Simulation of gate-based quantum computers with superconducting qubits

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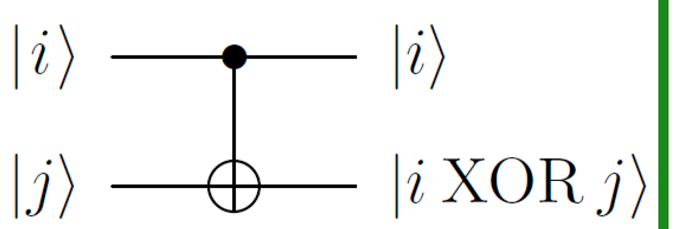
Gate-based quantum computing

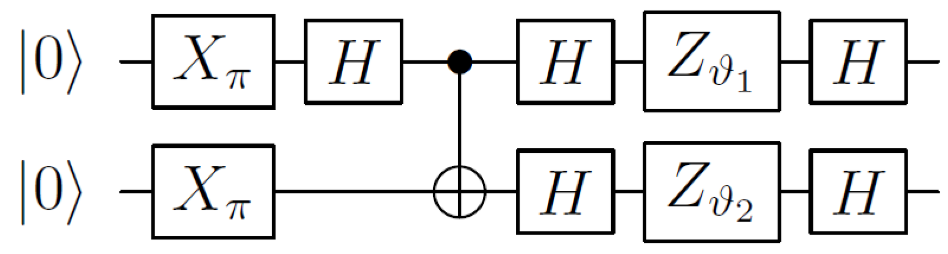
A quantum computer contains a set of two-level systems called qubits. Each qubit can be a complex superposition of the computational states $|0\rangle$ and $|1\rangle$. In the gate-based model of quantum computing, gates transform the qubits at each step. Examples for single-qubit gates are:

$\|0\rangle$	X_π	$\|1\rangle$	$\|1\rangle$	X_π	$\|0\rangle$
$\|0\rangle$	$X_{\pi/2}$	$\frac{\|0\rangle - i\|1\rangle}{\sqrt{2}}$	$\|1\rangle$	$X_{\pi/2}$	$\frac{-i\|0\rangle + \|1\rangle}{\sqrt{2}}$
$\|0\rangle$	Z_ϑ	$\|0\rangle$	$\|1\rangle$	Z_ϑ	$e^{i\vartheta}\|1\rangle$
$\|0\rangle$	H	$\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$	$\|1\rangle$	H	$\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$

A two-qubit gate such as the controlled-NOT (CNOT) is a conditional operation to entangle two qubits.



Quantum circuits consist of sequences of quantum gates.



All measurements of the quantum system ultimately produce a bit string by projecting each qubit to $|0\rangle$ or $|1\rangle$.

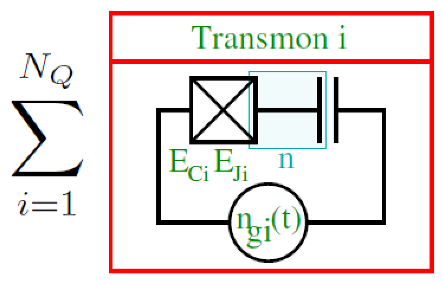
Superconducting qubit architecture

The architecture of the simulated quantum computer is defined by the system Hamiltonian

$$H = H_{\mathrm{CPB}} + H_{\mathrm{Res}} + H_{\mathrm{CC}}$$

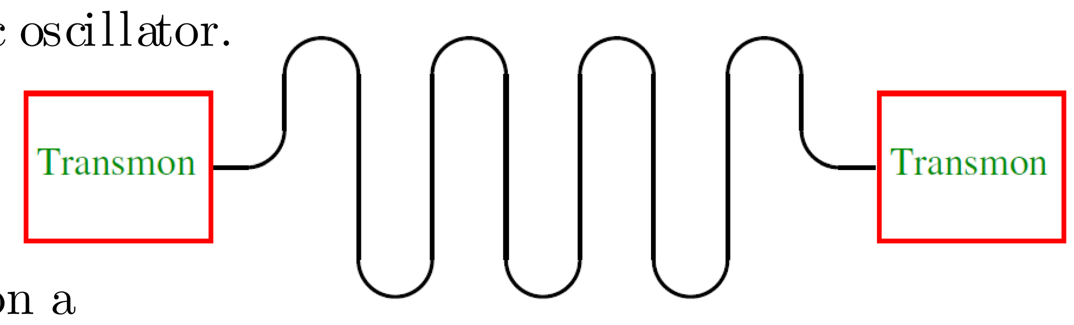
The qubits are given by the lowest eigenstates of Cooper Pair Boxes (CPBs) in the transmon regime [1].

$$H_{\mathrm{CPB}} = \sum_{i=1}^{N_Q} \left[E_{Ci}(\hat{n}_i - n_{gi}(t))^2 - E_{Ji}\cos\hat{\varphi}_i\right]$$



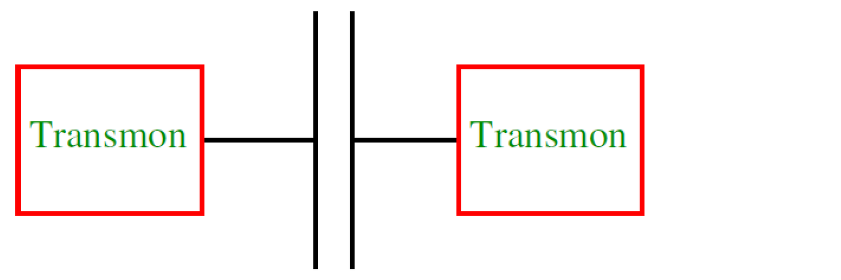
One way of coupling transmons is based on a transmission line resonator, modeled as a harmonic oscillator.

$$H_{\mathrm{Res}} = \omega_r \hat{a}^\dagger \hat{a} + \sum_{i=1}^{N_Q} g_i \hat{n}_i (\hat{a} + \hat{a}^\dagger)$$



Another way of coupling transmons is based on a capacitive electrostatic interaction.

$$H_{\mathrm{CC}} = \sum_{1\le i<j\le N_Q} E_{Ci,Cj}(\hat{n}_i - n^L_{Ci,Cj})(\hat{n}_j - n^R_{Ci,Cj})$$



Quantum gates are implemented by microwave voltage pulses.

$$n_{gi}(t) = \sum_j \Omega_{ij}(t)\cos(\omega_{ij}t - \gamma_{ij})$$

Transmon	$E_{Ci}/2\pi$	$E_{Ji}/2\pi$	$\omega_i/2\pi$	$\omega_r/2\pi$	$g_i/2\pi$
1	1.204	13.349	5.350	7	0.07
2	1.204	12.292	5.120	7	0.07

Simulation method

The time-dependent Schrödinger equation (TDSE)

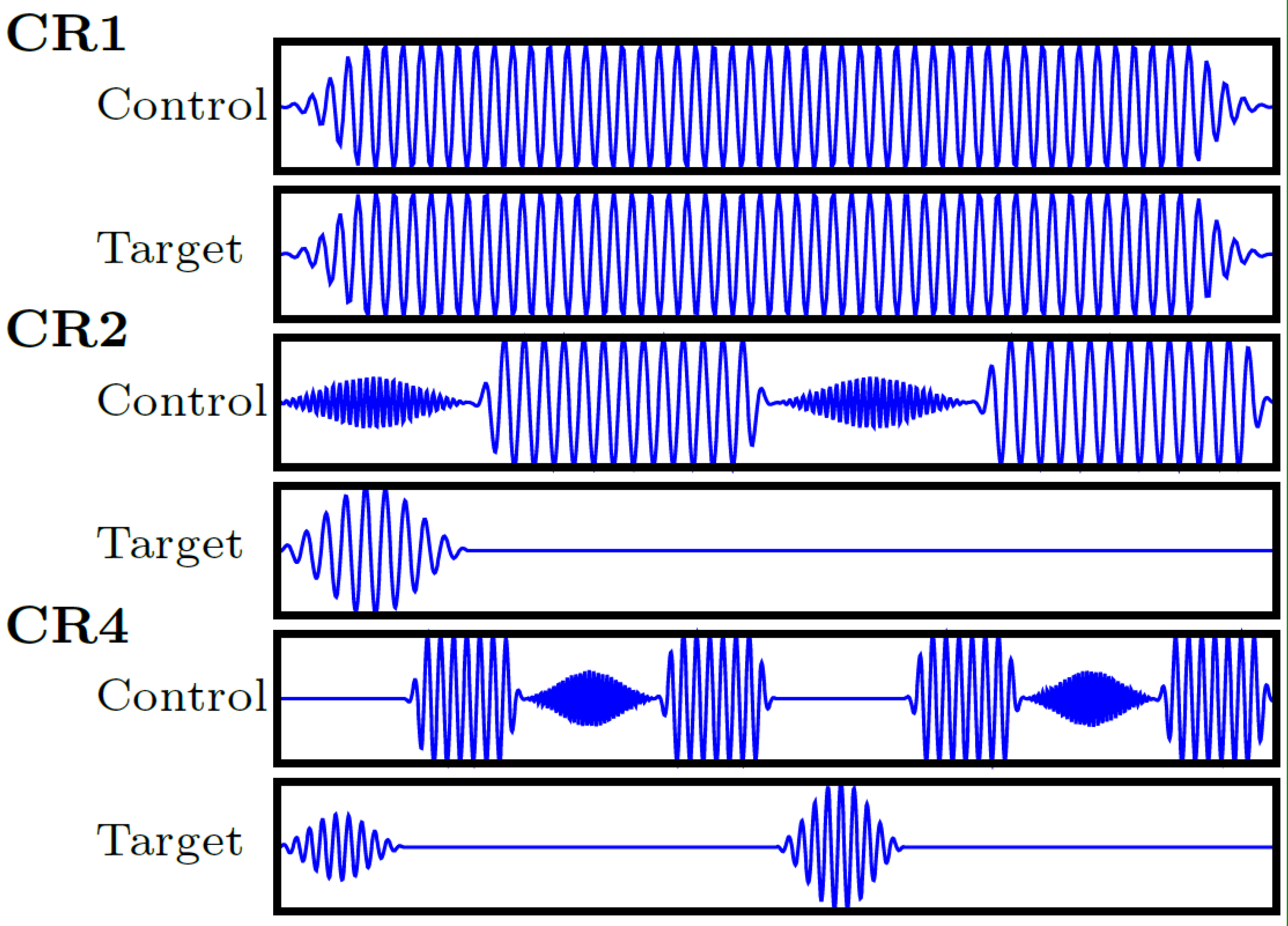
$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$$

is solved numerically using a Suzuki-Trotter product-formula algorithm [2] for the time-evolution operator

$$\mathcal{U}(\tau) = e^{-i\tau(H_1+\ldots+H_K)}$$
$$\approx e^{-i\tau H_1}\cdots e^{-i\tau H_K}$$

The goal is to find a pulse $n_{gi}(t)$ so that $\mathcal{U}(t)$ implements a certain quantum gate on the qubits. We use the Nelder-Mead algorithm to optimize the parameters of the pulse.

The CNOT gate is implemented in three different versions based on cross-resonance (CR) pulses [3].



Gate error metrics

Projection of the time-evolution operator $\mathcal{U}(t)$ on the qubit subspace gives the matrix M. Ideally, this matrix should be equal to the unitary quantum gate U.

$$\mathcal{G}_{ac}(|\psi\rangle\langle\psi|) = M\,|\psi\rangle\langle\psi|\,M^\dagger$$
$$\mathcal{G}_{id}(|\psi\rangle\langle\psi|) = U\,|\psi\rangle\langle\psi|\,U^\dagger$$

Average gate fidelity [4]

$$F_{\mathrm{avg}} = \int \mathrm{d}|\psi\rangle\ \langle\psi|\,\mathcal{G}_{ac}(\mathcal{G}_{id}^{-1}(|\psi\rangle\langle\psi|))\,|\psi\rangle$$

Diamond error rate [5]

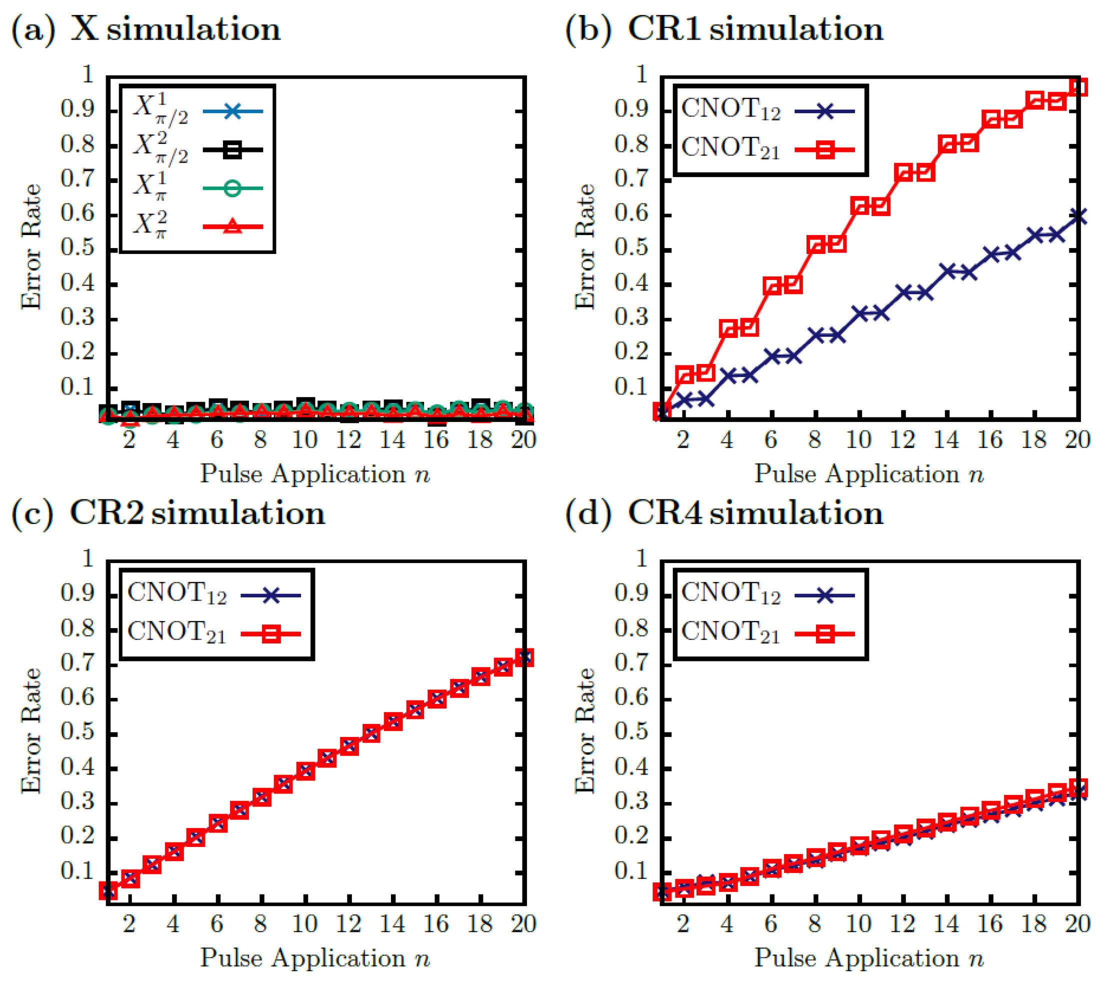
$$\eta_\diamond = \frac{1}{2}\left\|\mathcal{G}_{ac}\circ\mathcal{G}_{id}^{-1} - \mathbb{1}\right\|_\diamond$$

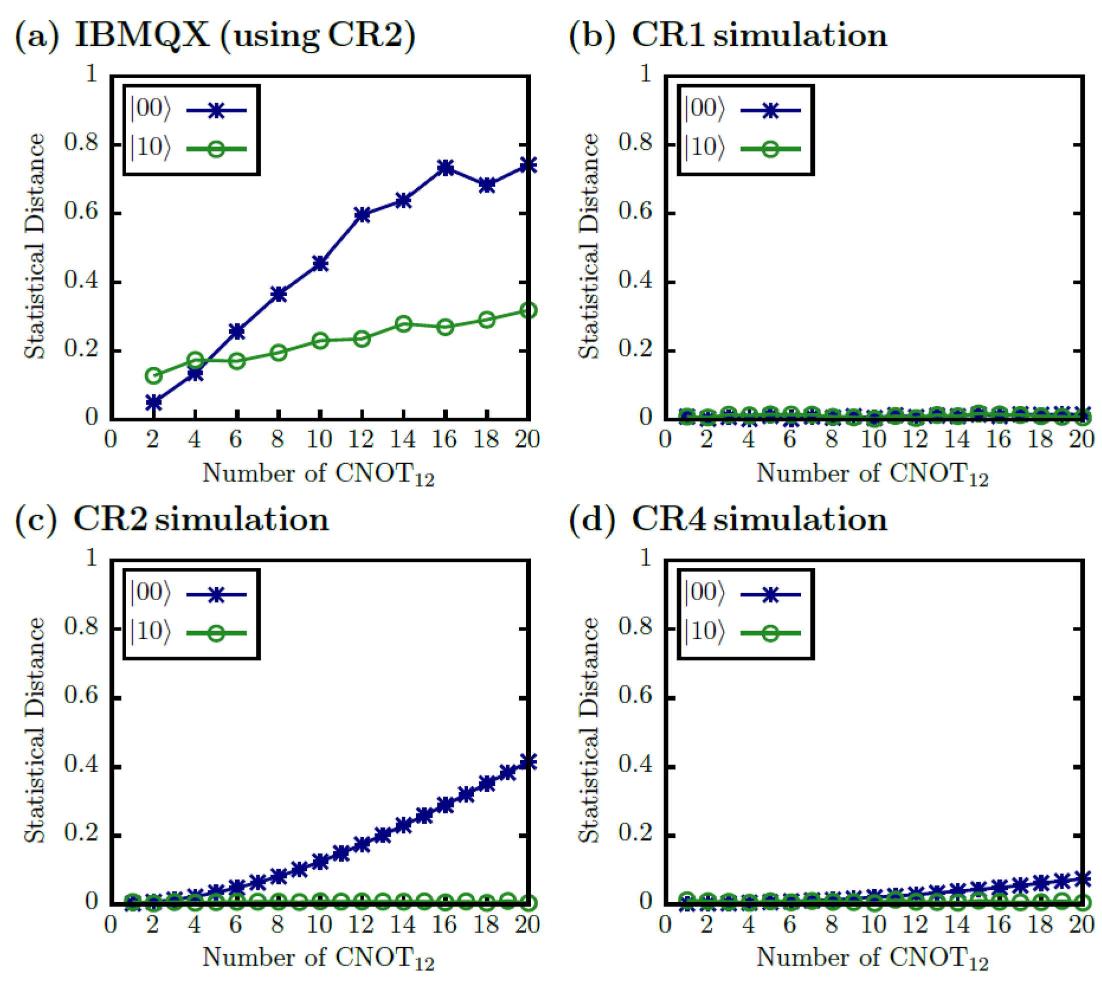
Unitarity [6]

$$u = \frac{d}{d-1}\int \mathrm{d}|\psi\rangle\ \mathrm{Tr}\left[\mathcal{G}'_{ac}(|\psi\rangle\langle\psi|)^\dagger\mathcal{G}'_{ac}(|\psi\rangle\langle\psi|)\right]$$

Simulation results

Gate	F_{avg}	$\eta_\diamond$	u
$X^1_{\pi/2}$	0.9946	0.027	0.990
$X^2_{\pi/2}$	0.9942	0.028	0.989
X^1_{π}	0.9949	0.020	0.990
X^2_{π}	0.9943	0.023	0.989
CR1$_{12}$	0.9842	0.029	0.969
CR1$_{21}$	0.9951	0.033	0.991
CR2$_{12}$	0.9943	0.048	0.991
CR2$_{21}$	0.9947	0.048	0.992
CR4$_{12}$	0.9934	0.049	0.989
CR4$_{21}$	0.9946	0.044	0.991





Conclusion: The gate metrics of the optimized pulses are nearly perfect and agree with experimental achievements [3]. However, in repeated applications or actual quantum circuits, the gates suffer from systematic errors. These can be observed in experiments [7,8]. Conceptually, the errors stem from the non-computational states. Although the gate metrics reveal them, they cannot replace the information of how well and how often a certain gate may be used in a quantum circuit. As this information is most important to eventual users of gate-based quantum computers, it should be reported separately.

References:

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