Dark Matter Properties from a Supercomputer

During the last decades, cosmology has delivered a grand wealth of groundbreaking discoveries. The understanding of the Universe at very large and very small scales advanced tremendously, both by experimental work, e.g. by the measurement of the accelerated expansion of the current Universe [1, 2] and by theoretical work, e.g. by the determination of the nature of the QCD phase transition in the early Universe [3]. Very often theoretical advances drive experimental ones, like the prediction of gravitational waves finally lead to "first light" for a gravitational wave observatory [4]. It also works the other way round just as frequently. For example high precision measurements revealed [5], that the usual type of matter accounts only for about 5 % of the total energy content of the Universe, with a factor 5 more attributed to Dark Matter and the rest to Dark Energy. In turn, theory has proposed several candidates for Dark Matter to be confirmed or ruled out by experiment.

A theoretically well motivated Dark Matter candidate is the axion particle. It explains a peculiar feature of the strong interaction: it is surprisingly symmetric under the P-transformation, the transformation which exchanges left and right. To see, what is surprising about this, let us go back in time to the last century.

In the first half of the 20th century most physicist thought, that all fundamental processes are symmetric under the P-transformation: left behaves the same as right. There is no way to distinguish the two from each other. A revolution came in the late 50's, when P-violation was found in an experiment, which was studying the weak interaction: a certain weak process happened many more times than its mirror image process. It turned out, that Nature does actually make a difference between left and right. The result was shocking for most physicists at that time. While many of them were still recovering, Pauli, one of the most brilliant among his contemporaries, immediately recognized, that the real problem, that has to be explained is:

> I am not so shocked about the fact, that GOD is left handed, but much more that as a left-handed he appears to be symmetric in his strong actions. [...] Why is the strong interaction left-right symmetric?[1]

Later Pauli's remark was laid on solid theoretical foundations. The theory of the strong interaction was developed, it is called Quantum-chromodynamics (QCD). It describes how quarks and gluons, the constituents of protons and neutrons, interact. It is possible to introduce a parameter into QCD, which violates the P-symmetry, it is usually called θ [6]. A-priori the value of θ can be any number, like many other fundamental parameters in particle physics. However experimentally it is found to be consistent with zero, with an extremely

[1] "Ich bin nicht so sehr durch die Tatsache erschüttert, dass der HERR die linke Hand vorzieht, als vielmehr durch die Tatsache, dass er als Linkshänder weiterhin symmetrisch erscheint, wenn er sich kräftig ausdrückt. Kurzum, das eigentliche Problem scheint jetzt in der Frage zu liegen: Warum sind starke Wechselwirkungen linksrechts symmetrisch?" See eg. in Martin Gardner: The Ambidextrous Universe, 1967.

good precision. The current bound on θ is $\theta \leq 10^{-10}$[7], thus QCD is P-symmetric to a very good approximation. Such a fine-tuning begs for an explanation.

A nice explanation was given by Peccei and Quinn [8]: instead of considering θ as a fixed parameter, they suggested to treat it as a dynamical field, whose value can change with space and time. Now, if the potential of this field is such that it has a minimum at $\theta = 0$, then the θ field evolves in time and relaxes to this minimal value, and effectively explains, why θ is small. Theorists have come up with all sorts of possible potentials, that have their minima at $\theta = 0$. The immediate consequence is, the existence of a very weakly interacting particle: the axion [9, 10]. Whether this is the correct explanation, and whether axions exist is not known. There are several experiments around the world looking for them, none of them successful yet.

It was realized soon after the proposal, that since axions couple weakly to ordinary matter, they are perfect candidates for Dark Matter. Even if they interact weakly, they could be produced in sufficient amount during the Big-Bang. Assuming that axions are the only source of Dark Matter, one can calculate their mass. For this we have to know how the axion potential looks like today and how it looked like during the Big-Bang.

In this project we calculated the axion potential from QCD for the whole history of our Universe. From this we gave an estimate on the axion's mass [11]. This can help to design future experiments looking for these particles.

Objectives, challenges and methods

Our goal was to calculate the axion potential for the whole history of our Universe. At early times the Universe was much hotter than now, and with expanding it cooled down rapidly to reach its current temperature today (0.235 meV). So what we needed is the temperature dependence of the axion potential. For the simplest axion model, the potential only receives contributions from QCD. To compute this, the equations of QCD had to be solved. As QCD is a highly non-linear theory and its coupling constant is not particularly small, a non-perturbative technique is required to work out its properties, for which we used the lattice discretization of QCD.

In performing these computations we faced two challenges. The first one is an algorithmic issue. Determination of the axion potential using a standard lattice QCD algorithm is analogous to the following simple problem: one has to determine the ratio of red and blue balls in a black bag, by randomly picking balls from the bag. See Figure 1/a for an illustration. The ratio is essentially the curvature of the axion potential. For small temperatures there are similar number of red and blue balls in the bag, by picking a few hundred one can give a very good estimate on the ratio. However as the temperature is increased, the ratio drops rapidly and one needs more and more random picks, which costs more and more CPU time. To calculate the potential

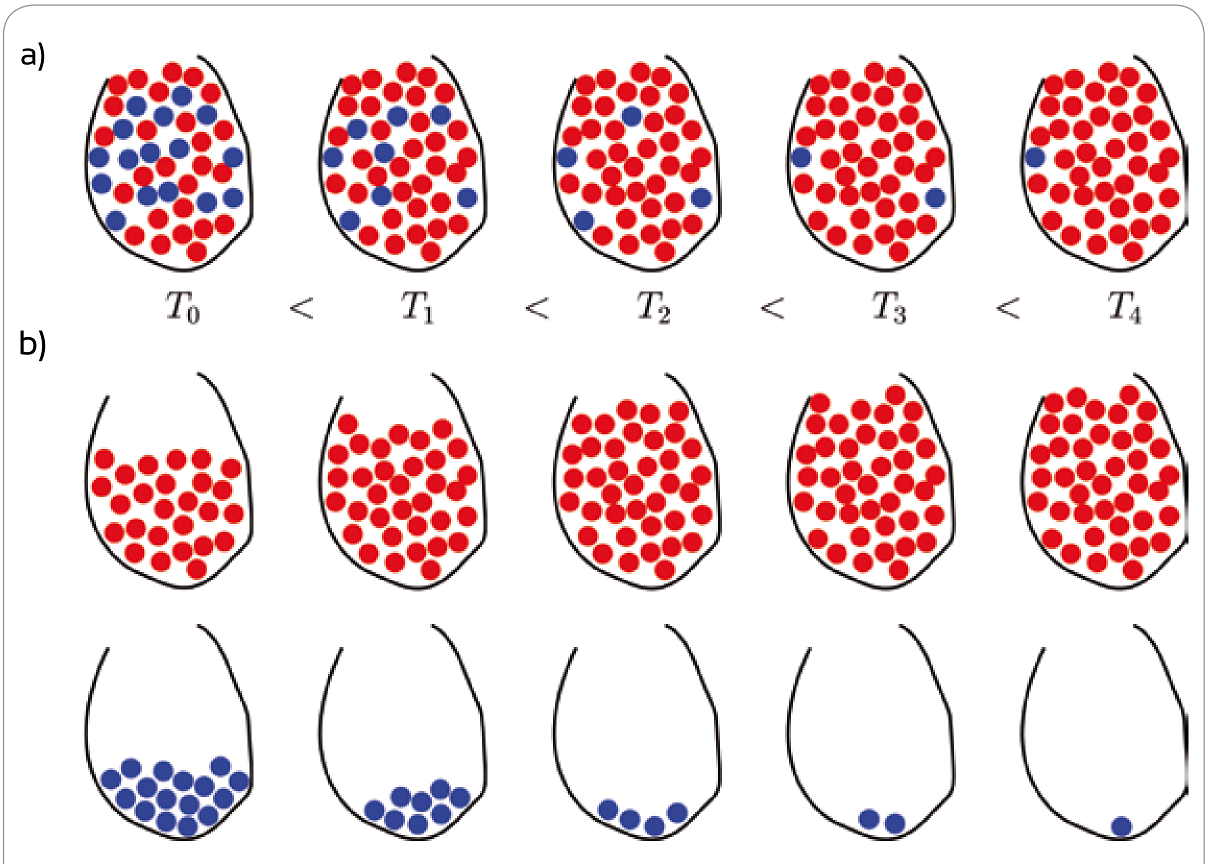

Fig. 1: Explanation of the standard simulation strategy (a) and our newly proposed procedure (b) to measure the axion potential for high temperatures. See the text for details.

using this standard approach in the interesting temperature region (~ 2 GeV) one needs about 10^{10} years of computational time even on a supercomputer.

We came up with an alternative procedure, see Figure 1/b. First at a relatively small temperature, let us call it T_0, where the standard approach was still feasible, we measured the ratio in the usual way. Then for higher temperatures we separated the balls into two bags, and carried out simulations with either only red or only blue balls. Then we have measured, how the number of blue/red balls were changing as the temperature was increased. Using these temperature differences plus the starting value of the ratio at T_0 we could then calculate the ratio at higher temperatures.

Another challenge was related to the large discretization artefacts in the axion potential. These artefacts are on the 10% level in typical lattice QCD simulations, and can be get rid of by performing the so-called continuum extrapolation procedure. Here one takes lattices with smaller and smaller lattice spacing and performs an extrapolation to the continuum limit. The axion potential turned out to have much larger errors, and the continuum extrapolation to be much more difficult than usual. Here we also designed a new procedure, to get rid of the large discretization artefacts and demonstrated in several cases the effectiveness of the procedure.

Beside the axion potential one also needs equations governing the expansion of the Universe to determine the mass of the axion. The expansion is governed by the visible matter content, for which the thermodynamical properties, pressure and energy density, had to be determined. In our paper we also calculated the QCD component of these. Here the challenge was to go up to a sufficiently high temperature, where the non-perturbative lattice QCD results could be connected to perturbation theory. We were able to reach a temperature of 1 GeV, after which we smoothly connected to known thermodynamics results from perturbation theory.

Results and outlook

In Figure 2 and Figure 3 we show the two main results of our work [11]. The first is the temperature dependence of the curvature of the axion potential. This is the first determination of this quantity from first principles in a range of temperatures relevant for axion cosmology with control over all errors. We extended significantly the reach of previous lattice determinations, and managed to give a result where all systematic errors were estimated. From the second plot one can read off the energy density and pressure of our Universe in a temperature range of five orders of magnitude.

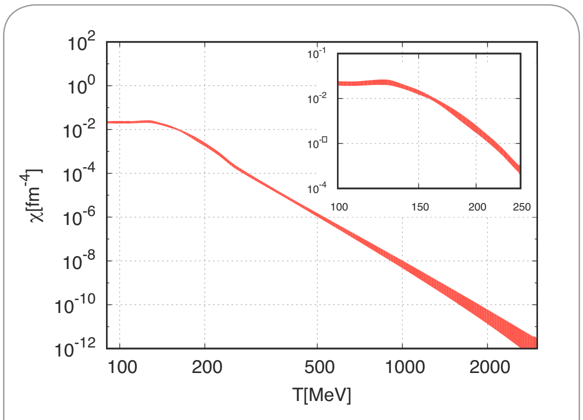

Fig. 2: The curvature of the axion potential as a function of the temperature.

Combining these two results one obtains the mass of the axion: mA = 28(1)μeV. This number assumes, that all Dark Matter is made of axions and also assumes the simplest cosmological production scenario. It is important to mention, that there exist more complicated axion production scenarios and also more complicated axion like particle models. The current best estimation of these scenarios increases the axion mass together with its uncertainty considerably: 50μeV ≤ mA ≤ 1500μeV. It is an important, though very non-trivial task to decrease the size of these uncertainties in the future.

The resulting value for the mass is an important hint for experimentalists how to design experiments looking for axion particles in the near future. If the experimental search succeeded, the axion would be the first confirmed constituent of Dark Matter and an evidence for physics of an unknown world.

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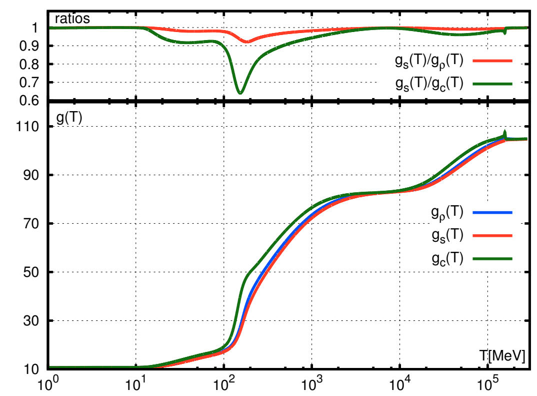

Fig. 3: Thermodynamical properties (energy, entropy density and speed of sound) of our Universe as the function of the temperature.

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