Conformal Invariance in Driven Diffusive Systems at High Currents

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We consider space-time correlations in driven diffusive systems which undergo a fluctuation into a regime with an atypically large current or dynamical activity. For a single conserved mass we show that the spatiotemporal density correlations in one space dimension are fully determined by conformal field theory with central charge $c = 1$, corresponding to a ballistic universality class with dynamical exponent $z = 1$. The full phase diagram for general atypical behavior exhibits the conformally invariant regime and, for atypically low current or activity, a region of phase separation. The phase transition line between these two regimes corresponds to typical behavior and the dynamics belongs to the Kardar-Parisi-Zhang universality class with dynamical exponent $z = 3/2$, except for a diffusive point with $z = 2$. The exact universal dynamical structure function is obtained in explicit form from the one-dimensional asymmetric simple exclusion process with periodic and open boundaries in the limit of maximal current.

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Introduction.—An intriguing question is whether a many-body system far from thermal equilibrium that undergoes some big atypical fluctuation can be understood in terms of an upscaled description of "normal" spatiotemporal behavior or whether it behaves in a qualitatively different fashion during this atypical fluctuation. The statistical properties of such fluctuations are also the fundamental object of interest in large deviation theory which provides—in equilibrium—the foundations of thermodynamics in terms of thermodynamic ensembles. Of course, one does not expect a unique answer to a question posed so generally, but some insight into large fluctuations without external trigger may be gained from considering a reasonably wide, but still well-defined class of model systems, viz. stochastic lattice gas models that have served as paradigmatic models for nonequilibrium phenomena in the last decades [1–4].

These models have a stochastic particle hopping dynamics that mimics noise, interactions between particles frequently include a hard-core repulsion, and a bulk driving field or boundary gradients maintain a fluctuating nonequilibrium steady state with a nonzero locally conserved mass current. The basic example is the asymmetric simple exclusion process (ASEP) in one dimension where lattice sites can be occupied by at most one particle. They jump randomly to their nearest neighbor sites, provided the target site is empty. Biased jump rates $w_0 e^f$ to the right and $w_0 e^{-f}$ to the left and/or open boundaries, where particle exchanges with reservoirs take place, create a nonequilibrium situation.

The fundamental question is whether in a driven lattice gas the spatiotemporal fluctuation patterns remain essentially unchanged during an untypical large fluctuation of the current (or more generally of the undirected jump activity of the particles), or whether correlations during such a fluctuation are qualitatively different from typical behavior.

The answer that we shall give is, in a nutshell, the following. (I) If a system with hard-core repulsion that is typically in a stationary state of spatially homogeneous density undergoes a large fluctuation into a regime of *low* current or jump activity then this is most likely realized by phase separation into domains of high and low density, respectively, and not by fluctuations in the random time at which particles attempt to jump (upscaling), as is the case, e.g., for simple random walkers. (II) Conversely, during a fluctuation into a regime of *high* current or jump activity, the homogeneous stationary density is maintained, but a qualitative change of density correlations typically realizes the atypical large fluctuation [5]. Going further, we assert that these space-time correlations are universal and can be predicted for nonequilibrium particle systems in one space dimension from conformal field theory for two-dimensional equilibrium critical phenomena [6,7].

The first answer can be understood by noting that in a region of high density fewer jumps will occur naturally due to the repulsive interaction, while in a low-density region fewer jumps occur trivially because of the smaller number of particles, thus optimizing the probability for such untypical behavior. This picture is well borne out by the powerful machinery of Macroscopic Fluctuation Theory [8], which demonstrates for the ASEP that phase separation sets in below some critical atypical current [9–13]. Such a dynamical phase transition was found also for an atypically low activity in the symmetric simple exclusion process (SSEP) which has no hopping bias [14].

On the contrary, for the until now poorly understood fluctuations with a large current or activity, such phase separation would be counterproductive and one expects a homogeneous bulk density as in the typical steady state. It is aim of this work to establish that in the stationary regime of any higher-than-typical current or activity one can go much beyond macroscopic fluctuation theory: In $1+1$ dimensions *all* spatiotemporal correlations are fully described by conformal field theory (CFT).

In particular, we predict that in a translation invariant setting with a single locally conserved current the dynamical structure function $S(k,l,t) = \langle (n_k(t) - \rho_k)(n_l(t) - \rho_l)\rangle$, i.e., the stationary space-time correlations of the local particle numbers $n_k(t)$ with local average density ρ_k, has the universal scaling form

$$S(x,t) \approx \frac{C_1}{v_L^2 t^2} \frac{1-\xi^2}{(1+\xi^2)^2} + \frac{C_2}{(v_L t)^{2K}} \frac{\cos[2(q^* x - \omega t)]}{(1+\xi^2)^K}, \quad (1)$$

with $x = k - l$, and the scaling variable $\xi = (x - v_c t)/(v_L t)$ indicating a dynamical exponent $z = 1$ rather than 2 or 3/2 for typical dynamics, and nonuniversal amplitudes C_i. The static critical exponent $K \geq 1$ depends on the particle interactions. The collective velocity v_c, the Luttinger liquid time scale v_L, the wave vector q^*, and the oscillation frequency ω can be computed from the generally complex dispersion relation as discussed below. The cosine term is reminiscent of similar behavior for fermions in one dimension and it is related to low-energy processes that connect, close to the Fermi level, right movers with left movers [15]. Higher-order correlations are fully determined by a modified CFT with central charge $c = 1$ of the Virasoro algebra, the difference to usual conformal invariance being the appearance of the collective velocity v_c in the Galilei shift of the space coordinate x and the time dependence of the oscillating part of the correlation function with frequency ω. Open boundaries are described by a corresponding boundary CFT.

Dynamical structure function in the conditioned ASEP.—Working in the ensemble where λ is conjugate to the current j and μ is conjugate to the activity a on which we condition, see Refs. [16,17], we prove these assertions for the ASEP [Fig. 1] in the limiting case of maximal current. For this case, the dynamical structure has been computed earlier for periodic conditions [18,19], but the oscillating contribution in Eq. (1) went unnoticed. Here we extract this oscillating part from the exact expression for finite time and finite lattice and also compute the dynamical structure function for open boundary conditions, which has not been considered before. We employ finite-size-scaling theory to identify the central charge and some critical exponents that characterize the associated CFT.

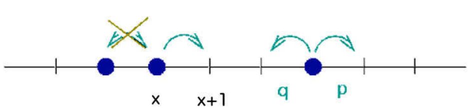

FIG. 1. Schematic representation of the asymmetric simple exclusion process. A particle hops to the neighboring site provided this target site is empty. In the regime of maximal current the jumps to the left do not contribute to the statistical properties of the conditioned process.

The starting point of our analysis is the exact mapping of the dynamics of the ASEP conditioned on an atypical current or hopping activity to a non-Hermitian Heisenberg spin-1/2 quantum chain in the ferromagnetic range

$$\tilde{H}(f,\lambda,\mu) = -\frac{1}{4}\sum_{k=1}^{L}(\tilde{g}_k - \Delta), \quad (2)$$

with (arbitrary) time scale $w_0^{-1} = 2e^{\mu}\cosh(f+\lambda)$ and

$$\tilde{g}_k = 2(1+\nu)\sigma_k^+\sigma_{k+1}^- + 2(1-\nu)\sigma_k^-\sigma_{k+1}^+ + \Delta\sigma_k^z\sigma_{k+1}^z. \quad (3)$$

The anisotropy parameter

$$\Delta = \frac{e^{-\mu}\cosh f}{\cosh(f+\lambda)} = \cos\left(\frac{\pi}{2K}\right) \geq 0 \quad (4)$$

fixes the Luttinger parameter K in Eq. (1) and $\nu = \tanh(f+\lambda)$ leads to an imaginary Dzyaloshinskii-Moriya interaction [20]. The case $\lambda = \mu = 0$ corresponds to typical dynamics with stationary current $j = \nu\rho(1-\rho)$ and activity $a = \rho(1-\rho)$ for particle density $\rho = N/L$. Positive λ and μ correspond to enhanced current and activity, respectively. The large deviation function is the lowest eigenvalue per site $\epsilon_0(f,\lambda,\mu)$ of Eq. (2).

In order to show that the non-Hermitian terms in Eq. (3) lead to the modifications of CFT described above we focus on maximal current $\lambda \to \infty$, leading to

$$H = -\sum_{k=1}^{L}\sigma_k^+\sigma_{k+1}^- \quad (5)$$

for periodic boundary conditions with L sites. Following Refs. [18,21], the maximal current is realized by two mechanisms: The trivial speed-up of the jump frequency (which is absorbed in the time scale w_0) and a nontrivial building up of correlations captured by the dynamical structure function.

In order to compute the dynamical structure function, including the oscillating part missing in Refs. [18,19], and uncover other hallmarks of conformal invariance, we take the standard approach by diagonalizing Eq. (5) in terms of Jordan-Wigner fermionic operators [22]. Fourier transformation with momentum p then yields

$$H = \sum_{p=1}^{L}(\mathcal{P}^-\epsilon_p\hat{c}_p^\dagger\hat{c}_p + \mathcal{P}^+\epsilon_{p-\frac{1}{2}}\hat{c}^\dagger_{p-\frac{1}{2}}\hat{c}_{p-\frac{1}{2}}) \quad (6)$$

in the graded Fock space built by the free fermion creation operators $\hat{c}_p^\dagger$ and annihilation operators $\hat{c}_p$ in the even particle sector and $\hat{c}^\dagger_{p-1/2}$, $\hat{c}_{p-1/2}$ in the odd sector. The single-particle energy is

$$\epsilon_p := -e^{-(2\pi i p/L)}. \tag{7}$$

Remarkably, the ground state for $0 \le N \le [L/2]$ particles, which is defined by having the lowest real part of the eigenvalues of Eq. (6), is the same as for the Hermitian case with ground state energy per site given by

$$-e_0 = \frac{v_F}{L \sin(\pi/L)} = v_F\left(\frac{1}{\pi} + \frac{\pi}{6L^2} + O(L^{-4})\right), \tag{8}$$

where $v_F = \sin(\pi\rho)$ is the Fermi velocity. The energy gaps, however, are in general complex. The lowest gap (with smallest real part) is given by $\delta = \epsilon_{(N+1)/2} - \epsilon_{(N-1)/2} = 2\sin(\pi/L)[\sin(\pi\rho) + i\cos(\pi\rho)]$ with real part.

$$\Re(\delta) = v_F \frac{2\pi}{L}\left[1 + O(L^{-2})\right]. \tag{9}$$

For the dynamical structure function at density ρ the Wick theorem gives

$$S_\rho(n,m,t) = \langle c_n^\dagger(t) c_m(0)\rangle\langle c_n(t) c_m^\dagger(0)\rangle - \langle c_n^\dagger(t) c_m^\dagger(0)\rangle\langle c_n(t) c_m(0)\rangle, \tag{10}$$

where the second part vanishes in the periodic system due to particle number conservation. The time-dependent operators are defined by $X(t) := e^{Ht} X e^{-Ht}$, which leads to $\hat{c}_p(t) = e^{-\epsilon_p t}\hat{c}_p$ and $\hat{c}_p^\dagger(t) = e^{\epsilon_p t}\hat{c}_p^\dagger$ and allows us to compute straightforwardly the basic correlators in Eq. (10).

In terms of the function

$$f_{L,N}(r,t) := \frac{1}{L}\sum_{p=1}^{N} e^{(-2\pi i/L)(p-(N+1)/2)r + \epsilon_{p-(N+1)/2} t}, \tag{11}$$

we find the exact result

$$\langle c_{n+r}^\dagger(t) c_n(0)\rangle_{L,N} = f_{L,N}(r,t), \tag{12}$$

$$\langle c_{n+r}(t) c_n^\dagger(0)\rangle_{L,N} = (-1)^r f_{L,L-N}(-r,t), \tag{13}$$

and, therefore,

$$S_\rho(r,t) = (-1)^r f_{L,N}(r,t) f_{L,L-N}(-r,t). \tag{14}$$

Now we study the thermodynamic limit $L, N \to \infty$, such that $\rho = N/L$ is finite. We recall the Fermi momentum $k_F = \pi\rho$ and introduce the continuum dispersion relation $\epsilon(p) := -e^{-ip}$ with its real and imaginary parts $\epsilon_1(p) := \Re[\epsilon(p)] = -\cos p$, $\epsilon_2(p) := \Im[\epsilon(p)] = \sin p$. By taking the derivative of the continuum dispersion relation with respect to the momentum p, one obtains the Fermi velocity $v_F = \epsilon_1'(k_F) = \sin k_F = \sin(\pi\rho)$ and the collective velocity $v_c = \epsilon_2'(k_F) = \cos k_F = \cos(\pi\rho)$. Notice that in the Hermitian case $\epsilon_2(p) = 0$ and, therefore, $v_c = 0$.

We also define the complex coordinate

$$z := ir + \epsilon'(k_F)t = i\tilde{r} + v_F t = v_F t(1 + i\xi), \tag{15}$$

where $\tilde{r} = r - v_c t$ and the phase angle

$$\varphi(r,t) := k_F r - \epsilon_2(k_F)t = k_F r - \sin(k_F)t. \tag{16}$$

Scaling analysis of Eq. (11) then yields

$$f_\rho(r,t) = \frac{e^{-i\varphi(r,t) - t\cos(k_F)}}{2\pi\bar{z}} + \text{c.c.} \tag{17}$$

and we arrive at

$$S_\rho(r,t) = \frac{1}{4\pi^2}\left(\frac{1}{z^2} + \frac{1}{\bar{z}^2} + \frac{2\cos[2\varphi(r,t)]}{z\bar{z}}\right) \tag{18}$$

$$= \frac{1}{2(\pi v_F t)^2}\left(\frac{1-\xi^2}{(1+\xi^2)^2} + \frac{\cos[2\varphi(r,t)]}{1+\xi^2}\right), \tag{19}$$

which is of the predicted form (1) with $v_L = v_F$ and $K = 1$. The main novelty is the oscillating part, which is relevant for maximal current and which is inaccessible to macroscopic fluctuation theory [5]. For nonmaximal large currents, corresponding to $K > 1$ in Eq. (1), the oscillating part is relevant for finite lattices, but it is suppressed in the scaling limit of macroscopic distances and times $\tau = t/a$ due to the microscopic time scale a that enters, after proper rescaling of the structure function, the oscillating part in Eq. (1) with an amplitude a^{2K-2} that vanishes as $a \to 0$.

As an independent fundamental test for conformal invariance of the strongly non-Hermitian problem we consider next open boundary conditions where particles are injected and extracted. This creates several technical difficulties for the exact treatment, viz. lack of periodicity, violation of particle conservation, and loss of the bilinear free-fermion property underlying the computations of the previous paragraphs. The latter problem, however, can be overcome by augmenting the lattice with auxiliary boundary sites 0 and $L+1$, which swap their state (empty or occupied) whenever a creation or annihilation event occurs. This leaves the dynamics of the exclusion process unchanged.

For maximal positive current the left-hopping rates are irrelevant and only injection to site 1 with rate αr and absorption into a reservoir at site L with rate βr need to be considered. The dependence on the boundary rates α and β can be removed by the similarity transformation $V = \prod_{k=1}^{L} u_k^{\hat{n}_k}$ with the choice $p = (2\alpha\beta)^{-(1/L+1)}$ and $u_k = \sqrt{2}\alpha p^k$. The transformed generator then reads in terms of Jordan-Wigner fermions

$$H = -\sum_{k=1}^{L-1} c_{k+1}^\dagger c_k - \frac{1}{\sqrt{2}}[c_1^\dagger(c_0 - c_0^\dagger) + (c_{L+1} + c_{L+1}^\dagger)c_L]. \tag{20}$$

For L even we define the Bogolyubov transformation

$$b_k = \frac{1}{\sqrt{2}}(c_k + (-1)^k c_{L+1-k}^\dagger), \quad 1 \le k \le L \tag{21}$$

$$b_0 = \frac{1}{2}(c_0 - c_0^\dagger + c_{L+1} + c_{L+1}^\dagger). \tag{22}$$

The equations of motion read $[H, b_k] = b_{k-1}$ with periodic boundary conditions, even though the original problem has no translation invariance. Subsequent Fourier transformation leads to $H = \sum_{p=0}^{L} \epsilon_p \hat{b}_p^\dagger \hat{b}_p$ with the single-particle "energies" $\epsilon_p = -e^{-(2\pi i p/L+1)}$.

Thus, we are back to a periodic problem, albeit with $L+1$ sites and only odd particle number which follows from the boundary conditions in Eq. (20). The ground state is given by populating all negative energy modes $\Re(\epsilon_p) = \cos(2\pi p/(L+1)) \leq 0$ and, therefore,

$$-e_0^{\text{open}} = \frac{1}{\pi} + \frac{1}{\pi L} + \frac{\pi}{24L^2} + O(L^{-3}). \tag{23}$$

For the lowest energy gap one finds

$$\Re(\delta^{\text{open}}) = \frac{2\pi}{L} + O(L^{-2}) \tag{24}$$

as in the periodic system at half-filling where $v_F = 1$.

The stationary density of the conditioned process is constant with $\rho_n = 1/2$ as one would expect for maximal current or activity. For the dynamical structure function one can repeat the analysis of the periodic system due to the pseudoperiodicity of Eq. (21). However, we need the full Wick theorem since the second part is nonzero due to the lack of particle number conservation. After some computations using the Bogolyubov transformation (21) and taking the thermodynamic limit one arrives at

$$S^{\text{open}}(n, m, t) = S_{1/2}(n-m, t) - S_{1/2}(n+m, t). \tag{25}$$

Conformal invariance.—The ground state of the *Hermitian* spin-1/2 Heisenberg quantum chain is known to be described by CFT with central charge $c = 1$ [6,7,23]. Intriguingly, the ground state results [Eqs. (8) and (9)] for the periodic system and Eqs. (23) and (24) for the open system can also be understood in terms of CFT even though Eqs. (6) and (20) are strongly non-Hermitian. After rescaling time by the Fermi velocity v_F (identified as such by the computation of the dynamical structure function) and using finite-size scaling theory for conformal invariance [24,25] the leading corrections (8) and (23) to the ground state energy correspond to central charge $c = 1$ of the Virasoro algebra. The real part $2\pi x/L$ of the lowest energy gap corresponds to the lowest critical bulk exponent $x = 1$ of a primary field of the corresponding CFT for the periodic case and $x = 2$ for the open boundary conditions, in complete analogy to the CFT describing the Hermitian XX chain at half-filling $\rho = 1/2$. The non-Hermitian nature of the time evolution enters only through the imaginary part of the energy gap, which yields the collective velocity $v_c = \cos(\pi\rho)$ and the oscillation frequency $\omega = \sin(\pi\rho)$.

The dynamical structure function can be understood in terms of CFT by adapting the standard splitting of the free-fermion operators $c_n(t)$ into right movers (with positive momentum) and left movers (with negative momentum) [15] to the non-Hermitian scenario. With $\varphi(n,t) = k_F n - \Im(\epsilon_{k_F})t$ and $\varepsilon = \Re(\epsilon_{k_F})$ this yields

$$c_n(t) = \frac{1}{\sqrt{2\pi}}[e^{-i\varphi(n,t)+\varepsilon t}\psi_{\tilde{n}}(t) + e^{i\varphi(n,t)-\varepsilon t}\bar{\psi}_{\tilde{n}}(t)] \tag{26}$$

and similar for $c_n^\dagger(t)$ with Galilei-transformed coordinate $\tilde{n} = n - v_c t$. For large n and t the two-point functions are predicted from CFT with central charge $c = 1$ to be

$$\langle\psi^\dagger(z)\psi(z')\rangle = \frac{1}{z - z'}, \qquad \langle\bar{\psi}^\dagger(\bar{z})\bar{\psi}(\bar{z}')\rangle = \frac{1}{\bar{z} - \bar{z}'}. \tag{27}$$

This then leads to the dynamical structure function with the static exponent $K = 1$ that we computed. The same scaling form (1) with a nonuniversal exponent $K > 1$ can be obtained via bosonization [15] for nonmaximal speeding-up, i.e., $\Delta > 0$. Since t is to be understood in units of the lattice spacing, the oscillating part thus becomes irrelevant even without spatial coarse-graining. The dynamical structure function for open boundaries can be understood from boundary conformal field theory with reflecting boundary conditions which yields nonvanishing correlators between the right and left movers which are of the form similar to Eq. (27), but with a $1/(z + z')$ dependence coming from reflection.

From Eq. (3) we read off the remarkable fact that at a critical activity parameter,

$$\mu_c = -\ln\left(\frac{\cosh(f+\lambda)}{\cosh f}\right), \tag{28}$$

the conditioned generator becomes the generator of an unconditioned ASEP with hopping asymmetry $f' = f + \lambda$. Then, for $\mu < \mu_c$ one has phase separated atypical behavior as shown in Ref. [14] for $f = \lambda = 0$ and in Ref. [19] for $f = 0$, whereas for $\mu > \mu_c$ one has—as elaborated above—conformal invariance; see Fig. 2. On the phase transition line $\mu = \mu_c$ one has typical stochastic dynamics with $z = 2$ for $\lambda = -f$ (symmetric simple exclusion process) and $z = 3/2$ for $\lambda \neq -f$ (ASEP), as predicted by nonlinear fluctuating hydrodynamics [26] for one-dimensional driven systems with one conservation law.

The same CFT with central charge $c = 1$ describes other quantum spin chains such as the spin-s Heisenberg chain in the gapless regime [27,28] with anisotropy $0 \leq \Delta < 1$ whose non-Hermitian generalization maps to the partial exclusion process [29] conditioned on an atypical current and activity. One has an energy gap for integer spin (Haldane conjecture) only for negative Δ. In fact, it opens up exactly at $\Delta = 0$ where the mapping to stochastic dynamics gets lost, as positivity of the transition rates implies $\Delta \geq 0$. The energy gap is finite in the ferromagnetic regime $\Delta > 1$, corresponding to phase separation in the particle system. These observations support the notion of universality of Eq. (1) and also confirm the dynamical phase transition to a phase-separated regime at low current or activity.

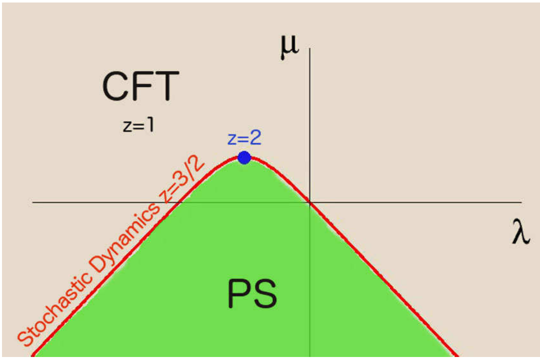

FIG. 2. Phase diagram of the conditioned ASEP as a function of current parameter λ and the activity parameter μ for positive drive $f > 0$. The exact phase transition line that separates the conformally invariant regime CFT from the phase-separated regime PS is given by Eq. (28).

Conclusions.—The phase diagram of atypical behavior in d-dimensional diffusive systems far from thermal equilibrium can be explored by studying the critical behavior of $d + 1$-dimensional equilibrium models. In $1 + 1$ dimension with a single conserved species there is a dynamical phase transition from a dynamically critical conformally invariant regime for atypically fast dynamics through a phase transition line corresponding to typical dynamics in the diffusive or Kardar-Parisi-Zhang universality class to a phase-separated regime of atypically slow dynamics. Thus, large fluctuations in driven diffusive systems cannot be understood by simple upscaling, but are sustained by spatiotemporal correlations that are qualitatively different from typical behavior. The dynamical phase transition can be studied further in terms of the large deviation function $\epsilon_0(f, \lambda, \mu)$. For the SSEP ($f = 0$) we obtain from Ref. [30] that $\partial_\mu^2 \epsilon(0, 0, \mu)$ diverges $\propto \mu^{-1/2}$ at the critical point $\mu = 0$ (for fixed $\lambda = 0$), while $\partial_\lambda^2 \epsilon(0, \lambda, 0)$ is continuous at $\lambda = 0$ (for fixed $\mu = 0$), but $\partial_\lambda^3 \epsilon(0, \lambda, 0)$ has a jump discontinuity.

For systems with more than one conservation law the scenario is more complex. It has been shown recently that typical behavior is described by dynamical critical exponents $z_i > 1$ given by the ratios of neighboring Fibonacci numbers F_{i+1}/F_i, starting with $F_2 = 1$ and $F_3 = 2$ [31]. Also, quantum systems for n conserved species that would describe the critical behavior in the regime of high current or activity may be governed by a CFT with central charge $c > 1$. For low current or activity one still expects phase separation, which, however, may exhibit richer behavior than for $n = 1$.

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[1] H. Spohn, *Large Scale Dynamics of Interacting Particles* (Springer, Berlin, 1991).
[2] G. M. Schütz, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. Lebowitz (Academic, London, 2001), Vol. 19, p. 1.
[3] B. Derrida, J. Stat. Mech. (2007) P07023.
[4] A. Schadschneider, D. Chowdhury, and K. Nishinari, *Stochastic Transport in Complex Systems* (Elsevier, Amsterdam, 2010).
[5] R. L. Jack, I. R. Thompson, and P. Sollich, Phys. Rev. Lett. **114**, 060601 (2015).
[6] J. L. Cardy, *Scaling and Renormalization in Statistical Physics* (Cambridge University Press, Cambridge, England, 1996).
[7] M. Henkel, *Conformal Invariance and Critical Phenomena* (Springer, Berlin, 1999).
[8] L. Bertini, A. De Sole, D. Gabrielli, G. Jona Lasinio, and C. Landim, Rev. Mod. Phys. **87**, 593 (2015).
[9] T. Bodineau and B. Derrida, Phys. Rev. E **72**, 066110 (2005).
[10] C. P. Espigares, P. L. Garrido, and P. I. Hurtado, Phys. Rev. E **87**, 032115 (2013).
[11] T. Bodineau and B. Derrida, J. Stat. Phys. **123**, 277 (2006).
[12] V. Belitsky and G. M. Schütz, J. Phys. A **46**, 295004 (2013).
[13] A. Lazarescu, J. Phys. A **48**, 503001 (2015).
[14] V. Lecomte, J. P. Garrahan, and F. Van Wijland, J. Phys. A **45**, 175001 (2012).
[15] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).
[16] R. Chetrite and H. Touchette, Ann. Henri Poincaré, **16**, 2005 (2015).
[17] R. L. Jack and P. Sollich, Eur. Phys. J. Spec. Top. **224**, 2351 (2015).
[18] V. Popkov and G. M. Schütz, J. Stat. Phys. **142**, 627 (2011).
[19] H. Spohn, Phys. Rev. E **60**, 6411 (1999).
[20] G. M. Schütz, in *Springer Proceedings of the International School and Workshop on Nonlinear Mathematical Physics and Natural Hazards*, edited by B. Aneva and M. Kouteva-Guentcheva (Springer, Cham, 2015).
[21] V. Popkov, D. Simon, and G. M. Schütz, J. Stat. Mech. (2010) P10007.
[22] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).
[23] R. J. Baxter, *Exactly Solved Models in Statistical Mechanics* (Academic, New York, 1982).
[24] I. Affleck, Phys. Rev. Lett. **56**, 746 (1986).
[25] H. W. J. Blöte, J. L. Cardy, and M. P. Nightingale, Phys. Rev. Lett. **56**, 742 (1986).
[26] H. Spohn, J. Stat. Phys. **154**, 1191 (2014).
[27] F. C. Alcaraz and A. Moreo, Phys. Rev. B **46**, 2896 (1992).
[28] F. C. Alcaraz and A. L. Malvezzi, J. Phys. A **30**, 767 (1997).
[29] G. Schütz and S. Sandow, Phys. Rev. E **49**, 2726 (1994).
[30] C. Appert-Rolland, B. Derrida, V. Lecomte, and F. van Wijland, Phys. Rev. E **78**, 021122 (2008).
[31] V. Popkov, A. Schadschneider, J. Schmidt, and G. M. Schütz, Proc. Natl. Acad. Sci. U.S.A. **112**, 12645 (2015).