equiv \frac{v_{\rm os}}{c} \equiv \frac{p_{\rm os}}{m_e c} \equiv \frac{eE_0}{m_e \omega c} \qquad (16)$$

Relativistic intensity

The laser intensity I_L and wavelength λ_L are related to E_0 and ω by:

$$I_L = \frac{1}{2} \varepsilon_0 c E_0^2; \quad \lambda_L = \frac{2\pi c}{\omega}$$

Substituting these into (16) we find :

$$I_L = \frac{2\pi^2 \varepsilon_0 m^2 c^5}{e^2} \frac{a_0^2}{\lambda_L^2}$$

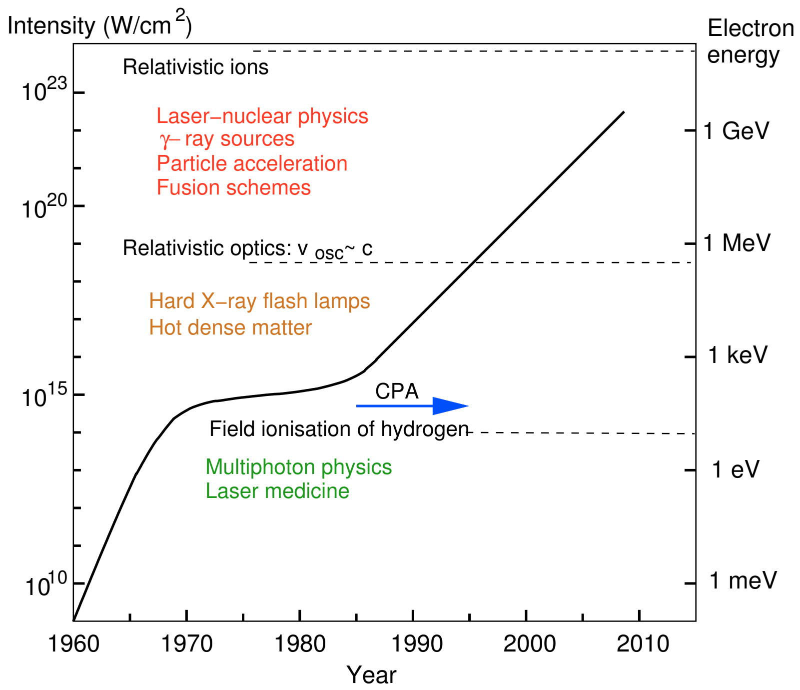
$$\simeq 1.37 \times 10^{18} a_0^2 \lambda_\mu^2 \text{ Wcm}^{-2} \qquad (17)$$

Exercise

where $\lambda_\mu = \frac{\lambda_L}{\mu m}$.

Implies that we will have **relativistic electron velocities**, or $a_0 \sim 1$, for $I_L \geq 10^{18}$ Wcm^{-2}, $\lambda_L \simeq 1\ \mu$m.

Laser technology progress: chirped pulse amplification



Further reading

1. F. F. Chen, *Plasma Physics and Controlled Fusion*, 2nd Ed. (Springer, 2006)
2. R.O. Dendy (ed.), *Plasma Physics, An Introductory Course*, (Cambridge University Press, 1993)
3. J. D. Huba, *NRL Plasma Formulary*, (NRL, Washington DC, 2007) `http://www.nrl.navy.mil/ppd/content/nrl-plasma-formulary`

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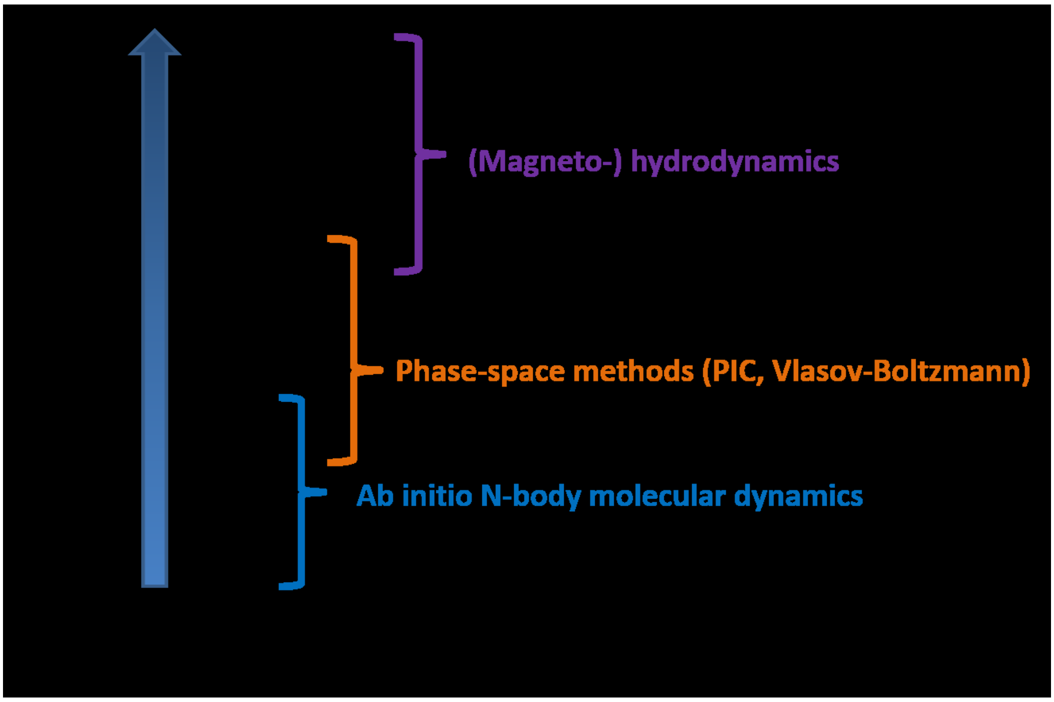
Lecture 2: Wave propagation in plasmas

- Plasma models
- Fluid equations
- Electromagnetic waves
  - Dispersion
  - Relativistic self-focussing
- Langmuir waves
- Ponderomotive force
- Wakefield excitation
- Wave breaking amplitude
- Electron acceleration
- Further reading

Model hierarchy (laser-plasmas)



Nonlinear wave propagation

The starting point for most analyses of nonlinear wave propagation phenomena is the Lorentz equation of motion for the electrons in a *cold* ($T_e = 0$), unmagnetized plasma, together with Maxwell's equations.

We also make two assumptions:

1. The ions are initially assumed to be singly charged ($Z = 1$) and are treated as a immobile ($v_i = 0$), homogeneous background with $n_0 = Zn_i$.
2. Thermal motion is neglected – justified for underdense plasmas because the temperature remains small compared to the typical oscillation energy in the laser field: $k_B T_e \ll m_e v_{\rm os}^2$.

Lorentz-Maxwell equations

Starting equations (SI units) are as follows

$$\frac{\partial \mathbf{p}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{p} = -e(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \tag{18}$$

$$\nabla\cdot\mathbf{E} = \frac{e}{\varepsilon_0}(n_0 - n_e), \tag{19}$$

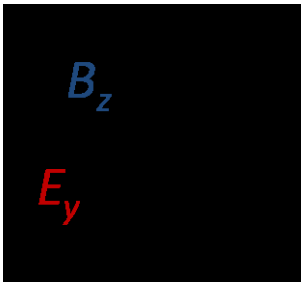
$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{20}$$

$$c^2\nabla\times\mathbf{B} = -\frac{e}{\varepsilon_0}n_e\mathbf{v} + \frac{\partial \mathbf{E}}{\partial t}, \tag{21}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{22}$$

where $\mathbf{p} = \gamma m_e \mathbf{v}$ and $\gamma = (1 + p^2/m_e^2c^2)^{1/2}$.

Electromagnetic waves



To simplify matters we first assume a plane-wave geometry like that above, with the transverse electromagnetic fields given by $\mathbf{E}_L = (0, E_y, 0)$; $\mathbf{B}_L = (0, 0, B_z)$.
From Eq. (18) the transverse electron momentum is then simply given by:

$$p_y = eA_y, \tag{23}$$

Exercise

where $E_y = \partial A_y/\partial t$.
This relation expresses conservation of canonical momentum.

The EM wave equation I

Substitute $\boldsymbol{E} = -\nabla\phi - \partial\boldsymbol{A}/\partial t$; $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ into Ampère Eq.(21):

$$c^2\nabla \times (\nabla \times \boldsymbol{A}) + \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = \frac{\boldsymbol{J}}{\varepsilon_0} - \nabla\frac{\partial\phi}{\partial t},$$

where the current $\boldsymbol{J} = -en_e\boldsymbol{v}$.
Now we use a bit of vectorial magic, splitting the current into rotational (solenoidal) and irrotational (longitudinal) parts:

$$\boldsymbol{J} = \boldsymbol{J}_\perp + \boldsymbol{J}_{||} = \nabla \times \boldsymbol{\Pi} + \nabla\Psi$$

from which we can deduce (see Jackson!):

$$\boldsymbol{J}_{||} - \frac{1}{c^2}\nabla\frac{\partial\phi}{\partial t} = 0.$$

The EM wave equation II

Now apply Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$ and $v_y = eA_y/\gamma$ from (23), to finally get:

EM wave

$$\frac{\partial^2 A_y}{\partial t^2} - c^2\nabla^2 A_y = \mu_0 J_y = -\frac{e^2 n_e}{\varepsilon_0 m_e \gamma}A_y. \quad (24)$$

The nonlinear source term on the RHS contains two important bits of physics:

$$n_e = n_0 + \delta n \quad \rightarrow \quad \text{Coupling to plasma waves}$$

$$\gamma = \sqrt{1 + \boldsymbol{p}^2/m_e^2c^2} \quad \rightarrow \quad \text{Relativistic effects}$$

Dispersion properties: EM waves

For the moment we switch the plasma oscillations off ($n_e = n_0$) in Eq.(24) and look for plane wave solutions $A = A_0 e^{i(\omega t - kx)}$. The derivative operators become: $\frac{\partial}{\partial t} \to i\omega$; $\frac{\partial}{\partial x} \to -ik$, yielding a

Dispersion relation

$$\omega^2 = \frac{\omega_p^2}{\gamma_0} + c^2k^2 \qquad (25)$$

with associated

Nonlinear refractive index

$$\eta = \sqrt{\frac{c^2k^2}{\omega^2}} = \left(1 - \frac{\omega_p^2}{\gamma_0\omega^2}\right)^{1/2} \qquad (26)$$

where $\gamma_0 = (1 + a_0^2/2)^{1/2}$, and a_0 is the normalized oscillation amplitude as in (16).

Linear propagation characteristics ($a_0 \ll 1; \gamma_0 \to 1$)

Underdense plasmas

From the dispersion relation (25) a number of important features of EM wave propagation in plasmas can be deduced.

For *underdense* plasmas ($n_e \ll n_c$):

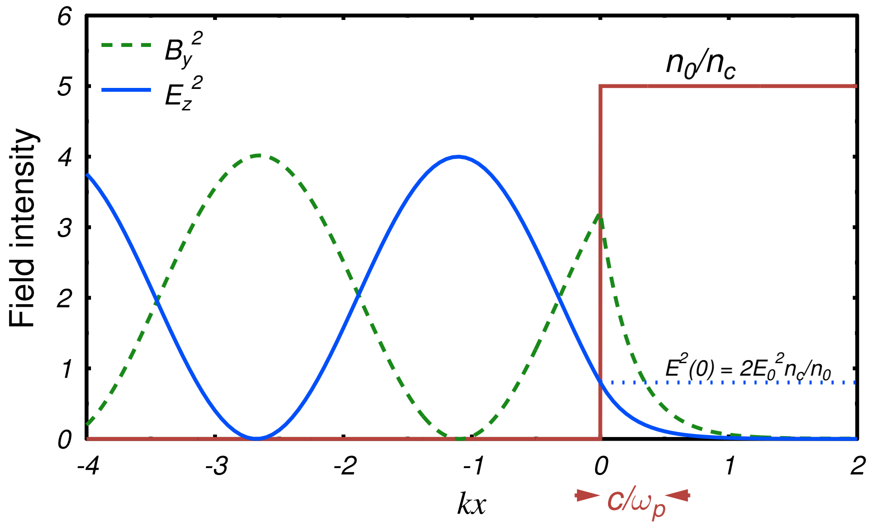
$$\text{Phase velocity} \quad v_p = \frac{\omega}{k} \simeq c\left(1 + \frac{\omega_p^2}{2\omega^2}\right) > c$$

$$\text{Group velocity} \quad v_g = \frac{\partial\omega}{\partial k} \simeq c\left(1 - \frac{\omega_p^2}{2\omega^2}\right) < c$$

Propagation characteristics (2)

Overdense plasmas

In the opposite case, $n_e > n_c$, or $\omega < \omega_p$, the refractive index η becomes imaginary. The wave can no longer propagate, and is instead attenuated with a decay length determined by the **collisionless skin depth** c/ω_p.



Nonlinear refraction effects

Real laser pulses are created with focusing optics & are subject to:

1. diffraction due to finite focal spot σ_L: $Z_R = 2\pi\sigma_L^2/\lambda$
2. ionization effects $dn_e/dt \Rightarrow$ refraction due to radial density gradients
3. relativistic self-focusing $\Rightarrow \eta(r) = \sqrt{\left(1 - \frac{\omega_p^2(r)}{\gamma_0(r)\omega^2}\right)}$
4. ponderomotive channelling $\Rightarrow \nabla_r n_e$
5. scattering by plasma waves $\Rightarrow k_0 \rightarrow k_1 + k_p$

All nonlinear effects important for laser powers $P_L > 1\,TW$

Focussing threshold – practical units

Litvak, 1970; Max *et al.*1974, Sprangle *et al.*1988

Relation between laser power and critical power:

$$\begin{aligned} P_L &= \left(\frac{m\omega c}{e}\right)^2 \left(\frac{c}{\omega_p}\right)^2 \frac{c\epsilon_0}{2} \int_0^\infty 2\pi r a^2(r) dr \\ &= \frac{1}{2}\left(\frac{m}{e}\right)^2 c^5 \epsilon_0 \left(\frac{\omega}{\omega_p}\right)^2 \tilde{P}, \\ &\simeq 0.35 \left(\frac{\omega}{\omega_p}\right)^2 \tilde{P}\ \mathrm{GW}, \quad \text{where} \quad \tilde{P} \equiv \pi a_0^2(\omega_p^2 \sigma_L^2/c^2) \end{aligned}$$

The critical power $\tilde{P}_c = 16\pi$ thus corresponds to:

Power threshold for relativistic self-focussing

$$P_c \simeq 17.5 \left(\frac{\omega}{\omega_p}\right)^2 \mathrm{GW}. \tag{27}$$

Focussing threshold – example

Critical power

$$P_c \simeq 17.5 \left(\frac{\omega}{\omega_p}\right)^2 \mathrm{GW}, \tag{28}$$

Example

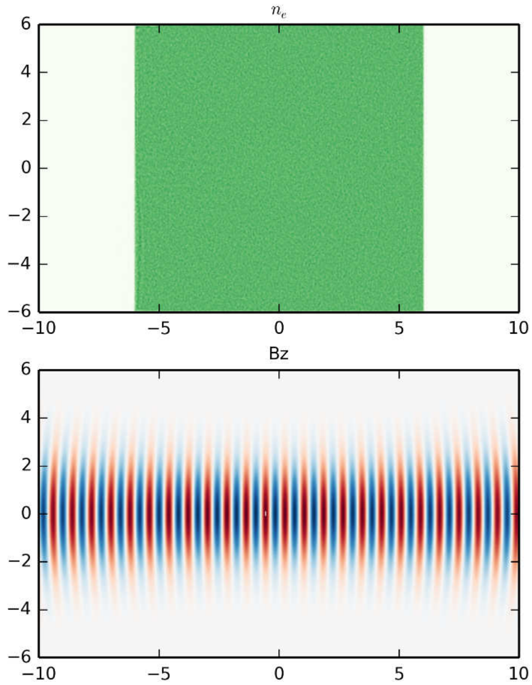
$$\lambda_L = 0.8\mu m, \quad n_e = 1.6 \times 10^{20}\ \mathrm{cm}^{-3}$$

$$\Rightarrow \frac{n_e}{n_c} = \left(\frac{\omega_p}{\omega}\right)^2 = 0.1$$

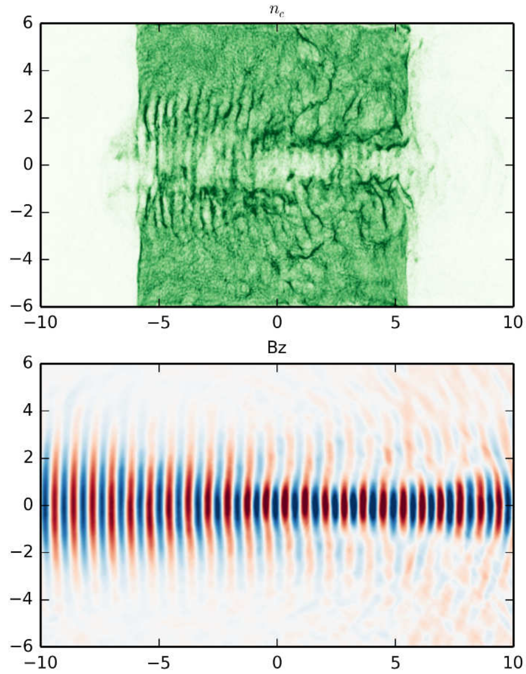
$$\Rightarrow P_c = 0.175\mathrm{TW}$$

Propagation examples: long pulses

i) $P_L/P_c \ll 1$



ii) $P_L = 2P_c$



Plasma waves

Recap Lorentz-Maxwell equations (18-22)

$$
\begin{aligned}
\frac{\partial \mathbf{p}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{p} &= -e(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \\
\nabla\cdot\mathbf{E} &= \frac{e}{\varepsilon_0}(n_0 - n_e), \\
\nabla\times\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
c^2\nabla\times\mathbf{B} &= -\frac{e}{\varepsilon_0}n_e\mathbf{v} + \frac{\partial \mathbf{E}}{\partial t}, \\
\nabla\cdot\mathbf{B} &= 0
\end{aligned}
$$

Electrostatic (Langmuir) waves I

Taking the *longitudinal* (x)-component of the momentum equation (18) and noting that $|\boldsymbol{v} \times \boldsymbol{B}|_x = v_y B_z = v_y \frac{\partial A_y}{\partial x}$ from (23) gives:

$$\frac{d}{dt}(\gamma m_e v_x) = -eE_x - \frac{e^2}{2m_e\gamma}\frac{\partial A_y^2}{\partial x}$$

We can eliminate v_x using Ampère's law $(21)_x$:

$$0 = -\frac{e}{\varepsilon_0} n_e v_x + \frac{\partial E_x}{\partial t},$$

while the electron density can be determined via Poisson's equation (19):

$$n_e = n_0 - \frac{\varepsilon_0}{e}\frac{\partial E_x}{\partial x}.$$

Electrostatic (Langmuir) waves II

The above (closed) set of equations can in principle be solved numerically for arbitrary pump strengths. For the moment, we simplify things by *linearizing the plasma fluid* quantities:

$$\begin{aligned} n_e &\simeq n_0 + n_1 + \ldots \\ v_x &\simeq v_1 + v_2 + \ldots \end{aligned}$$

and neglect products like $n_1 v_1$ etc. This finally leads to:

Driven plasma wave

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_p^2}{\gamma_0}\right) E_x = -\frac{\omega_p^2 e}{2m_e\gamma_0^2}\frac{\partial}{\partial x} A_y^2 \qquad (29)$$

The driving term on the RHS is the *relativistic ponderomotive force*, with $\gamma_0 = (1 + a_0^2/2)^{1/2}$.

Plasma (Langmuir) wave propagation

Without the laser driving term ($A_y = 0$), Eq.(29) describes linear plasma oscillations with solutions

$$E_x = E_{x0} \sin(\omega t),$$

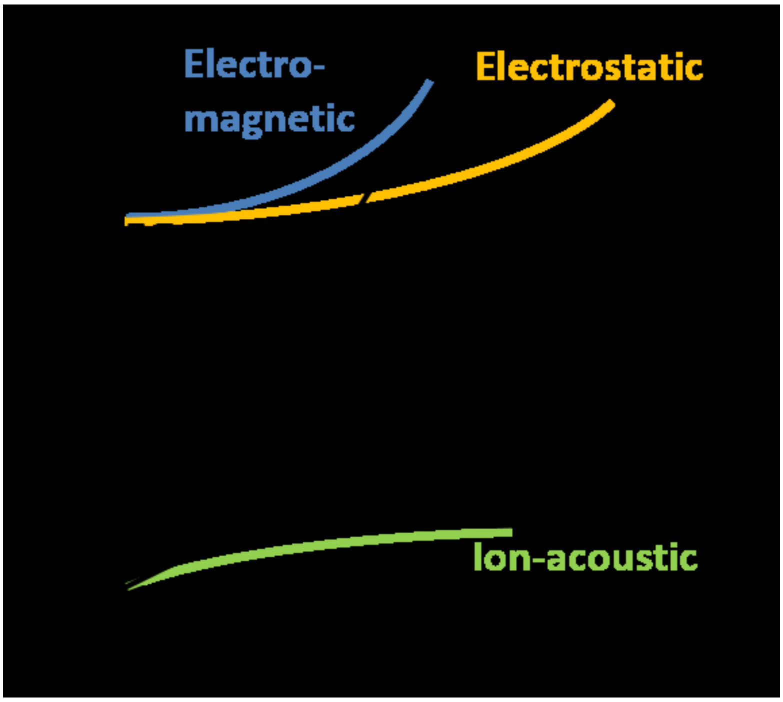
giving the dispersion relation:

$$-\omega^2 + \omega_p^2 = 0. \qquad (30)$$

The linear eigenmode of a plasma has $\omega = \omega_p$.
To account for a finite temperature $T_e > 0$, we would reintroduce a pressure term ∇P_e in the momentum equation (18), which finally yields the Bohm-Gross relation:

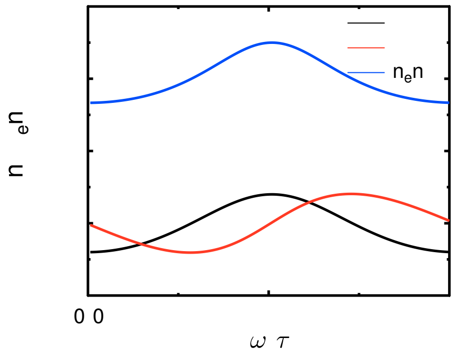
$$\omega^2 = \omega_p^2 + 3v_t^2 k^2. \qquad (31)$$

Summary: dispersion curves



Numerical solutions – linear Langmuir wave

Numerical integration of the electrostatic wave equation on slide 5 for $v_{\mathrm{max}}/c = 0.2$



NB: electric field and density 90^o out of phase

Coupled cold plasma fluid equations: summary

Electromagnetic wave

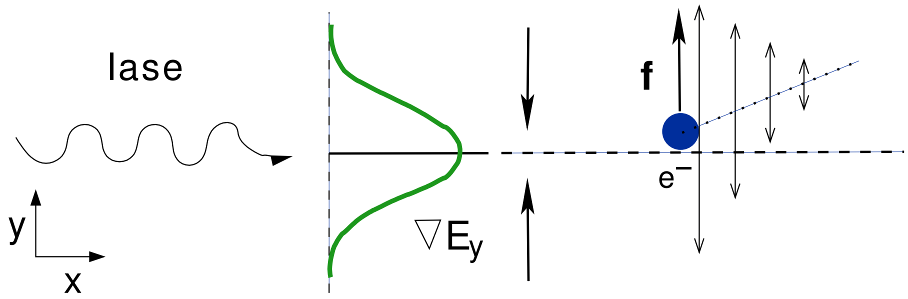
$$\frac{\partial^2 A_y}{\partial t^2} - c^2\nabla^2 A_y = \mu_0 J_y = -\frac{e^2 n_e}{\varepsilon_0 m_e \gamma} A_y$$

Electrostatic (Langmuir) wave

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_p^2}{\gamma_0}\right) E_x = -\frac{\omega_p^2 e}{2 m_e \gamma_0^2}\frac{\partial}{\partial x} A_y^2$$

Ponderomotive force

- Single electron oscillating slightly off-centre of focused laser beam:



- After 1st quarter-cycle, sees lower field
- Doesn't quite return to initial position

⇒ Accelerated away from axis

DEMO

`fpond.py`

Ponderomotive force: non-relativistic

In the limit $v/c \ll 1$, the equation of motion (18) for the electron becomes:

$$\frac{\partial v_y}{\partial t} = -\frac{e}{m} E_y(\boldsymbol{r}). \tag{32}$$

Taylor expanding electric field about the current electron position:

$$E_y(\boldsymbol{r}) \simeq E_0(y)\cos\phi + y\frac{\partial E_0(y)}{\partial y}\cos\phi + ...,$$

where $\phi = \omega t - kx$ as before.
To lowest order, we therefore have

$$v_y^{(1)} = -v_{\rm os}\sin\phi; \quad y^{(1)} = \frac{v_{\rm os}}{\omega}\cos\phi,$$

where $v_{\rm os} = eE_L/m\omega$.

Ponderomotive force: non-relativistic (contd.)

Substituting back into Eq. (32) gives

$$\frac{\partial v_y^{(2)}}{\partial t} = -\frac{e^2}{m^2\omega^2} E_0 \frac{\partial E_0(y)}{\partial y}\cos^2\phi.$$

Multiplying by m and taking the cycle-average yields the

ponderomotive force

$$f_p \equiv m\overline{\frac{\partial v_y^{(2)}}{\partial t}} = -\frac{e^2}{4m\omega^2}\frac{\partial E_0^2}{\partial y}. \tag{33}$$

on the electron.

Relativistic ponderomotive force

Rewrite Lorentz equation (18) in terms of the vector potential $\boldsymbol{A}$:

$$\frac{\partial \boldsymbol{p}}{\partial t} + (\boldsymbol{v}.\nabla)\boldsymbol{p} = \frac{e}{c}\frac{\partial \boldsymbol{A}}{\partial t} - \frac{e}{c}\boldsymbol{v}\times\nabla\times\boldsymbol{A}. \qquad (34)$$

Make use of identity:

$$\begin{aligned} \boldsymbol{v}\times(\nabla\times\boldsymbol{p}) &= \frac{1}{m\gamma}\boldsymbol{p}\times\nabla\times\boldsymbol{p} \\ &= \frac{1}{2m\gamma}\nabla\mid p\mid^2 - \frac{1}{m\gamma}(\boldsymbol{p}.\nabla)\boldsymbol{p}, \end{aligned}$$

separate the timescales of the electron motion into slow and fast components $\boldsymbol{p} = \boldsymbol{p}^s + \boldsymbol{p}^f$ and average over a laser cycle,

$$\boldsymbol{f}_p = \frac{dp^s}{dt} = -mc^2\nabla\overline{\gamma}, \qquad (35)$$

Exercise

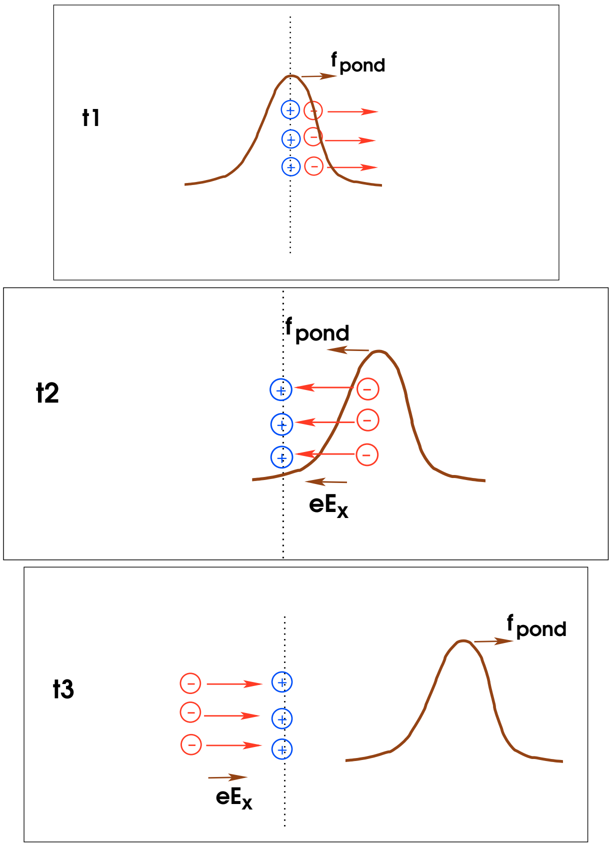
where $\overline{\gamma} = \left(1 + \frac{p_s^2}{m^2c^2} + \frac{1}{2}a_0^2\right)^{1/2}$.

Ponderomotively driven plasma waves

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_p^2}{\gamma_0}\right)E_x = -\frac{\omega_p^2 e}{2m_e\gamma_0^2}\frac{\partial}{\partial x}A_y^2$$

Note that RHS can either be short laser pulse (**laser wakefield**) OR particle beam (**plasma wakefield**).

Driven plasma waves: wakefield excitation



Resonance condition

The amplitude of the longitudinal oscillation will be enhanced if the pulse length is roughly matched to the plasma period:

$$\tau_L \simeq \omega_p^{-1}.$$

Example

What plasma density do we need to match a 100 fs pulse?

$$\omega_p \simeq 5 \times 10^4 n_e^{1/2} \quad s^{-1}$$

Matching condition:

$$n_e \simeq 4 \times 10^{14} \tau_{\rm ps}^{-2} \ \text{cm}^{-3}$$

For 10 fs, need $n_e = 4 \times 10^{18}$ cm^{-3}.

DEMO

`wake.py`

Maximum field amplitude - wave-breaking limit

For relativistic phase velocities, find

$$E_{\mathrm{max}} \sim m\omega_p c/e$$

– wave-breaking limit – Dawson (1962), Katsouleas (1988).

Example

$$\begin{aligned} m_e &= 9.1 \times 10^{-28}\text{g} \\ c &= 3 \times 10^{10}\text{cms}^{-1} \\ \omega_p &= 5.6 \times 10^4 (n_e/cm^{-3})^{1/2} \\ e &= 4.8 \times 10^{-10}\text{statcoulomb} \\ E_p &\sim 4 \times 10^8 \left(\frac{n_e}{10^{18}\ \text{cm}^{-3}}\right)^{1/2} \text{V m}^{-1} \end{aligned}$$

Electron acceleration by wakefields

- Conventional synchrotrons and LINACS operate with field gradients limited to around 100 MVm^{-1}.
- Plasma is already ionized; can theoretically sustain a field 10^4 times larger, given by:

$$E_p = \frac{m_e c \omega_p}{e}\varepsilon \simeq n_{18}^{1/2}\varepsilon \ \mathrm{GV\,cm}^{-1}, \quad (36)$$

where n_{18} is the electron density in units of 10^{18} cm^{-3}.

Laser-electron accelerator

Tajima & Dawson, 1979

Laser-driven wakefields must propagate with velocities approaching the speed of light ($v_p = v_g < c$).
Plasma wave has a phase velocity:

$$v_p = c\left(1 - \frac{\omega_p^2}{\omega_o^2}\right)^{\frac{1}{2}} \simeq c\left(1 - \frac{1}{2\gamma_p^2}\right), \quad (37)$$

where $\gamma_p = \omega_0^2/\omega_p^2$.

Acceleration length

A relativistic electron ($v \simeq c$) trapped in such a wave will be accelerated over at most half a wavelength in the wave-frame, after which it starts to be decelerated.
Effective acceleration length:

$$
\begin{aligned}
L_a = \frac{\lambda_p c}{2(c - v_p)} &\simeq \lambda_p \gamma_p^2 \\
&= \frac{\omega^2}{\omega_p^2} \lambda_p \\
&\simeq 3.2\, n_{18}^{-3/2} \lambda_{\mu\mathrm{m}}^{-2} \ \text{cm}. \qquad (38)
\end{aligned}
$$

Maximum energy gain

Combine Eq. (36) and Eq. (38) to obtain the maximum energy gain:

$$
\begin{aligned}
\Delta U &= eE_p . L_a \\
&= e \left(\frac{m \omega_p c}{e} \right) \varepsilon \frac{\omega^2}{\omega_p^2} \frac{2\pi c}{\omega_p} \\
&= 2\pi \left(\frac{\omega}{\omega_p} \right)^2 \varepsilon\, mc^2 \\
&\simeq 3.2\, n_{18}^{-1} \lambda_{\mu\mathrm{m}}^{-2} \ \text{GeV}. \qquad (39)
\end{aligned}
$$

Plasma accelerator physics

In principle, TW laser is capable of accelerating an electron to 5 GeV in a distance of 5 cm through a plasma with density 10^{18} cm^{-3}, but many limiting factors: diffraction, instabilities. GeV milestone reached September 2006 (Berkeley Lab). Much more to come in forthcoming lectures:

- Plasma wakefield (Osterhoff)
- Laser wakefield (Kim)
- Nonlinear waves (Bulanov)
- Plasma-based particle acceleration (Lu)
- Beam dynamics (Suk)
- Betatron radiation (Chen)

Further reading

1. J. Boyd and J. J. Sanderson, *The Physics of Plasmas*
2. W. Kruer, *The Physics of Laser Plasma Interactions*, Addison-Wesley, 1988
3. J. D. Jackson, *Classical Electrodynamics*, Wiley 1975/1998
4. J. P. Dougherty in Chapter 3 of R. Dendy *Plasma Physics*, 1993
5. E. Esarey, C. B. Schroeder, W. P. Leemans, *Physics of laser-driven plasma-based electron accelerators*, Rev. Mod. Phys. **81**, 1229-1285 (2009)

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JÜLICH FORSCHUNGSZENTRUM

Physics of High Intensity Laser-Plasma Interactions

7th Asian Summer School and Symposium on Laser-Plasma Acceleration and Radiation
Shanghai, 17-23 July 2016 | Paul Gibbon

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Lecture 3: Interaction with Solids

Short pulse vs. long pulse interactions

Long-pulse interaction physics (Inertial Confinement Fusion – ns lasers):

- Collisional heating and creation of long scale-length plasmas
- Laser reflected at critical density surface
- Fast (keV) particles produced at 'high' intensities (10^{16} Wcm^{-2})

Femtosecond pulses

- Pulse duration typically $<$ ion motion timescale (hydrodynamics)
- Huge intensity range $> 10^8$
- No single interaction model possible

Creation of plasma surface layer via ionization

Example

Al has 3 valence electrons; 6 more can be released for a few hundred eV. The electron density is given by:

$$n_e = Z^* n_i = \frac{Z^* N_A \rho}{A}. \tag{40}$$

effective ion charge:	$Z^* = 9$
atomic number:	$A = 26$
Avogadro number:	$N_A = 6.02 \times 10^{23}$
mass density:	$\rho = \rho_{\rm solid} = 1.9$ g cm^{-3}
electron density:	$n_e = 4 \times 10^{23}$ cm^{-3}
critical density (Eq. 14):	$n_c \simeq 1.1 \times 10^{21} \lambda_\mu^{-2}$ cm^{-3}
density contrast ($\lambda = 1\ \mu$m):	$n_e/n_c \simeq 400$

Heating

Target is *initially* heated via electron-ion collisions to 10s or 100s of eV depending on the laser intensity. The plasma pressure created during heating causes ion blow-off (ablation) at the **sound speed**:

$$\begin{aligned} c_s &= \left(\frac{Z^* k_B T_e}{m_i}\right)^{1/2} \\ &\simeq 3.1 \times 10^7 \left(\frac{T_e}{\text{keV}}\right)^{1/2} \left(\frac{Z^*}{A}\right)^{1/2} \text{cm s}^{-1}, \end{aligned} \quad (41)$$

where k_B is the Boltzmann constant, T_e the electron temperature and m_i the ion mass.

Expansion

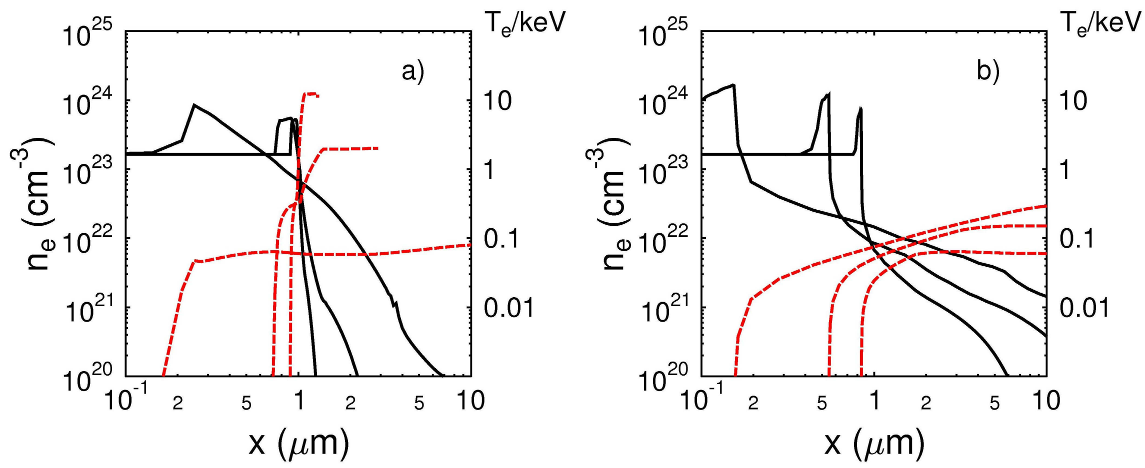
Because of ion ablation, density profile formed is *exponential* with scale-length:

$$\begin{aligned} L &= c_s \tau_L \\ &\simeq 30 \left(\frac{T_e}{\text{keV}}\right)^{1/2} \left(\frac{Z^*}{A}\right)^{1/2} \left(\frac{\tau}{100\text{ fs}}\right) \text{ nm} \end{aligned} \quad (42)$$

Example

100 fs Ti:sapphire pulse on Al foil heats the target to a few hundred eV $\rightarrow$ plasma with scale-length L/λ= 0.01–0.1. (cf: 100-1000 for ICF plasmas).

Expansion examples: 100fs vs 100ps laser pulse



Collisional absorption

Modelled using **Helmholtz wave equations**: standard method for electromagnetic wave propagation in an inhomogeneous plasma – see books by Ginzburg or Kruer.

Start from Maxwell's equations with small field amplitudes and a non-relativistic fluid response including collisional damping:

$$m\frac{\partial \mathbf{v}}{\partial t} = -e(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}) - m\nu_{ei}\mathbf{v}, \tag{43}$$

where ν_{ei} is the electron-ion collision frequency - see Lecture 1, Eq.(8).

Physically arises from binary collisions, resulting in a frictional drag on the electron motion - often called **inverse bremsstrahlung**.

Derivation of Helmholtz wave equations Exercise

The relevant EM wave equations for $\boldsymbol{E}$ and $\boldsymbol{B}$ are obtained in the usual way by taking the curl of the Faraday and Ampère equations (20, 21) respectively, to give:

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \boldsymbol{J}}{\partial t} + \nabla(\nabla\cdot\boldsymbol{E}), \quad (44)$$

$$\nabla^2 \boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = -\frac{4\pi}{c}\nabla\times\boldsymbol{J}. \quad (45)$$

The right-hand sides of each equation represent the source terms of the EM waves in the plasma.

Linearized wave equations

Assume that all field and fluid quantities have a harmonic time-dependence $\exp(-i\omega t)$, where ω is the laser frequency:

$$f(\boldsymbol{x},t) = f_0(\boldsymbol{x}) + f_1(\boldsymbol{x})e^{-i\omega t} + f_2(\boldsymbol{x})e^{-2i\omega t} + \ldots,$$

which results in the following simplifications:

$$\begin{aligned} \frac{\partial}{\partial t} &\rightarrow -i\omega \\ n_e &\rightarrow n_o + n_1 \\ \boldsymbol{J} &\rightarrow -en_0\boldsymbol{v}_1 \\ (\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) &\rightarrow \boldsymbol{E}_1. \end{aligned}$$

Ohm's Law

Inserting these approximations into the Lorentz equation (43) allows us to solve for $\boldsymbol{v}_1$, namely:

$$\boldsymbol{v}_1 = \frac{-i}{\omega + i\nu_{ei}} \frac{e\boldsymbol{E}_1}{m}.$$

This immediately gives us the induced plasma current

$$\boldsymbol{J}_1 = -en_o\boldsymbol{v}_1 = \sigma_e \boldsymbol{E}_1, \tag{46}$$

where σ_e, the AC electrical conductivity, is

$$\sigma_e = \frac{i\omega_p^2}{4\pi\omega(1 + i\tilde{\nu})}. \tag{47}$$

Note $\tilde{\nu} = \nu_{ei}/\omega$.

General wave equation

Substituting expression (46) for $\boldsymbol{J}_1$ into the RHS of the wave equation (44) for $\boldsymbol{E}_1$ gives us a general expression for the electric field:

$$\nabla^2 \boldsymbol{E}_1 + \frac{\omega^2}{c^2}\boldsymbol{E}_1 = \frac{\omega_p^2}{c^2}\frac{\boldsymbol{E}_1}{1 + i\tilde{\nu}} + \nabla(\nabla\cdot\boldsymbol{E}_1). \tag{48}$$

Dispersion relation

For a planar, transverse EM wave propagating in a uniform plasma we have $\nabla \to i\boldsymbol{k}$, and $\boldsymbol{E}_1$ perpendicular to $\boldsymbol{k}$, so that $\nabla\cdot\boldsymbol{E}_1 = 0$. In this limit we recover the standard linear dispersion relation:

$$-k^2 + \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2(1+i\tilde{\nu})}\right) = 0. \qquad (49)$$

Dielectric constant

From this we identify the dielectric constant of the propagation medium

$$\varepsilon \equiv \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2(1+i\tilde{\nu})} = 1 + \frac{4\pi i\sigma_e}{\omega}.$$

Can be readily generalized to a non-uniform plasma by allowing permittivity $\varepsilon(x)$ to vary in space.

One-dimensional density gradient

Consider plasma density with a gradient in one direction, so that

$$\varepsilon(x) \equiv n^2(x) = 1 - \frac{n_0(x)/n_c}{(1 + i\tilde{\nu}(x))}, \quad (50)$$

where $n(x)$ is the local refractive index, n_0 the equilibrium electron density and n_c the critical density of the EM wave, and $\tilde{\nu} = \nu_{ei}/\omega$.

Absorption in steep density profiles: skin effect

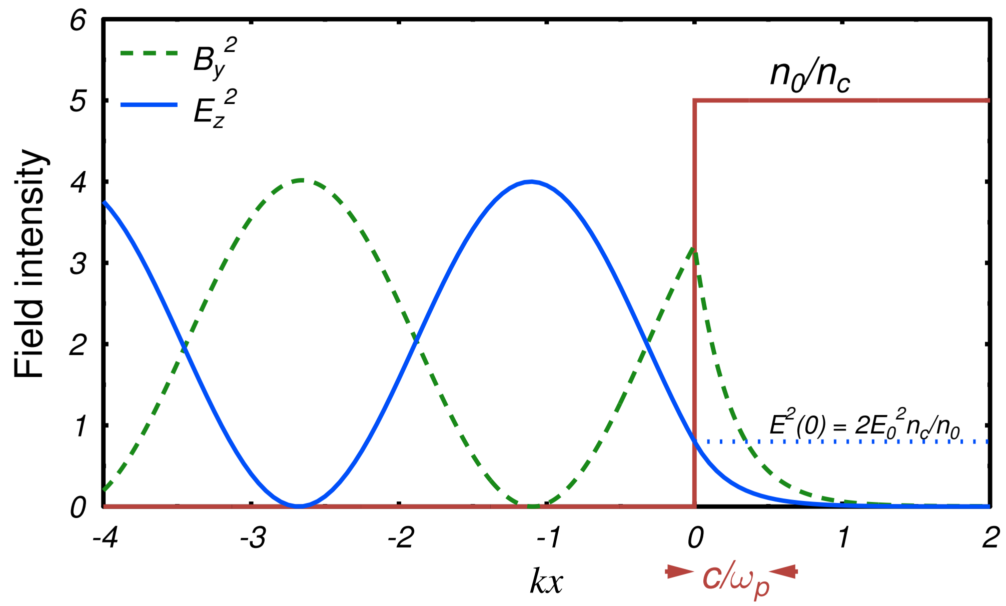
Density profile can be approximated by a Heaviside step function:

$$n_0(x) = n_0 \Theta(x),$$

giving a dielectric constant (cf: dispersion relation Eq. **??**):

$$\varepsilon(x) \equiv \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2(1 + i\nu_{ei}/\omega)} \Theta(x). \quad (51)$$

Solution for laser field I



Solution for laser field II Exercise

For normally incident light, the transverse electric field has the solution

$$E_z = \begin{cases} 2E_0 \sin(kx\cos\theta + \phi), & x < 0 \\ E(0)\exp(-x/l_s), & x \geq 0 \end{cases} \tag{52}$$

where $l_s \simeq c/\omega_p$ is the collisionless skin-depth, $k = \omega/c$, E_0 is the amplitude of the laser field and ϕ a phase factor.

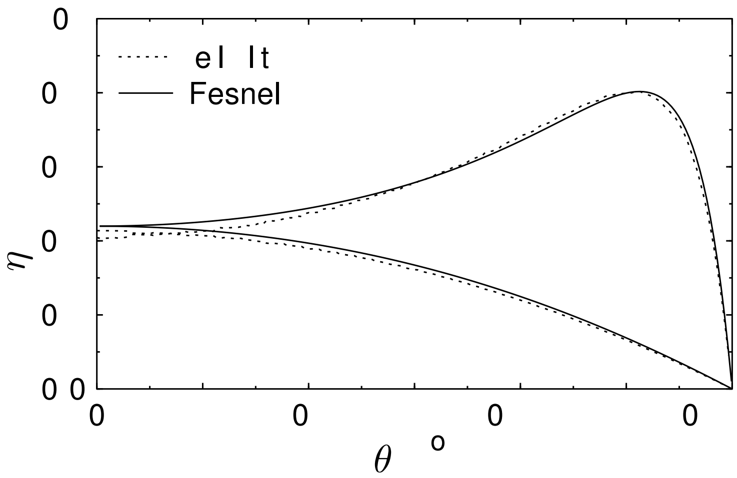
Matching vacuum and solid solutions at the boundary $x = 0$ gives:

$$\begin{aligned} E(0) &= 2E_0 \frac{\omega}{\omega_p}\cos\theta \\ \tan\phi &= -l_s \frac{\omega}{c}\cos\theta. \end{aligned}$$

Reflectivity: Drude model

Example

Al: $Z^* = 3$, $n_e \simeq 2 \times 10^{23}$ cm^{-3}, $\lambda_L = 0.8\ \mu$m , $n_e/n_c \simeq 100$.



Angular absorption for a step-profile with $n_e/n_c = 100$ and $\nu/\omega = 5$ calculated from the Fresnel equations (solid) and numerically from the Helmholtz wave equations (dotted).

Collisional frequency turn-off

quiver velocity correction

Effective collision frequency strongly reduced by quiver motion in laser field

$$\nu_{\rm eff} \simeq \nu_{ei} \frac{v_{te}^3}{(v_{os}^2 + v_{te}^2)^{3/2}}. \qquad (53)$$

A temperature of 1 keV corresponds to a thermal velocity $v_{te} \simeq 0.05$, so collisional absorption starts to turn off for irradiances $I\lambda^2 \geq 10^{15}$ Wcm$^{-2}\mu$m^2.

DEMO

`helmholtz.py`

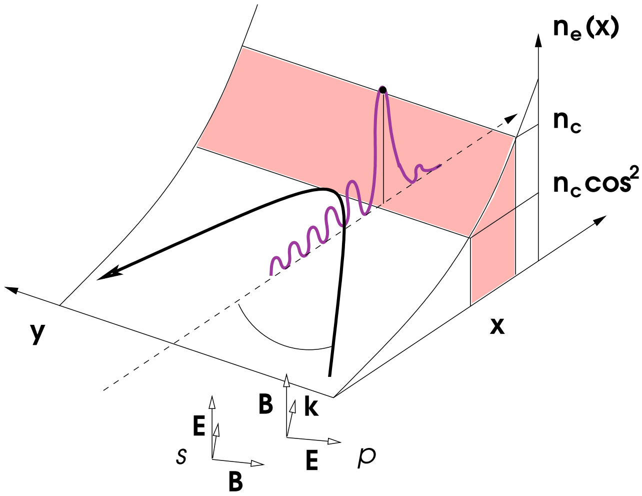
Collisionless absorption mechanisms

What other absorption mechanisms can couple laser energy to a hot, solid-density target?

1. Resonance absorption ($L/\lambda_L \gg v_{os}/\omega$) – Denisov (1957)
2. Anomalous (collisionless) skin effect – Weibel (1967)
3. 'Vacuum heating' – Brunel (1987), Gibbon (1992)
4. Relativistic $\boldsymbol{j} \times \boldsymbol{B}$ heating – Kruer (1988), Wilks (1992)
5. Anharmonic resonance – Mulser (2008)
6. Zero Vector Potential – Baeva (2010)

All of these mechanisms will generate **fast electrons** with energies $T_h \sim$ keV–MeV.

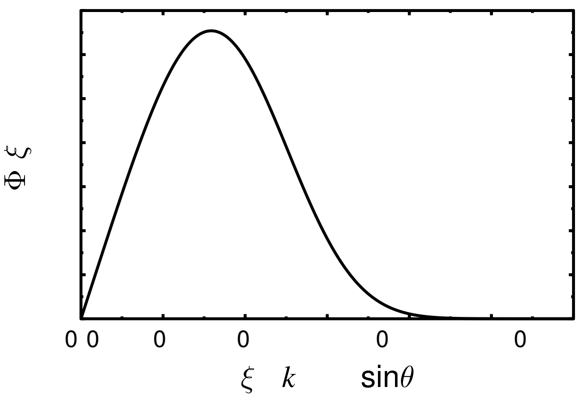
Collisionless resonance absorption



Standard picture of resonance absorption: a *p*-polarized light wave tunnels through to the critical surface ($n_e = n_c$) and drives up a plasma wave. This is damped by particle trapping and wave breaking at high intensities.

Resonance absorption: Denisov function

Self-similar dependence on the parameter $\xi = (kL)^{1/3} \sin\theta$, $kL \gg 1$.



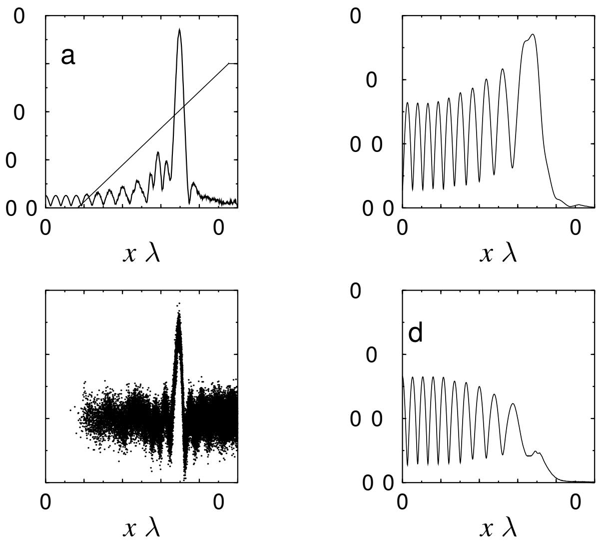
To a good approximation, $\phi(\xi) \simeq 2.3\xi \exp(-2\xi^3/3)$ and the fractional absorption is given by

$$\eta_{\rm ra} = \frac{1}{2}\Phi^2(\xi)$$

.

Kinetic simulation of resonance absorption

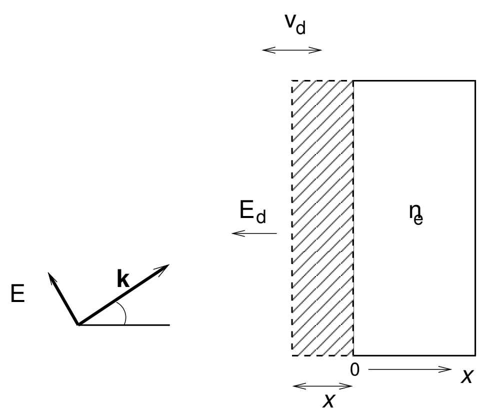
Particle-in-Cell



PIC simulation of resonance absorption for parameters $\theta = 9^o$, $v_{os}/c = 0.07$, $L/\lambda = 5\,(kL = 10\pi)$, and $n_e^{\max}/n_c = 1.5$: a) Density profile and normal electric field b) parallel electric field, c) particle momenta, d) laser magnetic field.

Vacuum heating: Brunel model

Resonance absorption not possible if oscillation amplitude exceeds the density scale length L, i.e. if $v_{\rm os}/\omega > L$.
Capacitor approximation: magnetic field of the wave is ignored; assume laser electric field E_L has some component E_d normal to the target surface given by $E_d = 2E_L \sin\theta$.



Capacitor model of the Brunel heating mechanism.

Brunel model IV

Comparing absorbed power

$$P_a \simeq \frac{\varepsilon_0}{4\pi}\frac{e}{m\omega}E_d^3$$

to the incoming laser power

$$P_L = \varepsilon_0 c E_L^2 \cos\theta/2$$

we obtain the

Brunel absorption rate

$$\eta_a \equiv \frac{P_a}{P_L} = \frac{4}{\pi}a_0\frac{\sin^3\theta}{\cos\theta}, \quad (54)$$

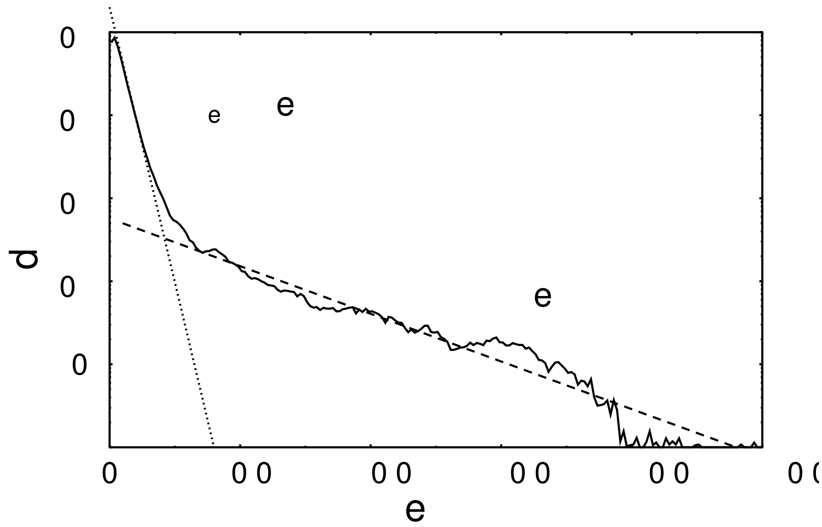
where $a_0 = v_{\rm os}/c$.

Expect more absorption at large angles of incidence and higher laser irradiance, $I\lambda^2 \propto a_0^2$.

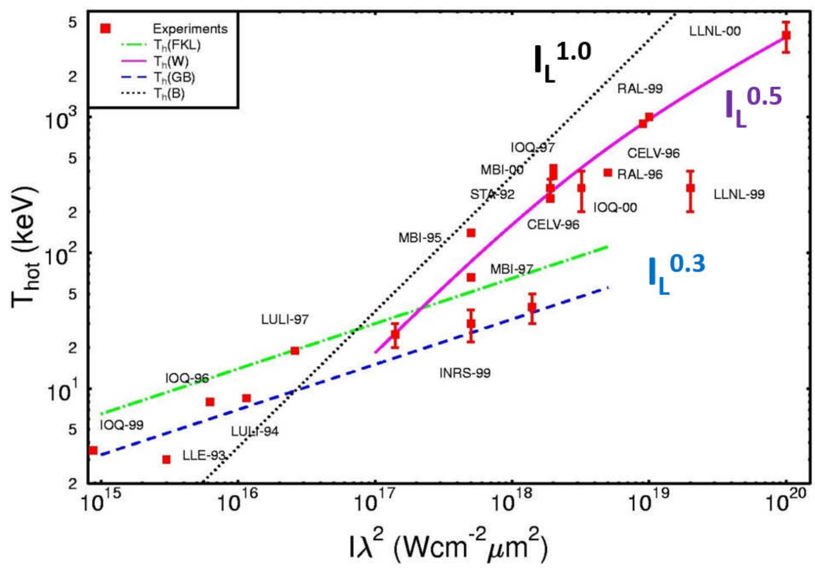
Vacuum heating

Hot electron generation

Typical signature of collisionless laser heating – bi-Maxwellian electron spectrum with characteristic temperatures T_c and T_h.



Hot electron temperature scaling



Hot electron temperature measurements in femtosecond laser-solid experiments (squares) compared with various models: long pulse; Brunel; Gibbon & Bell; Wilks.

How can you tell what absorption processes are acting?

- Angular dependence: peak abs $> 45\,\text{deg} \rightarrow L/\lambda > 1$, resonance abs
- Peak at high incidence angles (Fresnel-like) means high laser contrast, sharp density profile $\rightarrow$ Brunel mechanism

- Hot electrons directed along laser axis $\rightarrow$ relativistic mechanism
- 2π hot electron spread $\rightarrow$ rough target surface or hole boring

Ion acceleration

Direct acceleration in laser field inefficient, since

$$\frac{v_i}{c} \simeq \frac{eE_L}{m_i \omega c} = \frac{m_i}{m_e} a_0 \leq \frac{a_0}{1836}$$

Get relativistic ion energies for

$$\begin{aligned} a_0 &\sim 2000 \\ \text{or} \qquad I\lambda_L^2 &\geq 5 \times 10^{24}\ \text{Wcm}^{-2}\mu\text{m}^2 \end{aligned}$$

Therefore need means of transmitting laser energy to ions over many cycles $\rightarrow$ exploit electrostatic field in plasma.

Mechanisms

1. Coulomb explosion: clusters; ponderomotive channelling in gas jets
2. Electrostatic sheath formed by hot electron cloud (TNSA)
3. Collisionless shock formation: hole boring
4. Light sail: radiation pressure on mass-limited target

Sheath model

Electrostatic plasma expansion into vacuum: ions initially at rest ($n_i = n_{i0}$), hot electrons described by Boltzmann distribution:

$$n_e = n_{e0} \exp(e\phi / T_h)$$

where $n_{e0} = Zn_{i0}$, and ϕ satisfies Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0}(n_e - Zn_i)$$

Ion expansion is described by continuity and momentum equations:

$$\begin{aligned} \left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}\right) n_i &= -n_i \frac{\partial v_i}{\partial x} \\ m_i \left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}\right) v_i &= -Ze \frac{\partial \phi}{\partial x} \end{aligned} \tag{55}$$

Self-similar solution

If the plasma stays quasineutral everywhere ($n_e \simeq Zn_i$), then Eqs. (55) have a self-similar solution in x/t:

$$\begin{aligned} Zn_i &= n_{e0}\exp(-x/c_st - 1) \\ v_i &= c_s + x/t \\ e\phi &= -T_e(\frac{x}{c_st} + 1) \end{aligned} \quad (56)$$

where $c_s = \sqrt{ZT_e/m_i}$ is the ion sound speed.
Max ion velocity is

$$v_f = 2c_s \log(\tau + \sqrt{\tau^2 + 1}) \quad (57)$$

where

$$\tau = \frac{\omega_{pi}t}{\sqrt{2e}}; \quad \omega_{pi} = \left(\frac{Z^2e^2n_{i0}}{\varepsilon_0 m_i}\right)^{1/2}$$

Energy spectrum

Ion energy spectrum is given by:

$$\frac{dN}{dU} = \frac{n_{i0}c_st}{(2UU_0)^{1/2}} e^{-(2UU_0)^{1/2}} \quad (58)$$

where $U_0 = Zk_BT_h$.

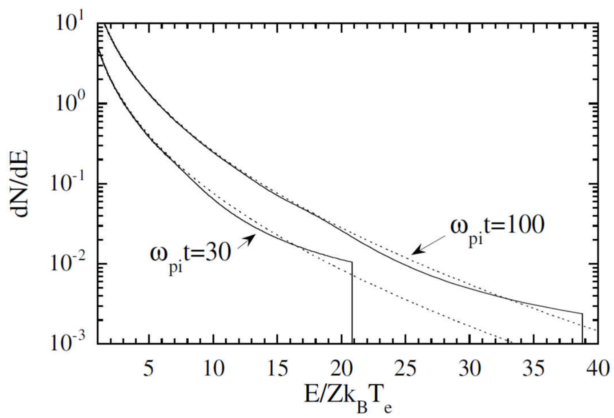


Figure 1: Ion energy spectrum from Mora expansion model

Hole boring

On the front side of an overdense plasma, the ponderomotive force (recall Eq. 35) will displace and compress electrons into the target, creating an electrostatic field acting on the ions.

Momentum balance across the collisionless, electrostatic shock thus formed implies:

$$u_i = 2u_s = 2\left(\frac{I_L}{m_i n_i c}\right)^{1/2} = 2\sqrt{\frac{Z}{A}\frac{m_e}{m_p}\frac{n_c}{n_e}} \qquad (59)$$

where u_s is the velocity of the shock front. This leads to a quasi-monoenergetic component in the ion spectrum. See: Denavit, PRL (1992); Wilks, PRL (1992); Macchi, PRL (2005)
Relativistic HB formula: Robinson, PPCF (2009).

Light sail acceleration

A mass-limited target, such as a nm-thick foil, allows nearly complete displacement of electrons, thereby maximizing the ES field. A simple capacitor model suffices to determine the threshold intensity for this scenario.
The charge separation field of a foil with thickness d is:

$$\Delta E = \frac{e}{\varepsilon_0} n_e d$$

This is balanced by the net laser field at the surface, $2E_L = 2m_e\omega c a_0/e$, leading to the matching condition:

$$a_0 \simeq \pi \frac{n_e}{n_c}\frac{d}{\lambda_L} \qquad (60)$$

Under these conditions, find max ion energy $U_i \sim t^{1/3}$ – see Esirkepov, PRL (2004).

Advanced topics

High-density plasmas also offer a rich variety of particle and radiation sources – for example:

- Gigagauss magnetic fields (Mima)
- Laboratory astrophysics (Bulanov)
- Proton acceleration (Shen)
- Terahertz radiation (Li)

Reading material

1. M. C. Levy, S. C. Willks, M. Tabak, S. B. Libby, M. G. Baring, *Petawatt absorption bounded*, Nat. Commun. **5**, 4149, (2014) DOI: 10.1038/ncomms5149
2. A. Macchi, M. Borghesi, M. Passoni, *Ion acceleration by superintense laser-plasma interaction*, Rev. Mod. Phys. **85**, 770-793 (2013)
3. U. Teubner, P. Gibbon, *High-order harmonics from laser-irradiated plasma surfaces*, Rev. Mod. Phys. **81**, 445-479 (2009)
4. P. Gibbon, *Short Pulse Laser Interactions with Matter: An Introduction*, Imperial College Press, 2005

Prerequisites for running Python demos

- Scientific Python download site: `http://www.scipy.org/install.html`
- Minimum packages (usually present or post-installable in a Linux distribution): python ($\geq$ 2.7), ipython, numpy, scipy, matplotlib $\geq$ 1.2)
- Recommended package bundle for Windows users: anaconda (all inclusive + Spider IDE)
- Execution: python wake.py (Linux/command shell) or run from Spider GUI (Windows)