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Towards smoke and fire simulation with grid adaptive FEM:

Verification of the flow solver.

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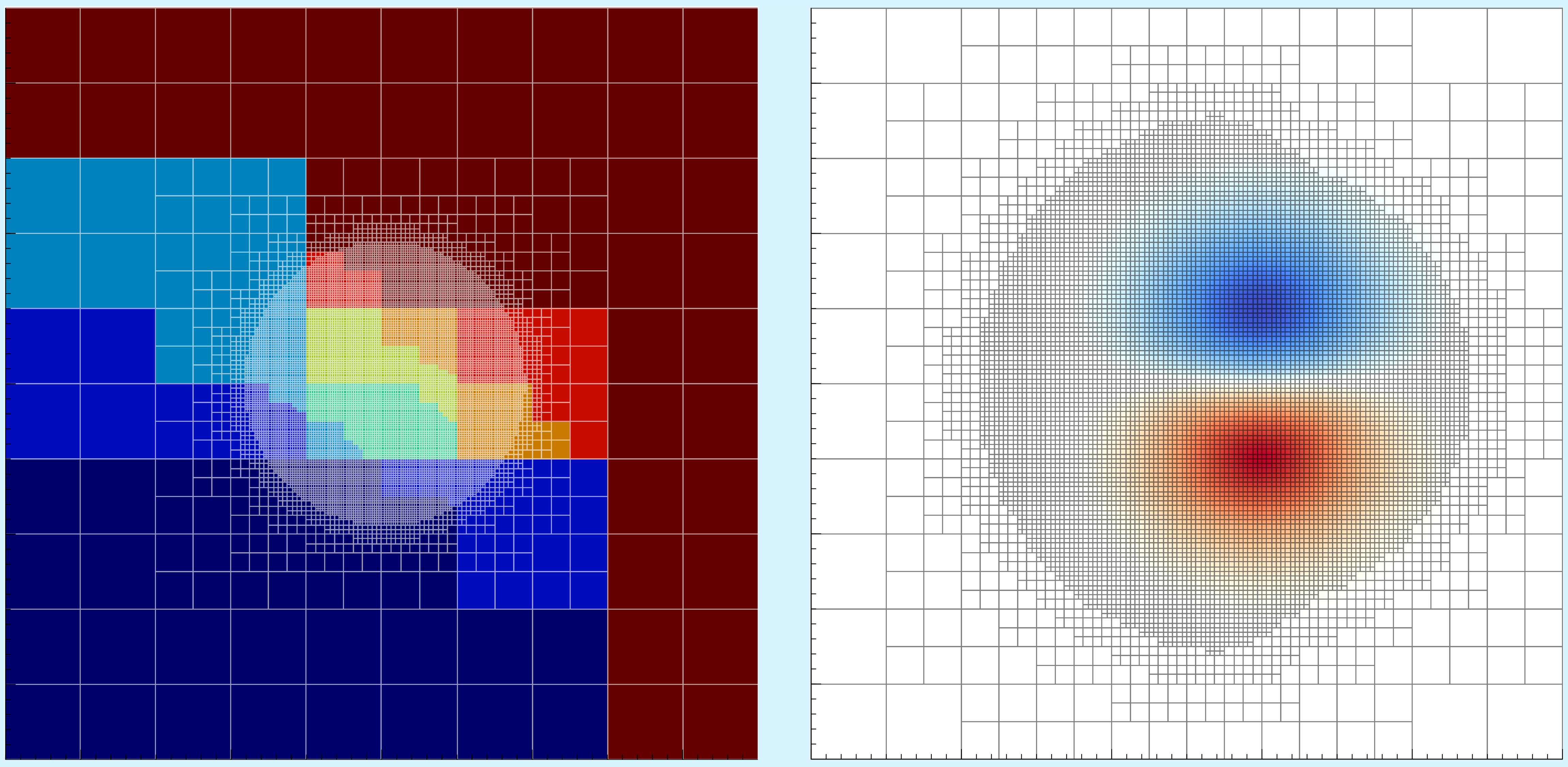


Figure: Demonstration of adaptive mesh refinement using the Gould vortex example [1]. The mesh of corresponding simulation runs are displayed a few steps after initialization along with the decomposed domain, here on eight cores, (left) and the smaller region of interest with the velocity magnitude a few steps later (right).

In fire safety science and engineering, the Fire Dynamics Simulator (FDS, [1]) is a well established tool, which encounters limits at complex geometries and large–scale problems because of static, structured meshes.

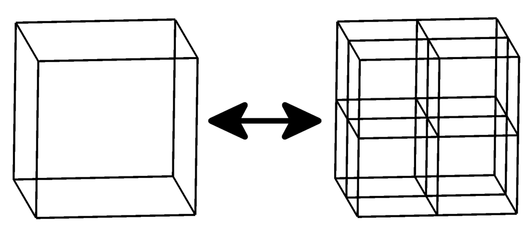
To overcome these, a new approach using the Finite Element Method (FEM) with unstructured meshes and adaptive methods with a dynamic mesh resolution will be presented.

The underlying model is presented along with its numerical implementation. First attempts in the verification of the flow solver are presented and compared to FDS. A demonstration of adaptive mesh refinement and the scaling on the JURECA [2] supercomputer is shown.

Adaptive Mesh Refinement

Solutions of PDEs in FEM require

- domain decomposition in cells of length h
- shape functions of polynomial degree p



Adjustment of parameters locally where necessary as compromise between accuracy and runtime. Our refinement criterion:

- Reference: Norm of velocity gradient $||\nabla \mathbf{u}||$.
- Flag top/lower 30% for refinement/coarsening at each step.

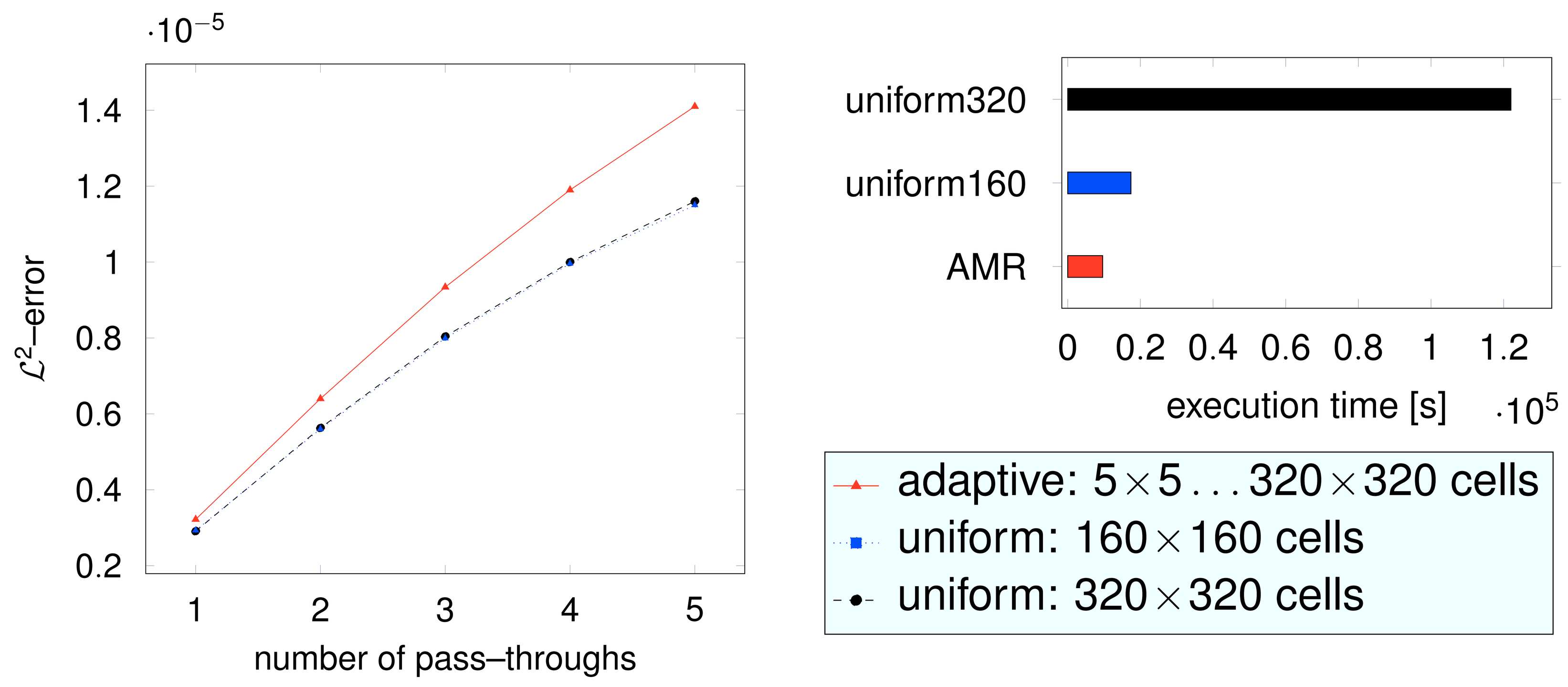


Figure: Demonstration of adaptive mesh refinement using the Gould vortex example [1]. Global $\mathcal{L}^2$–errors at each periodic pass–through (left). Execution time of the simulation, run in serial on a common desktop computer (right).

Model equations

Incompressible Navier–Stokes equations ($\mathrm{Ma} < 0.3$) for smoke propagation with buoyancy force following Boussinesq:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \rho_0^{-1} \nabla p - \nabla \left(2\,\nu\,\epsilon_{ij}(\mathbf{u})\right) = \beta \Delta T \mathbf{g} \quad (2)$$

$$\partial_t T + (\mathbf{u} \cdot \nabla) T - 2\, c_{\mathrm{p}}^{-1}\, \nu\, \epsilon_{ij}(\mathbf{u}) : \nabla \mathbf{u} - \nabla \cdot \left(\mathrm{Pr}^{-1} \nu\, \nabla T\right) = \gamma \quad (3)$$

Turbulence modeling with Smagorinsky–Lilly:

$$\nu = \nu_{mol} + \nu_{turb} \qquad \text{with} \qquad \nu_{turb} = (C_s h)^2 \left|\left|\epsilon_{ij}(\mathbf{u})\right|\right|_2 \quad (4)$$

with strain rate tensor $\epsilon_{ij}(\mathbf{u}) = \frac{1}{2}\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathrm{T}}\right]$.

Numerical implementation with semi–implicit time marching scheme of first order and FEM for space discretization of arbitrary order using the deal.II library [3].

Verification

Determination of convergence order with Richardson extrapolation [1]:

$$\ln\left(\frac{f_3 - f_2}{f_2 - f_1}\right) = p \ln(r) \quad (5)$$

In FDS, final velocity u_x in the domain center is chosen as the reference f. For comparison with FEM however, we need to take into account the final $\mathcal{L}^2$–error according to the Bramble–Hilbert lemma:

$$||\mathbf{u} - \mathbf{u}_h||_{\mathcal{L}^2} \leq C\, h^{k+1} \left(||\mathbf{u}||_{\mathcal{H}^{k+1}} + ||p||_{\mathcal{H}^k}\right) \quad (6)$$

For the convergence analysis, the McDermott testcase [1] is chosen.

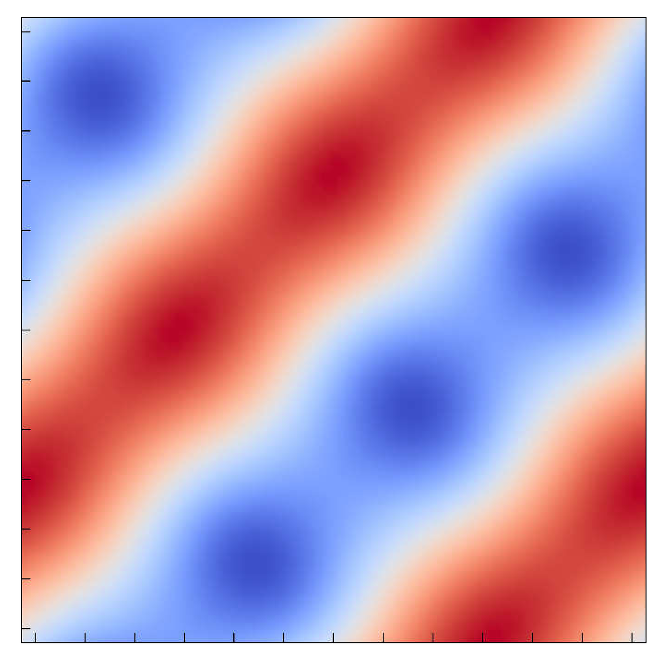


Figure: Velocity magnitude of the stationary McDermott solution [1].

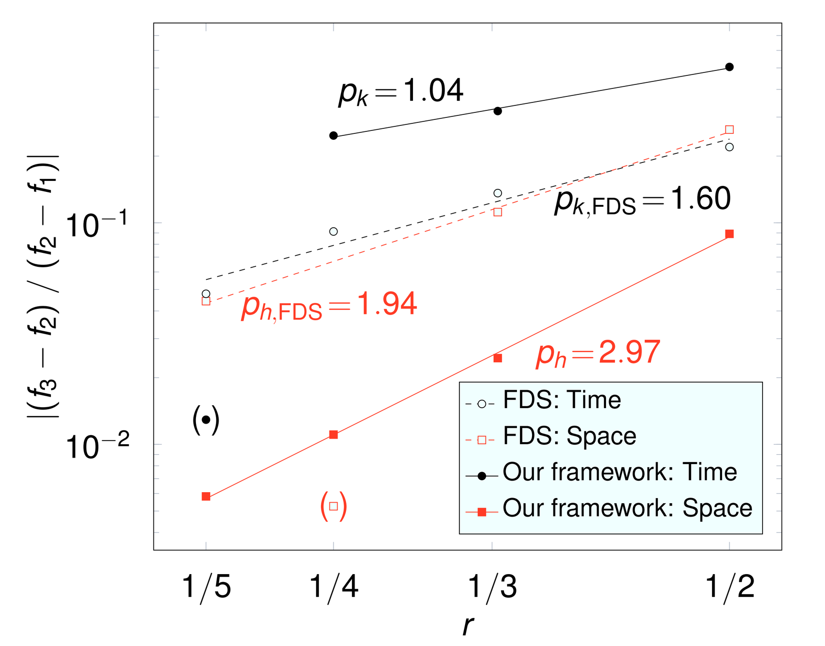


Figure: Time and space conv. plots. For FEM, lowest order Taylor–Hood elements were used, thus expecting space conv. order of 3. To consider different total amounts of time steps, the FDS velocities are time–averaged for time conv.

Scaling

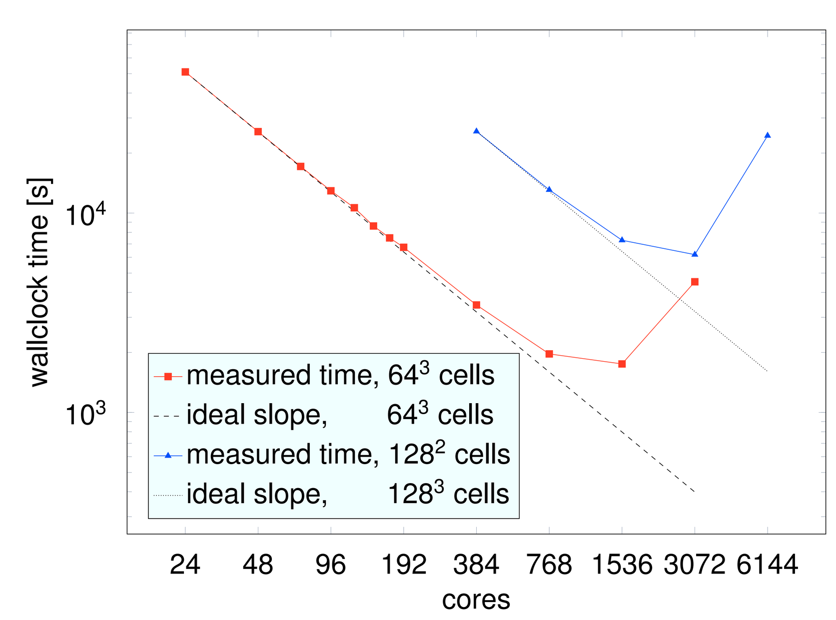


Figure: Strong scaling on JURECA [2] for a uniform mesh in 3D.

References

[1] K. McGrattan et. al., Fire Dynamics Simulator Technical Reference Guide Volume 2: Verification, NIST Special Publication, 1018–2.

[2] JURECA: General–purpose supercomputer at Jülich Supercomputing Centre. Journal of large–scale research facilities, 2:A62.

[3] W. Bangerth et. al., deal.II – a General Purpose Object Oriented Finite Element Library, ACM Trans. Math. Softw., 33(4):24/1–24/27.

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